# Screening Using a Menu of Contracts in Imperfectly Competitive Markets with Adverse Selection

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**Abstract**

I develop a model of screening with imperfect competition. The model has a unique equilibrium in pure strategies, which I characterise in closed form. It allows me to analyse a contractual externality and derive a sufficient statistic for it. I apply my model to credit markets and show that contrary to conventional wisdom, increasing capital requirements, increasing the Federal Reserve rate, or decreasing competition can increase lending. I provide an empirical application in the context of consumer credit and show that too many different maturities are offered due to the externality. The model parameters are identified by linear regression of prices on quantities controlling for contract market shares.

# 1 Introduction

In many markets, firms use menu of contracts to make their customers reveal their private information through their choices (i.e., screening). Examples includes the insurance market (Handel, Hendel, and Whinston 2015, Einav, Finkelstein, and Tebaldi 2019), the mortgage market (Taburet, Polo, and Vo 2024) and the market for credit cards (Nelson 2020).

However, the empirical literature abstracts from screening with menus (Einav, Finkelstein, and Mahoney 2021). This stems in large part from theoretical frameworks that assume extreme market structures (e.g., Stiglitz 1977, Rothschild and Stiglitz 1976) and face issues of equilibrium non-existence.

This paper bridges theory and empirical research on adverse selection. I incorporate imperfect competition in an otherwise standard screening model à la Rothschild and Stiglitz (1976). Following the empirical industrial organization literature, I model competition using a discrete choice approach as McFadden (1981). This approach has convenient analytical properties, and yields demand functions that nest various degrees of competition, from perfect competition to monopoly. This framework allows me to analyze how incentives to screen, information rents, and welfare vary with competition. I discuss the identification and estimation of the model parameters for a situation in which an econometrician has access to data on menus of contracts and choices for one firm (e.g., Hertzberg, Liberman, and Paravisini 2018); or when they only observe the contracts chosen for the full market (e.g., credit register data as in Crawford, Pavanini, and Schivardi 2018). I also provide a working example in the context of consumer credit using data from Hertzberg, Liberman, and Paravisini (2018).

Giving firms incomplete market power has three main effects on the analysis. First, it solves the equilibrium non-existence problem. The model delivers a unique pure strategy Nash equilibrium, which I characterize in closed form.[1] The closed-form characterization is important for empirical applications because it allows me to derive sharp moment conditions and makes the exclusion restrictions transparent.

Second, the competition between firms creates a contractual externality, for which I derive a sufficient statistic. This externality results from the fact that when lenders screen borrowers using a menu, they change the composition of their competitors' borrowers. The equilibria can thus be inefficient in the second-best sense, so an informationally constrained social planner can create a menu that makes both firms and their clients better off. The existence of the externality does not depend on the use of a particular equilibrium refinement such as

1. To derive this result, I need to impose a capacity constraint on firms as Azevedo and Gottlieb (2017), but only when the market is close to perfect competition.

Wilson (1980) or Riley (1979).

Third, I show that policies affecting competition, or the cost of originating a contract have unintended consequences due to the presence of competition and screening. For instance, in the context of credit markets, increasing capital requirements, increasing the Federal Reserve rate, or decreasing competition can increase lending.

To be close to the empirical application, I present my model in the context of credit markets. I provide micro-foundations for the functional forms used and develop a unifying framework for screening in the credit markets. I allow for screening based on maturity, loan size, interest rate collateral, and fees. However, the framework is general enough to capture labour markets, insurance markets or other financial markets.[2]

In the baseline model version, a contract comprises a loan size and an interest rate. Borrowers have quasi-linear utility over the present value of the loan. They have private information about their willingness to pay (WTP) for each dollar borrowed, their demand elasticity and their default probabilities. Their WTP is positively correlated with their default probabilities. In that context, screening is achieved by setting the interest rate above the low-default borrower WTP for larger loans. That way, high-default borrowers self-select into higher-rate contracts. Screening thus allows to potentially restore perfect information pricing; However, it also requires that low-default borrowers get a lower loan than high-default borrowers, which is not necessarily what would happen in the first best.

To model borrowers' choice of lender, I assume that part of borrowers' utility contains a borrower-bank-specific random shock following a continuous probability distribution and entering the utility additively (as in, for instance, McFadden 1981). This shock can be interpreted as borrowers having heterogeneous preferences over lenders' characteristics (e.g., distance from the closest branch as in Hoteling 1929) or as borrowers being imperfectly informed about the contracts offered by each bank (Varian 1980). I consider the situation in which the random shock is uncorrelated to borrowers' preferences for contract characteristics so that banks cannot use their menus to sort borrowers on their random shock realization. When the shocks follow an extreme value distribution, its variance parameterizes the product demand elasticity. In the limit case in which the variance of the shock tends to infinity, each lender behaves like a monopolist. When the variance tends to zero, borrowers' demand elasticity becomes infinite, as in the perfect competition case. I allow the variance of the shock to be correlated with the willingness to pay. This provides additional price discrimination

2. A key requirement is that contracts are effectively exclusive; see, for instance, Attar, Mariotti, and Salanié 2011 for non-exclusive contracts under perfect competition. Search costs may prevent borrowers from getting multiple contracts in practice, which allows firms to screen.

incentives and makes the model more realistic.

Unlike Rothschild and Stiglitz (1976), a pure strategy equilibrium always exists in my model. In Rothschild and Stiglitz (1976), the equilibrium does not exist when the relative number of low-default borrowers is high enough. I show that in my model, deviations that break the equilibrium in Rothschild and Stiglitz (1976) attract relatively too many high-default borrowers to be profitable. This is because those deviations are more valuable to high-default borrowers, and lenders cannot attract the whole market under imperfect competition. In the spirit of Einav, Finkelstein, and Cullen (2010), I provide a graphical intuition to illustrate why having an imperfectly elastic demand restores the pure strategy equilibrium.[3]

The equilibrium is constrained inefficient. The intuition is the following. Maintaining customers' incentives to self-select is costly as it may require distorting contracts relative to the first best. When the distortions are too high, it is more efficient to pool borrowers instead. Yet, if pooling contracts are offered and competition is high, a competitor can take advantage of a pooling contract by introducing a product that steals the most profitable customers only (cream skimming). Cream skimming can be ex-post inefficient because lenders do not internalize how their screening strategies (e.g., cream skimming) change the types of borrowers selecting competitors' products — and thus the cost of lending via those products. The reasoning is the same as in Rothschild and Stiglitz (1976). However, contrary to Rothschild and Stiglitz (1976), I can compute the equilibrium and characterize the size of this friction. I show that the equilibrium is inefficient when competition is high, and the marginal total surplus per dollar lent in the market for low-default borrowers is higher than the difference between borrowers' willingness to pay (i.e., the information rent in Rothschild and Stiglitz 1976).

I show that when the equilibrium is constrained inefficient, decreasing competition can be a Pareto improvement. It lowers' incentives to implement possibly inefficient cream-skimming deviations and thus restores lenders' ability to pool. A lower level of competition can be beneficial to low-default borrowers as it lowers their credit constraint at the cost of a higher mark-up. It is also beneficial to high-default borrowers as competition can lower the interest rate of their contracts due to pooling. Lenders are also better off because market

3. When competition is large enough, I must add a capacity constraint to the model. Formally, I assume that lenders cannot serve more than half of the total market. This can be interpreted as an extreme case of a lending cost that is increasing in the number of customers. This assumption is needed because the lender objective function becomes convex when markets are competitive. Indeed, deviations from screening attract bad borrowers in high proportion first, but once most of the bad borrowers have been attracted, the good borrowers become easier to attract relative to the bad borrowers. With this condition, the model equilibrium under perfect competition is the same as the one using Azevedo and Gottlieb (2017) equilibrium concept.

power allows them to can extract more surplus from borrowers. The same result holds when the policy increases lenders' cost of originating low loan-size contracts and decreases the cost of originating high loan contracts, but only if competition is low enough. Examples of such policies are capital requirements in credit markets.

I develop a guide on how to bring the model to the data using either proprietary data on menu of contracts and choices for one firm, or for credit register data. Those two situations are the most common in practice (See, for instance, Hertzberg, Liberman, and Paravisini (2018) and Crawford, Pavanini, and Schivardi (2018) for the credit market or Handel and Kolstad (2015) for the insurance market).

To illustrate the approach, I provide an application using data from an online lending platform (Lending Club) as in Hertzberg, Liberman, and Paravisini (2018).[4] Adapting my theoretical framework, I develop the first structural model of screening with maturity. The parameters are identified and estimated using a two-step approach. I first calculate average default probabilities conditional on contract terms and observable borrower characteristics. I then use those estimates to construct the present value of the loan and regress it on loan size and relative contract market shares. The regression identifies the key model parameters and allows the decomposition of the equilibrium interest rate into a break-even price, a perfect information markup and an information premium or discount. The information premium divided by the perfect competition markup of the contract designed for high-default borrowers is a sufficient statistic for the existence of the contractual externality.

The regression is the empirical counterpart of the model equilibrium present value formula. It is an identity regression, so the coefficients do not need to have a causal interpretation. Unobserved demand heterogeneity can, however, bias the result. I use the borrowers' risk category as an instrument for relative market share. The exclusion restriction is that the average search costs of borrowers choosing high-maturity contracts are uncorrelated with observable risk categories.

As Hertzberg, Liberman, and Paravisini (2018), I find that high-default borrowers self-select into higher maturity loans. Using the structural model, I can also characterize the distortions caused by adverse selection and imperfect competition. I show that prices are close to the first best case. Borrowers choosing high-maturity contracts are less price elastic but get an asymmetric information discount that mitigates the competition channel. The

4. Although contracts are not formally exclusive in this market, the fact that Hertzberg, Liberman, and Paravisini (2018) finds that screening with menus happens in practice shows that using Rothschild and Stiglitz (1976) framework (exclusive contracts) instead of Attar, Mariotti, and Salanié (2011) (non-exclusive contracts) is a valid approach in this context.

information rent is equal to 15 per cent of the loan's present value. There are substantial distortions in maturity. Using the sufficient statistic I show that borrowers would be better off being pooled with a 60-month maturity contract instead of being screened with 36-month and 60-month maturity options.

The rest of the paper is structured as follows. Section 2 provides a literature review. I describe the model setup in section 3. Then, in section 4, I provide intuition about the incentives to screen, I prove the existence and uniqueness of the equilibrium and solve for the model in closed form in section 5. In section 6, I provide comparative statics and discuss the potential implications of various widely used policy interventions. Section 7 discusses extensions and how to bring the model to the data. Section 8 present the empirical application. Section 9 concludes.

# 2 Literature Review

There is a growing empirical literature on adverse selection. A first strand of the literature developed tests for adverse selection using reduced form approaches (Hertzberg, Liberman, and Paravisini 2018, Karlan and Zinman 2009, Chiappori and Salanié 2002). To go beyond testing for adverse selection, another strand of the literature uses models to estimate the cost of adverse selection. Following Einav, Finkelstein, and Cullen (2010), the empirical literature is based on Akerlof (1978) model, which considers a market for a single contract with exogenous characteristics (see, for instance, Einav, Finkelstein, and Mahoney 2021 for a review). Papers in credit markets have focused on the situation in which lenders can only choose interest rates (see Crawford, Pavanini, and Schivardi (2018) for a structural empirical framework), or the case in which lenders can pay a fixed cost to learn about borrowers type (Yannelis and Zhang 2021). My paper differs from those two by looking at a situation in which lenders can screen their customers by designing their menu of contracts. The closest paper is Taburet, Polo, and Vo (2024), which uses a structural model of screening for default probability. The empirical part of this paper complements Taburet, Polo, and Vo (2024) by offering alternative modelling that allows the recovery of the model parameters without the need to estimate demand and default elasticities. Those elasticities are problematic because of the selection of unobservables. Furthermore, the model can be estimated in a situation where only data on an individual firm is available. Finally, the modelling assumption also allows me to analyse the theoretical properties of the model. Due to its low computational burden, this paper can accommodate a wider range of counterfactual simulations, such as changes in competition.

The paper contributes to the theoretical literature on adverse selection and on the role of contract terms and prices as screening devices. The vast majority of the literature assumes either perfect competition (Rothschild and Stiglitz 1976) or monopoly (à la Stiglitz 1977). Recent examples of perfect competition models are Azevedo and Gottlieb (2017) for a model solution based on an equilibrium refinement, and Guerrieri, Shimer, and Wright (2010) for a pure strategy characterization based on the assumption that the principal can match with at most one agent. The use of refinements emerged because the mixed strategy equilibrium is difficult to compute even in stylized frameworks (see Farinha Luz (2017) for the first numerical characterization in a Rothschild and Stiglitz (1976) model).

A more recent literature studies screening under imperfect competition. Chade and Swinkels (2021) shows existence and uniqueness and characterizes the equilibrium in a situation where the principal has heterogeneous costs of producing a good of a given quality and uses the menu to screen agents with heterogeneous willingness to pay. In contrast, this paper focuses on a situation that is closer to the banking market, where the cost heterogeneity is on the agent, not the principal. Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2019) is closer to my framework. Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2019) uses a search model à la Burdett and Judd (1983) to model imperfect competition in an otherwise standard screening model in goods of different quality (lemon market). This paper complements Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2019) analysis on the interaction of competition and adverse selection trade-off by highlighting the contractual externality. I also focus on a different set of comparative statistics relevant to credit markets. The theoretical analysis is significantly different as my demand system is continuous. This implies that I can use first-order conditions to analyze the model properties and deliver closed-form solutions.[5]

5. Papers such as Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2019) rely on a mixed strategy numerical characterization. Their model has to be solved in pure strategy because of their modelling of demand. By modelling borrowers as some being infinitely price elastic while others being completely inelastic, Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2019) end is a situation similar to Rothschild and Stiglitz (1976) in which both pooling deviations attract the same proportion of good and bad borrowers as in the full market. As discussed in the introduction, this creates, in some situations, incentives to deviation from both pooling and screening, preventing the model from being solved using pure strategies.

# 3 Model Set-up

## 3.1 General considerations

I consider a 2-period model with two groups of agents: borrowers and lenders. I also refer to the second group as banks. There is a finite number of banks $\mathbf{B} > 1$ indexed by $b \in [\![1, \mathbf{B}]\!]$. Borrowers have heterogeneous characteristics (age, income, risk aversion..). This heterogeneity implies that borrowers have different default probabilities and preferences over loan contracts. There are two types of borrowers indexed by $i \in \{G, B\}$. The number of type i borrowers is denoted $n_i$. I denote $j$ a given borrower. Banks offer menus of contracts. I index the contract offered by bank b and designed to be chosen by borrower i by the subscript ib or by c to simplify the notation.

**Timing:** At the beginning of the first period, each borrower makes a decision to enter or not the credit market. There is no entry cost, borrowers choose to participate if the utility they get from borrowing is higher than the one of not borrowing. Conditional on participation, a borrower chooses one loan contract from one lender.

Loans mature in the second period. Borrowers may default on their loans.

**Information structure:** There is asymmetric information in the economy: lenders do not perfectly observe borrowers' type (i.e. their preference and their default probabilities).

Whenever it is profitable and feasible, lenders use a menu of contracts to make borrowers reveal their type.

## 3.2 Borrowers

In this section, I model borrowers' decision to participate in the credit market as well as their choice of loan contract and repayment behaviour. I then provide a micro-foundation of the demand system.

**Information structure:** All parameters defined in this section are part of borrower i information set at the time they make their choice of contract and bank.

### 3.2.1 Choice of contract and bank

**Utility:** The utility of type i borrowers when borrowing an amount $L_c$ in period 1 via contract c requiring a repayment of $(R_c)$ at maturity is specified as:

$$u_i(L_c, R_c) := \alpha_i F(L_c) - \theta_i R_c, \; WTP_i := \frac{\alpha_i}{\theta_i} > 0 \tag{1}$$

$WTP_i F'(\cdot)$ is the borrowers' willingness to pay for each extra dollar borrowed. Without loss of generality, I assume that $WTP_G < WTP_B$. A high $WTP$ captures that the borrower derives a high utility level from housing, or less disutility from having a high interest rate. Heterogeneous $WTP$ coefficients are necessary for lenders to be able to screen using loan size L and the face value of the debt R. Screening is achieved by offering a contract whose pricing for an extra loan unit is in between the two borrowers' willingness to pay. That way, the larger loans are only purchased by the high WTP borrowers.

$\theta_i$ is borrower i survival probability. This interpretation of the coefficient $\theta_i$ implies that the loan is non-recourse. This is a way, among others, to rationalize adverse selection in the model.

I provide a micro-foundation for this utility form in section 3.2.2. The model can be extended to feature a finite number of contract characteristics X as long as they enter linearly in the utility.

**Choice of bank b contract c:** Borrower i chose the bank that offers him the best deal. Formally:

$$\max_{\{c\}} \{u_i(L_c, R_c) + \sigma_i^{-1} \epsilon_{jb}\}, \; \epsilon_{jb} \; iid, \; EV, \text{ independent of type} \tag{2}$$

$\sigma_i^{-1}\epsilon_{jb}$ is the main departure from the classic principal-agent model. $\epsilon_{jb}$ is a borrower-specific shock that is common across all contracts offered by the same bank. $\sigma$ drives the product elasticity (competition) and can be interpreted as the distance between the borrower and the closest bank branch as in, for instance, Hoteling (1929) or more generally about heterogeneous preferences over bank characteristics (brand name, customer services etc). The assumption that $\epsilon_{jb}$ draws are type-independent ensures that lenders cannot screen on borrower-specific draws. When $\sigma$ tends to infinity, borrowers only care about the contract features the banks offer (i.e perfect competition). When $\sigma^{-1}$ tends to 0, each bank behaves like a monopolist with their borrowers.[6].

**Participation constraint (PC):** Borrower i accept a contract if it provides them a higher level of utility than the one they would get if they do not take any loan. Formally, borrower i accepts the loan if:

$$u_i(L_c, R_c) \geqslant \bar{V}_i = \bar{V} \in \mathbb{R}^+ \tag{3}$$

The fact that $\epsilon_{ib}$ is not present here is in favour of the interpretation of $\epsilon_{ib}$ being a sunk cost that has to be paid to switch banks. This is done for simplicity of the exposition so that each borrower type has the same participation constraint. Using a nested logit approach, one could use the condition $E[u_i(L_c, R_c) + \sigma_i^{-1}\epsilon_{ib}]) = ln(\sum_c u_i(L_c, R_c)) \geqslant \bar{V}_i$. This participation constraint will not be important for the vast majority of the results as it will be non-binding.

**Survival probability:** The market is adversely selected:

$$Adverse\ selection:\ Corr(\theta_i, WTP_i) < 0 \tag{4}$$

Equation (4) state that borrower with a high willingness to pay $WTP_B > WTP_G$ are more likely to default $\theta_B < \theta_G$). The classic justification is that high-default borrowers are less sensitive to interest rate changes as they do not expect to repay the full face value of the debt. This is the case when for instance, $\alpha_G = \alpha_B$.

### 3.2.2 Possible micro-foundation borrowers' indirect utility

This section provides a micro foundation of the indirect utility function $u$. As I will look at mortgage policies in the last section of the paper, I use a mortgage micro-foundation.

Borrowers do not have any income in period 1. They can get a loan ($L_c$) to invest in a house of size ($H_1$) yielding the utility $F(H_1)$ in each period as long as they do not sell it. They can also consume ($C_1$), from which they derive utility ($u(C_1)$). The function $F$ and $u$ are increasing and concave with $F(0) = u(0) = 0$. Borrowers discount period 2 utilities with the discount factor $\delta$. In the second period, borrowers' income is either W with probability ($\theta$) or 0 with probability ($1 - \theta$). Borrowers use their income W to consume ($C_2$) and to

6. when $(\varepsilon_{i,j})_j$ are not all equal

repay the loan ($R_c$). When their income is equal to zero, they fire sell the house and get ($\gamma H$) to repay for the loan $R_c$. The borrower maximization problem is:

$$\max_{\{C\}} u(C_1) + F(H_1) + \delta\theta \overbrace{[u(C_2) + F(H_1)]}^{utility\ when\ not\ defaulting} + \delta(1-\theta) \overbrace{[u(max\{\gamma H_1 - R, 0\})]}^{utility\ when\ defaulting} \quad (5)$$

$$s.t.\ C_1 + H_1 = L_c \quad (6)$$

$$C_2 = W - R_c \quad (7)$$

Assuming for simplicity of the notation that fire selling the house in the second period is costly ($\gamma = 0$)[7], that borrowers prefer to invest in the house rather than consuming in the first period (i.e. $u'(0) < [1 + WTP(1-d)F'(\bar{L})]$, with $\bar{L}$ being the maximum loan size available), but that they prefer consuming rather than getting a new house in the second period (i.e. $u'(W) > F'(0)$) the indirect utility can be written:

$$u_i = [\delta\theta + 1]F(L_c) - \delta\theta u(W - R_c) \quad (8)$$

The expression implies that high default borrowers are more likely to have a high willingness to pay for loan as they do not expect to repay the full face value of the debt (captured by $\frac{1}{\delta\theta}$). When the utility of consumption is linear, the same indirect utility as in equation (1) is obtained (up to a normalization of the R coefficient). This assumption is consistent with Hertzberg, Liberman, and Paravisini (2018) empirical findings that the self-selection in the consumer lending market seems to be driven by private information on the income process rather than risk aversion.

### 3.2.3 Extension: Loan-to-Value fees, Maturity and Collateral

The loan size L and the repayment R can also be interpreted as LTV and maturity, respectively. The following paragraph presents a formal argument for the LTV and its maturity.

**Loan-to-Value and fees:** The model presented in section 3.2.2 can be extended by

7. In that particular case, borrowers would be better off keeping the house (and lender would be no worse off). Selling the house upon default can be, however rationalized by the use of the house as collateral to prevent borrower from filling for default even when income is equal to W (see appendix C).

allowing borrowers to have income (A) in the first period of the contract to allow for down payments (D) and fees (f). The utility thus becomes:

$$u_i = [\delta\theta + 1]F(L + D) + u(A - D - f) - \delta\theta u(W - R) \tag{9}$$

In that case, we get that high-default borrowers are less willing to put their own wealth into their houses. This justifies the fact that, under screening, high LTV loans will be selected by borrowers with unobservably high default probabilities.

When the marginal cost of lending $mc$ is greater than 1, then the optimal contract is such that the lender puts as much down payment as possible (i.e., $D = A$). The loan size L in the equation (1) can thus also be interpreted as Loan-to-Value.

**Maturity:** The full model with maturity is presented in the appendix B. This section just focuses on the main equation. When the default rate is constant in each period, the borrower value function can be written as:

$$u_i = \frac{F(L + D) - \theta C[1 - e^{-\rho T}]}{\rho} - D - f \tag{10}$$

$\frac{F(L)}{\rho}$ is the present value of the loan. $\frac{C}{\rho}[1 - e^{-\rho T}]$ is the present value of the debt for borrowers. $\rho$ is the discount rate. $\theta$ is the per-period survival probability. $T$ is the maturity. $\theta C := \theta r + (1 - \theta)[\gamma K + \Delta]$ is the expected per period loan cost for borrowers, with $r$ as the monthly payment, $\gamma K$ the collateral value and $\Delta$ as a cost of defaulting beyond the loss of collateral.

The supply side is identical to the one presented in the following section but set in continuous time. Lenders offer differentiated loan products and maximize their expected profits. I show in appendix B that it is optimal for the lender to set very high monthly payments (infinite) and minimise the contract's maturity so that the borrower's probability of paying the non-monetary cost is minimal. If there is a limit on how high the monthly payment can be — if, for instance, repayment cannot be higher than disposable monthly income — it is optimal to set the monthly payment at the limit and extract surplus with the longer maturity.

**Collateral:** Appendix C, which states the conditions for collateralized debt to arise as the optimal contract. There are two critical assumptions. The first assumption states lenders cannot observe the cash flow (it that is is costly to do so as in Townsend 1979), so borrowers can lie about their income. The second assumption is and that the bank can use collateral and seize it upon default if this is more efficient than spending the verification cost. For the collateral to be seized upon default only, the bank must value it less than the borrower. This assumption is adapted Lacker (2001) to fit mortgage markets (see Appendix C). Papoutsi, Paravisini, Rappoport, and Taburet (2024) shows that empirically this assumption holds in the market for corporate loans.

### 3.3 Lenders

**Information structure:** Lenders do not observe individual borrowers' type $(WTP_i, \theta_i, \sigma_i)$ but they know the population distribution. They can use the revelation principle to design a menu of contracts to make borrower reveal their private information through their choices. I assume that $\epsilon_{ib}$ is independent of other borrower parameters $(WTP_i, \sigma_i, \theta_i)$, so banks cannot screen borrowers on their $\epsilon_{ib}$ draw. The model is thus the classic textbook principal-agent model but with demand elasticities that are not infinite or null.

**Net Present Value of Lending:** Lender b net present value of lending to borrower i via a contract ib is denoted $NPV_{ib}$:

$$NPV_{ib} := \theta_i R_{ib} - mcL_{ib} \tag{11}$$

Where $mc$ is the marginal cost of lending via contract ib. $\theta_i R_{ib}$ is the present value of the loan. The use of collateral as in the mortgage mico-foundation presented in the previous section would give us the additional term capturing the amount recovered upon default $(1-\theta_i)min\{\gamma H, R_{ib}\}$, where $\gamma$ is the value of the house upon default. For risk discrimination to be relevant, I consider the case in which the price of house upon default $\gamma$ is low enough so that the collateral is not enough to repay the face value of the debt. Without loss of generality, I assume $\gamma = 0$ for notation simplicity.

**Demand:** Conditional on participation (i.e., $PC_i$ satisfied), borrower i chooses bank b according to equation (2). Formally, the borrower j of type $i$ chooses bank b if (denoting

$(PC_{ijb})$ the condition):

$$PC_{ijb}: \; u_i(L_{ib}, R_{ib}) \geqslant u_i(L_{cd}, R_{cd}) + \sigma_i^{-1}[\epsilon_{jb} - \epsilon_{jd}], \forall cd \in \{G, B\} \times [\![1, \mathbf{B}]\!], \forall j \in [\![1, n_G + n_B]\!] \tag{12}$$

The expected number of borrowers that come to lender b is thus:

$$N_i^b(u_i(L_{ib}, R_{ib})) = E[\sum_{j=1}^{n_i} \mathbf{1}_{PC_{jb} \; satisfied}] \tag{13}$$

$$= \overbrace{n_i}^{market \; size} \overbrace{\frac{exp(\sigma_i u_i(L_{ib}, R_{ib}))}{\sum_{x \in \mathbf{B}} exp(\sigma_i u_i(L_{ix}, R_{ix}))}}^{Probability \; i \; chooses \; b} \tag{14}$$

$\sigma_i \rightarrow \infty$ captures the perfect competition case. In that case, if contract from lender b $(L_{ib}, R_{ib})$ is epsilon better than the one of its competitors, it attracts the full market (i.e., $\frac{exp(\sigma_i u_i(L_{ib},R_{ib}))}{\sum_{x \in B} exp(\sigma_i u_i(L_{ix},R_{ix}))} \rightarrow 1$). $\sigma_i \rightarrow 0$, then each lender behaves as a monopoly. The market share of lender b is $\frac{n_i}{B}$ no matter what contract it offers.

**Lender maximisation problem:** Bank b maximize its expected profits subject to borrowers' incentive compatibility constraints and participation constraint:

$$\max_{\{(L_{ib}, R_{ib}) \in \mathcal{F}\}} \sum_{i \in \{G,B\}} \overbrace{N_i^b(u_i(L_{ib}, R_{ib}))}^{Demand} \qquad \overbrace{NPV_i}^{Expected \; profit \; on \; loan} \qquad |WTP_i, \theta_i, \sigma_i] \tag{15}$$

$$s.t. \; (IC_i): \; u_i(L_{ib}, R_{ib}) \geqslant u_i(L_{jb}, R_{jb}) \; \forall i, j \in \{G, B\} \tag{16}$$

$$(P_i): \; u_i(L_{ib}, R_{ib}) \geqslant \bar{V} \tag{17}$$

For the problem (15) to be well defined when the function F is linear, I assume that contracts' characteristics are bounded. Formaly, lenders need to offer contracts within the following set $\mathcal{F} := \{(L_c, R_c) : L_c \in [0, \bar{L}], \; R \geqslant 0\}$ with $\bar{L} > 0$. $\bar{L}$ is the maximum house size available in the market. Alternatively, this constraint can be written as a maximum Loan-to-Value constraint to capture existing regulations. I use a constraint on L rather than a constraint on the borrower's second-period income (and thus R) so that the house size is fixed in the first best and independent of the competition level.

**Assumptions A1:** In order to get closed-form solutions, I assume that F is linear on $[0, \bar{L}]$ and equal to $F(L) = L$.

To make the use of menus relevant, I consider the situation in which both market segments have positive NPV $\alpha_i - mc > 0 \ \forall i$ and $\theta_i > 0$. If only the low WTP borrower had positive NPV, then the model will be similar to Akerlof (1978), in which only one contract is offered. Similarly, if only the high WTP borrowers have a positive NPV, then banks exclude the low WTP borrowers using one contract.

# 4 Optimal Menu Design

In this section, I analyze how each lender set contract terms under under perfect information, I then focus on the imperfect information case. In contrast with other screening models (for instance, Rothschild and Stiglitz 1976, Lester, Shourideh, Venkateswaran, and Zetlin-Jones 2019), my model has a continuous demand system. Based on this, I propose a new intuitive decomposition of the economic forces at play in screening models without relying on numerical methods or equilibrium refinements.

## 4.1 Contracts when $WTP_i$ observable

The first order conditions of problem (15) without the incentive compatibility constraint ($IC_B$) yield:

***Proposition 1: Banks' contract under perfect information***
*Each bank uses product characteristics ($L_i$) to maximize the surplus of lending ($S_i(L_i) := (\alpha_i - mc)L_i$), then uses the interest rates to split the surplus between itself and the borrowers.*

Dropping the b index for clarity of the notation, the optimal contract $(L_i^{PI}, R_i^{PI})$ is:

**Characteristics:**

$$L_i^{PI} := \bar{L} \tag{18}$$

**Pricing:**

$$R_i^{PI} := \begin{cases} \overbrace{\frac{mc_i L_i^{PI}}{\theta_i}}^{fair\ price} + \overbrace{\frac{N_i}{-\partial_R N_i}}^{\text{“}markup\text{”}} & \text{if } \ u_i \geqslant \bar{V}_i \\ WTP_i L_i^{PI} - \frac{\bar{V}_i}{\theta_i} & \text{Otherwise} \end{cases} \tag{19}$$

**Utility:**

$$u_i = \begin{cases} \overbrace{S_i(L_i^{PI})}^{Lending\ \ surplus} - \overbrace{\frac{N_i}{-\partial_R N_i}}^{\text{“}markup\text{”}} & \text{if } \ u_i \geqslant \bar{V}_i \\ \bar{V}_i & \text{Otherwise} \end{cases} \tag{20}$$

with $S_i(L_i) := (\alpha_i - mc)L_i$ being the surplus generated by the lending activity.

Equation (18) states that the optimal contract allows borrowers to get the biggest house possible $\bar{L}$. This is because lending generates positive NPV ($\alpha_i - mc > 0, \ \forall i$).

As shown in section 6, in equilibrium, the right-hand side part of the equation (19) does not depend on rates as the mark-up $\frac{N_i}{\partial_R N_i}$ simplifies to $\frac{1}{\sigma_i(1-B^{-1})\theta_i}$.[8] Let us describe the different economic forces at play.

The upper right-hand side of the equation (19) captures the classic extensive and intensive margin channels driving the interest rate level. The first term ($\frac{mcL_i^{PI}}{\theta_i}$) is the fair price. That is the price at which banks break even. The second term is a "markup". It is equal to the inverse price semi-elasticity. The numerator of the "markup" captures the intensive margin: by increasing R, banks earn more on each borrower. The denominator ($\partial_R N_i$) captures the impact of the extensive margin on pricing: by increasing R the bank loses customers.

Figure (1) summarizes the model equilibrium. I plot the borrower indifference curves (Blue for B borrowers and green for G borrowers) and lenders' break-even conditions on each market segment. To make the graph more legible, I consider the situation in which lenders have to pay a fixed cost of originating the contracts (i.e., $mcL = f \in \mathbf{R}^+$).

8. Indeed, under the logit formulation the r terms cancel each other to become $\frac{1}{\sigma_i(1-n_iN_i)\theta_i}$. The parameter $\sigma$ drives the product elasticity. When sigma is high, a lot of the surplus is given back to the borrower. $(1-n_iN_i)$ captures the fact that the price elasticity under a logit depends on the number of competitors. In a symmetric equilibrium, this term will be equal to the relative number of borrowers of type i over the total number of banks.

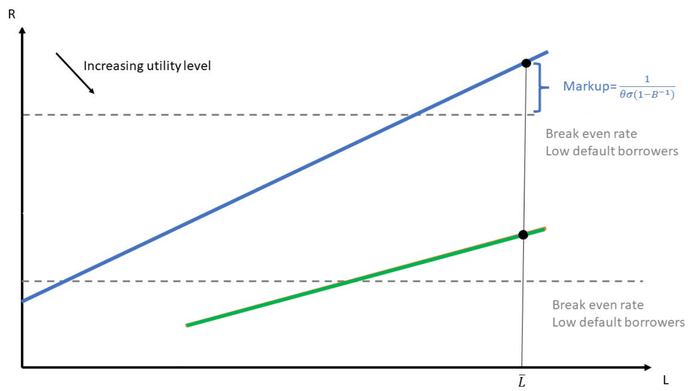


Figure 1: Equilibrium contracts under perfect information

**Impact of additive separability:** Due to the utility of loan size and interest rate being additively separable, absent asymmetric information, imperfect competition does not distort loan size away from its first best value. The reason is that, under perfect information, banks set the contract characteristics that maximise the lending surplus and use interest rates to split the surplus between borrowers and lenders. This separability assumption may not be valid when the loan is used to buy a good that is a complement to consumption. I do not analyze this case in the model as I want to focus on the distortions related to the contractual externality only. An empirical structural model should however consider how this assumption impacts the counterfactual simulations.

## 4.2 Contracts when $WTP_i$ unobservable

Now, let us focus on the perfect information case and how it impacts banks' contracts.

As shown in the previous section, given assumption A1 stated in section 3.3: (i) borrowers get offered the same loan size in the first best, but (ii) banks would like to price them differently due to borrowers' different default probability or price elasticity. As a result, the first best contracts are not incentive-compatible. All else equal, borrowers always choose the cheaper product.

Banks, thus, must distort the first best contracts to maintain borrowers' incentives to self-select. To do so, lenders can use two tools: interest rates (via cross-subsidies) and loan size (via credit rationing). This section analyses this trade-off. Figures 2 and 3 give a visual representation of how incentives are maintained using each of these margins.

As in the textbook principal-agent model, the system of IC can be simplified. The simplifications are summarized in the following Lemma 1, 2 and 3.

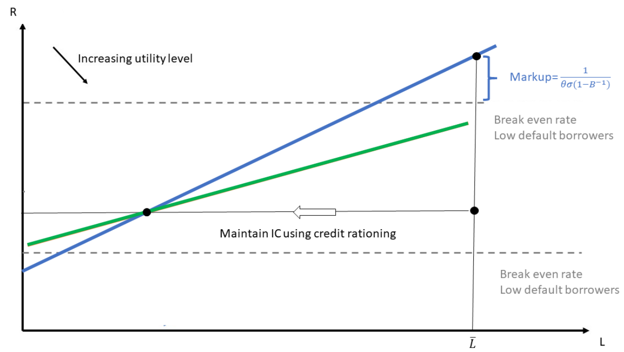


Figure 2: Tool 1 to maintain IC: Use credit rationing

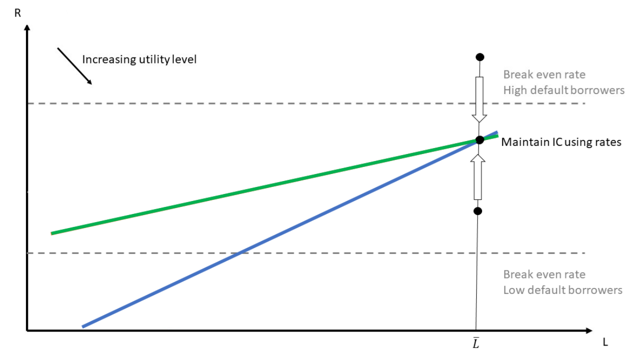


Figure 3: Tool 2 to maintain IC: Use interest rates

***Lemma 1: At least one IC is biding.*** *The IC constraint of B borrower is binding when competition is high enough or if $\theta_B\sigma_B < \theta_G\sigma_G$. When competition is high enough, no participation constraints are binding.*

Proof: Appendix D.

The assumption $\theta_B\sigma_B < \theta_G\sigma_G$ makes sure that the high WTP borrower is always the one that benefits from pretending to be the other type. This is because when this condition is satisfied, their perfect information interest rate is always higher. This assumption makes presenting the problem easier as one does not have to track which IC constraint is binding.

Solving for the problem 15 using Lemma 1, we get:

***Lemma 2: No distortion at the top. Credit rationing at the bottom.***
*(i) The loan size of B borrowers is equal to its first best value.*
*(ii) The loan size of G borrowers is lower than the first best. The credit rationing is proportional to the interest rate spread. Formally:*

$$L_B = \bar{L} \tag{21}$$

$$(IC_B) : \Delta L_G := \bar{L} - L_G = \frac{R_B^{II} - R_G^{II}}{WTP_B} \in [0, \bar{L}] \tag{22}$$

**Proof:** Appendix D

The intuition behind those results can be grasped from Figures 2 and 3. The no distortion at the top results from the fact that lowering the B borrower's loan size does not help with self-selection. It makes the B borrower worse off and makes the G contract even more attractive. Credit rationing works because B borrowers are less willing to pay for loan size. Equation (22) is the B borrower incentive compatibility. It states that screening works when the average cost of one extra dollar is above $L^{PI}$ is priced at the B borrower's willingness to pay. That way, only the B borrowers find the extra amount of borrowing attractive.

***Lemma 3: Interest rate distortions.***

*Relative to the first best, the interest rate of B borrowers is lower, and the one of the G borrowers is higher. The pricing has the form:*

$$R_i := \begin{cases} \overbrace{\frac{mcL_i}{\theta_i}}^{fair\ price} + \overbrace{\frac{N_i}{-\partial_R N_i}}^{\text{“}mark\ up\text{”}} [1 + \overbrace{I_i}^{Asymmetric\ information\ discount/premium} & \text{if } \ u_i \geqslant \bar{V}_i \\ \tilde{W}TP_i L_i - \bar{V}_i & \text{Otherwise} \end{cases} \tag{23}$$

$$I_G > 0, I_B < 0 \tag{24}$$

**Proof:** (i) Use lemma 2 and Lemma 1. (ii) solve for the problem using the Lagrangian.

Equation (23) states that the price can be written as in the perfect information case (fair price and markup) with an extra additive term. I call this extra term the asymmetric information discount or premium (I). The I term enters positively for the G borrower (i.e. premium) and negatively for the B borrowers (i.e. discount). Absent $I_i$, the spread between rates would be higher, implying that banks would have to distort $L_G$ more intensively. $I_i$ thus provide information about the incentives to screen: setting high $I$ allows the spread between rates to be lowered, thus lowering the product distortion on the G market segment. In the extreme case, high I imply that banks offer just one (pooling) contract.

The interest rate formula (23) is a key novelty of the paper. Papers in the literature do not use a continuous demand system and thus cannot derive an equation like this one and rely on numerical methods to interpret the results. The formula captures channels that are absent from perfect competition and monopoly models. Under perfect competition, only the fair price $\frac{mcL_i^I}{\theta_i}$ is present. Under monopoly, the participation constraint is binding for one borrower, and the price of the other borrower is (partially) driven by this outside option. The extra two terms (i.e., the markup ($\frac{N_i}{\partial_R N_i}$) and the asymmetric information premium or discount $I_i$) are thus specific to the imperfect competition case and endogenously determined. Analysing ($I_{G,B}$ ) thus allows us to study how different changes in the economic environment affect incentives to use both interest rates and loan size distortion to screen.

**Banks' trade-off:** $I_i$ summarizes the distortion in the i market segment relative to the perfect information case. It captured the profit loss relative to the perfect information case, given the other lender contracts. It can thus be used to analyse the trade-off faced by lenders.

An increase in $|I_B|$ lowers the B market segment profits compared to the perfect information case but allows for an increase in G market segment profits by relaxing the incentive compatibility constraint. An increase in $I_G$ increases G interest rate distortion but lowers the credit rationing level.

***Proposition 2: Banks' incentives to screen*** *Banks have incentives to screen (i.e., to use lower cross-subsidies) when the surplus generated by lending in the G market segment is low and when screening is costly. The asymmetric information premium and discount are given by:*

$$I_B := -\frac{n_G}{n_B}\frac{\overbrace{\alpha_G - mc}^{Benefit\ of\ pooling:\ More\ surplus}}{\underbrace{WTP_B - WTP_G}_{Cost\ of\ pooling:\ Information\ rent}} > 0 \tag{25}$$

$$-I_G := \frac{\alpha_G - mc}{WTP_B - WTP_G} > 0 \tag{26}$$

I show in the next section (section 5) that an interior solution ($i.e.\ L_G^{II} \in [0, \bar{L}]\ and\ u_i \geqslant \bar{V}$) where the participation constraints are not binding exists when competition is high enough. To gain intuition about incentives to screen, I focus on the interior solution in the subsequent paragraphs.

Equation (25) captures the following trade-off. By lowering high default borrower interest rates above their perfect information value, lenders are able to relax the incentive compatibility constraint and extract the surplus generated ($\alpha_G - mc$ per dollar lent). The benefit is proportional to the number of low-default borrowers they have ($N_G$). However, to be able to lend one dollar more and maintain incentives, lenders have to provide a discount of $WTP_B - WTP_G$. This can be seen using the incentive compatibility constraint of the B borrower in terms of utility ($IC_B:\ u_B = u_G + (WTP_B - WTP_G)L_G$). $(WTP_B - WTP_G)L_G$ is referred to as the information rent in the literature. If a lender wants to provide utility $u_G$ to low-default borrowers, lenders must design a B contract that gives them an extra amount of utility proportional to the differences in willingness to pay ($(WTP_B - WTP_G)L_G$).

Equation (25) captures the following trade-off. Increasing the interest rate allows to lend more and up to ($\alpha_G - mc$ per dollar lent). However, to maintain incentives, there is a limit to how much they increase the surplus they extract. This is captured by $\frac{1}{(WTP_B - WTP_G)}$.

# 5 Equilibrium: Existence and Uniqueness

In this section, I solve for the equilibrium contracts. In contrast with other screening models (for instance Rothschild and Stiglitz 1976, Lester, Shourideh, Venkateswaran, and Zetlin-Jones 2019), my model features a unique pure strategy equilibrium. This existence result allows the analysis of the impact of screening when banks interact with each other without using equilibrium refinements. In that context, I show that there is a screening externality which leads to excessive screening (i.e., not enough cross-subsidization).

**Non-existence result in the literature:** In the textbook perfect competition model of screening (i.e., Rothschild and Stiglitz 1976), pooling contracts cannot be offered. This is because of cream-skimming: a competitor can take advantage of the pooling contract being offered and introduce a new contract with a smaller loan size and price it so that it only steals the most profitable borrowers. Yet, offering screening contracts cannot be an equilibrium when the number of low-default borrowers is large enough. In that case, a pooling deviation exists as low-default borrowers prefer to be pooled as it allows them to get a lower credit constraint at the cost of a small interest rate increase. This intuition is summarized in Figures 4 and 5.

To overcome this non-existence result, one can use other equilibrium concepts as in, for instance, Riley (1979), Bisin and Gottardi (2006) or Wilson (1980). The two first equilibrium concepts restore the existence of the screening equilibrium, while the third one restores the pooling equilibrium. Another way to overcome this issue is to change the modelling. For instance, Guerrieri, Shimer, and Wright (2010) assumes that the principal can match at most

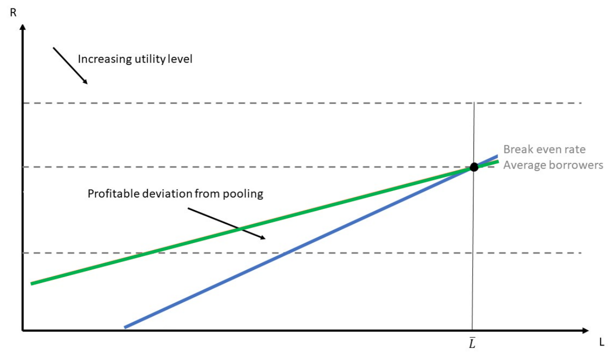


Figure 4: Perfect competition case: Deviation from pooling always exists

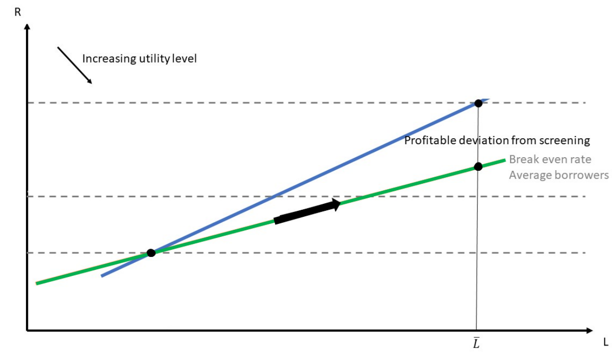


Figure 5: Perfect competition case: Deviation from screening always when the share of good borrower is high

one borrower. Finally, allowing for mixed strategies also solves the non-existence problem (see Dasgupta and Maskin (1986) for a proof that a mixed strategy equilibrium exists, and Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2019) or Farinha Luz (2017) for a numerical solution to those mixed strategies).

**In my model:** The following proposition states the conditions under which the capacity constraint is needed and characterizes the equilibrium.

***Definition 1: Capacity constraint***
*A lender has a capacity constraint if they cannot serve more than a certain number of customers $\bar{N}$. I denote by $k_i$ the maximum share of customers that the firm can serve if they only serve type i borrowers (i.e., $\{k_i : k_i \cdot n_i = \bar{N}\}$). I denote $k = \max_{i \in \{G,B\}} k_i$.*

For simplicity of the notation, we now assume that the price elasticity of both borrowers is the same ($\sigma := \sigma_G(1 - \frac{n_G}{B}) = \sigma_B(1 - \frac{n_B}{B})$).

***Proposition 3: Equilibrium existence, uniqueness and characterization***
***Existence and uniqueness:*** *There exists a unique pure strategy equilibrium as long as the demand elasticity $\sigma$ are lower than $\bar{\sigma} := \frac{1}{[WTP_B - mc]\bar{L} - 1}$ or as long as lenders have a capacity constraint $k < \frac{1}{2}$.*

***Characterization:*** *The equilibrium is symmetric and is given by replacing $N_i$ by $\frac{n_i}{\mathbf{B}}$ and*

$\frac{N_i}{\partial_{R_i} N_i}$ *by* $\frac{1}{\sigma_i(1-\frac{n_i}{\mathbf{B}})\theta_i}$ *in the formulas (22),(23), (91) and (26).*

*Proof:* See appendix E.

In my model, deviations from screening are not profitable when the demand elasticities are low enough. The reason is that deviations from screening attract too many high default borrowers to be profitable. This is because they do not attract the whole market and are relatively more valuable to costly borrowers. The intuition behind why deviation from screening is more valuable to high-default borrowers is the following. Pooling deviations (i.e., using cross-subsidies) require increasing the G contract loan size, which is relatively more valuable to the high-default borrowers. A heuristic derivation of this argument is given in the appendix F.

A visual representation of this argument is provided in Figure 5. The most profitable perfect competition pooling deviation implies moving along the G borrower indifference curve. This deviation is thus more attractive to B borrowers. Under perfect competition, this attracts the full market segment. The break-even rate is thus below the pooling contract deviation when there are enough low-default borrowers. Under imperfect competition, those deviations attract too many high default borrowers to be profitable.

All in all, the equilibrium under imperfect competition features just enough cross-subsidy so that any pooling deviations are not profitable as they attract a relative number of types of agents so that the existence condition in Rothschild and Stiglitz (1976) is satisfied. Deviations using cream-skimming strategies (as in the deviation from pooling case) are also unprofitable because they attract too few borrowers from other banks.

When competition is high enough, pooling deviations become profitable as lenders can attract the full market segment. So, the argument about pooling deviation being relatively more valuable to high default borrowers is not valid anymore. To prevent this from happening, I introduce a capacity constraint.

Using Lemma 2-3-4 and proposition 3, one can derive the level of credit rationing. Given (22), the level of credit rationing is proportional to the interest rate spread. Formally:

***Lemma 4: Equilibrium level of credit rationing (interior solution)****: Abusing* *the notation I use $\sigma_G$ and $\sigma_B$ as a shortcut for $\sigma_G(1-\frac{n_G}{\mathbf{B}})$ and $\sigma_B(1-\frac{n_B}{\mathbf{B}})$.*

$$L_G = \overbrace{\frac{1}{S_{BG}}[S_B\bar{L}}^{perfect\ competition\ distortion\ level} + \underbrace{(\frac{1}{\sigma_G\theta_G} - \frac{1}{\sigma_B\theta_B})}_{competition\ channel} + \overbrace{\frac{I_G}{\sigma_G\theta_G} + \frac{|I_B|}{\sigma_B\theta_B})}^{cross-subsidy\ channel} \tag{27}$$

$$S_{BG} := WTP_B - \frac{mc}{\theta_G} \tag{28}$$

$$S_i := WTP_i - \frac{mc}{\theta_i},\ i \in \{G, B\} \tag{29}$$

Equation (27) captures the three channels driving credit rationing. I use the notation $WTP_i = \frac{\alpha_i}{\theta_i}$ to distinguish the elements in the formula that depend on borrowers' preferences from the one that comes from the supply side.

The first part of the formula (i.e., $\frac{1}{\alpha_B - mc\frac{\theta_B}{\theta_G}}(\alpha_B - mc)\bar{L}$) captures the level of distortion absent any mark-up or cross-subsidy. $(\alpha_B - mc)\bar{L}$ is the utility of B borrowers under perfect competition when the contract is priced at marginal costs ($R_B = \frac{mc}{\theta_B}$). $\alpha_B - mc\frac{\theta_B}{\theta_G}$ is the utility derived from B borrowers per dollar lent when choosing the contract designed for G borrowers under perfect competition ($R_G = \frac{mc}{\theta_G}$). The loan size $L_G = \frac{(\alpha_B - mc)\bar{L}}{\alpha_B - mc\frac{\theta_B}{\theta_G}}$ makes the B borrower indifferent between the two contacts when each contract is priced at the break even price.

The second term captures the distortion level once we add the perfect information markups. $(\frac{1}{\sigma_G\theta_G} - \frac{1}{\sigma_B\theta_B})$ is the difference between the perfect information markups of the G and B contracts when the competition level is $(\sigma_G, \sigma_B)$. A higher spread makes the G contract less attractive for B borrowers which relaxes the credit constraint.

The last term captures how much cross-subsidies are used to relax the credit rationing.

# 6 Analysis of the Equilibrium Contracts

In this section, I do a positive and normative analysis of the equilibrium contracts.

## 6.1 Positive Analysis of the Equilibrium Contracts

This section provides comparative statics with respect to competition ($\sigma$), adverse selection ($\frac{\theta_G}{\theta_B}$), changes in house prices ($\alpha$), and changes in the marginal costs of lending ($mc$). I use those comparative statistics to discuss how monetary policy and capital requirements affect contract terms. I focus on an interior solution (i.e., $L_G < \bar{L}$ and the participation not binding) as changes in $L_G$ provide intuition about the different forces behind pooling and screening incentives.

***Result 1: Competition.*** *Let us denote* $\Delta\sigma := \frac{\sigma_G}{\sigma_B}$ *and look and changes in competition levels* ($\sigma = \sigma_B$)*, holding the ratio* $\Delta\sigma$ *constant.*

*(i) Lender screen when competition is high enough (i.e.,* $\sigma \in [\bar{\sigma}, \infty)$*).*

*(ii) When competition decreases* ($\sigma \downarrow$)*, the amount of credit rationing and the interest rate spread decreases as long as the cross-subsidy effect dominates the markup one (i.e.,* $MU + CS > 0$*). The switching point between screening and pooling* ($\bar{\sigma}$) *is defined by:*

$$\frac{1}{\bar{\sigma}} := \frac{\bar{L}[S_{BG} - S_G]}{MU + CS} \tag{30}$$

$$MU := \frac{1}{\theta_B} - \frac{1}{\Delta\sigma\theta_G} < 0 \tag{31}$$

$$CS := (\frac{I_G}{\Delta\sigma\theta_G} + |I_B|) > 0 \tag{32}$$

The intuition behind result 1 (i) is presented in section 5, which discusses the existence of the equilibrium and how it is compared to classic perfect competition models such as Rothschild and Stiglitz (1976).

Result 1 (ii) derives from the following considerations. First, from Lemma 3 equation (22), we know that the level of credit rationing is proportional to the interest rate spread between contracts. Second, changes in interest rates spread can be decomposed into perfect information markups spread and asymmetric information discounts or premiums spreads. Indeed, under perfect information, banks can increase rates by approximately $\frac{1}{\theta_i} d\sigma^{-1}$ as competition decreases. However, under imperfect information, lenders increase the rate less strongly for high WTP borrowers as they want to provide information rent to relax the incentive compatibility constraint. Thus as competition increases, the spread between interest rate decreases as long as $\frac{1}{\sigma_G\theta_G} \geqslant \frac{1}{\sigma_B\theta_B}$ or $\frac{1}{\sigma_G\theta_G} < \frac{1}{\sigma_B\theta_B}$ and the incentive to provide an information rent dominates (i.e, $MU + CS > 0$).

Equation (30) states that lenders tend to pool more often ($\bar{\sigma} \uparrow$) when there are more profits to be made in the G market segment and when asymmetric information discounts and premiums are effective in dealing with the distortions. Those channels are captured by the CS term in the denominator. The term $MU < 0$ captures the spread between perfect information markups. When $|MU|$ is high, the perfect competition contract is less incentive-compatible, making it more likely to screen. The numerator captures how large the distortion would be absent any cross-subsidies or markups. It is a function of the spread between defaults and the loan size. The higher those elements, the more cross-subsidies are required to maintain pooling. High levels of cross-subsidies are only feasible when competition is low.

Overall, this first result goes against the standard argument that lowering competition leads to lower loan sizes due to monopoly distortions. I summarize Result 1 in Figure 6, where I plot the various types of equilibrium that arise when competition and the number of borrowers varies. To focus on risk discrimination, the figure is plotted for the case in which $\frac{\sigma_B}{\sigma_G} = 1$ and varies the level of product elasticity ($\sigma$) while keeping the ratio of product elasticities constant.

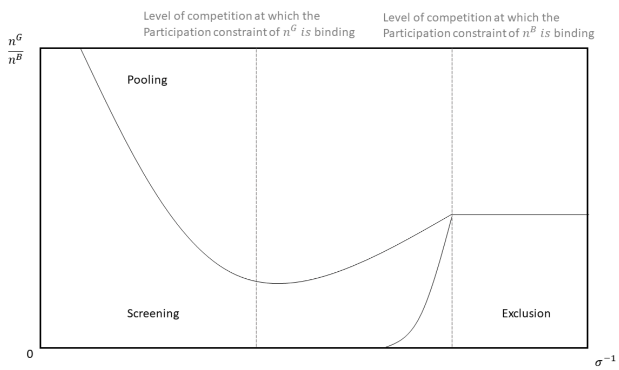


Figure 6: Equilibrium regions with the capacity constraint

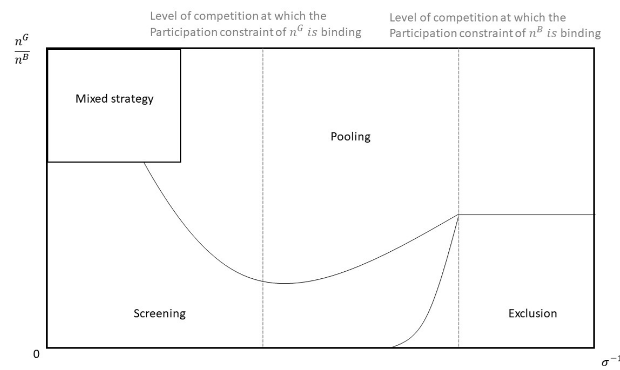


Figure 7: Equilibrium regions without the capacity constraint

***Result 2: Default probabilities (Adverse Selection).*** *An increase in adverse selection measured as the survival probability spread ($\frac{\theta_G}{\theta_B}$) has different effects depending on whether the changes come from default probability in the B or G market segment.*

*(i) An increase in $\frac{\theta_G}{\theta_B}$ caused by a decrease in G default probability increases credit rationing under high competition levels. Under low competition levels, it relaxes credit rationing if the cross-subsidy channel is higher than the perfect competition channel and the competition chanel. When demand elasticities are the same (i.e., $\sigma := \sigma_B\theta_B = \sigma_G\theta_G$), so that the competition channel is absent, the cross-subsidy channel dominates if and only if $\frac{1}{1+\theta_G^{-1} - \frac{WTP_G}{WTP_B}} \geqslant \frac{mc}{WTP_B}$. The competition switching point is $\sigma = \frac{CS(\varepsilon_{CS} - mc)}{mc \cdot L^{PC}}$ where $L^{PC}$ is the perfect competition loan size and $\varepsilon$ is the cross-subsidy semi elasticity to $\theta_G^{-1}$.*

*(ii) An increase in $\frac{\theta_G}{\theta_B}$ caused by an increase in B default probability decreases credit rationing under high competition levels. Under low competition levels, it relaxes credit rationing if the cross-subsidy channel is higher than the perfect competition channel and the*

*competition chanel. When demand elasticities are the same (i.e., $\sigma := \sigma_B\theta_B = \sigma_G\theta_G$), so that the competition channel is absent, the cross-subsidy channel dominates if and only if $\frac{WTP_B}{1+\frac{1}{\theta_B}-\frac{WTP_G}{WTP_B}}\frac{\theta_G}{\theta_B} \geqslant \frac{mc}{WTP_B}$.*

Proof: See Appendix G.

There are three forces at play. To illustrate them, let us consider a situation in which the default probability of the B market segment increases

First, under a high level of competition, the default probability increase must be fully passed through the B contract interest rates. In turn, the loan size of the G contract must be lowered to maintain incentives to self-select. If borrowers' willingness is an increasing function of default probabilities, there is an additional force that pushes the credit constraint in the opposite direction. The increase in default probability increases borrowers' willingness to pay for loan size. As a result, for a given level of credit rationing, the low default contract thus becomes relatively less attractive. This effect allows lenders to decrease the credit constraint. When borrowers willingness to pay is $WTP_i = \frac{\alpha_i}{\theta_i}$, the later effect dominates.

Second, the decrease in the default probability of B borrowers makes them more sensitive to interest rate changes. Thus, the perfect information markup must decrease. As a result, the spread between interest rates and credit constraints decreases. This allows lenders to decrease the credit constraint level.

Third, the decrease in the default probability of the B borrowers increases the potential profits in this market. This creates incentives to distort the market segment G to extract more surplus from contracts designed for market B.

I summarize Result 2 in Figure 9 for the case in which the cross-subsidy channel dominates. When adverse selection decreases due to a decrease in B default probability, the equilibrium region moves from the dotted lines to the solid lines in figure 9.

***Result 3: Willingness to pay (House Prices and Help-to-Buy).***

*(i) A decrease in house prices (i.e., $\alpha_G = \alpha_B$ increases) decreases the interest rate and loan size spread between contracts. This effect is stronger under low levels of competition as long as the lender is not pooling (i.e., $\sigma > \bar{\sigma}$).*

*(ii) A government intervention to increase the value of small size loan via a help-to-buy scheme (i.e., $\alpha_G$ increases) increases the credit rationing and the interest rate spread under high competition. It decreases it under low competition (i.e., $\sigma < \bar{\sigma}_\alpha$) if and only if*

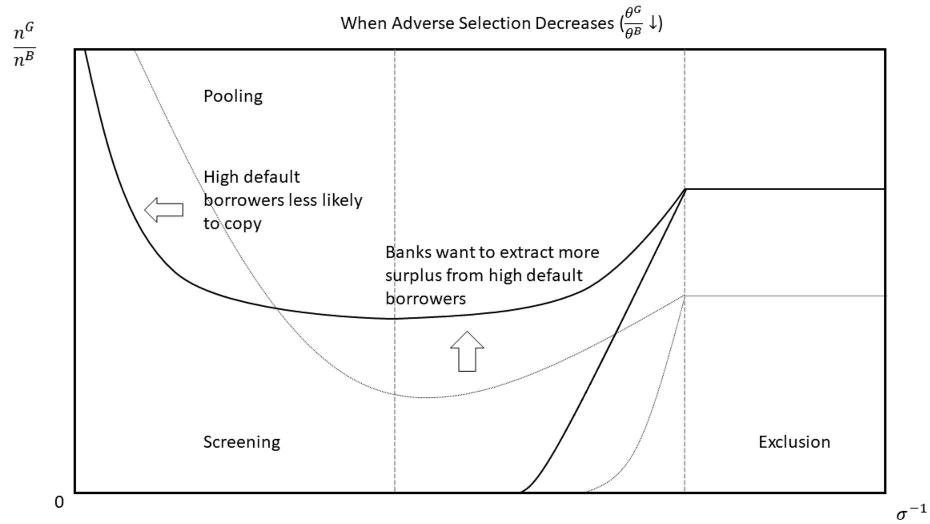


Figure 8: Equilibrium regions when adverse selection decreases

*its* $CS\frac{mc}{\alpha_G} > CS + MC$*. The competition threshold is then* $\bar{\sigma}_\alpha := \frac{-MU - CS[1-\frac{mc}{\alpha_G}]}{PC} > 0$*, where* $PC := S_B\bar{L}$ *denotes the utility of B borrowers when getting the perfect competition B contract.*

Proof: See Appendix G

The intuition for the result (i) is the following. Decreasing house prices increase borrowers' willingness to pay for each dollar lent. This lowers credit constraints via two channels. First, the price change makes credit rationing more costly for high-default borrowers. All else constant, the G contract becomes less attractive, so the perfect competition credit rationing level decreases. Second, credit rationing becomes more costly for lenders as the G market segment generates more surplus. This creates incentives to use asymmetric information discounts and premiums more intensively to lower credit rationing.

There are two channels driving the help-to-buy result. First, holding contract terms constant, the G contract becomes more attractive to B borrowers. This makes the level of cross-subsidy required to completely undo credit rationing higher. Second, the help-to-buy policy makes the G market segment more profitable, which creates incentives to increase the cross-subsidy level. In the formula $CS\frac{mc}{\alpha_G} > CS + MC$, $CS + MC$ captures the first channel while $CS\frac{mc}{\alpha_G}$ captures the second channel. $\frac{mc}{\alpha_G}$ is the strength of the marginal impact of an increase in help to buy on the cross-subsidy.

***Result 4: Marginal costs (Monetary policy, risk weights).***

*(i) Increasing the marginal cost of lending via monetary policy or risk weights increases the interest rate and loan size spread between contracts under all competition levels.*

*(ii) Increasing the marginal cost of lending via for instance capital requirements of low loan contracts decreases the credit rationing level and interest rate spread under high competition but increases it it under low competition (i.e.,* $\sigma < \sigma_{mc_G} := \frac{CS[\frac{S_{BG}}{S_G}-1]-MU}{PC}$*).*

*(iii) Increasing the marginal cost of lending of high loan contracts increases the amount of credit rationing.*

Proof: See Appendix G

Again, let us decompose the effect into a perfect competition and imperfect competition channel. Under perfect competition, increasing the marginal cost of G contracts makes the G contract more attractive. As a result, the incentive compatibility constraint is relaxed and lenders can increase the G loan size. Increasing the marginal cost of B contracts has the opposite effect, as it tightens the incentive compatibility constraints. When both marginal costs increase, the second effect dominates as interest rates increase relatively more in the B contract because they are scaled up by default probabilities.

Under imperfect competition, there is an additional effect. The increase in the G marginal cost decreases the surplus in the G market. As a result, this lowers incentives to cross-subsidize.

## 6.2 Normative Analysis: Screening Externality and Pareto Improvement

In this subsection, I show that excessive screening may be caused by a contractual externality. Indeed, an informational-constrained social planner would choose a menu that is a Pareto improvement over the decentralised equilibrium. This implies that there is room for policy interventions. I analyze how competition policies, monetary policy, and capital requirements affect this externality.

**Contractual externality:** To get a lower bound on the contractual externality, I show that there exists a menu of contracts that makes all borrowers and lenders better off. I do so by solving for the optional contract of an informationally constrained social planner.

Formally, the social planner maximizes firms' profits subject to the same incentives compatibility constraints as the firms and subject to providing at least as much utility to borrowers as the market contracts for a given competition level (denoted $(u_B^*, u_G^*)$). That is, the problem is the same as (15) but with constant market shares ($n_i$) and the participation constraints being equation to $u_i^*$ instead of $\bar{V}$. The constant market shares assumption shuts down the contractual externality. The social planner problem can be written:

$$\max_{\{(L_B,L_G)\in[0,\bar{L}]^2\}} {}^{i} \overbrace{n_i}^{Market\ size} \overbrace{[(WTP_i - \frac{mc}{\theta_i})L_i - u_i^*]}^{profits\ on\ type\ i\ market\ segment}$$
$$s.t.\ (IC_B) : u_B^* = u_G^* + (WTP_B - WTP_G)L_G$$

Using the first-order conditions, I derive the following Proposition:

**Proposition 4: Sufficient Statistic for the Contractual Externality.** *Banks should pool borrowers together at $\bar{L}$ if and only if:*

$$\frac{\alpha_G - mc}{(WTP_B - WTP_G)} \frac{n_G}{n_B} \frac{1}{\theta_G} > 1 \tag{33}$$

*However, except under monopoly, the competitive equilibrium switching point is different than the social planner one (33). For instance, according to Result 2, when both participation constraints are not binding, banks pool borrowers if and only if: $\sigma \in [0, \bar{\sigma}]$.*

Proposition 4 shows the conditions under which the equilibrium is inefficient in the second-best sense. I show in the empirical section how to recover it.

The friction emerges for the following reasons. Maintaining customers' incentives to self-select is costly as it may require distorting contracts relative to the first best. When the distortions are too high, it is more efficient to pool borrowers instead. Yet, if pooling contracts are offered and competition is high, a competitor can take advantage of a pooling contract by introducing a product that steals the most profitable customers only (cream skimming). The friction arises because lenders do not internalize how their screening strategies (e.g., cream skimming) change the types of borrowers selecting competitors' products — and thus the cost of lending via those products.

This friction exists only when pooling is a Pareto improvement over screening so that cream-skimming deviations are ex-post-inefficient. Equation (33) captures the condition for the latter to be true. The numerator is the benefit of increasing $L_G$ while the denominator represents the costs. Increasing the loan size allows the social planner to generate more profits $(WTP_i - \frac{mc}{\theta_i})$ per G borrowers. However, to maintain incentives to self-select, lenders have to decrease the interest rate on the B market segment, leading to a loss of $(WTP_B - WTP_G)$ per B borrower.

Equation (33) could be used to test for the existence of the contractual externality. One can use a revealed preference approach to recover borrowers' willingness to pay and relative contracts' shares to get the relative numbers of borrowers. The marginal cost can be recovered either using accounting data or by estimating it using the interest rate first order conditions as standard in the industrial organisation literature.

**Effect of Policy interventions:** To analyze if policy interventions have a positive impact on welfare, I use the following lemma:

***Lemma 5: Equilibrium utility and profit levels (Interior Solution)***:

$$u_B = (\alpha_B - mc)\bar{L} - \pi_B \tag{34}$$

$$u_G = (\alpha_B - mc)L_G - \pi_G \tag{35}$$

$$\Pi_B := n_B\pi_B = n_B\frac{1}{\sigma_B\theta_B}[1 - \frac{\alpha_G - mc}{WTP_G - WTP_B}\frac{n_G}{n_B}] \tag{36}$$

$$\Pi_G := n_G\pi_G = n_G\frac{1}{\sigma_G\theta_G}[1 + \frac{\alpha_G - mc}{WTP_G - WTP_B}] \tag{37}$$

The equations (34) and (35) state that borrowers' utility is the surplus generated by the loan minus the lender profits per borrower. Equations (36) and (36) state that the lender profits are the markup plus or minus the asymmetric information premium or discount.

**Changes to competition:** I analyse how changes in competition $(\sigma^{-1})$ affect the welfare of the different market participants. For the welfare results to be independent of the welfare function used, I then look if there exists a situation in which a decrease in competition leads to Pareto improvements.

Decreases in competition decrease the interest rate of the B contract and increase the B borrower utility when the asymmetric information discount is higher than the pure markup effect (i.e., $\frac{\alpha_G - mc}{WTP_G - WTP_B}\frac{n_G}{n_B} > 1$).[9] This happens when the surplus in the B market segment

9. Because of the linearity assumption of the utility, the condition implies that lenders make negative profits on the B market segment. It is not necessarily true otherwise.

is large (i.e., $\frac{\alpha_G - mc}{WTP_G - WTP_B}$ large) and when the cost of screening is low (i.e., $WTP_G - WTP_B$ low) and the relative number of G borrower is high.

A decrease in competition increases G borrowers' rate but also relaxes the credit rationing. It increases welfare when the increase in price is compensated by the increase in loan size (i.e., $\frac{S_G}{S_{BG}}[MU + CS] > \frac{1}{\Delta\sigma\theta_G} + \frac{1}{\Delta\sigma\theta_G}[\frac{S_G}{WTP_B - WTP_G}]$). This happens when the relative number of G borrowers is high or if the G market segment is relatively more elastic.

A decrease in competition has an ambiguous effect on lenders' profits. It increases lenders' profits if incentives to provide an information rent information rent are low enough $n_G \frac{S_G}{WTP_B - WTP_G}[\frac{1}{\Delta\sigma\theta_G} - \frac{1}{\theta_B}] + \frac{n_B}{\theta_B} + \frac{n_G}{\Delta\sigma\theta_G} > 0$. For instance, when interest rate elasticities are the same (i.e., $\sigma_G\theta_G = \sigma_B\theta_B$), a decrease in competition always increases profits. When the interest rate elasticity of the G market segment is high, and the benefit of screening is large ($\frac{S_G}{WTP_B - WTP_G} > n_B$ and $\Delta\sigma$ large enough ), a decrease in competition can decrease profits. This is because a decrease in competition creates incentives to distort more the the B market segment to extract surplus on the G market segment. However, since the G market segment is relatively more elastic, lenders start competing more on this market. Overall, the latter effect dominates.

As a result when the number of high-borrowers is high enough, when the G market's surplus is high enough, then a decrease in competition is a Pareto improvement.

**Changes to the marginal cost of lending:** I analyse how changes in the marginal cost of lending ($mc$) affect the welfare of the different market participants. For the welfare results to be independent of the welfare function used, I then look if there exists a situation in which a decrease in competition leads to Pareto improvements.

Let us denote $mc_i$ the marginal cost of lending via i contracts.

An increase in $mc_G$ makes interest rates in the B market higher. The surplus in the G market lower, which makes the surplus in the B market relatively higher. It gives incentives to provide less information rent to B borrowers. A decrease in $mc_B$ is passed through lower interest rates. This increases the welfare of B borrowers.

The effect of an increase in $mc_G$ on G borrower is ambiguous. An increase in $mc_G$ is partially passed through interest rates. It also lowers the relative surplus in the G market segment, which gives incentives to increase the asymmetric information discount (i.e., the interest rate) so that more profits can be made in the B market segment. The rate increase also decreases scream-skimming incentives, so lenders can increase the cross-subsidy and the amount lent in the G market segment without the threat of losing their customers. Overall, welfare can increase. $mc_B$ changes have no impact.

Increases in $mc_G$ positively impact profits if the interest rate elasticity is higher in the G market segment. This is because an increase in $mc_G$ provides incentives to provide lower information rents. $mc_B$ changes have no impact on lenders' profits in my model as the cost is fully passed through borrowers.

As a result, an increase in $mc_G$ coupled with a decrease in $mc_B$ can be a Pareto improvement policy. The extreme case scenario is equivalent to banning contracts with lower than $\bar{L}$ loan size, which prevents cream-skimming. This is a Pareto efficient policy when cream skimming is ex-post inefficient

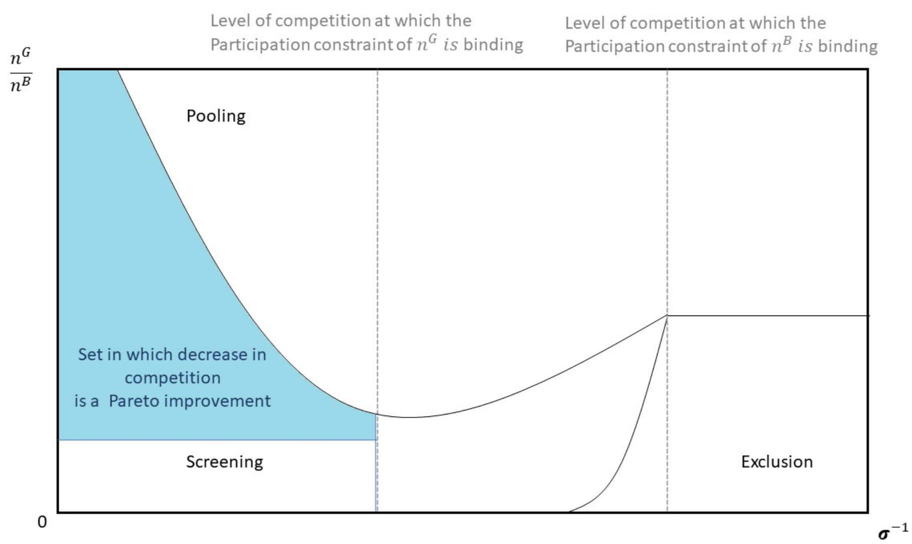


Figure 9: Pareto set

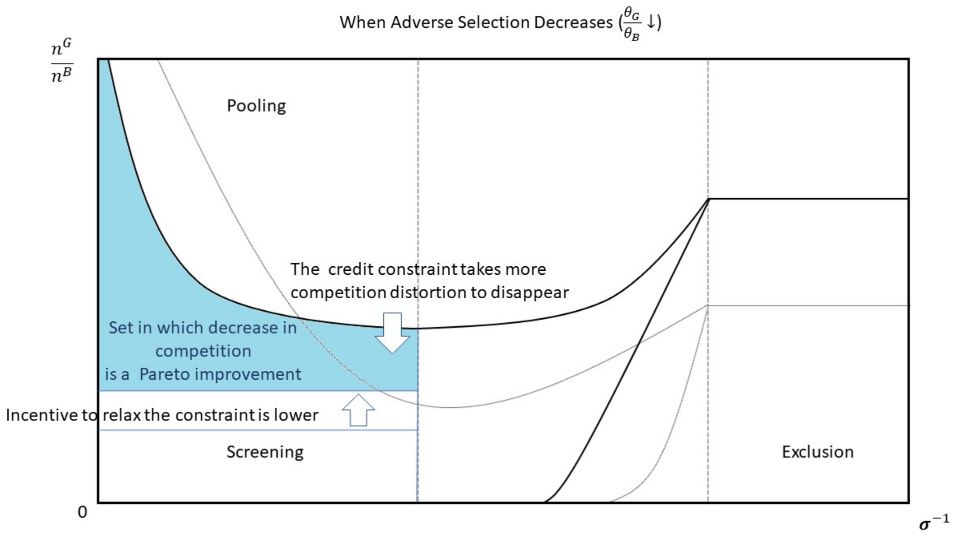


Figure 10: Pareto set when adverse selection decreases

# 7 Bringing the Model to the Data: A Guidebook

This section provides a guide on how to apply the model to the data. It discusses the motivating empirical facts that guide the modelling assumptions.

For the framework to cover a wide range of empirical settings, I show in Appendix K how to extend the model to include multiple borrower types and contract characteristics.

## 7.1 Identification and Estimation

I consider two cases. First, a situation where the econometrician has data on menus and customer choices for an individual firm. This is the most common scenario in which menus are observable (see, for instance, Hertzberg, Liberman, and Paravisini (2018) for the credit market or Tebaldi (2024) for the insurance market). I refer to this as the proprietary data case.

Second, I discuss a situation where the econometrician has data on multiple firms but can only see the equilibrium contracts chosen by each customer. This is the data constraint faced when using credit register data.

### 7.1.1 First Case: Proprietary Data on a Single Firm

The estimation is based on a three-step procedure.

**Step 1: Default Probabilities $(1-\theta_i)$.**

I assume that the per period survival probability $\theta_i$ is not affected by maturity, loan size and interest rate other than through monthly payments. That is, borrowers default if their disposable income $mp_i$ is lower than the monthly payment $M_i$. I consider that disposable income is observable by the lender. In practice, lenders have a good sense of this value through credit checks.

The assumption implies that the income process is the most important source of heterogeneity (and of private information) among borrowers. The modelling is consistent with the empirical evidence in Hertzberg, Liberman, and Paravisini (2018) showing that the unobserved heterogeneity driving borrowers' behaviour in the consumer lending market is due to private information about the income process rather than a difference in risk aversion or interest rate risk. It is also consistent with the Bank of England NMG household survey that documents that the main reasons for defaulting are an increase in other bills or unexpected expenses/costs (21 percent), being made redundant, unemployed or getting a cut in wage (67 percent), or an increase in loan payment (10 percent).

A convenient implication of this assumption is that it allows recovering the key model parameters without having to estimate default elasticities (see Step 2). Identifying default elasticities relies on comparing borrowers who are observationally equivalent but chose different contracts. Because of the selection on unobservables, finding the right control group is problematic when lenders screen.

Formally, denoting $mp_{it}$ the random variable capturing the disposable income process of borrower i at time t, the borrower survival probability at maturity $T_i$ is:

$$S_i := E[\mathbf{1}_{\{\exists t<T_i : M_i < mp_{it}\}} | T_i, X_i, i \text{ chose contract } c] \tag{38}$$

$X_i$ are borrowers' characteristics or economic environment variables that are observable by the lender, that affect the income variance, and that are legal to use to make acceptance and rejection or pricing decisions.

For each observationally equivalent borrower choosing contract c (indexed by j $\in [\![1, n_c]\!]$), one can estimate the average survival probability ($\hat{S}_i$) using realised default at maturity:

$$\hat{S}_i := \sum_{j=1}^{n_c} \mathbf{1}_{\{j \ did \ not \ default\}} \tag{39}$$

Using average default probabilities at maturity is the most common approach in structural works (see, for instance, Crawford, Pavanini, and Schivardi 2018 or Benetton 2018).

Using a constant hazards rate model, and denoting $T_i$ the loan maturity in months, this yields the following constant monthly survival probability:

$$\hat{\theta}_i := e^{-\hat{\lambda}_i}, \ with \ \lambda_i : \ \hat{S}_i = e^{-\hat{\lambda}_i T_i} \tag{40}$$

The present value of a loan is then $[1 - e^{-\lambda T}]M \approx \lambda T M$. The model can be extended to any distribution process. The constant hazards rate model is convenient when screening with maturity (see empirical application). It is consistent with assuming that the income process follows the same Bernoulli distribution in each period with values $mp_i$ and $\delta < M_i$. $\delta$ captures the bad shock described in the Bank of England NMG household.

**Step 2: Borrower preferences ($\alpha_i$).**

Most menus offer various maturity options, allowing borrowers to reach any given monthly payment for any given loan size. I consider that borrowers max out their maximum monthly payment capacity ($mp_i$). In a model without saving this is the monthly income net of irreducible costs (e.g., rents, some type of consumption). This feature arises naturally from a model with linear utility and default being costly (see the following footnote[10]). It is consistent with empirical evidence (Argyle, Nadauld, and Palmer 2020) showing that borrowers use a monthly payment rule when making their decisions.

Together with the default model used in Step 1, this approach allows to avoid estimating default elasticities to contract terms and makes the model empirically tractable. Depending on the setting and the data available, it may be preferable to depart from this assumption. The interested reader willing to do so can refer to, for instance, Taburet, Polo, and Vo (2024).

I use a revealed preference approach to recover borrowers' preferences. The identification leverages the idea that if a borrower chooses a contract with a loan size of $1,000 while they had access to a contract a loan size of $1,100 for an interest rate increase of, say, 100 bps,

10. For instance with the following model: $\max_L \alpha L - p(rL > W)rL - (1-p)c$, $c > r\bar{W}$ and W follows a Bernoulli with its maximum value being $\bar{W}$.

it must be that their willingness to pay for loan size is lower than 100 bps. Figure (11) provides a visual representation of this argument. I plot the internet rate of the contract as a function of loan size (i.e., the pricing schedule) and the borrower indifference curve. When a continuum of contracts is offered, the optimality condition implies that the slope of the indifference curve (i.e., the willingness to pay) must be tangent to the pricing schedule at the contract chosen.

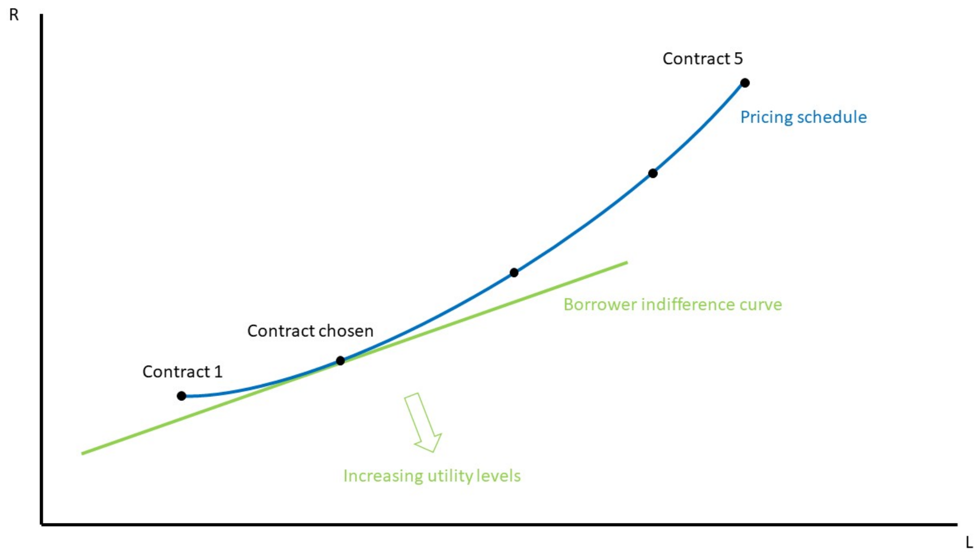


Figure 11: The slope of the pricing schedule at the contract chosen allows us to recover the borrower's willingness to pay $\frac{\alpha_i}{\theta_i}$

Formally, borrower I choose the contract designed for them (contract i) if its indirect utility is higher than the one if they were to choose similar contracts (i-1 and i+1). Considering contracts that yield the same monthly payment and denoting G the probability distribution function of $\alpha$, the indirect utility inequalities yield:

$$u_i(R_i, L_i) \geqslant u_i(R_j, L_j) \ \forall j \implies a := \frac{R_i - R_{i+1}}{L_i - L_{i+1}} \leqslant \frac{\alpha_i}{\theta_i} \leqslant b := \frac{R_i - R_{i-1}}{L_i - L_{i-1}} \tag{41}$$

Relative contract market shares ($\frac{n_i}{\sum_c n_c}$) and the variables (a, b) are observable by the econometrician. So an estimate of G is:

$$\hat{G}(a \leqslant \frac{\alpha}{\theta} \leqslant b) = \frac{n_i}{\sum_c n_c} \tag{42}$$

The key advantage of choosing contracts which yield the same monthly payment is that survival probability $\theta$ stays constant across contract choices. This allows fixing the burden of payment channels and avoids having to estimate default elasticities.

Exogenous variation in a or b allows the recovery of the full distribution non-parametrically. One can use variation in contract terms coming from either cross-sectional or time series variation. For instance, assuming that the distribution of borrower type stays constant over time exogenous variation in the spread between contracts' interest rates allows the recovery of the distribution of borrowers' preferences to default ratio.

Parametric assumptions on G can help with power when the number of observations is small. Moral hazard, as long as its effects are homogeneous across types, can be dealt with by looking at variations of contracts other than contract i. For instance, Taburet, Polo, and Vo (2024) uses variation in the contract-specific marginal cost of lending to recover the preference heterogeneity in a model with moral hazard and adverse selection. The advantage of this approach is that it does not require the use of contract fixed cost to justify that only a discrete number of contracts is offered in the data. It is well known in the literature that the fixed cost makes counterfactual simulation with an endogenous number of contracts ill-behaved (see for instance Wollmann 2018). Appendix L proposes an alternative approach based on an approximation of the pricing schedule into a continuous function as in Figure (11).

**Step 3: Demand elasticities ($\sigma_i$).**

The approach used in this section leverages the fact that proprietary data offers information on marginal costs or that it is common practice in banking to consider that the deposit rate is the marginal cost of lending. The empirical application of section 8 showcase how this approach can be generalized to recover both the demand elasticity, the marginal cost and the information rent.

Given an estimate of borrower's parameter $(\alpha_i, \theta_i)$, and conditional on observing the cost of lending ($mc$) (for instance, equal to the deposit rate) one can construct the asymmetric information premium and discount using the equilibrium conditions from proposition 3:

$$\hat{\theta}_i \hat{I}_i := [\frac{1}{n_i} \frac{\alpha_i - mc}{\Delta WTP_i} 1_{i>1}] - [\frac{1}{n_{i-1}} \frac{\alpha_{i+1} - mc}{\Delta WTP_{i+1}}] 1_{i<x} \tag{43}$$

The demand elasticity of each group is recovered using the loan present value equilibrium conditions:

$$\theta_i R_i = \hat{m}c\text{Ł}_i + \sigma_i[1 + I_i] \implies E[\hat{\sigma}|c] := R_i - \frac{\hat{m}c\text{Ł}_i}{E[\hat{\theta}_i|c]}[1 + \hat{I}_i]^{-1} \tag{44}$$

By assuming a symmetric equilibrium, I avoid estimating demand elasticities. Similarly to the default elasticities, demand elasticities are problematic to estimate when there is a selection of unobservables, even with credit register type of data. This is because the econometrician only observes the contract chosen and thus has to estimate the contract offered by competitors using observational equivalent borrowers.

**Discussion: Heterogeneity within contracts.**

With those three steps, I recover borrowers' preferences ($\alpha_i$), default ($1-\theta_i$) and elasticity ($\sigma$) conditional on contract choice. If lenders pool borrowers, then there may be substantial borrower heterogeneity within a contract.

To learn more about this heterogeneity without having to impose parametric assumptions, one can use the fact that the pooling contract was optimal. For instance, if the econometrician observes that only one contract with loan size $\bar{L}$ is concerned that distribution has at least two borrowers, they can deduce that the $L_G$ implied by the equation (27) is above $\bar{L}$. Conditional on, for instance, $(\sigma, \theta)$, this inequality provides a bound on the value of $\theta$ within borrowers that chose this sample.

### 7.1.2 Second Case: Credit Register Data

When data on menus are unavailable, one has to predict the contract that could be offered using observational equivalent borrowers. Crawford, Pavanini, and Schivardi (2018) uses this approach and predicts the contracts offered by competitors

In that situation, data on lenders' marginal cost of lending are often not available. It is then common practice in industrial organisations to estimate the demand elasticity using, for instance, a logit model for bank choice and use equation (44) can then be used to recover marginal costs instead of demand elasticities ($\sigma$).

In the context of m model, one can use equation (100) to recover the utility conditional on bank choice ($u^*_{i,b}$) and then use a logit model with a utility of the form: $u^*_{i,b} + \mu_b + \sigma\varepsilon_{ib}$, with $\varepsilon_{ib}$ iid and extreme value distributed and $\mu_b$ is a bank fixed effect.

# 8 Empirical Application

## 8.1 Data and Setting

To illustrate the approach, I use data from a key paper on screening in credit markets: Hertzberg, Liberman, and Paravisini (2018). The data is on an online lending company Lending Club (henceforth LC) from 2013. This is an ideal institutional setting for studying screening on borrowers' private information. It provides all the borrower information observed by lenders at the time of origination. The menu offered and the borrower's choice are also observable.

LC is an online lending platform that offers unsecured loans for amounts between \$1,000 and \$35,000. LC was the largest online lending platform in the U.S. in that period. It originated \$4.4B in consumer loans across 45 states. Prosper Marketplace, its nearest rival, originated \$1.6B in the same year.

The borrowing process is the following. Prior to the borrower selecting a loan amount or maturity, LC assigns one of 25 risk categories to them using a proprietary credit risk assessment algorithm. The algorithm uses the hard information in a borrower's credit report (e.g., FICO score, outstanding debt, repayment status) and income. Conditional on the risk category, borrowers get offered the same menu. They can chose how much to borrow and chose a maturity of 36 or 60 months. The interest rate only depends on the risk category, not on the loan size or maturity. For low loan size amounts, only the low-maturity contract is available. I observe the risk category, the menu available to the borrower and their choice. Figure 12 plots the menu available to a borrower categorised as C1 in February 2013.

## 8.2 Adapting the Model to the Setting

For the model to be able to deliver a continuum of loan size I consider the following extensions. For a given risk category, I consider that there are N types of borrowers (i.e., N combinations of $(mp_i, \frac{\alpha_i}{\theta_i}, \sigma_i))$ where $mp_i$ is the disposable income $\frac{\alpha_i}{\theta_i}$ is the willingness to pay for a dollar, and $\sigma_i$ is the demand elasticity.

Borrowers have linear utility on loan size so that, conditional on maturity, they borrow as much as possible until their monthly payment capacity ($mp_i$) is maxed out. For each borrower, I can thus construct the following incentive compatibility constraint:

$$\alpha_i L_i - PV(mp_i, T_i) \geqslant \alpha_i L_j(mp_i, M_j) - PV_i(mp_i, M_j) \tag{45}$$

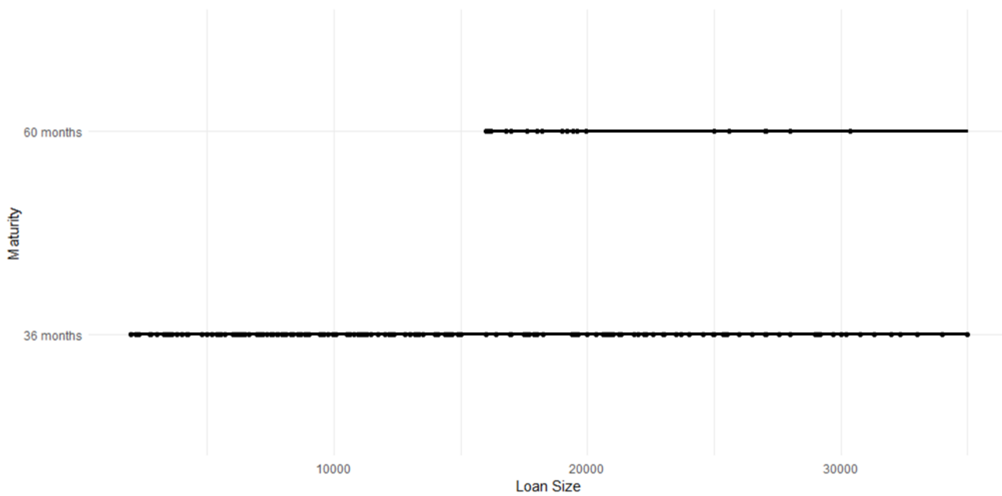


Figure 12: Maturity as a function of loan size (investment grade C1 in February 2013)

$L_j(mp_i, M_j)$ is the loan size that would be chosen by the borrower if they were to choose a contract with the other maturity. $PV(mp_i, T_i)$ is the present value of the loan as a function of maturity $T_i$.

Using the annuity formula, one can show that the heterogeneity in $mp$ is irrelevant as it cancels out in the incentive compatibility constraints. This feature allows us to abstract away from $mp$ and focus on the two contract cases only. [11]

The fact that LC offers the same interest rate independently of loan size is consistent with the assumption that the distribution of maximum monthly payment $mp_i$ is independent of the distribution of willingness to pay and $\frac{\alpha_i}{\theta_i}$ and demand elasticities , $\sigma_i$ conditional on risk category. This is the modelling approach I use for this application. I recover the distribution of $mp$ using the observed distribution of monthly payments. I am agnostic about the number of types $(\frac{\alpha_i}{\theta_i}, \sigma_i)$. Appendix (M) shows that the equilibrium conditions of the model with an arbitrary number of borrower types but with LC choosing to offer two optimally, and the optimality conditions with two borrower types are identical.

11. In continuous time, the principal is $L = mp \cdot [1 - \frac{e^{-r_i T_i}}{r_i}]$, and the present value is $mp \cdot e^{-\lambda_i M_j}$. $\lambda_i$ is the hazard rate so that $e^{-\lambda_i}$ is the monthly survival probability. Formally, I linearize $e^{-\lambda_i M_j}$ and using the change of variable $\tilde{L}_i := [1 - \frac{e^{-r_i T_i}}{r_i}]$.

As I do not observe maturity above 60 months, I assume that their cost is such that longer than 60 month loans are not profitable. This assumption plays the same role as the maximum loan size $\bar{H}$ in the baseline model.

## 8.3 Empirics

I apply the approach discussed in section 7.1.1. Given the variation in menus generated by risk categories, I can use only step 1 and a generalized version of step 3 to recover the key model parameters. Nonetheless, I present the results of step 2 at the end of the section to show how they can be used to get information about the distribution of willingness to pay and that the results are plausible.

**Default Probability Estimates (Step 1):** I start by calculating the average default probability on long and short-maturity conditional on the borrower's investment grade and the monthly payment-to-income ratio (Denoted $PI_i$). I run the following regression:

$$1 - Default_i = \delta_i^{gt} + \beta \frac{Montly\ Payment_i}{Income_i} + \alpha \mathbf{1}_{\{T_i = 60\ Month\}} + e_i \tag{46}$$

$Default_i$ is a dummy equal to 1 if the loan is reported as charged off after the loan should have been completely repaid. $\delta_i^{gt}$ are investment grade-time fixed effects. The superscript g indexes the grade and t the time. $\frac{Montly\ Payment_i}{Income_i}$ is the monthly payment to yearly income ratio. I normalize the standard deviation to one. $\alpha$ is the coefficient of interest. This coefficient captures whether high default borrower self-selects into different loans.

I define $\theta_i$ as the monthly survival probability. Formally, denoting $\lambda_i$ the hazard rate:

$$\theta_i := e^{-\lambda_i} \text{ where } \lambda_i := -\frac{log(E[1 - Default_i | g_i, T_i, \frac{Montly\ Payment_i}{Income_i}])}{T_i} \tag{47}$$

**Marginal costs, Demand elasticicities, asymmetric information premium and discounts (Generalized Step 3):**

I use the equilibrium condition to recover the marginal cost, the asymmetric information premium and discounts and the demand elasticity. Instead of inverting the equation like suggested in section (7.1), I leverage the variation in the relative market share of the contracts for each investment grade.

I run the following two regressions. For the high maturity contract, regress the present


Figure 13: Decomposition of the present value of the average loan (normalized by loan size)

value of the loan on loan size, a constant, and the relative contract shares ($\frac{n_{36}}{n_{60}}$ where 36 stands for the low maturity contract). For the high maturity contract, I regress the present value of the loan on the loan size and a constant. I parametrize the principal of the loan R as a linear approximation of the continuous time annuity formula ($\rho R := M[1 - e^{\rho T}] \approx \rho m T$ where m is the monthly payment T is the maturity so that $\theta M \rho T$ is the approximate loan present value). All coefficients on the right and side should be interpreted as divided by $\rho$. This implies that the equation identifies the relative value of each component of the pricing equation. Formally I run the two regressions on a given month:

$$E[\theta_i | g_i, T_i = 60] R_i = mc_{T_i} \text{Ł}_i + \sigma_{T_i} + \beta \frac{n_{36g}}{n_{60g}} + e_i \tag{48}$$

$$E[\theta_i | g_i, T_i = 36] R_i = mc_{T_i} \text{Ł}_i + \alpha + e_i \tag{49}$$

$(mc_{T_i}, \sigma_{T_i}, \beta, \alpha)$ are the parameters to estimate. The error term $e_i$ captures the heterogeneity in demand elasticity $\sigma_i - \sigma$ and information rent $I_g \sigma_i - I\sigma$ where $I_g := \frac{\alpha_{36g} - mc}{WTP_{60g} - WTP_{36g}}$. I allow for the marginal cost to vary at the contract-month level, so it is unlikely that the error term contains heterogeneity in costs.

The coefficient $\beta$ is the information rent $\sigma_{60} I$. I recover $\hat{I}$ as $\frac{\hat{\beta}}{\hat{\sigma}_{60}}$. The coefficient $\alpha$ captures the perfect information markup and the asymmetric information premium $\sigma_{36}[1+I]$. I recover $\hat{\sigma}_{36}$ as $\frac{\hat{\alpha}}{1+\hat{I}}$.

Equations (48) and (49) are identity regressions. Thus the independent variable impact should not be interpreted as causal. However, allowing for the error terms to come from borrower unobservable heterogeneity implies the following exclusion restrictions. The linearity of the equation allows the use of the full set of tools developed in the reduced-form literature. This is one main advantage of this model compared to a theory-oriented framework such as Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2019). It also makes the exclusion restrictions transparent.

The variation in relative market shares across investment grades allows us to identify $\beta$. The identifying assumption is that the relative market share variations $(\frac{n_{36g}}{n_{60g}})$ are uncorrelated with preference heterogeneity $WTP_{60g} - WTP_{36g}$ and uncorrelated with the average demand elasticity $\sigma_B$ of borrowers choosing 60-month maturity contracts. As the investment grade goal is to predict default (i.e., the borrower cost), not demand elasticities, this assumption is plausible.

The variation in loan size within a group allows for recovery of the marginal cost $mc$. Conditional on the market segment, the loan size variation should be uncorrelated with deviation from the average demand elasticities $\sigma_{T_i}$ (i.e., search costs). To mitigate the endogeneity of concerts, I use a granular instrument approach. That is, I instrument loan size using the residual from a loan size regression controlling for contract characteristics and observable borrowers' characteristics. The underlying assumption is that the search cost heterogeneity is uncorrelated with the residual demand. The results are very similar with and without the instrument.

## 8.4 Results

Table 1 reports the regression results of the default regressions. The average default probability is 15 percent. Borrowers that chose high maturity contracts are $\alpha$ is 4% more likely to see their loan charged off. The implied average hazard rate is $4 \cdot 10^{-3}$ and the one of borrowers with high maturity contract is $2.7 \cdot 10^{-3}$. The hazard rate allows taking into account the fact that with constant default probabilities, longer loan are more likely to be charged-off.

The results of the present value regressions are reported in Tables 2 and 3. All coefficients are significant and have the sign predicted by the theory. Figure (13) provides a visual decomposition of the present value of the loan. All else equal, high-maturity contracts are more expensive to originate (the marginal cost is 1.5 per dollar lent instead of instead of 1.2). Borrowers that self-select into low maturity contracts are less likely to default (from step 1). They are also very elastic, so they do not pay any markups. This is consistent with the fact that searching costs are very low in the online lending market. Borrowers that self-select into high-maturity contracts are more likely to default (from step 1). They are less price elastic. Their perfect information markup would be 0.2 dollars per dollar borrowed. However, because of adverse selection they get an information rent that cancels this markup.

Overall, Due to screening, the price are close the the perfect competition perfect information case. However, low-default borrowers get a lower maturity compared to what they should get under perfect information. To understand whether there is too much screening, I use the social planner framework described in section 6.2.

**Competition: Sufficient Statistic.** Using proposition 4, I find that: $\frac{n_{36g}}{n_{60g}} I = 1.32 > E[\theta_i|g_i, T_i = 60] = 0.99$. This result implies that there is excessive screening. Due to the contractual externality low default borrower get a lower maturity than what they should get under perfect information but also lower maturity than what they should get in the second best scenario (informationally constrained social planner). Decreasing competition or increasing the marginal cost of lending via low maturity contracts can thus lead to a Pareto improvement.

**Distribution of willingness to pay (Step 2):** To test if the inequality constraint assumption delivers reasonable results, I implement step 2. I use equation (41) to recover the distribution of borrowers' willingness to pay. For each borrower, I calculate the implied loan size that would give them the same monthly payment if they were to change maturity. I consider only borrowers for whom the alternative combination of loan size and maturity is feasible, given the menus offered by LC.

For a given investment grade, the inequality allows me to recover one lower bound for the willingness to pay off high-maturity borrowers. I use the interest rate variation across borrower investment grades to recover the shape of the distribution of willingness to pay. The identifying assumption is that the distribution of marginal utility of loan size ($\alpha \sim G$) does not change with investment grades.

Figure (14) report the distribution of willingness to pay. The average willingness to pay is 1.6. Interpreting the value in terms of a two-period model, the results imply that for each dollar borrower in period 0, a borrower is willing to pay up to 1.5 dollars in period 2.

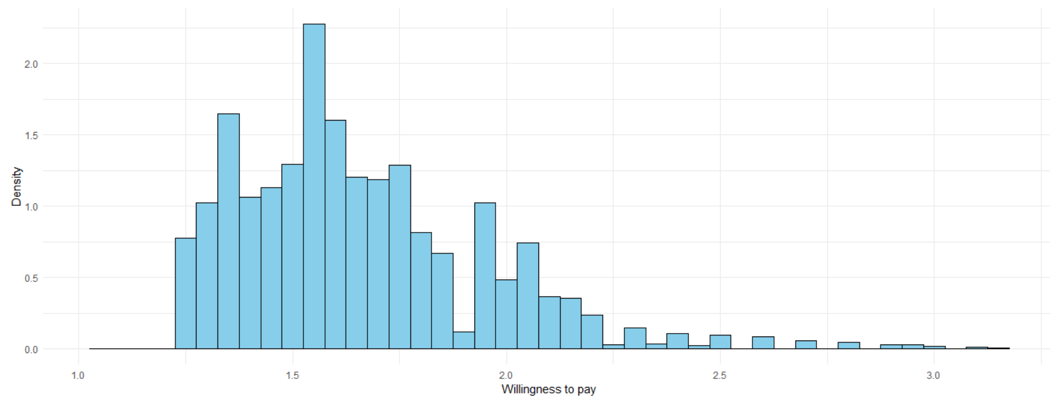


Figure 14: Borrowers' willingness to pay (all market segment pooled)

# 9 Conclusion

This paper characterizes the contractual externality that arises in screening models a la Rotchild Stiglitz. It shows that decreasing competition or using legal tools to change firms' marginal cost of originating contracts can lead to Pareto improvements.

I apply the model to credit markets where lenders screen using Loan-to-Values (LTV) and rates or loan size and maturity. Papoutsi, Paravisini, Rappoport, and Taburet (2024) shows that screening with LTV is empirically relevant in the mortgage market and Hertzberg, Liberman, and Paravisini (2018) shows that screening with maturity is relevant in the consumer loan market.

Using this banking framework, I derive new policy implications for capital requirements and monetary policy. My model implies that policies increasing the marginal cost of lending can increase aggregate lending and decrease the spread between loan sizes. When lenders screen with maturity, quantitative easing is likely to have the effect discussed in this paper as it heterogeneously impacts lenders' costs of originating contracts with different maturities. When lenders screen with Loan-to-Values, capital requirements —such as the one used in the Basel Accords — are relevant policies that should be investigated through the lens of this framework.

Finally, this paper closed the bridge between theoretical work and empirical work on adverse selection. The framework provides closed-form solutions for the equilibrium contracts in screening models with imperfect competition. I do so without the use of equilibrium concept refinements. This allows me to derive sharp moment conditions that can be used to recover the model parameters.

The tractability of the model relies on imposing symmetric equilibrium. This assumption opens the possibility of looking at counterfactuals —such as changes in competition and changes in capital requirements — that would be otherwise retractable of ill-behaved using standard empirical industrial organisation assumptions. For this reason, papers such as Papoutsi, Paravisini, Rappoport, and Taburet (2024) focused on characterising the contractual externality using the perfect information benchmark or a social planner benchmark in its counterfactual exercise.

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# A Tables

Table 1: Survival Probabilities

| | Loan is fully paid at maturity |
|---|---|
| Term: 60 months | -4*** |
| | (0.0118) |
| Monthly payment to income ratio | -2*** |
| | (0.01445) |
| Subgrade fixed effect | Yes |
| Average number of default at maturity | 15 |

Note: Coefficients are reported in percent. Standard errors are in parentheses. ***p < 0.01, **p < 0.05, *p < 0.1

Table 2: Present value regression (60 month maturity)

| | Present Value |
|---|---|
| Intercept | $4e^{3}$*** |
| | ($1.5e^{2}$) |
| Loan Size | 1.5*** |
| | ($5.5e^{-3}$) |
| Market share ratio | -9.3 $e^{3}$*** |
| | ($1.3e^{2}$) |
| Average present value | 34,438 |
| $R^2$ | 0.98 |

Note: Coefficients are reported in percent. Standard errors are in parentheses. ***p < 0.01, **p < 0.05, *p < 0.1

Table 3: Present value regression (36 month maturity)

| | **Present Value** |
|---|---|
| Intercept | 82** |
| | (25.4) |
| Loan Size | 1.2*** |
| | ($1.5e^{-3}$) |
| Average present value | 16,940 |
| $R^2$ | 0.98 |

Note: Coefficients are reported in percent. Standard errors are in parentheses. ***p < 0.01, **p < 0.05, *p < 0.1

# B Model with Maturity

## B.1 Borrower

A borrower derives utility $F(z)$ during the time interval $\Delta t$. With probability $(1 - \theta\Delta t)$ the borrower can repay the monthly loan payment $rL$ during that period. With probability $(1 - \theta)\Delta t$, the firm defaults and loses collateral $cK$.

Denoting $V(t)$ the value function of the borrower at time t when he did not default and $\tilde{V}(t)$ when he defaults, denoting the discount factor by $\rho$, the value function is:

$$V(t) = F(z)\Delta t + \frac{\Delta t \lambda(t)}{1 + \rho\Delta t}\{[\tilde{V}(t + \Delta t) - cK]\} + \frac{1 - \lambda(t)\Delta t}{1 + \rho\Delta t}[V(t + \Delta t) - rL] \tag{50}$$

Assuming constant default probability, constant hazard rate $\lambda$ and outside option, taking the limit when $\Delta t \to 0$ and using the boundary condition $V(T) = \bar{V}(L) = V(T) = F(L)\int_0^\infty e^{-\rho t}dt = \frac{1}{\rho}F(L)$ and using the discount factor $\rho$ for the period below the maturity, we get for $t < T$:

$$V = \frac{F(L)}{\rho}e^{-\rho T} + \frac{F(L) - R}{\rho}[1 - e^{-\rho T}] \tag{51}$$

$\frac{F(L)}{\rho}$ is the present value of lending. $e^{-\rho T}$ is the probability of survival until maturity times the discount rate. $R := \theta rL + \theta[cK + \Delta V]$ is the expected per period cost of the loan for borrowers.

I parameterize $\tilde{V} = V - \delta$ so that $\Delta V$ is a constant equal to $\delta$. This implies that lenders expect the next period value functions $\tilde{V}(t + \Delta t)$ and $V(t + \Delta t)$ to satisfy an equilibrium condition $\tilde{V} = V - \delta$. This can be interpreted as borrowers renegotiating the debt upon default and getting their previous value function minus some amount $\delta$.

## B.2 Lender

Each Lender maximizes expected profits:

$$\max_{\{r,L,K,T\}} \Phi(V(t))\{\frac{(1-\lambda)rL + \lambda cK}{\rho}[1 - e^{-\rho(T-t)}] - mcL\} \tag{52}$$

Using equation (10) to express the problem in terms of promised utility. This yields:

$$\max_{\{V,K,T,L\}} \Phi(V)\{\frac{F(L)}{\rho} - V - \lambda\frac{\Delta V}{\rho}[1 - e^{-\rho(T-t)}] - mcL\} \tag{53}$$

So, lenders want to minimize borrowers' default to prevent borrowers from paying the cost of defaulting $\Delta V$. With exogenous default, lenders want to set maturity to zero and monthly payment to infinity.

If there is a limit to how high the monthly payment can be (for instance, if monthly payment rL cannot be above monthly income W), then the lender sets rL to W and extracts surplus using interest rate rather than maturities.

# C Conditions for collateralized debt to be the optimal contract

The assumptions are:

**ASSUMPTION 1:** *Ex post Private information.* In this paper, I model this assuming that banks cannot observe the second period cash flow (W) of the borrower,[12] borrowers can thus lie about their income and hide it from the bank. The bank can spend some amount to verify it.

12. or it is costly to do so

**ASSUMPTION 2:** *The house can be used as collateral (i.e. housing is observable). Using collateral to deal with ex post private information is less costly than verifying cash flows. The borrowers values the house more than the bank.* This assumption makes sure that in the optimal mortgage contract, the bank ask for cash instead of housing when possible. In this paper, the reason for the borrower to prefer the house more than the bank, is that borrowers value house more than its selling price and that banks have a utility over cash rather than house.

At t=2, borrowers observe privately their income realization ($\tilde{W}$) and choose whether to fill for default or not. Borrowers default when they cannot repay ( i.e. $\delta H + W - R < 0$) or when its better for them to strategically default (i.e. $H + u(W - R) < u(K + W)$ or $u(\delta H - R + W) < u(K + W)$. K is the amount that the bank give back (or ask) after seizing the house and selling it. K has thus to be lower or equal to $\delta H - R$ so that all inequalities are satisfied. In our model, since borrower value more cash upon default than banks ($\alpha > 1$), the constraint is binding $K = \delta H - R$. Notice that, the bank wants to prevent strategic default when the borrowers is in negative equity ($\delta H < R$) it has to punish the the borrower by seizing more than the house ( i.e. $K = \delta H - R$ can be negative).

# D Lemmas

***Lemma H1: At least one IC constraint is binding.*** *Given the assumption that the sets* $\bar{N}PV_i := \{f : WTP_{if} > \frac{mc_f}{\theta_i}\}$, $\underline{N}PV_i = \{f : WTP_{if} < \frac{mc_f}{\theta_i}\}$ *and* $NPV_i = \{f : WTP_{if} = \frac{mc_f}{\theta_i}\} = \varnothing$ *are the same for all borrower i, the first best contracts are not incentive compatible except on the sets* $\{(\sigma, \theta, V) : r_j(X^*) = r_i(X^*), \forall(i, j)\}$, $X^*$ *defined in equation (**??**). One incentive compatibility contraint at least is thus binding.*

**Proof Lemma 1:** The two ICC can be written $(WTP_a - WTP_b)' L_a \geqslant \bar{u}_a - \bar{u}_b \geqslant (WTP_a - WTP_b)' L_b$ when $L_a = L_b$ as in the FB, this implies that $\bar{u}_a - \bar{u}_b = (WTP_a - WTP_b)' X^*$ which is the case when $r_a(\bar{X}) = r_b(\bar{X})$

***Lemma 1 bis: When both borrowers have a positive NPV, there is always at least one contract with*** $X = \bar{X}$***.***

Proof Lemma H1 bis: if it is not binding, then the other one is binding from Lemma 1.

If that is the case the FoC on $L_G$ of the promised utility problem gives that $L_B = L_G = \frac{\bar{u}-u}{\overline{WTP}-\underline{WTP}}$. In that case both constraints are binding. However, if $L_G = \frac{\bar{u}-u}{\overline{WTP}-\underline{WTP}} < \bar{X}$ the banks can do a Pareto improvement by setting $L_B = \bar{X}$.

**Proof Lemma 2:**

(i) Formally, the no distortion at the top result is shown by solving for the maximization problem in its promised utility form and noticing that $(IC_B)$ does not depend on $L_B$ nor $D_B$. Intuitively, the self-selection problem comes from high-default (high WTP) borrowers wanting to pretend to be low-default (low WTP) borrowers to get a cheaper loan. If the bank offers a contract with distorted B loan size ($L_B < \bar{L}$), there is a Pareto improvement that leads to either more profits in the B market segment or relaxes the IC constraint. For instance, increasing $L_B$ up to $\bar{L}$ and increasing interest rate by $WTP_B(L_B - \bar{L})$ increases bank profits without effecting the incentive compatibility constraints.

(ii) To illustrate how screening works, and let us rewrite the incentive compatibility constraints. Let us consider that $\sigma_B \leqslant \sigma_G$ so that $(IC_B)$ is always binding. $(IC_B)$ states that borrower B (i.e. the one that wants to pretend to be of the other type) must be indifferent between his contract and the contract chosen by borrower G. $IC_B$ can be written:

$$(IC_B) : R_B - R_G = WTP_B(L_B - L_G) \tag{54}$$

$(IC_G)$ can be written as a monotoniticy constraint:

$$(IC_G) : L_B \geqslant L_G \tag{55}$$

Equation (54) and (55) implies that screening is achieved by making B borrowers self-select into a high-interest rate, high loan size contract. They do so because each unit of loan increase above $L_G$ is priced at B borrowers' willingness to pay (i.e. $\frac{R_B-R_G}{L_B-L_G} = WTP_B > WTP_G$). That way, only B borrowers find the cost of increasing their loan size from $L_G$ to $L_B$ cheap enough.

***Lemma 2: When both IC constraints are binding, bank pool borrowers.***

Proof Lemma H2: for a given $\bar{u}_i$ the IC implies: $(WTP_{ac} - WTP_{bc})L_b^b = (WTP_{ac} - WTP_{bc})L_a^b$, this equation is satisfied for $L_a^b = L_b^b$. Since the surplus of the profit of the bank is ${}_iS_i(L_i) - \bar{u}_i$ and the maximum surplus is generated by $L_i = \bar{X}$ then the profit is maximized when $L_a^b = L_b^b = \bar{X}$. This is the unique maximum under the conditions of Lemma 1.

***Lemma 3: Which characteristics X are used to screen.*** *Bank use interest rate and the characteristics $(L_f)_f$ to screen. In order to screen, the bank will favor the non binding characteristics c that have the lowest $\tilde{\lambda}_c$ value. $\tilde{\lambda}_c$ is defined as:*

$$\tilde{\lambda}_c := \frac{\overbrace{WTP_{bc} - \frac{mc}{\theta_b}}^{Surplus\ Increases}}{\underbrace{(WTP_{ac} - WTP_{bc})}_{IR\ Increases}} \tag{56}$$

Proof Lemma H3: solve for the optimization problem using the lagrangian. As long as the lower bound $\underline{X}$ on banks preferred characteristics for screening is low enough, banks screen using only one product characteristics and rates. When this condition is not satisfied, Banks use their preferred screening device until it reach the bound, then it moves to the second preferred and so on.

***Simplification:*** *That $\sigma_G \geqslant \sigma_B$, $(IC_B)$ is always binding and $(IC_B)$ can be written as a (non-binding) monotonicity constraint $L_B \geqslant L_G$. When borrowers have the same outside option of not borrowing (i.e. $\bar{V}_G = \bar{V}_B$), the participation constraint of borrower B ($u_B \geqslant \tilde{V}_B$) is redundant.*

*The maximization problem defined in (15) can thus be written:*

$$maL_{\{L_i \in [0,\bar{X}], \bar{u}_i \in \mathbb{R}\}} \sum_{i \in \{G,B\}} \overbrace{n_i \cdot N_i^b(\bar{u}_i) \cdot 1_{\bar{u}_i \geqslant \bar{V}_i}}^{Demand} \quad \overbrace{\theta_i [\underbrace{WTP_i L_i - \frac{mcL_i}{\theta_i}}_{surplus:\ S_i(L_i)} - \bar{u}_i]}^{expected\ profit\ on\ each\ loan:\ \theta R - mc' X} \tag{57}$$

$$s.t.\ (IC_B) : \bar{u}_B = \bar{u}_G + \underbrace{(WTP_B - WTP_G)}_{>0} L_G$$

Proof: Lemmas appendix (D). Lemma H3 in the appendix (D) shows how to select f. The maximization problem () derives from the problem ($problemPU1$). I use the fact that ICB is binding and write the problem in terms of promised utility $\bar{u}$ to make it similar to the monopoly case as in $Stiglitz$(1977).

I postpone the discussion of how screening works to section (4.1) and (4.2). Here I discuss the role of the assumptions used in Lemma 1.

Assumption (ii) allows banks to screen perfectly borrower's type, assumptions (i)-(iii) makes the problem similar to the textbfook model of screening in which at least one participation constraint is binding. Assumptions (v)-(vi) and $\sigma_G \geqslant \sigma_B$ insure that the binding IC is the high default borrower. (v)-(vi) are made in order to simplify the exposition. It makes sure that the borrower type that benefits from pretending to be the other type is always the B type. Indeed, under high level of competition, borrower B benefits from the lower price due to low default. Under low level of competition, he benefits from the lower price due to the lower WTP and higher price elasticity of the other borrower. This is done to simplify the analysis, drivers behind the screening incentives do not depends on this assumption.

As shown in Lemma H3 in Appendix (D), banks screen using only one product characteristics ($L_c \in X$) and rate to screen. I index this characteristic by f. As explained in the section (4.2), this characteristic must satisfies $(WTP_B - WTP_G) > 0$. I provide in the appendix the conditions to determine which characteristic is used to screen. This conditions depends on how good is the screening device (i.e. borrowers' willingness to pay for the screening characteristics is very different) and costly it is to distort the product characteristic used to screen (i.e. how much surplus is lost by unit of characteristic distortion).

# E Existence and Uniqueness

Let us denote Borrower utility by:

$$u(L, R) := WTPL - \theta R \Longleftrightarrow \theta R = WTPL - u \tag{58}$$

We use the following change of variable:

$$u := WTPL - \theta R \Longleftrightarrow \theta R = WTPL - u \tag{59}$$

The bank's objective function for a particular market is thus:

$$\Pi(L, u) := \Phi(u)\pi(L, u) \tag{60}$$

$$\pi(L, u) := (WTP - mc)L - u \tag{61}$$

For the lender maximization problem to be well-behaved, the Hessian matrix of $\Pi(L, u)$ needs to be negative definite. Using Sylverster's Criterion, we need to show that the leading principial minors of the hessian matrix of $-\Pi(L, u)$ have positive determinant:

The conditions to be satisfied are :

$$\pi\{(1-2\phi)\} < \frac{2}{\sigma} \ and \quad -[(\Phi_u\pi + \Phi\pi')\Phi(WTP - mc)]^2 < 0 \tag{62}$$

The second condition is always satisfied.

We separate the case where competition is low enough (a) from the one where competition is high (b). If (b) is satisfied, we do not need the assumption on (a) for the Nash equilibrium to exist.

(a) **Condition when competition is low enough:** The left hand side is bounded above by: $\pi\{(1-2\phi)\} < \{[WTP - mc]\bar{L} - u\}(1-2\phi) < \{[WTP - mc]\bar{L} - \bar{u}\} < \{[WTP - mc]\}\bar{L}$. For the las inequality, we used the fact that utilities cannot be negative because of a participation constraint ($i.e., u \geqslant 0$).

The inequality implies there exists a level of competition $\sigma > \bar{\sigma} := \frac{1}{[WTP-mc]\bar{L}-1}$ such that the Nash equilibrium exists and is unique.

**Condition when competition is high:**

Because of the participation constraint, individual profits are bounded by $c := [WTP - mc]\bar{L}$, then we have:

$$\partial_u\Pi(u) < \partial_u\Phi c \implies \pi < c + \frac{1}{\sigma(1-\phi)} \tag{63}$$

If there is a capacity constraint k, then:

$$\pi < c + \frac{1}{\sigma(1-k)} \tag{64}$$

The market share is bounded by one so that $((1-2\phi) < 1)$.

We thus have $\pi\{(1-2\phi)\} < \frac{1}{\sigma(1-k)} + c < \frac{2}{\sigma}$.

Hence, k should be lower than $\frac{1-c\sigma}{2-c\sigma}$ this is decreasing for $\sigma$ large enough so we have $k < \frac{1}{2}$.

# F Intuition why the capacity constraint works

Small deviations from screening to pooling attract a high proportion of high default borrowers ($\bar{n}$). Indeed, starting from the screening candidate, we get:

$$\frac{\Delta \bar{u}}{\Delta u} \cdot \frac{\Phi(\bar{u})(1-\Phi(\bar{u}))}{\Phi(u)(1-\Phi(u))} \tag{65}$$

$$\implies \frac{u^n - u^o + (\frac{\bar{WTP}}{\bar{\theta}} - \frac{WTP}{\theta})L}{u^n - u^o} \cdot \frac{\bar{n}}{n}\frac{(1-\bar{n})}{(1-n)} \tag{66}$$

$$\implies \frac{\bar{n}}{n} < \frac{u^n - u^o + (\frac{\bar{WTP}}{\bar{\theta}} - \frac{WTP}{\theta})L}{u^n - u^o} \cdot \frac{\bar{n}}{n} < \frac{u^n - u^o + \overbrace{(\frac{\bar{WTP}}{\bar{\theta}} - \frac{WTP}{\theta})L}^{>0}}{u^n - u^o} \cdot \frac{\bar{n}}{n}\frac{(1-\bar{n})}{(1-n)} \tag{67}$$

Local deviations are not profitable, but large ones are. The capacity constraint limits the ability to do non-local deviations and thus restores the pure strategy equilibrium existence.

Condition for screening inefficient:

$$\frac{n_G}{n_B} IR_B := \frac{n_G \theta_G}{n_B \theta_B} \frac{WTP_G - \frac{mc}{\theta_G}}{WTP_B - WTP_G} > 1 \tag{68}$$

Equilibrium $L_G$ amount:

$$L_G = \delta[\bar{L} + \frac{1}{\sigma}\tilde{\delta}\frac{1}{WTP_B - WTP_G}(1 + \frac{\theta_G n_G}{\theta_B n_B})] = \delta[\bar{L} + \frac{1}{\sigma}\frac{\tilde{\delta}}{WTP_B - WTP_G} + \frac{IR}{\sigma[WTP_B - \frac{mc}{\theta_B}]})] < \bar{L} \text{ for } \sigma \text{ h} \tag{69}$$

with: $\delta := \frac{WTP_B - \frac{mc}{\theta_B}}{WTP_B - \frac{mc}{\theta_G}}$ is the strength of the AS problem. $\tilde{\delta} := \frac{WTP_G - \frac{mc}{\theta_G}}{WTP_B - \frac{mc}{\theta_B}}$ is the relative profit measure.

When $\sigma = \frac{2}{A_B \bar{L}}$, $IR = 1^+$:

$$L_G = \delta[1 + \frac{\tilde{\delta}}{WTP_B - WTP_G}\frac{A_B}{2} + \frac{1^+}{2})] < \bar{L} \text{ For } WTP_B - WTP_G \text{ large enough or } A_B \text{ low enough (this i} \tag{70}$$

Given the condition for the function to be well behaved and screening to be inefficient, there exist a zone where banks screen. We can thus analyse the interior solution to understand the screening property trade-off. The level of inefficiency should be low enough so that the bound on the degree of competition is low enough so that banks to not have enough freedom to pool borrowers.

# G Proof Results

**Result 2:** (i)

$$\partial_{\theta_G^{-1}} L_G < 0 \iff mcL_G < \frac{1}{\sigma}[-\partial_{\theta_G^{-1}} CS - \partial_{\theta_G^{-1}} MC] \tag{71}$$

$$\iff mc\sigma \frac{(WTP_B - \frac{mc}{\theta_B})\bar{L}}{WTP_B - \frac{mc}{\theta_G}} < [\underbrace{-\partial CS}_{cross-subsidy\ channel} - \overbrace{mc\frac{CS}{WTP_B - \frac{mc}{\theta_G}}}^{perfect\ competition\ channel} - \underbrace{\partial MU}_{competition\ channel} - \overbrace{mc\frac{MU}{WTP_B -}}^{perfect\ competition} \tag{72}$$

$-\partial MU - mc\frac{MU}{WTP_B - \frac{mc}{\theta_G}}$ is negative. $-\partial CS$ is positive $-mc\frac{CS}{WTP_B - \frac{mc}{\theta_G}}$ is negative so the difference can be positive or negative. It is negative when the no cross-subsidy effect is stronger than the cross-subsidy effect.

To focus on the cross-subsidy channel, let us consider that borrowers have the same elasticity (i.e., $\sigma := \sigma_B\theta_B = \sigma_G\theta_G$) so that the competition channel is absent. In that case, we have:

$$\frac{\partial CS}{CS} = -\frac{WTP_B}{WTP_B - WTP_G} \tag{73}$$

$$\implies \frac{\partial CS}{CS} - \frac{mc}{WTP_B - \frac{mc}{\theta_G}} \geqslant 0 \iff \frac{1}{1 + \theta_G^{-1} - \frac{WTP_G}{WTP_B}} \geqslant \frac{mc}{WTP_B} \tag{74}$$

This implies that a decrease of $\theta_G$ has a positive impact when competition is low. It has a negative impact when competition is high if $\frac{WTP_B}{1+\theta_G^{-1} - \frac{WTP_G}{WTP_B}} \geqslant mc$.

The right-hand side captures the perfect competition channel. When the marginal costs are high, changes in survival probabilities have a high impact on the perfect competition spread between rates. When $WTP_B$ is high, the high default borrower is less likely to copy the low default contract, so the perfect competition channel is relatively less important.

The left-hand side captures the strength of the cross-subsidy channel. When the spread between preferences is high ($\frac{WTP_G}{WTP_B}$), or the survival probability of the low default borrower is low ($\theta_G$), the cross-subsidy channel is low, making the channel less likely to dominate the marginal cost effect.

(ii)

$$\partial_{\theta_B} L_G > 0 \iff 0 < \frac{mc}{\theta_G} L_G + \frac{1}{\sigma}\{\partial_{\theta_B} CS + \partial_{\theta_B} MC\} \tag{75}$$

We have: $\partial_{\theta_B^{-1}}(\theta_B MC) > 0$ $\partial_{\theta_B^{-1}}(\theta_B CS) < 0$. To focus on the cross-subsidy channel, let us consider that borrowers have the same elasticity (i.e., $\sigma := \sigma_B\theta_B = \sigma_G\theta_G$) so that the competition channel is absent. In that case, we have:

$$\frac{\partial_{\theta_B} CS}{CS} = -\frac{WTP_B}{WTP_B - WTP_G}\frac{1}{\theta_B} \tag{76}$$

$$\implies -\frac{\partial_{\theta_B} CS}{CS} - \frac{mc}{\theta_G}\frac{1}{WTP_B - \frac{mc}{\theta_G}} > 0 \iff \frac{WTP_B}{1 + \frac{1}{\theta_B} - \frac{WTP_G}{WTP_B}}\frac{\theta_G}{\theta_B} \geqslant \frac{mc}{WTP_B} \tag{77}$$

**Result 3:** (i) $\partial_\alpha L_G > 0 \iff L_G < \bar{L} + \underbrace{\frac{mc}{\alpha^2}\frac{1}{\sigma}\frac{\alpha}{WTP_B - WTP_G}[\frac{\theta_B}{\theta_G} + \frac{n_B}{n_G}]}_{>0}$

(ii) $\partial_{\alpha_G} L_G > 0 \iff \frac{\alpha_G}{\theta_B} L_G < \frac{1}{\sigma} CS \implies \bar{\sigma}_\alpha := \frac{CS[\frac{mc}{\alpha_G} - 1] - MU}{PC}$, with $PC := S_B \bar{L}$

**Result 4:** (i) $\partial_{mc} L_G > 0 \iff \frac{1}{\theta_G} L_G - \frac{1}{\theta_B}\bar{L} - \frac{1}{\theta_G}\frac{CS}{S_G} > 0$ $\sigma_{mc} := \frac{CS[\frac{S_{BG}}{S_G} - 1] - MU}{PC - \frac{\theta_G}{\theta_B}\bar{L}\cdot S_{BG}} < 0$ if $CS[\frac{S_{BG}}{S_G} - 1] - MU > 0$ with $S_{BG} := WTP_B - \frac{mc_G}{\theta_G}$ and $S_G := WTP_G - \frac{mc_G}{\theta_G}$.

(ii) $\partial_{mc_G} L_G > 0 \iff \frac{1}{\theta_G} L_G - \frac{1}{\theta_G \sigma}\frac{CS}{S_G} > 0$ so $\sigma_{mc_G} := \frac{CS[\frac{S_{BG}}{S_G} - 1] - MU}{PC}$ with $S_{BG} := WTP_B - \frac{mc_G}{\theta_G}$ and $S_G := WTP_G - \frac{mc_G}{\theta_G}$.

(iii) $\partial_{mc_B} L_G < 0 \iff \frac{-\frac{\bar{L}}{\theta_B}}{WTP_B - \frac{\theta_G}{mc_G}} < 0$ wich always holds.

# H Participation constraint binding

## H.1 When the low WTP participation constraint is binding[13]

**$\underline{\theta} \geqslant \bar{\theta}$ and as long as $0 \leqslant \underline{L}^*(\sigma) \leqslant L^*$ (Screening)**

$$\bar{C} \begin{cases} \bar{L}^* = L^* \\ \bar{R} = \bar{\alpha} L^* - \bar{u} \end{cases}$$

$$\underline{C} \begin{cases} \underline{L}^*(\sigma) = \frac{\bar{u} - O^{NB}}{\bar{\alpha} - \underline{\alpha}} \\ \underline{R} = \underline{\alpha} \underline{L}^*(\sigma) - O^{NB} \end{cases}$$

With $\bar{u} = [\bar{\alpha} - \frac{r}{\bar{\theta}}]L^* - \sigma^{-1} + \sigma^{-1} \frac{\underline{n}}{\bar{n}} \frac{\bar{\theta}}{\underline{\theta}} [\underline{\alpha} - \frac{r}{\underline{\theta}}] \frac{1}{\bar{\alpha} - \underline{\alpha}}$

When $L^*(\sigma) \geqslant L^*$ (Pooling):

$$C \begin{cases} L = L^* \\ R = \underline{\alpha} \underline{L}^*(\sigma) - O^{NB} \end{cases}$$

When $L^*(\sigma) \leqslant 0$ (Exclusion):

$$C \begin{cases} L = L^* \\ R = \bar{\alpha} \bar{L}^* - \bar{u} \end{cases}$$

$$O \leqslant \bar{u} \leqslant (\bar{\alpha} - \underline{\alpha}) L^* + O$$

$$\bar{u} = \bar{\alpha} L^* - \frac{r}{\bar{\theta}} L^* - \sigma^{-1}$$

## H.2 When the low and high WTP participation constraint is binding: Monopoly case

**Under monopoly**, we have[14] :

13. it binds first
14. The reason why screening does not arises in the monopoly case is because $L^* < \frac{W}{\underline{\alpha}}$, W being the maximum amount the borrower can pay in the next period

$$\bar{L}^M = L^* \tag{78}$$

$$\underline{L}^M = \begin{cases} 0 & \text{When S satisfied with a strict inequality:} \\ (0, L^*) & \text{When S satisfied with an equality:} \\ L^* & \text{Otherwise} \end{cases} \tag{79}$$

The rate are:

$$\underline{R}^M = \begin{cases} 0 & \text{When S satisfied with a strict inequality:} \\ \underline{\alpha} L - O & \text{Otherwise} \end{cases} \tag{80}$$

$$\bar{R}^M = \begin{cases} \bar{\alpha} L^* - O & \text{When S satisfied with a strict inequality:} \\ \bar{\alpha} L^* - O - \underbrace{(\bar{\alpha} - \underline{\alpha})\underline{L}}_{Information\ Rent} & \text{Otherwise} \end{cases} \tag{81}$$

The bank does exclude market participant, When[15]:

$$S: \quad \underbrace{\bar{N}(\bar{\alpha} - \underline{\alpha})\bar{\theta}}_{increase\ lending\ lowerICC} \quad \geqslant \quad \underbrace{\underline{N\theta}(\underline{\alpha} - r)}_{increase\ lending\ increases\ profits}$$

By excluding market participant, the bank is able to charge the high willingness to pay a higher price, but it losses the potential profits from lending to low WTP borrowers.

Under the case in which S is satisfied, we have Pooling until when competition is high enough so that $\underline{L}^*(\sigma) \leqslant L^*$ (Pooling)

# I Poof propositions

**Proposition 4** $\partial_\delta L_G = \underbrace{\frac{\frac{mc_2}{(\theta_B+\delta_B)^2}}{WTP_B - \frac{mc_2}{\theta_G}} L_{90}}_{\text{"perfect competition" effect}} - \underbrace{IR_B \frac{1}{\theta_B + \delta_B}}_{\text{"monopoly" competition effect}}$

PROOF (ii): $u_i = S(L_i) - \frac{1}{\sigma} - IR_G 1_{i=G} + IR_B 1_{i=B}$. For borrowers G, $S(L_G)$ increases when $L_G$ increases. In the other market segment: $\partial_\delta u_B = -\partial_\delta R_B = \frac{1}{\theta_B+\delta_B}[\underbrace{\frac{mcL_{95}}{\theta_B + \delta_B}}_{fair\ price} - \underbrace{IR_B}_{Information\ rent}]$

15. The general condition when the outside option is no 0 is: $\bar{N}[\bar{\theta}(\bar{\alpha} - \underline{\alpha})L^* + \bar{O}^{NB} - \underline{O}^{NB}] \geqslant \underline{N}[\underline{\theta}[\underline{\alpha} - \underline{O}^{NB})L^* - rL^*]]$

# J Policy Interventions

In this section, I analyze the positive and normative impact of marginal cost policies (i.e. capital requirements), preferences and marginal costs policies (i.e. government guarantee schemes).

For simplicity, as they do not bring anything to the policy analysis, I assume that the price elasticity of both borrowers are the same ($\sigma := \sigma_G(1 - \frac{n_G}{B}) = \sigma_B(1 - \frac{n_B}{B})$).

The key point of those policies is that they are designed differently for each specific product. For tractability, it will be useful to model those policies as a piecewise linear function over some product characteristics. As a result, it is convenient to also introduce discontinuities in the marginal costs at the same thresholds so that the first best products are different for each borrower. This assumption makes sure that the demand is continuous. Formally, We consider a situation in which the marginal cost of lending at high X is higher above a certain threshold $\bar{T}$:

$$mc(X) = mc_1 + 1_{X>\bar{T}}(mc_2 - mc_1) \tag{82}$$

Borrowers have preference such that:

$$WTP_G \geqslant \frac{mc_1}{\theta_G},\ WTP_G < \frac{mc_1}{\theta_G} \tag{83}$$

$$WTP_B > \frac{mc_2}{\theta_B},\ WTP_B \geqslant \frac{mc_2}{\theta_B} \tag{84}$$

To enlighten the impact of the poly on the distortions, I will consider that they are not large enough to change the above ordering.

### J.0.1 Policy experiment 1: Effect of changes in capital requirements

Capital requirements are often based on Loan-to-Values ratios (for instance in Basel III). For this reason, I model capital requirements base on LTV. I consider that the marginal cost have now the form:

$$mc_c = mc - \omega^l \frac{1}{LTV} 1_{LTV<L\bar{T}V} - (\omega^h - \omega^l)\frac{1}{LTV} 1_{LTV\geqslant L\bar{T}V} \tag{85}$$

$\omega\frac{1}{LTV}$ being the capital requirements and $\omega$ capturing how the capital requirements vary with the loan leverage. A positive $\omega^l$ (or $\omega^h$) implies that capital requirements are increasing in LTV when LTV is below (above) a threshold. Using the fact that $LTV := \frac{L}{H}$, this specific functional form is equivalent to redefining the $\gamma$ parameter as $(1-\theta_i)(\gamma - \frac{\omega}{(1-\theta_i)})H$ in our our previous model.

**Assumptions 5:** Let us assume that the increase in capital requirements is not high enough to make the NPV of lending negative.

For simplicity of the exposition, Let us assume that $\frac{1}{\epsilon} < \frac{mc}{\theta}$ so that the optimal contract features screening on loan size and a maximum amount of deposit $D$. This last assumption do not impact the results.

Using Proposition 2 and Proposition 2, the equilibrium distortions can be written:

$$L_G = \underbrace{\frac{\tilde{W}TP_B^L - \frac{\tilde{m}c^L+\omega^h}{\theta_B}}{\tilde{W}TP_B^L - \frac{\tilde{m}c^L+\omega^l}{\theta_G}}(\bar{L}-\bar{D})+}_{Fair\ price\ effect} \underbrace{\frac{AI_B - AI_G}{\tilde{W}TP_B^L - \frac{\tilde{m}c^L+\omega^l}{\theta_G}}}_{Asymmetric\ information\ discount\ effect} \leqslant \bar{L} - D_G^{PI}\ when\ \sigma\ high\ enough \tag{86}$$

$$\begin{aligned}&Asymmetric\ information\ discount\ given\ to\ type\ B:\\ &IR_B := \frac{1}{\sigma}\frac{\tilde{W}TP_G - \frac{mc^L+\omega^l}{\theta_G}}{(\tilde{W}TP_B - \tilde{W}TP_G)}\frac{\theta_G}{\theta_B}\frac{n_G}{n_B} > 0\end{aligned} \tag{87}$$

$$\begin{aligned}&Asymmetric\ information\ premium\ paid\ by\ type\ G:\\ &IR_G := \frac{1}{\sigma}\frac{\tilde{W}TP_G - \frac{\tilde{m}c^L+\omega^l}{\tilde{\theta}_G}}{(\tilde{W}TP_B - \tilde{W}TP_G)} > 0\end{aligned} \tag{88}$$

***Proposition 5:*** *(i) And increase in low LTV capital requirements ($\omega^l$) have an ambiguous effect on contract characteristics distortion. Under high competition the distortion tend to increases. Under low competition, it decreases.*
*(ii) The welfare of the low LTV market segment changes in the same direction as the low LTV. The welfare in the other market segment is ambiguous; welfare tends to decrease following the policy intervention when competition is low; it increases otherwise.*

PROOF: Appendix I

(i) The ambiguous result is due to two opposing effects. I call the first effect the "Fair price effect". Providing a government guarantee to high X loan will lower the cost of high X lending. Under a high level of competition, the decrease in the cost of lending must be passed through to the high X interest rate. This, in turn, relaxes the distortion in the other market segments as borrowers shopping in high X markets then have fewer incentives to choose low X contracts.

The second effect goes into an opposite direction. I call it the "Asymmetric information discount effect". Under a low level of competition, the decrease in the cost of lending to the high X market segment increases profits in this market. As shown in Proposition 2, this creates incentives to distort the other market segments to enable extracting more surplus from high X loans.

(ii) The effect on the high X is ambiguous because, under low level of competition, the increase in profits makes banks less willing to provide an information rent as they want to extract more surplus from this market segment.

The same two channels do not appear when changing $\omega^h$) in my model. This is because of the kink in the housing utility function. Without this, both channels would be present as well.

### J.0.2 Policy experiment 2: Effect of the UK mortgage guarantee scheme (Guarantee of high LTV loans)

As shown in Figure (**??**) the COVID-19 pandemic has led to a reduction in the availability of high loan-to-value (X) mortgage products. This is particularly true for mortgage buyer willing to put 5% of deposits. In order to help those borrowers climb the property ladder, the government has introduced a government guarantee scheme. This scheme provides a guarantee to lender that compensates them for a portion of their losses in the event of foreclosure. This scheme was available to any first time buyers as long as their property value was less that £600, 000 and their loan had an X above 91%.[16]

**Effect on demand:** Using the micro-foundation in section **??**, we have that the willing-

16. Details on the government guarantee scheme is provided at: "https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment$_{data/file}$/965665/210301$_{Budget_S}$

ness to pay for loan size is driven by:

$$_i = 1 + \frac{1}{\theta_i + (1-\theta_i)es} \tag{89}$$

es is a positive parameter driving how costly it is to default. As defaulting is more likely to happen for low survival probability borrowers (i.e. low $\theta_i$), the government guarantee is likely to be more beneficial to those borrowers and thus increase the spread between borrowers' willingness to pay. As the preferences are now a function of the policy and thus of the product chosen, we use the notation $WTP_i^j$ for the preference of borrower j when choosing contract i. We consider that choosing a non-guaranteed contract lowers the willingness to pay by $\gamma$. We allow for the effect to differ based on the contract chosen and denote it $WTP_i^j := \gamma_i^j WTP_j$.

**Effect on supply:** Let us consider the following scenario: the government guarantee scheme is beneficial to banks I model this as an increase $g$ in the survival probability for loans:

$$\theta R + \underbrace{g^l R}_{extra\ profits\ from\ government\ guarantee} 1_{LTV<L\bar{T}V} - (g^h - g^l)\frac{1}{LTV}1_{LTV\geqslant L\bar{T}V}$$

With those assumptions, the equilibrium distortion can be written:

$$L_G = \underbrace{\frac{\gamma_B^B WTP_B - \frac{\tilde{m}c^L}{\theta_B+g^h}}{\gamma_G^B WTP_B - \frac{\tilde{m}c^L}{\theta_G+g^l}}\bar{L}}_{Fair\ price\ effect} + \underbrace{\frac{AI_B - AI_G}{\gamma_G^B WTP_B - \frac{\tilde{m}c^L}{\theta_G+g^l}}}_{Asymmetric\ information\ discount\ effect} \leqslant \bar{L} - D_G^{PI}\ when\ \sigma\ high\ enough \tag{90}$$

$$Asymmetric\ information\ discount\ given\ to\ type\ B:$$

$$IR_B := \frac{1}{\sigma}\frac{\gamma_G^G WTP_G - \frac{\tilde{m}c^L}{\theta_B+g^l}}{(\gamma_G^B WTP_B - \gamma_G^G WTP_G)}\frac{\theta_G+g_G}{\theta_B+g_B}\frac{n_G}{n_B} > 0 \tag{91}$$

$$Asymmetric\ information\ premium\ paid\ by\ type\ G:$$

$$IR_G := \frac{1}{\sigma}\frac{WTP_G - \frac{\tilde{m}c^L}{\theta_G}}{(\gamma_G^B WTP_B - \gamma_G^G WTP_G)} > 0 \tag{92}$$

***Proposition 6:*** *(i) An increase in the government guarantee has an ambiguous effect on contract characteristics distortion. Under high competition, X increases. Under low competition, X decreases. The bigger is the effect on the relative preferences, the more likely X increases.*
*(ii) The welfare of the low X market segment changes in the same direction as the low X. The welfare in the other market segment is ambiguous; welfare tends to decrease following the policy intervention when competition is low; it increases otherwise.*

PROOF: Appendix I

# K Extensions: Many borrowers, many contract features

## K.1 Many Borrower Type, uni-dimensional case

Let us consider x borrower type. I denote them with the index $i \in \mathcal{N}^*$, with $WTP_i > WTP_j,\ \forall i < j$. The incentive compatibility constraints can be written as a set of IC and a monotonicity constraint:

$$(ICs):\ u_1 \geqslant u_2 + \Delta WTP_1 L_2 \geqslant u_3 + \Delta WTP_2 L_3 ... \geqslant u_x + \Delta WTP_{x-1} L_x \tag{93}$$

$$(Ms): L_i \geqslant L_j, \forall i > j \tag{94}$$

Let us focus on a situation where competition is high so that we have an interior solution (i.e., $L_i < \bar{L},\ \forall i > 1$). At equilibrium, the optimal contract is :

$$L_1 = \bar{L} \tag{95}$$

$$L_{i-1} - L_i = \frac{R_{i-1} - R_i}{WTP_{i-1}} \in [0, \bar{L}],\ i > 1 \tag{96}$$

$$R_i = \frac{mcL_i}{\theta_i} + \frac{1}{\sigma_i\theta_i}\{[1 + \frac{1}{n_i}\frac{\alpha_i - mc}{\Delta WTP_i}1_{i>1}] - [\frac{1}{n_{i-1}}\frac{\alpha_{i+1} - mc}{\Delta WTP_{i+1}}]1_{i<x}\} \tag{97}$$

## K.2 Many Borrower Type, multi-dimensional case

Let us denote $X_i$ a vector of non-interest rate characteristics of the contract designed for borrower i, the incentive compatibility constraints are:

$$(ICs):\ u_1 \geqslant u_2 + \Delta WTP_1 X_2 \geqslant u_3 + \Delta WTP_2 X_3 ... \geqslant u_x + \Delta WTP_{x-1} X_x \tag{98}$$

$$(Ms): L_i \geqslant L_j, \forall i > j \tag{99}$$

The difficulty comes from having to guess and track which ICC is binding. The simplest case is when the ordering of borrowers and willingness to pay is the same along each characteristic X. This is a natural case when the only source of unobservable heterogeneity comes from default probabilities. An alternative solution is to assume that lenders maximize the worst-case scenario as in Carroll (2017). In that case, the optimum for the principal is to screen along each contract term separately.

C section discusses the conditions for collateralized debt to be the optimal contract.

# L Borrower preference approximation

Using data on menus, I approximate the pricing schedule $(L_c, r_c, T_c)_c$ where r is the interest rate, and T is the maturity using a spline function. Doing this interpolation allows using first-order conditions to derive the moment equation. I discuss alternative options in the paragraph labelled Heterogeneity within contracts.

Formally, I first calculate the present value of the loan $R_{i,c} = \sum_{t=1}^{T} \theta_i^t T_i$ for each contract c holding the monthly payment $(T_i)$ constant. Then, I find the coefficients $(\gamma_1, \gamma_2, \gamma_3)$ such that $R_i(L) := \hat{\gamma}_{1,i} + \hat{\gamma}_{2,i} L + \frac{\hat{\gamma}_{3,i}}{2} L^2$ best match the data points $(R_{i,c})$.

I recover borrowers' preferences using a revealed preference approach. For a given monthly payment and an implied pricing schedule, I have:

$$u_i^* := \max_L u(L) := \alpha_i L - \theta_i R_i(L) \implies L_i = \frac{\alpha_i}{\hat{\gamma}_{3,i}\theta_i} - \frac{\hat{\gamma}_{2,i}}{\theta_i} \tag{100}$$

For each contract c, I can thus recover borrowers' average preference by inverting equation (100) and using the average default estimates:

$$E[\hat{\alpha}_i|c] = L_i\hat{\gamma}_{2,i}(1 + E[\hat{\theta}_i|c]) - \hat{\gamma_{2,i}} \tag{101}$$

Equation (101) states that the slope of the pricing schedule at the contract chosen allows us to recover the borrower's willingness to pay $\frac{\alpha_i}{\theta_i}$. Figure 5 provides a visual representation of the identification strategy. An identification threat would be the slope of the pricing schedules, which varies due to unobserved contract characteristics such as covenant. In that case, the econometrician is not identifying the willingness to pay for loan size but the willingness to pay for both covenant and loan size. The paragraph labelled Heterogeneity within contracts discus how to deal with this issue.

# M Empirical Application Model

I consider that there are N types of borrowers (i.e., N combinations of $(mp_i, \frac{\alpha_i}{\theta_i}, \sigma_i)$) where $\bar{m}p_i$ is the maximum monthly payment the borrower can repay. I construct for each borrower type an incentive compatibility constraint:

$$L_i - \frac{\theta_i}{\alpha_i} mp_i T_i \geqslant L_j(mp_i, M_j) - \frac{\theta_i}{\alpha_i} mp_i M_j \tag{102}$$

Where $L_j(mp_i, M_j)$ is the loan size that would be chosen by the borrower if they were to choose a contract with the other maturity.

$$\sum_{i=1}^{l} n_i\phi(u_i)\theta_i[R_l - \frac{mc}{\theta_i}L_h] + \sum_{i=h}^{n} n_i\phi(u_i)\theta_i[R_h - \frac{mc}{\theta_i}L_h] \tag{103}$$

$$with\ u_i := WTP_i L_i - R_i \tag{104}$$

$$\{h := l+1, l\} : u_h(L_h, R_h) := u_l(L_l, R_l) + (WTP_h - WTP_l)L_i \tag{105}$$

Using the average utility in each market segment ($\bar{u} := u_i - \Delta_i L_i$ with $\bar{\Delta}_i := (\bar{WTP} - WTP_i)$):

$$\sum_{i=1}^{l} n_i \phi(\bar{u} + \bar{\Delta}_i L_h)\theta_i[(W\bar{T}P - \frac{mc}{\theta_i})L_h - \bar{u}] + \sum_{i=h}^{n} n_i \phi(\underline{u} + \Delta_i L_l)\theta_i[(\underline{WTP} - \frac{mc}{\theta_i})L_h - \underline{u}] \tag{106}$$

$$\bar{u} + (WTP_h - W\bar{T}P)L_h = \underline{u} + (WTP_l - \underline{WTP})L_l + (WTP_h - WTP_l)L_l \tag{107}$$

Replacing the constraint and taking the first-order conditions with respect to ($\bar{u}$ and $\underline{u}$), one gets the same first-order conditions than in the two investor model.

When only two maturity options are offered, there exists a unique type $\bar{\alpha}$ that is indifferent between the two contracts. The model can thus be written as:

$$\max_{\{M,L\}} mp[\int_0^{\bar{\alpha}} \phi(u(\alpha))\{T_i\lambda(\alpha) - mcL_i dG(\alpha)\} + \int_{\bar{\alpha}}^{\infty} \phi(u(\alpha))\{M_j\lambda(\alpha) - mcL_j dG(\alpha)\}] \tag{108}$$

$$where\ \bar{\alpha} := \frac{L_j - L_i}{M_j - T_i} \tag{109}$$

DO THE FOC WITH RESPECT TO M

This can be written in term of the representative borrower $\bar{u} := \int_{\bar{\alpha}}^{\infty} u(\alpha)dG(\alpha)$ and $\underline{u}$ for the other representative borrower:

$$\max_{\{M,L\}} mp[\phi(\underline{u})\{T_i E[\lambda|\underline{u}] - mcL_i\} + \phi(\bar{u})\{M_j E[\lambda|\underline{u}] - mcL_j\}] \tag{110}$$