Bank Competition, Information Choice and Inefficient Lending Booms

Job Market Paper

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Abstract

Are inefficient lending booms the downside to more bank competition? In this paper, I develop a simple model of borrower screening in which increasing competition between lenders generates increasingly pronounced credit cycles: when interest rates are low and liquidation values of collateral are high, the economy experiences an inefficient expansion of credit and the average quality of new loans falls. Once the lending boom goes bust, the model predicts a flight to quality that goes along with excessive credit rationing. The mechanism is the following: "Inside" banks who have private information on borrower quality face the problem that their loan offers are observable by uninformed "outside" lenders, which gives rise to two distinct effects: First, for a given precision of borrower screening, the informed incumbent finds it optimal to "poison the well" for uninformed entrants by lowering credit standards and making loans to some projects with negative expected net present value. Second, when borrower screening is chosen endogenously, I find that during booms competition reduces screening precision to inefficiently low levels. The combination of both effects generates procyclicality in lending and offers an explanation for the poor allocation of credit in lending booms.

Keywords: information choice, banking competition, poaching, lending standards
JEL-classification: G21, E44, D82

1 Introduction

Over the past two decades, many countries worldwide have undertaken a process of deregulation[1] of their financial sectors that has turned tightly regulated oligopolistic banking landscapes into much more competitive industries. It is widely believed that these reforms have provided credit-constrained firms and self-employed workers with better access to bank credit. However, the increasing occurrence of credit-driven boom-bust cycles[2] raises the question whether this increase in competition may have come at a cost: anecdotal evidence suggests that in episodes of strong economic outlook, credit may actually have become *too easy* to obtain. In good times, so the narrative goes, competitive pressure drives banks to reduce their screening effort and lower their lending standards, resulting in the build-up of large positions of poorly screened assets in their balance sheets that lay the foundations for the next financial crisis.

In this paper, I develop a theory of inefficient lending cycles that are driven by banking competition. In a model of borrower screening, I obtain two main results: first, more competition can incentivate informed incumbent banks to knowingly take bad projects into their loan portfolio. This is optimal because it protects them against the entry of uninformed lenders who could poach their customers. Instead of raising welfare by reducing markups, more bank competition exerts detrimental downward pressure on lending standards. Since in equilibrium banks offer credit to some borrowers despite of a negative screening evaluation, they also fail to make best use of their information which leads to my second main result: if banks choose their screening precision endogenously, I find that competition reduces borrower screening to inefficiently low levels. The combination of these two results provides me with a powerful theory of the credit cycle.

The key to my findings is to acknowledge that just like all other information centric industries, banks may face difficulties in protecting their private information when assessing credit. Specifically, I assume in my model that the lending decisions of privately informed banks can be observed by uninformed competitors. This could for example be the case if banks are legally required to present their loan offers in writing. Loan-approved borrowers could then use the written offer to credibly signal their positive evaluation to an uninformed outside lender in the attempt to receive a better offer.

When actions are observable by competitors, an informed incumbent bank must strike a careful balance between optimizing her portfolio quality and protecting herself from competition: the more wisely she chooses her loan portfolio, the more positive information will be conveyed by every loan approval, and the more profitable it becomes for uninformed outside lenders to enter the incumbent's market and poach loan-approved customers. If the informed incumbent "poisons the well" by making less prudent choices for her own

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loan portfolio, she will clearly suffer from more defaulting loans but may be rewarded with the successful deterrence of entry.

I show that the distortions in screening and lending decisions that arise from banks' optimal response to informational free-riding problems reproduce the known stylized facts about credit procyclicality remarkably well. When the economy emerges from a recent recession, the composition of the borrower pool is sufficiently poor that competitive threats barely matter at all. Banks then screen very precisely to mitigate adverse selection, and the average quality of loans that are originated in such times is high. As economic conditions improve, competitive threats begin to distort banks' lending decisions: lending standards fall, and credit grows at an increasing pace because banks start to admit some negative net present value projects into their loan pool in order to keep competition away. At the same time, they reduce the precision of borrower screening because they can no longer appropriate the full returns to their costly information acquisition effort. In a booming economy, the average borrower becomes credit-worthy. The reduced level of screening precision then contributes further to the expansion of inefficient lending because the combination of high prior belief and imprecise screening information leaves many poor projects in the portfolio that is financed. In line with empirical evidence, my model predicts that the worst loans are made right at the peak of the boom, and that low policy rates can exacerbate inefficient lending. Once the boom goes bust, the situation reverses. If screening precision requires time to build, the bust phase will be characterized by a credit crunch and a flight to quality: as screening remains impaired from the effects of competition during the preceding boom phase, only exceptionally good borrowers succeed in generating a sufficiently positive signal to receive credit.

Finally, my model also emphasizes the important role of project liquidation values for the lending cycle. In recessions, expected liquidation values are low. This makes banks hesitant to lend because the option of repossessing defaulted projects offers them little protection against risk. The opposite is true when liquidation values are high: not only will those high values reduce banks' incentives to screen thoroughly (Manove et al., 2001), but in the presence of competition they can also exacerbate the inefficiencies in bank lending: this is because diminishing risk encourages uninformed outside banks to compete more aggressively, and incumbents respond to this threat of competition by making more bad loans.

My work is related to several strands of literature. First, it contributes to the literature on credit screening. Imperfect credit-worthiness testing in competitive markets has been analyzed, among others, by Broecker (1990) and Riordan (1993). However, they focus on externalities between banks that arise from credit rejection decisions which exacerbate adverse selection within the applicant pool. In sharp contrast, I analyze here a setting of contestable markets where banks' lending decisions have no impact on the composition of competitors' applicant pools, but where informational spillovers limit the ability of lenders to fully appropriate the returns to costly private information gathering. Lending standards are softened in this type of competition as a result of cross-subsidization between supra- and submarginal projects, which is somewhat similar to de Meza and Webb (1987).

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Ruckes (2004) and Direr (2008) analyze credit screening models based on Broecker (1990) in which the choice of information is endogenous, just as it is here. The scope of their analysis, however, is different from mine: they are interested in the screening activity of banks in the *absence* of any informational spillovers whereas I study the very consequences of such an imperfection.

To the best of my knowledge, the endogenous screening choice in the presence of informational spillovers between competing lenders has so far only been analyzed by Ogura (2006). In his model, lenders use observational learning to update their beliefs regarding the credit-worthiness of a borrower from a competitor's lending decision in the previous period which leads to reduced screening. There are, however, substantial differences: my model here is a story of informed inside banks who adjust the composition of their loans in the face of competing uninformed outside lenders, whereas his model analyzes the lending decisions of equally informed transactional lenders that are confined to financing only at most one single customer each. This assumption precludes the use of cross-subsidization strategies to deter entry from informational free-riders, an idea that lies at the heart of this paper. Thus, he finds that increasing competition reduces the credit risk taken by every individual bank, which is the opposite of my conclusions.

The idea that adverse selection can constitute an effective barrier to bank entry goes back to Dell'Ariccia et al. (1999). They show that the rejection decisions from Bertrand competition between two informed lenders can deteriorate the applicant pool for a third entrant so much that entry is blockaded.

This paper is also related to the literature on the impact of bank competition on information and lending standards. Dell'Ariccia and Marquez (2006) present a model of a credit boom driven by increases in competition, but their booms, whilst possibly increasing financial fragility, are surplus-increasing and thus suggest substantially different policy than the inefficient booms which I discuss here. Both Marquez (2002) and Hauswald and Marquez (2006) address information in a banking system under increasing levels of competition, where the latter present the idea that increases in competition reduce returns to information; however, in their model information acquisition nevertheless remains socially excessive, which is not the case here.

Finally, my model also adds to the literature that explores the economic mechanisms behind lending booms. Rajan (1994) develops the idea that in the presence of thriving competitor banks, short-horizon reputational concerns of bank managers can promote inefficiently lax lending standards. In Lorenzoni (2008), lenders' failure to internalize the general equilibrium price impact of their liquidation of collateral gives rise to excessive lending, which is complimentary to the idea of informational problems that I pursue here.

The paper is structured as follows: in section 2, I present the model. I discuss the planner's second-best benchmark in section 3 before solving for the competitive equilibrium in section 4. Section 5 discusses the relationship between the model's predictions, existing empirical findings and possible policy measures, whereas section 6 explores the robustness the model to changes in screening technology and the inclusion of aggregate uncertainty. Section 7 concludes.

2 Model

2.1 Setup

The economy in my model comprises of two islands, with a unit mass of entrepreneurs and a financial intermediary on each of these islands.

Every entrepreneur $i \in [0, 1]$ is endowed with a risky project. The project requires one unit of investment, and may either succeed and yield a perfectly verifiable payoff of $R > 1$, or fail and yield its liquidation value, $r < 1$:

$$X_i = \begin{cases} R & \text{with probability } p_i \\ r & \text{with probability } 1 - p_i \end{cases} \tag{1}$$

The quality of projects is heterogeneous in the sense that some projects are more likely to succeed than others: the success probability p_i of entrepreneur i's project is drawn independently from a uniform probability distribution with mean $\bar{p}$:

$$p_i \sim U\left(\bar{p} - \frac{\varepsilon}{2}, \bar{p} + \frac{\varepsilon}{2}\right) \tag{2}$$

I assume that this distribution is the same for both islands and is publicly known whereas an individual project's success probability p_i is only known to the entrepreneur. Entrepreneurs have zero initial wealth, so they need to borrow an amount of 1 against the state-contingent promise of repayment of (D_i, d_i) from a bank in order to develop their project. I assume that all entrepreneurs apply for a loan irrespectively of their type[3], and that they do not have any signaling devices such as collateralization or self-financing at their disposal. Banks are risk-neutral and can access an unlimited amount of financing through the interbank market at a cost of ρ. I will interpret ρ directly as the monetary policy rate, and assume that $r < \rho < R$. Before coming to a decision on a loan, banks can screen every *domestic* project (i.e. every project on the same island) by using all soft and hard information available to them to generate an informative but imperfect private signal about its idiosyncratic probability of success.

To this end, they resort to an imperfect credit-worthiness test. The precision of this test depends on a parameter $\lambda \in [0, 1]$ which captures the extent to which the lender's institutional setup and business strategies are well-aligned with the purpose of generating reliable credit-relevant information and channeling it to the decision-making loan officers. In the case $\lambda = 0$ the test does not generate any useful information at all, whereas for $\lambda \to 1$ the test reveals every borrower's type perfectly. By making its choice of λ, a bank endogenously selects its optimal degree of exposure to asymmetric information problems in its lending activities. The cost associated with holding a given level λ of precision in credit testing are primarily fixed cost which are independent of the number of screened customers and are described by a strictly convex cost function $c(\lambda)$ that satisfies $c' > 0$, $c'' > 0$, $\lim_{\lambda \to 0} c(\lambda) = 0$ and $\lim_{\lambda \to 1} c(\lambda) = \infty$; I assume that once these fixed cost are

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[3]This could, for example, be ensured by postulating that entrepreneurs derive an additional non-verifiable rent from running their projects.

paid, the screening of an individual domestic applicant is costless and yields an imperfect signal of his true type p_i. I refer to the signal's random variable as σ_i, and denote its realization as s_i. Active screening only works with domestic entrepreneurs; I assume that the screening of an entrepreneur from a different island always results in a completely uninformative signal.

Whilst the key results in this paper generalize to a wide class of screening technologies I postulate for now that the following specific test technology is in place: with probability λ, the credit-worthiness test generates a signal realization $s_i = p_i$ that is identical to the project's true success probability whereas with probability $1 - \lambda$ it yields a totally uninformative value s_i that is randomly drawn from p_i's prior distribution (2). In sharp contrast to the signal structure analyzed by Ruckes (2004), the bank does not know whether a given signal realization represents a "real" or an uninformative draw. I show in the appendix that Bayesian updating of beliefs given the observation of a signal realization of s_i results in a posterior expectation of

$$E[p_i | \sigma_i = s_i] = \lambda s_i + (1 - \lambda)\bar{p} \tag{3}$$

Note that the posterior expected value of p_i is simply a convex combination of the observed signal realization and the prior mean $\bar{p}$ whereby the test's precision determines the relative weight given to each of them. A more precise test will therefore move beliefs about p_i further away from the prior mean and create more dispersion in posterior expectations. As we shall see later, this is the first key ingredient for understanding the central properties of the model.

2.2 Competition

The second key ingredient to the model is competition between asymmetrically informed lenders where an imperfection of the market gives rise to informational spillovers. Specifically, I assume that legislation prohibits charging fees to a loan applicant for the review of a loan application before actually closing the loan[4] and that any loan offer must be made in writing. Entrepreneurs with a favorable loan evaluation by their domestic bank can then use the written offer to credibly signal their positive domestic evaluation to the uninformed outside lender on the other island. In this way, the uninformed bank on the other island can attempt to poach customers which the domestic bank considers creditworthy. However, a cost disadvantage limits the ability of the outside bank to compete for customers that do not reside on its island: I assume that for every loan made to an off-shore customer, the lender incurs a cost of $\gamma > 0$. Whilst there are several possible interpretations[5] to this charge, I prefer to think of it as the extra cost from monitoring the execution of a project that does not reside on the same island. In my analysis, I use

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[4]For example, this is the case under the German law.

[5]The model can also be reinterpreted as the reduced second period of a two-period game where in the first period, banks build customer relationships by providing non-credit banking services, e.g. cashless payment services. γ would then represent the switching cost of obtaining credit from a bank different from the one chosen for payment services in the first period.

γ to parametrize changes in the strength of competition: $\gamma \to \infty$ will result in monopoly, whereas $\gamma \to 0$ generates the highest possible level of competition.

2.3 Timing

. I now let the two banks compete with each other. In particular, I assume the following structure and timing of the game:

1. every bank $j \in \{1,2\}$ chooses its publicly observable level of screening effort λ^j, pays screening cost $c(\lambda^j)$ and obtains private signals $\sigma_{i,\lambda}$ for all domestic projects $i \in [0,1]$.

2. both banks then make their domestic loan offers: they choose a domestic *loan portfolio* comprising of a set $\mathcal{P}_j$ of projects to be offered a loan, and agreed repayment terms (D_i, d_i) for every project $i \in \mathcal{P}_j$ in case of success or failure, respectively.

3. each bank observes the domestic loan offers made on the other island $j' \neq j$ and decides whether and under which state-contingent repayment terms $(O_i^{j'}, o_i^{j'})$ it offers outside credit to the other islands' loan-approved entrepreneurs.[6]

4. entrepreneurs choose the loan offer with the lowest expected repayment terms; when two offers leave them indifferent, they stay with their domestic bank.

I solve the game by backward induction. Before doing so, I will however make a quick detour and analyze the information precision and credit allocation that a social planner who is constrained to using the same screening technology would choose.

3 The Social Planner's Solution

3.1 Constrained Optimal Portfolio Choice

Since I solve backwards, I first take λ as exogenously given. The constrained optimal portfolio is easily found by observing that the expected surplus π_i from a single project is monotonically increasing in its expected probability of success and reads

$$E[\pi_i | \sigma_{i,\lambda} = s_i] = E[p_i|s_i]R + (1 - E[p_i|s_i])r - \rho \tag{4}$$

Equating this expression to zero yields that a project with an expected success probability of

$$q \equiv \frac{\rho - r}{R - r} \tag{5}$$

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[6]Note that only domestically loan-approved entrepreneurs have incentive to apply for a loan from the outside bank, and that the outside lender will never find it profitable to offer credit to entrepreneurs without domestic loan approval.

will contribute exactly zero surplus, and every project with a higher expected success probability will generate positive surplus. The constrained optimal portfolio must therefore include all projects that have an expected success probability above the cut-off threshold q, and deny financing to all those projects that fall below it. As we shall see, all important expressions in the paper can be written in terms of this cut-off threshold q, so it deserves a brief discussion at this point: holding R and r constant, it is obvious that q lies between 0 and 1 when the interbank rate ρ moves within its assumed boundaries, $r < \rho < R$. Taking R and ρ constant and varying r it is also easy to see that higher liquidation values r decrease the probability of success q that is required to break even.

With the cut-off threshold q at hand, it is straightforward to calculate the volume and average success probability of the constrained optimal portfolio. It is useful to first derive the cumulative distribution function of $E[p_i|\sigma_{i,\lambda}]$,

$$\mathcal{E}_\lambda(q) \quad \equiv \quad \mathrm{Prob}\left(E[p_i|\sigma_{i,\lambda}] \leq q\right) \tag{6}$$

which can be obtained using equation (3):

$$\mathcal{E}_\lambda(q) = \mathrm{Prob}\left(\sigma_{i,\lambda} \leq \frac{q - (1-\lambda)\bar{p}}{\lambda}\right) = \begin{cases} 0 & \text{if } q \leq \bar{p} - \frac{\lambda\varepsilon}{2} \\ \frac{q-\bar{p}}{\lambda\varepsilon} + \frac{1}{2} & \text{if } \bar{p} - \frac{\lambda\varepsilon}{2} < q < \bar{p} + \frac{\lambda\varepsilon}{2} \\ 1 & \text{if } q \geq \bar{p} + \frac{\lambda\varepsilon}{2} \end{cases} \tag{7}$$

The constrained efficient portfolio's mass of credit is then simply $m_\lambda^{SB} = 1 - \mathcal{E}_\lambda(q)$, whilst the portfolio's average success probability can be calculated as

$$\mathcal{A}_\lambda^{SB} = \frac{\int_q^1 p \, \mathrm{d}\mathcal{E}_\lambda(p)}{\int_q^1 \mathrm{d}\mathcal{E}_\lambda(p)} \tag{8}$$

The results are conveniently summarized in the following proposition:

Proposition 1. *The constrained efficient portfolio contains all projects with expected success probability above the cut-off $q = \frac{\rho - r}{R - r}$ and none below. The optimally financed mass of projects reads*

$$m_\lambda^{SB} = \begin{cases} 1 & \text{if } q \leq \bar{p} - \frac{\lambda\varepsilon}{2} \\ \frac{1}{2} - \frac{q-\bar{p}}{\lambda\varepsilon} & \text{if } \bar{p} - \frac{\lambda\varepsilon}{2} < q < \bar{p} + \frac{\lambda\varepsilon}{2} \\ 0 & \text{if } q \geq \bar{p} + \frac{\lambda\varepsilon}{2} \end{cases} \tag{9}$$

and the projects in the portfolio attain an average success probability of

$$\mathcal{A}_\lambda^{SB} = \bar{p} + \frac{\lambda\varepsilon(1 - m_\lambda^{SB})}{2} \tag{10}$$

Proof. see appendix. □

Remembering that the limit $\lambda \to 1$ yields the perfect information case, we can now directly compare the second-best choice with imperfect testing precision $\lambda < 1$ to the outcomes of a (clearly unattainable) first-best world in which project types are perfectly observable:

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Corollary 1.1. *The second-best allocation when project types are noisily observed compares to the allocation when types are perfectly known as follows:*

- *If $\bar{p} < q$, strictly less projects are developed than in the first-best with perfect information. Moreover, the size of the constrained efficient portfolio is increasing in λ.*

- *If $\bar{p} > q$, the amount of projects that are developed exceeds the perfect information case. The size of the constrained efficient portfolio is decreasing in λ.*

In other words, imprecise information drives a wedge between the second- and the first-best amounts of financing whereby the sign of the wedge depends on whether the average project is credit-worthy: in exceptionally good times (i.e. $q < \bar{p}$), noisy information results in the financing of *more* projects than under perfect observability of types, whilst in normal times (i.e. $\bar{p} > q$), noisy information leads to *less* investment.

3.2 Constrained Optimal Screening Precision

Having determined the planner's optimal project portfolio decision, the next step is to ask how much screening precision λ the social planner would optimally choose to acquire. For this purpose I first calculate the gross surplus as a function of q and λ:

$$
\begin{aligned}
\Pi_\lambda^{SB} &= \int_q^1 pR + (1-p)r - \rho \, \mathrm{d}\mathcal{E}_\lambda(p) \\
&= (R-r) \int_q^1 \left(p - \tfrac{\rho - r}{R-r} \right) \mathrm{d}\mathcal{E}_\lambda(p) \\
&= m_\lambda^{SB} (R-r) \left(\mathcal{A}_\lambda^{SB} - q \right)
\end{aligned}
\tag{11}
$$

Note the very intuitive structure of this expression: for every financed project, the planner obtains a surplus that is equal to the risky part $R-r$ of the project's payoffs multiplied by the margin by which the portfolio's average success probability exceeds the zero-surplus cut-off q. Substituting previous results leads to

$$
\Pi_\lambda^{SB} = \begin{cases} (R-r)(\bar{p}-q) & \text{if } q \leq \bar{p} - \frac{\lambda\varepsilon}{2} \\ (R-r)\frac{(2\bar{p}-2q+\lambda\varepsilon)^2}{8\lambda\varepsilon} & \text{if } \bar{p} - \frac{\lambda\varepsilon}{2} < q < \bar{p} + \frac{\lambda\varepsilon}{2} \\ 0 & \text{if } q \geq \bar{p} + \frac{\lambda\varepsilon}{2} \end{cases}
\tag{12}
$$

The three regions in this equation are the same as for the constrained-efficient project mass in equation (9): the first region of the equation describes the case in which projects are so good and information is so imprecise that the size of the constrained optimal portfolio is 1. Region two stands for the intermediate case in which $0 < m_\lambda^{SB} < 1$; finally, the third region is relevant whenever screening precision is so low and average projects are so bad that it is impossible to identify any project with positive gross surplus at all.

The constrained efficient information choice λ_{SB}^* maximizes surplus net of information cost:

$$\max_\lambda \quad \Pi_\lambda^{SB} - c(\lambda)$$
$$\text{s.t.} \quad \lambda \geq 0 \tag{13}$$

Although the exact shape of the solution to this problem depends on the specific functional form of $c(\lambda)$ about which I have not made any assumption so far, its main properties are independent of this choice:

Proposition 2. *If the planner's information choice attains a nonzero level of information λ_{SB}^*,*

- *λ_{SB}^* is strictly increasing in the risky part $R - r$ of the projects' payoffs, and $\lim_{r \to R} \lambda_{SB}^* = 0$.*

- *λ_{SB}^* as a function of q follows an inverse-U shape. It reaches its maximum when the cut-off quality q coincides with the average quality of the pool: $q = \bar{p}$.*

Proof. see appendix. □

These results are quite intuitive: screening is only useful in the presence of risk, and the acquisition of a very precise signal pays off most when it resolves a maximal amount of ambiguity. This corroborates the observation of Ruckes (2004) that there is low incentive to screen when the average quality of the project pool is either very good or very bad because in either case the signal obtained by screening will change investment decisions only for a very small percentage of the project portfolio.

I provide a graphical representation of the results in this section in Figure 1. Let me go over this figure in detail because it not only illustrates the planner's solution but also paves the way for the subsequent discussion of the competitive equilibrium. We have found that when the average borrower is credit-worthy and information is relatively imprecise (i.e., $\bar{p} - q \geq \frac{\lambda \varepsilon}{2}$), the volume of lending grows until every project is financed. Let me draw the set of all (q, λ) pairs for which this applies in the diagram and label it as area ①. Since within this parameter region every entrepreneur receives a loan, screening is useless there. We can therefore conclude that the curve of optimal information choice $\lambda_{SB}^*(q)$ will either remain outside this region or drop to zero wherever it overlaps with it. The same holds for the opposite case where information is relatively imprecise and the average project makes an expected loss: no projects will be financed, the mass of credit is 0 and there are no returns to information acquisition within this region either. I mark the corresponding area as ②.

Wherever the constrained-optimal screening precision $\lambda_{SB}^*(q)$ is not zero, its graph must thus lie inside the white triangular area: these are the only combinations of economic state q and information precision λ for which screening can deliver positive value. Whilst the findings of the previous propositions are independent of the functional form of the
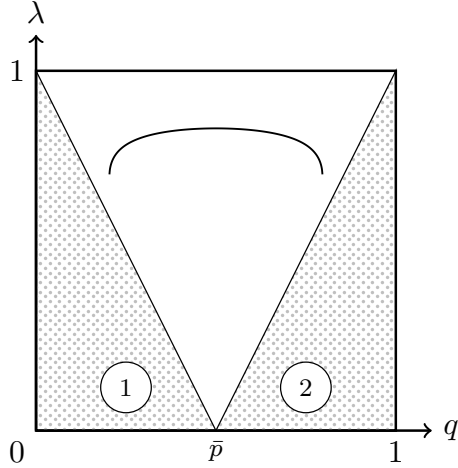
Figure 1: The planner's constrained optimal information choice.

The parameters of the plot are $\varepsilon = 1$, $\bar{p} = \frac{1}{2}$, $r = \frac{3}{4}$, $R = 2$ and $c(\lambda) = c_0 \frac{\lambda}{1-\lambda}$ with $c_0 = \frac{1}{250}$. Areas 1 and 2 which are shown in gray shading correspond to the cases handled in the first and third line of equation (9), respectively, where screening has no value because everybody, or nobody, is financed.

cost function $c(\lambda)$, we need to make such choice in order to draw a graph. I use here the specification $c(\lambda) = c_0 \frac{\lambda}{1-\lambda}$ which yields closed-form solutions[7] for both the planner's choice and the competitive case.[8]

Looking at the graph of $\lambda^*(q)$ we can see that for the chosen parameters the interior solution indeed covers a wide range of q, but as q moves further away from $\bar{p}$, the payoff to screening diminishes so much that it eventually becomes unprofitable to screen.

4   Competitive Equilibrium

4.1   Monopoly

Before discussing the full equilibrium under competition, it is instructive to look at the special case of monopoly. We can model this case within the existing setup by assuming that the remote monitoring cost γ are sufficiently high that no bank can ever profitably offer outside credit to projects outside its own island. This is the case for all possible values of q whenever $\gamma > (R - r)\varepsilon/2$.

It is easy to see that a profit-maximizing monopolistic lender will replicate the planner's choices perfectly: monopoly power enables the bank to extract the full surplus from all entrepreneurs by lending to them against a state-contingent repayment of (R, r). In order to maximize profits, the monopolist will choose the surplus-maximizing level λ_{SB}^* of

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[7]See the appendix for details.

[8]Note that this specific cost function implies that marginal cost of information acquisition are strictly positive everywhere, even for $\lambda \to 0$: $c'(0) = c_0$. Thus, if c_0 is sufficiently large, holding any positive amount of information will be suboptimal and the surplus maximization problem (13) yields the corner solution $\lambda_{SB}^* = 0$ for all values of q. I avoid this situation by assuming that c_0 sufficiently low, i.e. $c_0 < \frac{\varepsilon}{8}$, that there exists an interior solution $\lambda_{SB}^* > 0$ for some neighborhood of $q = \bar{p}$.

testing precision and (assuming that the lender observes the same signal realizations as the planner) give loans to exactly the same set of projects that the planner would choose to develop. Despite the fact that all surplus is given to the monopolistic lender, the allocation is therefore constrained Pareto optimal. It is noteworthy that this optimality result depends crucially on the monopolist's ability to extract all project surplus. The assumed exogenous nature of project size plays here an important role because it enables the monopolistic lender to perfectly know each entrepreneur's individual ability of repayment. If entrepreneurs would instead choose the size of their projects endogenously while facing decreasing returns to scale, monopolistic pricing would result in inefficiently small projects that are no longer constrained efficient. Similarly, if a regulator were to limit the monopolist's surplus extraction by curbing the maximum repayment on each unit of funds to some value $D_{\max} < R$, the resulting portfolio of financed projects would remain inefficiently small since the corresponding cut-off $\hat{q} = \frac{\rho - r}{D_{\max} - r}$ up to which the monopolist provides funding would lie higher than q.

4.2 Portfolio Allocations under Competition

I now study the optimal portfolio choices of the two lenders when they compete against each other. Under competition, lenders must be careful in how they use the private information that they have obtained in the screening process because their decisions will become known to their uninformed competitor. If their loan approval conveys sufficiently good news about the quality of a borrower, the uninformed rival bank can enter the market as an outside lender, undercut the incumbent's loan offer and poach some or all of its loan-approved customers. However, the outside lender's cost disadvantage of γ when lending to the other island puts a limit to this poaching activity: entry is not profitable if a poached borrower earns the outside lender in expectation less than this extra cost γ. This motivates the following definition:

Definition Denote with $\Pi[\mathcal{P}]$ the gross profit that accrues to a domestic lender in the absence of competition from holding a given domestic loan portfolio $\mathcal{P}$, and denote the volume of loans in the portfolio as $|\mathcal{P}|$. Then, a loan portfolio $\mathcal{P}$ shall be called *noncontestable* if its gross surplus per loan is less or equal to γ: $\Pi[\mathcal{P}] \leq \gamma|\mathcal{P}|$.

In equilibrium, every bank will choose a noncontestable portfolio: otherwise, the competing bank could profitably poach the entire set of customers by undercutting the domestic bank's offer for every domestically loan-approved entrepreneur by a small $\delta > 0$: $(O_i^j, o_i^j) = (D_i - \delta, d_i)$. The domestic lender's profits would drop to zero and be strictly lower than if it had chosen some noncontestable portfolio which yields lower but positive gross profits.[9] It must be pointed out that noncontestability is a necessary, but not a sufficient condition to guarantee that a loan portfolio does not attract entry: it only secures that an entrant financing the *entire* loan portfolio will suffer losses. If the variation in repayment terms (D_i, d_i) of individual borrowers carries any information on borrower

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[9]We will see that such profitable noncontestable portfolio exists for every $\gamma > 0$.

quality, the entrant might still be able to profitably target a subset of projects that are identified as more lucrative than others. Such scenario is no concern if the incumbent's portfolio is by chance not only noncontestable but also *symmetric*, i.e. every loan offer in the portfolio has the same repayment terms $(D_i, d_i) = (D, d)$. Then there exists no profitable possibility of entry, and the incumbent wins the full surplus of the portfolio. As the following lemma shows, choosing a symmetric portfolio does not need to be inferior to choosing a non-symmetric one:

Lemma 3. *Let $\mathcal{P}$ be a (not necessarily symmetric) noncontestable loan portfolio. Then there always exists a symmetric noncontestable portfolio $\mathcal{Q}$ for which the incumbent's payoff in the competition game is at least as high as when choosing $\mathcal{P}$.*

Proof. see appendix. □

A direct consequence of the lemma is that if we manage to find within the class of all symmetric noncontestable portfolios some portfolio $\mathcal{Q}^*$ that is profit-maximizing in the sense that no other symmetric noncontestable portfolio generates higher surplus, it must be an equilibrium of the game: to see this, observe that if any arbitrary portfolio $\mathcal{R}$ could generate a strictly higher payoff for the incumbent, such portfolio would be noncontestable for sure (otherwise, as we know, payoff would be zero). But then the lemma assures us that a symmetric noncontestable portfolio that generates at least the same payoff as $\mathcal{R}$ exists, thus generating an immediate contradiction to the maximality of $\mathcal{Q}^*$. Therefore, $\mathcal{R}$ can not have existed in first place, and we have

Corollary 3.1. *Any symmetric portfolio $\mathcal{Q}^*$ that maximizes gross surplus subject to the constraint of noncontestability,*

$$\max_{\mathcal{Q}=(\mathcal{S},\,(D,d))} \quad \Pi[\mathcal{Q}]$$
$$s.t. \quad \Pi[\mathcal{Q}] \leq \gamma|\mathcal{Q}| \tag{14}$$

is an equilibrium allocation of credit under competition.

The maximality of $\Pi[\mathcal{Q}^*]$ also makes it clear that even though the equilibrium portfolio $\mathcal{Q}^*$ is not necessarily unique, the equilibrium payoffs are always uniquely determined. The same is true for credit mass and average success probabilities: those two quantities are uniquely pinned down by the constraint whenever it is binding, whereas any possible indeterminacy that can arise if the constraint does not bind are eliminated by the previously made assumption that no project with zero expected surplus receives funding.

As a solution to the maximization problem for symmetric portfolios I find the following equilibrium allocations:

Proposition 4. *Let $\mathcal{F}$ denote the portfolio of the full unit measure of projects financed against state-contingent repayment promise of (R, r). Then, under competition the following portfolio allocations constitute an equilibrium:*

- *If the monopolistic portfolio is noncontestable, it is an equilibrium. This is the case if information is sufficiently imprecise, $\lambda \leq \frac{2(q-\bar{p})}{\varepsilon} + \frac{4\gamma}{\varepsilon(R-r)}$, and the threshold q is sufficiently high that $\mathcal{F}$ is noncontestable, $q > \bar{p} - \frac{\gamma}{R-r}$. The bank then acts as a monopolist and reproduces the constrained optimal allocation by lending to entrepreneurs whose expected success probability exceeds $q = \frac{\rho-r}{R-r}$ against state-contingent repayment of (R, r).*

- *If information is sufficiently precise that a monopolistic portfolio choice would attract competition, i.e. $\lambda > \frac{2(q-\bar{p})}{\varepsilon} + \frac{4\gamma}{\varepsilon(R-r)}$, but the full portfolio $\mathcal{F}$ remains noncontestable, $q > \bar{p} - \frac{\gamma}{R-r}$, it is an equilibrium to finance all positive net present value (NPV) projects and some projects with negative NPV by offering credit at terms (R, r) to projects whose expected success probability lies above $\hat{q} = \bar{p} - \frac{\lambda\varepsilon}{2} + 2\left(q - \bar{p} + \frac{\gamma}{R-r}\right)$, whereby $\hat{q} < q$.*

- *If both the monopolistic portfolio and the full portfolio $\mathcal{F}$ are contestable, i.e. $q \leq \bar{p} - \frac{\gamma}{R-r}$, it is an equilibrium to offer credit to every domestic entrepreneur against a state-contingent repayment of (D, r), with $D = \frac{\gamma+\rho-r(1-\bar{p})}{\bar{p}}$.*

Proof. see appendix. $\qquad\qquad\square$

This important result deserves several comments:

First, the surprising finding is that more competition has virtually zero impact on the cost of borrowing until every available project receives a loan. Before this point, an increase in competition (i.e. fall in γ) only lowers banks' lending standards and prompts them to take projects with negative net present value into their loan portfolios. The central force behind this result is that it is more profitable for the bank to deter entry by reinforcing the entrant's adverse selection problem than to do so by lowering its terms of repayment: imagine for a moment that the bank had instead reduced its repayment terms for its mass m of borrowers to some (D, r) with $D < R$ but had not lowered its lending standards. Its portfolio must then satisfy the noncontestability constraint with equality which bounds profits to $\gamma|m|$. This is no equilibrium because the bank can do better: it can extract the full surplus (R, r) of those m projects and use the proceeds to finance exactly as many additional negative NPV projects n as to restore noncontestability, $\Pi[m \cup n] \leq \gamma(|m| + |n|)$. The larger size of the project portfolio relaxes the noncontestability constraint and thereby allows the bank to raise its profits by $\gamma|n|$. This mechanism works until the bank runs out of bad borrowers that could be used to further poison its pool: only then, it optimally reacts to competition by offering lower repayment terms. Note that my assumption of fixed refinancing cost contributes to the *complete* absence of any price response: if we had considered a richer model in which

careless lenders incur higher refinancing cost, banks might choose an optimal mix of more aggressive pricing and lower lending standards in order to defend themselves against the entry of competitors.

Another important observation is that the game's competitive equilibrium is essentially a contestable market: the incumbent's informational advantage and the cost disadvantage of the entrant make it impossible for the entrant to actually ever poach any customers from the incumbent. It is the mere threat of entry that drives all the changes in allocations although in equilibrium entry never occurs.

Finally, the fact that the game has an equilibrium in pure strategies stands in sharp contrast to most credit screening games in the literature for which the nonexistence of pure strategy equilibria has been established (see e.g. Broecker, 1990; von Thadden, 2004). This is, however, not really unexpected given my substantially different setup: the sequential nature of bidding together with the observability of actions ensure that the entrant can correctly infer the incumbent's true valuation for the pool of loan-approved borrowers and hence does not face any winner's curse problem when submitting a bid on this pool.

I now proceed to discuss the equilibrium credit volume which can be calculated from the results of proposition 4:

Corollary 4.1. *In equilibrium, the volume of credit is the following:*

$$
m_\lambda^E(q, \gamma) = \begin{cases} 1 & \text{if } 0 \leq \lambda \leq \frac{2(\bar{p}-q)}{\varepsilon} \text{ and } \bar{p} - \frac{\gamma}{R-r} < q < \bar{p} \\ 0 & \text{if } 0 \leq \lambda \leq \frac{2(q-\bar{p})}{\varepsilon} \\ \frac{1}{2} - \frac{q-\bar{p}}{\lambda \varepsilon} & \text{if } \frac{2|q-\bar{p}|}{\varepsilon} < \lambda \leq \frac{2(q-\bar{p})}{\varepsilon} + \frac{4\gamma}{\varepsilon(R-r)} \\ 1 - \frac{2\left(q - \bar{p} + \frac{\gamma}{R-r}\right)}{\lambda \varepsilon} & \text{if } \frac{2(q-\bar{p})}{\varepsilon} + \frac{4\gamma}{\varepsilon(R-r)} < \lambda < 1 \text{ and } q > \bar{p} - \frac{\gamma}{R-r} \\ 1 & \text{for all } \lambda \in [0,1) \text{ if } q \leq \bar{p} - \frac{\gamma}{R-r} \end{cases} \tag{15}
$$

Whilst this expression looks rather complex at first glance, it is actually quite easy to understand. The first three lines represent the absence of competition: they are practically the same expressions that we had found for the planner's benchmark and the monopolistic case. The only difference is that the domain where the monopolistic case applies has shrunken, and there is a new domain of (q, λ) values where threat of entry is relevant. The new area is represented by the ultimate two lines: line four describes the case in which the bank cross-subsidizes some negative NPV loans in order to deter entry. Line five finally stands for the case in which the bank cross-subsidizes *all* negative NPV projects and additionally reduces repayments in order to attain a noncontestable portfolio.

To illustrate in which situations banks are exposed to competition, it is useful to make a similar graphical representation of the (q, λ) plane as in the planner's solution. Figure (2) plots the five different domains of the above function, numbered by the corresponding line number. Areas ① and ② are the same as in Figure 1. We can see that threat of entry (as indicated by the dark shaded areas ④ and ⑤) affects banks that have very precise information and / or operate under very favorable conditions such that the cut-off q is very low. As one can see from the expressions in lines 4 and 5 of equation (15), these areas grow as competition becomes stronger: in the limiting case $\gamma \to 0$, they cover almost the entire (q, λ) plane with the exception of area ②.

How do the comparative statics of credit allocations under threat of entry differ from those in the monopolistic regime? The following proposition holds the answer:

Proposition 5. *Let q be such that in contestable market equilibrium some but not all projects receive credit, i.e. $\bar{p} - \frac{\gamma}{R-r} < q < \bar{p} + \frac{\lambda \varepsilon}{2}$, and let competition be strong enough to potentially impact lenders within this regime, i.e. $\frac{\gamma}{R-r} < \frac{\lambda \varepsilon}{2}$. Then,*

- *the lower the chosen level λ of information acquisition, the more sensitive is the volume of lending $m_E(q, \gamma)$ to changes in both monetary policy rate ρ and project values (R, r).*

- *holding constant the level of λ, the sensitivity $\frac{\partial m_E}{\partial q}$ of credit volume to changes in monetary policy rate and project values is twice as high under threat of competition than if the domestic bank can act as a monopolistic lender.*

- *the average default rate $\mathcal{D}_\lambda = 1 - \mathcal{A}_\lambda$ in a lender's portfolio increases as monetary policy rate falls and expected liquidation values rise. The sensitivity $\frac{\partial \mathcal{D}_\lambda}{\partial q}$ to such changes is independent of λ, but is twice as high under threat of competition than under monopolistic lending.*

Proof. see appendix. $\qquad \square$

In other words, the presence of competition makes credit more volatile. An improvement in economic conditions will lead to both faster credit growth and faster deterioration of portfolio quality.

4.3 Information Choice under Competition

As the final step in solving the competition game backwards, I now find the equilibrium choices of screening precision that maximize total profits in a competitive environment. The gross profit function $\Pi_\lambda^E(q, \gamma)$ that is needed for this purpose follows immediately
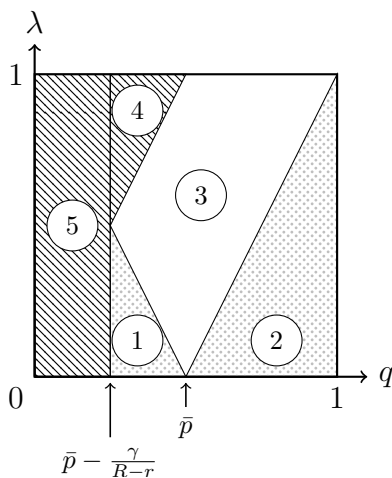

Figure 2: Competitive and monopolistic domains of lending.

from the results in the last section: for all those (q, λ) combinations for which there is no threat of entry, gross profits are exactly the same as under monopoly. In the remaining parameter ranges, the threat of competition limits equilibrium gross profits to $\gamma \cdot m_E$.

The equilibrium screening intensity λ_E^* then solves the problem

$$\begin{aligned} \max_{\lambda} \quad & \Pi_\lambda^E - c(\lambda) \qquad\qquad (16) \\ \text{s.t.} \quad & \lambda \geq 0 \end{aligned}$$

Even without a specific cost function, one can infer the impact of competition on screening directly from an inspection of marginal returns:

Proposition 6. *The equilibrium screening intensity λ_E^**

- *is equal to the constrained efficient screening intensity λ_{SB}^* whenever the monopolistic portfolio at this screening level does not attract competition, i.e. for all $q > \bar{p} - \frac{\gamma}{R-r}$ for which $\lambda_{SB}^*(q) \leq \frac{2(q-\bar{p})}{\varepsilon} + \frac{4\gamma}{\varepsilon(R-r)}$.*

- *falls below the constrained efficient level when competition is sufficiently strong to pose a threat of entry, i.e. $q > \bar{p} - \frac{\gamma}{R-r}$ and $\lambda_{SB}^*(q) > \frac{2(q-\bar{p})}{\varepsilon} + \frac{4\gamma}{\varepsilon(R-r)}$.*

- *is zero if competitive threats result in the financing of all projects regardless of their type, i.e. for all $q \leq \bar{p} - \frac{\gamma}{R-r}$.*

Proof. see appendix. □

This underinvestment in screening is my second key result. The intuition is clear: Whenever threat of entry prompts banks to "poison" their loan portfolios with some negative net present value loans, there is less incentive to acquire costly screening precision ex-ante. After all, banks disregard some of the information that is obtained by screening when they intentionally make bad loans. This is most apparent in the extreme case in which q is so low that banks finance all projects regardless of their type: under such conditions, screening is entirely useless.

I illustrate the result graphically in Figure 3 with a plot of λ_E^* over q for two different levels of competition. The cost function is the same as in the planner's case, $c = c_0 \frac{\lambda}{1-\lambda}$. In the graph on the left-hand side, competition is weak and generates little impact. Borrower poaching is only profitable under the best possible economic conditions, i.e. when the cutoff q is exceptionally low. Only in such exceptionally good economic states, the equilibrium choice of screening precision (shown as the solid line) will fall below the constrained-optimal level (dashed line). In all other times, the screening and lending choices of the bank and the social planner coincide. As the graph on the right-hand side of Figure 3 shows, the advent of more competition changes the situation dramatically: screening incentives are eroded over a wide range of economic states q, and for nearly all except the worst states of the economy, banks will choose a screening precision far below the constrained efficient level.

4.4 From Inefficiently Low Screening to Lending Booms

What are the consequences of inefficiently low screening for the total amount of lending? To answer this question, I evaluate the credit mass $m_\lambda^E(q, \gamma)$ at the equilibrium level $\lambda_E^*(q)$ of screening precision. Let me move the discussion immediately to the most interesting case:

Imagine that average project characteristics are good enough as to render the average project of the pool credit-worthy, i.e. $\bar{p} - \frac{\gamma}{R-r} < q < \bar{p}$. Then, as we have seen in corollary 1.1, the second-best amount of investment already exceeds the first-best benchmark of perfect information. Moreover, banks that hold *less* precise information than the planner will lend even *more* because their signal is too imprecise to actually push many bad projects below the zero expected surplus cut-off q. To make things worse, this is not even where lending under competition[10] actually ends! The equilibrium financing cut-off lies below q because (as shown in proposition 4) banks add additional loans of negative net present value to their portfolio in order to protect themselves against poaching of their clients. The overall amount of credit in this situation is therefore excessive relative to both second-best and first-best levels and indeed deserves to be called an "inefficient lending boom":

$$m_{\lambda_E^*}^E(q, \gamma) > m_{\lambda_E^*}^{SB}(q) > m_{\lambda_{SB}^*}^{SB}(q) > m_{\lambda=1}^{FB}(q) \tag{17}$$

Figure 4 illustrates the equilibrium credit mass as a function of the economic conditions q using the same equilibrium screening levels that were already shown in Figure 3. We can see in the left-hand side picture that for a moderate amount of competition, credit becomes excessive only when economic conditions are particularly strong, i.e. q is very low. The right-hand side graph displays the behavior of credit as a function of q when there is more banking competition. We can observe that credit is excessive over a much wider range of economic conditions q. The lending boom is most pronounced for very low values of q, which are attained whenever the monetary policy rate ρ is low and the liquidation value r of failed projects is high.

But the most important factor that determines the extent of an inefficient boom is the amount of banking competition. In fact, it is possible to show that independently of the chosen screening cost function, the size of the lending boom is always greater when there is more competition:

Proposition 7. *Let the average project be credit-worthy, i.e. $q \in (\bar{p} - \frac{\gamma}{R-r}, \bar{p})$, and let competition be strong enough to pose threat of entry, i.e. $\gamma < \frac{R-r}{2}\left(\bar{p} - q + \frac{\varepsilon \lambda_{SB}^*(q)}{2}\right)$. Then, the size $m_{\lambda_E^*}^E$ of the inefficient lending boom is increasing in the level of bank competition:*

$$\frac{\mathrm{d}}{\mathrm{d}\gamma} m_{\lambda_E^*}^E(q, \gamma) < 0. \tag{18}$$

Proof. see appendix. □

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[10]i.e. for $\gamma < \frac{R-r}{2}\left(\bar{p} - q + \frac{\varepsilon \lambda_{SB}^*(q)}{2}\right)$
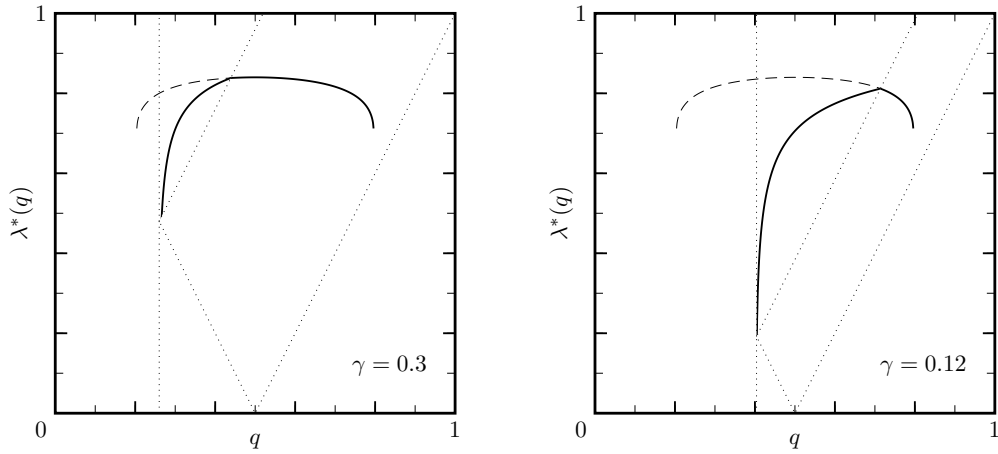
18

Figure 3: Screening precision λ_E^* as a function of q for two different levels of competition. The constrained efficient choice is indicated as a dashed line.
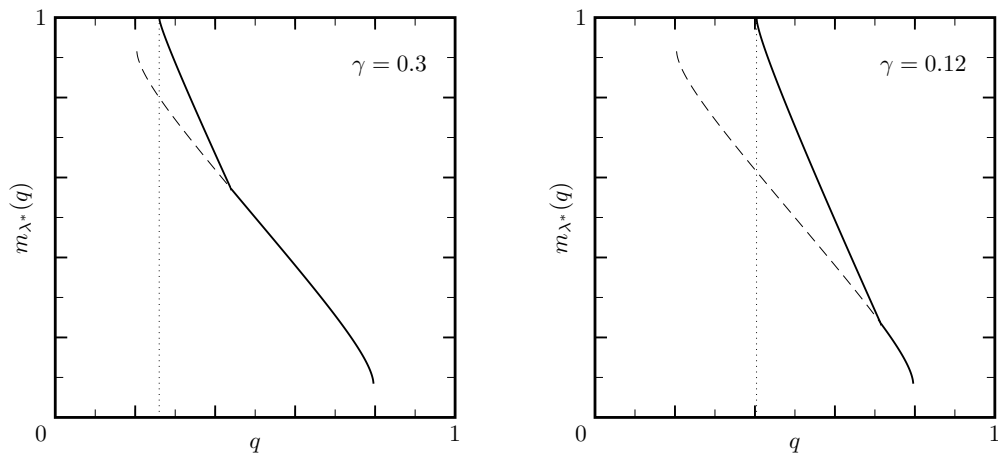

Figure 4: Credit mass $m_{\lambda^*}^E$ as a function of economic state q for two different levels of competition. The dashed line marks the constrained efficient amount.

5 Discussion and Implications

5.1 Predictions and Empirical Findings

With the central results in place, it is now time to take a step back and check how the model's properties and predictions relate to the existing empirical literature.

For a given level of competition, the model predicts the existence of procyclical lending which is driven by variations in bank screening. Although bank screening effort is not directly observable, there is empirical evidence in support of a lending cycle: Berger and Udell (2004) find that credit growth following a favorable change in economic conditions is more pronounced when a substantial amount of time has passed since the last wave of defaulted loans. They explain this observation with what they call the "institutional memory hypothesis": in good times, so their argument, loan officers become increasingly inefficient in recognizing bad credit risks until they experience a bust that allows them to "re-learn" their abilities. Whilst they argue that this deterioration of screening standards is an unintended consequence of problems of the organization like high staff turnover, my model offers the perspective that a rational bank may actually find it *optimal* to let its screening skills deteriorate in good times when there is a threat of outside competition. If banks' adjustments of screening precision λ are not instantaneous, for instance due to adjustment cost, this deterioration will be gradual and therefore observationally equivalent to the findings of Berger and Udell.

Equally in line with the screening cycle of my model are the findings of Ioannidou et al. (2009) and Jiménez et al. (2009): these two studies employ micro-level loan data for Bolivia and Spain, respectively, to investigate the impact of exogenous changes in monetary policy rates on the quality of new and existing loans. Consistent with the model's prediction that for lower values of ρ (and thus q) banks screen their loan applications less precisely and extend their lending towards inefficient projects, they observe that loans that were made in times of low monetary policy rates are substantially more prone to default.

The question whether adverse selection problems that are induced by incumbent lenders matter for the entry decisions of banks has been addressed empirically by Gobbi and Lotti (2004). They show in an analysis of the Italian retail loan market that higher potential earning opportunities as measured by spreads between loan and deposit rates do not attract competition from outside banks whereas earning opportunities in financial services that do not require private information do attract entry. Likewise, Bofondi and Gobbi (2006) observe that entrants in local Italian loan markets suffer from substantially higher default rates than incumbent banks, even more so if they do not have a local branch office. These findings corroborate the validity of my model's central assumption that entry in local credit markets is deterred by the inferior quality of outsiders' information relative to the incumbent lender.

But does available empirical evidence also support the model's message with respect to the effects of competition on asset selection and lending procyclicality? Before I answer

these questions, it is important to hold on and think for a moment how the degree of competition should be measured empirically. This is far from trivial, and the model itself with its contestable market structure provides a perfect example of where the problem lies. It is not necessary for a competitor to actually enter a market in order to have an effect on allocations: the mere threat of entry can already be enough. Studies that use bank concentration ratio or the number of active lenders in a market as a proxy for the degree of competition may thus measure something entirely different from what we were talking about so far: Claessens and Laeven (2004) warn that bank concentration can be a poor indicator of competition and suggest the use of a more sophisticated measure which quantifies the degree of *contestability* in a loan market. Given appropriate data, such measure can be constructed from the response of revenues earned to changes in input prices. Clearly, it would be desirable to use a measure of contestability instead of bank concentration in any empirical analysis related to the questions raised by my model, and results obtained with bank number or concentration may have to be treated with care.

With this caveat said, I now turn to the relevant findings regarding the effects of competition on lending. According to Micco and Panizza (2005), competition increases the procyclicality of lending: they analyze a panel of ninety-three countries for the 1990–2002 period and find a strong negative relationship between bank concentration and credit volatility. Although there are various alternative explanations for this increase in volatility, [11] the finding is clearly compatible with the view taken in my model.

The most imminent evidence that competition spurs inefficient lending and contributes to credit growth via insufficient screening comes from a recent paper of Dell'Ariccia et al. (2008) on loan rejections in the U.S. market for mortgages during the build-up of the U.S. subprime housing bubble: In an analysis of over 50 million loan-level observations they find that controlling for differences in economic conditions mortgages denial rates dropped faster in areas with a larger number of competitors. Even more important, they observe that the erosion of lending standards was most pronounced in loan markets in which banks were facing the entry of large outside lenders, which is a strong indication that excessive lending may have occurred in response to threat of entry.

5.2 Policy Implications

The unfortunate tendency of banks to lend too widely during booms has been often attributed to irrationality or overconfidence. By offering a rational perspective on these known facts, my model can serve as a framework for the discussion of macroprudential policies that are related to the procyclicality of lending.

5.2.1 Curtailing Lending Booms

Which advice does my model give to a policy maker who aims at curtailing an excessive lending boom and improving upon the distorted outcomes?

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[11]For example, it seems plausible that long-term borrower relationships are established more easily in concentrated markets which can also result in smoother funding (Petersen and Rajan, 1995).

The lending booms in my model arise from the incumbent bank's defense against the potential entry of competition which deteriorates its portfolio and screening choices. Thus, unless the mechanisms that give rise to informational spillovers can be somehow eliminated, a naive policy aiming at improving the one will deteriorate the other even more: for example, if a regulator would offer subsidies to information acquisition, this would result in higher information levels which would leave banks even more exposed to competition and force them to react with even more cross-subsidization of bad loans (see proposition 4). Returning to a monopolistic setting by creating barriers to entry (i.e., raising γ) may not be a good idea either, albeit for reasons which remain outside the model: as already mentioned, monopoly usually fails to attain Pareto optimality because monopolistic pricing fails to extract all surplus and renders projects inefficiently small. The model's prediction that return to monopoly raises surplus should therefore be taken with a grain of salt.

Hence, the primary focus of the policy maker should lie on actions that increase the cut-off parameter q. The most traditional way to attain more restrictive lending practices goes by raising the refinancing cost ρ: central banks usually try to limit the size of a lending boom by taking a more restrictive monetary policy stance.[12] As we can see from the definition of the cut-off $q \equiv (\rho - r)/(R - r)$, this measure is especially effective when the risky part $R - r$ of project payoffs is small because even minor changes in ρ then have major effect on q. Since the relevant trade-offs for the conduct of such policy are not modeled, the discussion of its welfare implications must remain incomplete: it is only possible to state that any policy that increases q without generating additional transfers reduces the surplus generated by the banking sector. In practice, the heterogeneity of credit markets may limit the ability of monetary policy to address lending booms effectively: in the model, we have analyzed a situation in which there was only one single credit market for which the average quality of projects was exogenously given as $\bar{p}$, so were applicant heterogeneity ε and project payoffs R, r. In reality, there are hundreds of distinct credit markets, e.g. car loans, mortgages, or loans to specific industries, which are all very different along each of these dimensions. Since monetary policy affects all of them, it may be impossible to fight a lending boom in some of these markets without creating a harmful contraction of lending in others.

An alternative policy measure that can be better targeted to certain credit markets is to selectively reduce the strength of the safety net that is offered to the lender by the option to repossess the project upon default of the borrower. This becomes especially relevant if project liquidation values r are relatively high. Banks may then become "lazy" in their screening activities because even a project run by a bad borrower will not expose them to the risk of substantial loss (Manove et al., 2001). It is easy to see how this problem arises within the model: a high value of r will not only result in a lower cut-off q and give rise to lower lending standards, but the risky part $R - r$ of the project will also be reduced, which by proposition 2 diminishes screening effort even independently from

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[12]see e.g. the criticism of Taylor (2007) of the lax monetary policy of the FED during the rise of the U.S. housing bubble.

the change in q.

What can be done in order to improve the situation? Imagine that there was some tax $\tau < r$ on repossessing the project of a borrower in default. Then, the lender could only extract a payoff of $\hat{r} \equiv r - \tau$ in case the project fails, and would choose a loan portfolio as if the cut-off q had the value $\hat{q} \equiv (\rho - \hat{r})/(R - \hat{r})$ which is clearly higher than q. In other words, lending standards would tighten and there would be less credit. Moreover, as expected project payoffs fall, banks would find themselves less exposed to competition, which would lead them to make less cross-subsidized loss-bringing loans. Finally, all these changes together with the elevated risky component of payoffs would increase banks' incentives to screen. If these measures are temporarily put in place during an episode of a massive lending boom (i.e. with q close to $\bar{p} + \frac{\gamma}{R-r}$), the reduction in inefficient lending plus the gains from tax proceeds can be so large that the new tax increases total welfare.

5.2.2 Dynamic Provisioning

Another particularly interesting case for which my model provides a new line of argumentation is the idea of *dynamic provisioning*. This macroprudential policy tool is being used in Spain since 2000 and has received revived interest in the light of the recent financial turmoil. The idea is relatively simple: if the regulator requires banks to accumulate a certain amount of *general* loan loss provisions during boom times, those provisions can be used during the downturn to compensate for the sharply increasing losses in the loan portfolio. But this policy tool has repeatedly come under heavy criticism because making loss provisions even before losses materialize is presumably not compatible with the principle of "true and fair value" accounting that is dictated by IFRS book keeping rules.

An implicit assumption which underlies this criticism of is that every loan on the bank's balance sheet should be considered as an asset of positive net present value at the time at which it is originated. If this was indeed the case, then the accumulation of forward-looking provisions could be rightfully criticized as some type of cookie-jar accounting which disguises the true value of the firm: loss provisions would be made for completely healthy loans without even knowing whether losses will actually be incurred. Dynamic provisioning would attain protection against risk in downturns only at the cost of sacrificing the ability of balance sheet information to provide a clear, precise and fully adequate picture about the state of the firm at a given moment in time.

If one adopts the view of bank competition and asset selection that I have developed in this paper, there is no reason for such criticism. If banks indeed find it optimal to mix projects with negative net present value into their portfolio in order to keep competitors out of their markets, the very same principles of true and fair value reporting command that provisions against those foreseeable losses must be made at the relevant moment, i.e. upon loan origination. This is exactly what a well-implemented dynamic provisioning policy can achieve: it signals that losses from negative NPV loans will certainly bind some of the future profits from other loans in the loan portfolio. There is no trade-off between dynamic provisioning and the ability of books to reflect the truthful state of the firm at

a given instant: such distortions would rather arise if traditional booking methods were followed because investors would be mislead into thinking that the bank holds a broad selection of equally profitable loans.

6 Extensions

6.1 Generalized Screening Technology

To which extent does the main result of the model – i.e. the finding of cross-subsidized lending under competition – depend on the chosen screening technology? Not much, as I will show: the driving force behind most results is the fact that credit-worthiness testing creates dispersion in posterior expectations – the more precise the test, the more of it (see Ganuza and Penalva, 2010).

Let me define a completely general credit-worthiness test as follows: for a given level of test precision λ, the test comprises of a family of cumulative distribution functions $\{S_\lambda(s|p_i)\}_{p_i \in [0,1]}$ which indicate for every possible realization of a project's true type p_i the probability of obtaining a signal less or equal to s. Together with the prior distribution $B(p)$, this induces a joint distribution from which one can calculate by Bayes' Law the posterior distribution of p conditional on observing a signal realization $\sigma_{i,\lambda} = s_i$. From this posterior distribution, one can then finally calculate the expected probability conditional on the observed signal, $E[p_i|s_i]$. Just as before, I denote the cumulative density function of the random variable $E[p_i|\sigma_i]$ under test precision λ as $\mathcal{E}_\lambda(p)$.

For any test specification that satisfies the following single-crossing assumption regarding $\mathcal{E}_\lambda(p)$, the model yields similar qualitative results:

Assumption 1. *Let $\lambda \in [0,1)$ and $\mu \in [0,1)$ with $\lambda > \mu$. Then, a credit-worthiness test with higher precision λ disperses posterior expectations further away from the prior $\bar{p}$ than a test with lower precision μ in the sense that*

$$
\begin{aligned}
\mathcal{E}_\lambda(p) &> \mathcal{E}_\mu(p) \text{ for all } p \in (0,\bar{p}), \text{ and} \\
\mathcal{E}_\lambda(p) &< \mathcal{E}_\mu(p) \text{ for all } p \in (\bar{p},1).
\end{aligned}
\tag{19}
$$

The results of the second-best benchmark as in proposition 1 then follow trivially from $m_\lambda^{SB}(q) = 1 - \mathcal{E}_\lambda(q)$: for exogenous λ, the constrained optimal amount of lending is higher than under perfect information if $q < \bar{p}$, and is lower than under perfect information if $q > \bar{p}$.

With a minor twist in order to accommodate discrete posterior distributions I can also solve for the optimal portfolio under competition: remember that we defined the average success probability of the set of all projects that lie above some cut-off z as $\mathcal{A}_\lambda(z) = \int\limits_z^1 p \, d\mathcal{E}_\lambda(p) / \int\limits_z^1 d\mathcal{E}_\lambda(p)$. This definition can cause problems when the posterior distribution is discrete since $\mathcal{A}_\lambda(z)$ would become a discontinuous function. By a change of variables, this problem is easily circumvented: I simply rewrite the average success

probability as a function of credit mass[13] m rather than cut-off probability z via the transformation $m = \mathcal{E}_\lambda(p)$ and define it as

$$\hat{\mathcal{A}}_\lambda(m) = \frac{1}{m} \int\limits_{1-m}^{1} \mathcal{E}_\lambda^{-1}(\mu)\,\mathrm{d}\mu \tag{20}$$

which makes it a continuous and decreasing function in m. Lemma 3 and corollary 3.1 continue to hold for this more general case, and one obtains an equilibrium portfolio under competition that bears much resemblance in its structure to what we have found before:

- If $\hat{\mathcal{A}}_\lambda(m_\lambda^{SB}) \leq q + \frac{\gamma}{R-r}$, the earnings per loan do not exceed γ and the monopolistic portfolio is noncontestable. In this case, the allocation will reproduce the second-best, and a state-contingent repayment terms for loans are (R, r).

- Under threat of entry $\hat{\mathcal{A}}_\lambda(m_\lambda^{SB}) > q + \frac{\gamma}{R-r}$, not every project will be financed as long as the full portfolio remains noncontestable, i.e. $q > \bar{p} - \frac{\gamma}{R-r}$. The financed mass of projects in this regime will be

$$m_\lambda^E(q) = \hat{\mathcal{A}}_\lambda^{-1}\left(q + \frac{\gamma}{R-r}\right) \tag{21}$$

  where $\hat{\mathcal{A}}_\lambda^{-1}(a) \equiv \sup\{m \in [0,1] : \hat{\mathcal{A}}_\lambda(m) = a\}$. Due to financing of negative-NPV projects, credit mass is excessive in this range, $m_\lambda^E(q) > m_\lambda^{SB}(q)$. Equilibrium repayment terms remain at (R, r), and gross profits amount to $\gamma \cdot m_\lambda^E(q)$.

- Finally, for $q \leq \bar{p} - \frac{\gamma}{R-r}$, every project is financed at reduced repayment terms such that total gross profit equals γ.

In other words, the equilibrium with cross-subsidized loans and excessive lending is also attained under a generalized screening technology.

6.2 Aggregate Uncertainty and Underprovision of Credit

As another robustness check, I ask how the model's outcomes are affected if the macroeconomic variables that determine the level of the cut-off q are not exactly known at the moment when the decision on the bank's screening precision λ has to be made. Understanding the choice of screening precision under aggregate uncertainty is very important: in reality, banks do face large fixed cost for changing their internal organizational structure which surely makes perfect adjustment of the bank's screening precision to the ex-post realized macroeconomic state prohibitively costly. It is therefore more in line with anecdotal evidence to assume that banks make their organizational choice based on their ex-ante expectations regarding the medium-term macroeconomic outlook and are unable to quickly adjust it when the ex-post realized state turns out to require substantially different levels of screening precision. A slightly modified version of the model can address this situation:

---

[13]It is understood that m stands for a mass of m projects selected in the order from best towards worse quality.

1. initially, the state of q is unknown, but it is known that it will be drawn from a cumulative distribution function $\mathcal{U}(q)$.

2. the rest of the setup follows exactly the baseline model: two islands, two banks; without knowing the actual realization of q, every bank chooses its screening precision $\bar{\lambda}$ and pays a cost of $c(\bar{\lambda})$ that becomes sunk immediately

3. after λ has been chosen, the actual level of the cut-off q is realized

4. banks choose their domestic portfolios, observe each other's domestic lending choices and attempt to poach customers as in the baseline model

Since the last steps of the game have remained untouched, the model's findings on portfolio choice for a given $\bar{\lambda}$ are unaffected by the modification; the new setup merely changes equation (16) which determines the optimal level $\bar{\lambda}_E^*$ of screening under competition. The new optimization problem then reads

$$
\max_{\bar{\lambda}} \quad \int \Pi_{\bar{\lambda}}^E(q, \gamma) - c(\bar{\lambda}) \, \mathrm{d}\mathcal{U}(q) \tag{22}
$$
$$
\text{s.t.} \quad \bar{\lambda} \geq 0
$$

The bank takes all possible realizations of q weighted by their probability of occurrence into account when optimizing its expected net profits. This creates two different effects: on the one hand, bank competition reduces in a now already familiar way the benefits of bank screening for a possibly large range of low-q states of nature. As long as the distribution $\mathcal{U}(q)$ assigns nonzero probability weight to those states of nature, the resulting ex-ante profit-maximizing level of screening $\bar{\lambda}_E^*$ will fall short of the amount that would be attained without competition. On the other hand, a new effect arises from the impossibility to condition information choice on the actual realization of q: The bank may ex-post encounter both situations in which its actual screening abilities *exceed* the level that would have been chosen if q had been known ex-ante, and situations in which the contrary is true and screening precision ends up being *too low* for the ex-post realization of q. We shall see in a bit that this has important consequences.

But first, I want to reflect for a moment about the relation between this generalized model and the baseline model that I have discussed before. What are the circumstances for which the one is more appropriate than the other? Here is how I think of it: the baseline model does well in describing financial intermediation in "normal" economic periods in which neither asset prices nor refinancing cost exhibit dramatic and totally unforeseen changes. The prediction error on q is then small enough to be negligible. The model with aggregate uncertainty is a generalization of that model and nests it as the special case in which the prior belief $\mathcal{U}(q)$ assigns an ex-ante probability of 1 to the ex-post realized state $q = v$. The additional value of the generalized model lies in its ability to also handle "abnormal times", i.e. rare but not entirely unanticipated macroeconomic phases of high volatility and uncertainty.

Let me use the following somewhat stylized example to illustrate the effects of such macroeconomic shocks: Consider that we play a sequence $t = \{1, 2, \dots\}$ of one-shot games of the general model with aggregate uncertainty whereby at any given point t in time, the cut-off parameter q_{t+1} in the next period is determined as follows: with relatively high probability ϕ it remains at its previous level q_t whereas with probability $1 - \phi$ a major change of economic conditions occurs that reassigns q_{t+1} to a random value anywhere within the possible range, $q_{t+1} \sim U(\bar{p} - \frac{\varepsilon}{2}, \bar{p} + \frac{\varepsilon}{2})$. This means that in the limit $\phi \to 1$ we return to playing a repeated one-shot game of the old model without aggregate uncertainty, whereas for $\phi \to 0$ the aggregate uncertainty regarding q_{t+1} is maximal in every period. The profit maximization problem (22) becomes

$$\max_{\bar{\lambda}_t} \quad \left[\phi \, \Pi^E_{\bar{\lambda}_t}(q_t, \gamma) + (1 - \phi) \int_{\bar{p} - \frac{\varepsilon}{2}}^{\bar{p} + \frac{\varepsilon}{2}} \Pi^E_{\bar{\lambda}_t}(q_{t+1}, \gamma) \, \varepsilon^{-1} \mathrm{d}q_{t+1} \right] - c(\bar{\lambda}_t) \qquad (23)$$

$$\text{s.t.} \quad \bar{\lambda}_t \geq 0$$

As Figure 5 shows, if the probability ϕ of retaining the last period state of q is not too low, the equilibrium information choice looks quite similar to the baseline model without aggregate uncertainty. Specifically, if there is a lending boom today (i.e. $q_t < \bar{p}$) and competition is sufficiently strong, tomorrow's ex-ante profit-maximizing screening precision $\bar{\lambda}^*_{E,t+1}$ is very low. This is because the probability of a regime change that could move q_{t+1} to some new value is rather low, and even if it were to occur, only some realizations of q_{t+1} will actually benefit from screening precision due to the presence of competition.
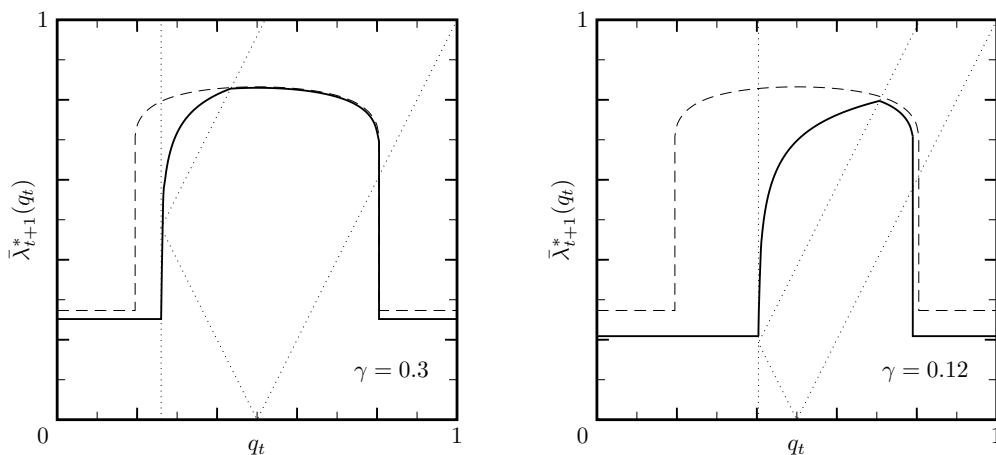

Figure 5: Information choice $\bar{\lambda}^*_{E,t+1}$ as a function of q_t for two different levels of competition in the presence of aggregate uncertainty ($\phi = 0.8$).

As in Figure 3, competition is stronger in the right figure than in the left one. The solid line displays the equilibrium information choice $\bar{\lambda}^*_{E,t+1}$ as a function of the current cut-off q_t. The second-best benchmark is shown with dashed lines. Note that in anticipation of an aggregate shock (with probability $1 - \phi = 0.2$) some information is also collected in states q_t in which information previously had no value.

But ex-post, there can be a very different story: let us think what happens if a macroeconomic regime change actually occurs and pushes the cut-off q_{t+1} to some relatively high value, i.e. $q_{t+1} > \bar{p} + \frac{\bar{\lambda}_{E,t+1}^* \varepsilon}{2} - \frac{2\gamma}{R-r} > \bar{p}$. Asset values are poor and refinancing cost are high enough as to give no incentive to any competitor to enter the market as an outside lender. In other words, the ex-ante threat of competition turns out to be irrelevant ex-post. One of the preemptive measures taken by the bank ex-ante to optimally respond to this threat remains in place, however, and impedes its lending activities: the bank's chosen screening precision $\bar{\lambda}_{E,t+1}^*$ is inefficiently low due to the ex-ante expectation of competition. Since the average project is not credit-worthy, we already know from corollary 4.1 that the imprecise information will prevent the bank from lending to as many borrowers as in the second-best benchmark.

In fact, due to the low precision of the screening signal, only borrowers of excellent quality will generate a signal that is sufficiently positive to secure their financing. The lack of precise information thus manifests empirically as a *flight to quality* combined with a sharp contraction in lending. This shows that one single cause – the lack of screening precision due to informational spillovers – has the potential to consistently explain two key pathologies that trouble most modern economies these days: the rise of inefficient credit booms, and the severe underprovision of credit once such economic boom goes bust.

7 Conclusions

In this paper, I have constructed a model of bank lending and bank competition that offers an information-centric explanation for credit booms. I find that excessive lending occurs when lenders asymmetrically acquire private information on borrowers and when informational spillovers make it possible for uninformed outside lenders to poach loan-approved customers. Lenders optimally react to the threat of entry by generating an adverse selection problem for the entrant through a socially inefficient widening of their lending activity to non-creditworthy customers and by simultaneously lowering their screening effort. This results in excessive credit and increases the procyclicality of lending. The model also offers a theoretical explanation for the mounting empirical evidence that low monetary policy rates prompt banks to lend to riskier and possibly inefficient projects.

Clearly, there is substantial room for further extension. For the purpose of clarity of exposition, I have made assumptions that cast the model in a very simple form and allow me to highlight the welfare-diminishing allocational effects of competition in the presence of informational spillovers. But this is clearly not the full story, and when employing the model to answer policy questions, the welfare-improving effects of competition must be put back into place: first, project size should be endogenous rather than fixed; as already mentioned earlier, endogenous project size under diminishing returns to investment would prevent the monopolist from extracting the full project surplus: monopolistic pricing would render projects inefficiently small, and the monopolistic case would no longer be constrained efficient. The second important assumption that deserves deeper thought is

whether a bank is really able to expand its lending to a very large set of projects (including inefficient ones) without incurring a penalty in the form of elevated refinancing cost. If "poisoning the well" by cross-subsidization of inefficient projects becomes increasingly costly as more and more loans are handed out, the bank's optimal response to competition by uninformed outside lenders must strike an optimal balance between inefficient excess lending and lower repayment terms. Competition would then put immediate downward pressure on loan prices and increase welfare by inducing larger project sizes. The question to which extent these welfare gains from competition are offset by its welfare-diminishing impact on lending standards that I have analyzed in this paper is an interesting avenue for both empirical and theoretical future research.

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A Appendix

A.1 Derivation of Posterior Beliefs after Screening

From the definition of the test, it is immediate that the cumulative density function of σ_i conditional on the project's actual quality p_i being p is given by a weighted combination of the cumulative of the random distribution and a step function,

$$P(\sigma_i \leq s | p_i = p) = (1 - \lambda)\left(\frac{1}{2} + \frac{s - \bar{p}}{\varepsilon}\right) + \lambda \mathcal{H}(s - p) \tag{24}$$

where $\mathcal{H}(\cdot)$ is the Heaviside step function that evaluates to zero if the argument is negative, and to one otherwise. The corresponding probability density function[14] then reads

$$P(\sigma_i = s | p_i = p) = (1 - \lambda)\frac{1}{\varepsilon} + \lambda \delta(s - p) \tag{25}$$

where $\delta(x)$ denotes the Dirac delta distribution that satisfies

$$\int_{-\infty}^{\infty} \delta(x) = 1 \text{ and } \delta(x) = \begin{cases} 0 & \text{for } x \neq 0 \\ +\infty & \text{for } x = 0 \end{cases}$$

By Bayes' Law,

$$P(p_i = p | \sigma_i = s) = \frac{P(\sigma_i = s | p_i = p)P(p_i = p)}{P(\sigma_i = s)} = (1 - \lambda)\varepsilon^{-1} + \lambda \delta(s - p) \tag{26}$$

and the conditional expectation becomes

$$\begin{aligned} E[p_i | \sigma_i = s] &= \int p\left[(1 - \lambda)\varepsilon^{-1} + \lambda \delta(s - p)\right] \mathrm{d}p \\ &= \int (1 - \lambda)\varepsilon^{-1} p \, \mathrm{d}p + \lambda s \\ &= (1 - \lambda)E[p] + \lambda s \end{aligned} \tag{27}$$

A.2 Proofs

Proof of Proposition 1:

The result for m_λ^{SB} follows trivially from $m_\lambda^{SB} = 1 - \mathcal{E}_\lambda(q)$. To verify the indicated average success probability $\mathcal{A}_\lambda^{SB}$, observe that the statement is true for $m_\lambda^{SB} \in \{0, 1\}$. Thus, we

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[14]strictly, this is not a function, but a distribution in the Schwartz sense

can assume $0 < m_\lambda^{SB} < 1$. Then,

$$
\begin{aligned}
\mathcal{A}_\lambda^{SB} &= \frac{1}{m_\lambda^{SB}} \int_q^1 p \, \mathrm{d}\mathcal{E}_\lambda(p) = \frac{1}{m_\lambda^{SB}} \int_q^{\bar{p}+\frac{\lambda\varepsilon}{2}} (\lambda\varepsilon)^{-1} p \, \mathrm{d}p \\
&= \frac{1}{m_\lambda^{SB}\lambda\varepsilon} \left[\frac{p^2}{2}\right]_q^{\bar{p}+\frac{\lambda\varepsilon}{2}} = \frac{1}{m_\lambda^{SB}\lambda\varepsilon} \left[\frac{(\bar{p}+\frac{\lambda\varepsilon}{2})^2}{2} - \frac{q^2}{2}\right] \\
&= \frac{\lambda\varepsilon}{2m_\lambda^{SB}} \left(\frac{\bar{p}-q}{\lambda\varepsilon} + \frac{1}{2}\right)\left(\bar{p}+q+\frac{\lambda\varepsilon}{2}\right) = \frac{\bar{p}+q+\frac{\lambda\varepsilon}{2}}{2} \\
&= \bar{p} + \frac{\lambda\varepsilon}{2} \left(\frac{q-\bar{p}}{\lambda\varepsilon} + \frac{1}{2}\right) = \bar{p} + \frac{\lambda\varepsilon}{2}(1-m_\lambda^{SB})
\end{aligned}
$$

$\square$

Proof of Proposition 2:

Marginal gross returns from information are

$$
\frac{\partial}{\partial\lambda}(R-r)\frac{(2\bar{p}-2q+\lambda\varepsilon)^2}{8\lambda\varepsilon} = (R-r)\left[\frac{\varepsilon}{8} - \frac{(\bar{p}-q)^2}{2\lambda^2\varepsilon}\right]
$$

which is always non-negative and attains a maximal value of $(R-r)\varepsilon/8$ when $q = \bar{p}$. Note that for $R \to r$ this goes to zero which proves that there will be no information acquisition in the limit of disappearing risk. $\square$

Proof of Lemma 3:

I show the lemma in three steps:

1. In equilibrium, the payoff from choosing any noncontestable symmetric portfolio $\mathcal{Q}$ is equal to its surplus $\Pi[\mathcal{Q}]$.

   This is obvious since, because of symmetry, no offer of the entrant can attract a subgroup of borrowers that is better than the average of $\mathcal{Q}$. From noncontestability it is clear that the entrant can not profitably enter the market even when the incumbent extracts all surplus.

2. If $\mathcal{P}$ is a (not necessarily symmetric) noncontestable portfolio, and $\mathcal{Q}$ is a symmetric noncontestable portfolio with $\Pi[\mathcal{Q}] \geq \Pi[\mathcal{P}]$, then by choosing the symmetric portfolio $\mathcal{Q}$ the incumbent obtains at least the same payoff in the contestable market game's equilibrium than by choosing $\mathcal{P}$.

   By the previous step, choosing $\mathcal{Q}$ yields a game payoff of $\Pi[\mathcal{Q}]$ for the incumbent. Thus, if the converse were true, there would exist some portfolio $\mathcal{P}$ which, if chosen, would yield for the incumbent in equilibrium a payoff strictly higher than $\Pi[\mathcal{P}]$. Since splitting a portfolio's profits between the two players whilst keeping repayments constant is already a zero sum game, the entrant would necessarily have to bear

negative profits; this holds even more when the necessarily lower repayment rates demanded by the entrant and his extra monitoring cost of γ per loan are considered. The entrant would do better by staying out of the market which contradicts that this is an equilibrium strategy.

3. If $\mathcal{P}$ is a (not necessarily symmetric) noncontestable portfolio, then there always exists a *symmetric* noncontestable portfolio $\mathcal{Q}$ with weakly higher surplus $\Pi[\mathcal{Q}] \geq \Pi[\mathcal{P}]$.

   Define $D_{\max} = \sup\limits_{i \in \mathcal{P}}\{D_i\}$, and denote the portfolio arising from financing every project $i \in \mathcal{P}$ at repayment rates $(D_{\max}, r)$ as $\hat{\mathcal{P}}$, and let $\hat{F}$ be the portfolio in which the full measure of *all* projects financed at repayment rates $(D_{\max}, r)$.

   If $\Pi[\hat{F}] > \gamma$, portfolio $\hat{F}$ can be made noncontestable by lowering repayment rates symmetrically to yield profits exactly equal to γ. The resulting profits are weakly higher than $\Pi[\mathcal{P}]$ since $\gamma \cdot 1 \geq \gamma|\mathcal{P}|$. The case in which $\pi[\hat{\mathcal{P}}] \leq \gamma$ is trivial: $\mathcal{Q} = \hat{\mathcal{P}}$. Thus it remains to show the statement for $\pi[\hat{\mathcal{P}}] > \gamma$ and $\pi[\hat{F}] \leq \gamma$.

   Denote the measure of loans *not* in $\mathcal{P}$ as N, index them arbitrarily with j (where $0 \leq j \leq |N|$), and denote as N_k the portfolio subset of loans in N with index no higher than k, financed at $(D_{\max}, r)$. Then, the function $k \mapsto \pi[\hat{\mathcal{P}} \cup N_k]$ is continuous and satisfies by construction $\pi[\hat{\mathcal{P}} \cup N_0] > \gamma$, and $\pi[\hat{\mathcal{P}} \cup N_{|N|}] \leq \gamma$. By the intermediate value theorem there exists some l with $0 < l \leq |N|$ such that $\pi[\hat{\mathcal{P}} \cup N_l] = \gamma$. Since $|\hat{\mathcal{P}} \cup N_l| \geq |\mathcal{P}|$, the symmetric noncontestable portfolio $\mathcal{Q} \equiv \hat{\mathcal{P}} \cup N_l$ must yield weakly higher surplus than $\mathcal{P}$.

Thus, let $\mathcal{P}$ be an arbitrary noncontestable portfolio. Then by the last step we can construct a noncontestable symmetric $\mathcal{Q}$ with the same or higher surplus, and due to the first two steps we know that the game payoff attained by playing $\mathcal{Q}$ is at least the same as the payoff from playing $\mathcal{P}$, q.e.d.

Proof of Proposition 4:

To prove the statement, I solve the maximization problem in corollary 3.1 for which I have already shown that every solution is an equilibrium portfolio choice in the game with competition. There are three cases:

1. If the solution to the unconstrained profit maximization problem is noncontestable, it is obvious that the monopolistic allocation maximizes eq (14). All projects with expected success probability above $q = \frac{\rho - r}{R - r}$ are financed at terms (R, r). This occurs whenever the success probability of the monopolistic portfolio satisfies

$$
\begin{aligned}
\mathcal{A}_\lambda^{SB} &\leq q + \frac{\gamma}{R - r} \\
\frac{\bar{p} + q}{2} + \frac{\lambda \varepsilon}{4} &\leq q + \frac{\gamma}{R - r} \\
\Rightarrow \lambda &\leq \frac{2(q - \bar{p})}{\varepsilon} + \frac{4\gamma}{\varepsilon(R - r)}
\end{aligned} \tag{28}
$$

2. If the noncontestability constraint in equation (14) binds (i.e. $\lambda > \frac{2(q-\bar{p})}{\varepsilon} + \frac{4\gamma}{\varepsilon(R-r)}$), it must hold with equality:

$$\max_{\mathcal{Q}=(\mathcal{S},(D,d))} \quad \Pi[\mathcal{Q}]$$

$$\text{s.t.} \quad \Pi[\mathcal{Q}] = \gamma|\mathcal{Q}| \tag{29}$$

Thus, any equilibrium has to maximize the size $|\mathcal{Q}|$ of the portfolio whilst maintaining an average profit per loan of exactly γ. Let me assume additionally that q is such that the portfolio $\mathcal{F}$ that comprises of all projects financed at (R,r) is noncontestable (i.e. $q \geq \bar{p} - \frac{\gamma}{R-r}$).

To solve this problem, I first characterize what the largest possible portfolio $\mathcal{Q}$ is that satisfies $\Pi[\mathcal{Q}] = \gamma|\mathcal{Q}|$ when repayment terms are exogenously fixed at some (D,d), where $D \leq R$ and $d \leq r$. To yield exactly γ per loan, the average probability of success in the portfolio must be equal to

$$\mathcal{A}_\lambda[\mathcal{Q}] = \frac{\int\limits_{\mathcal{S}\in\mathcal{Q}} p \, \mathrm{d}\mathcal{E}_\lambda(p)}{\int\limits_{\mathcal{S}\in\mathcal{Q}} \mathrm{d}\mathcal{E}_\lambda(p)} = \frac{\rho - d + \gamma}{D - d} \tag{30}$$

This condition is met by many symmetric portfolios that all have repayment terms (D,d). But the size of the portfolio can only be maximal if the portfolio is chosen in a pecking order that starts with the best projects and proceeds in the order of expected success probabilities towards worse ones: to see this, imagine that the portfolio selection would not follow a cut-off structure, and that some nonzero mass m of projects with average success probability π_m was denied financing whereas a subset of projects of equal size but worse average success probability $\pi_n < \pi_m$ was financed. Then, by exchanging in the portfolio the worse projects projects for the better ones, one can increase the nominator of eqn (30) by $m \cdot (\pi_m - \pi_n)$ whereas the denominator stays the same. This "excess" in success probability can be used to enlarge the portfolio size by financing an additional mass of previously unfinanced projects that have worse quality than the portfolio average, whereby the mass is chosen such as to restore the average success probability to its original value. This shows that a portfolio which does not respect the pecking order of "best loans come first" will not be maximal in size.

Thus, there will be a unique cut-off $\hat{q}$ that will be determined by meeting the average success probability condition:

$$\frac{2\bar{p} + 2\hat{q} + \lambda\varepsilon}{4} = \frac{\rho - d + \gamma}{D - d}$$

$$\Rightarrow \hat{q} = \frac{2(\rho - d + \gamma)}{D - d} - \bar{p} - \frac{\lambda\varepsilon}{2} \tag{31}$$

This expression makes it obvious that the cut-off is decreasing in D, so portfolio size is increasing in D; thus when D is also chosen, we must have $D = R$. Finally,

the derivative of (31) with respect to d yields $\frac{2(\rho-D+\gamma)}{(D-d)^2}$ which for $D = R$ (as it is the case here) is negative if $\gamma + \rho < R \Rightarrow \frac{\gamma}{R-r} + \frac{\rho-r}{R-r} < 1$, i.e. $q + \frac{\gamma}{R-r} < 1$. Note that the left side of the inequality is the average success probability from eq. (30). Thus the inequality holds, and the size of the portfolio is increasing in d, which implies $d = r$. We then have a cut-off $\hat{q}$ of

$$
\begin{aligned}
\hat{q} &= \frac{2(\rho - r + \gamma)}{R - r} - \bar{p} - \frac{\lambda\varepsilon}{2} \\
&= 2\left(q + \frac{\gamma}{R-r}\right) - \bar{p} - \frac{\lambda\varepsilon}{2}
\end{aligned}
\tag{32}
$$

3. If $\mathcal{F}$ is contestable, i.e. $q \leq \bar{p} - \frac{\gamma}{R-r}$, any noncontestable portfolio that maximizes size must have a size of 1 and yield gross profits of γ. For a symmetric portfolio with $D = \frac{\gamma + \rho - r(1-\bar{p})}{\bar{p}}$ and $d = r$, gross profits are exactly γ:

$$
\begin{aligned}
\Pi &= \bar{p}D + (1 - \bar{p})r - \rho \\
&= \gamma + \rho - r(1 - \bar{p}) + (1 - \bar{p})r - \rho \\
&= \gamma
\end{aligned}
\tag{33}
$$

Proof of Proposition 5:

The first two statements follow directly from taking the derivative of eq. (15) with respect to q for the monopolistic and the competitive regime,

$$
\frac{\partial m(q, \gamma)}{\partial q} = \begin{cases} \frac{-1}{\lambda\varepsilon} & \text{if } \bar{p} + \frac{\lambda\varepsilon}{2} - \frac{2\gamma}{R-r} < q < \bar{p} + \frac{\lambda\varepsilon}{2} \\ \frac{-2}{\lambda\varepsilon} & \text{if } \bar{p} - \frac{\gamma}{R-r} < q \leq \bar{p} + \frac{\lambda\varepsilon}{2} - \frac{2\gamma}{R-r} \end{cases}
\tag{34}
$$

Combining this result with $\frac{\partial \mathcal{D}_\lambda}{\partial m} = -\frac{\partial \mathcal{A}_\lambda}{\partial m} = \frac{\lambda\varepsilon}{2}$ yields

$$
\frac{\partial \mathcal{D}_\lambda}{\partial q} = \frac{\partial \mathcal{D}_\lambda}{\partial m} \cdot \frac{\partial m}{\partial q} = \begin{cases} -\frac{1}{2} & \text{if } \bar{p} + \frac{\lambda\varepsilon}{2} - \frac{2\gamma}{R-r} < q < \bar{p} + \frac{\lambda\varepsilon}{2} \\ -1 & \text{if } \bar{p} - \frac{\gamma}{R-r} < q \leq \bar{p} + \frac{\lambda\varepsilon}{2} - \frac{2\gamma}{R-r} \end{cases}
\tag{35}
$$

which proves the third statement.

Proof of Proposition 6:

To prove the statement, let's look at the gross profit function in the presence of competition: for $q > \bar{p} - \frac{\gamma}{R-r}$, it reads

$$
\Pi_\lambda(q, \gamma) = \begin{cases} \max\left\{0, (R-r)(\bar{p}-q)\right\} & \text{if } 0 \leq \lambda \leq \frac{2|q-\bar{p}|}{\varepsilon} \\ (R-r)\frac{(2\bar{p}-2q+\lambda\varepsilon)^2}{8\lambda\varepsilon} & \text{if } \frac{2|q-\bar{p}|}{\varepsilon} < \lambda \leq \frac{2(q-\bar{p})}{\varepsilon} + \frac{4\gamma}{\varepsilon(R-r)} \\ \gamma\left[1 - \frac{2\left(q-\bar{p}+\frac{\gamma}{R-r}\right)}{\lambda\varepsilon}\right] & \text{if } \frac{2(q-\bar{p})}{\varepsilon} + \frac{4\gamma}{\varepsilon(R-r)} < \lambda < 1 \end{cases}
\tag{36}
$$

whilst for $q \leq \bar{p} - \frac{\gamma}{R-r}$ gross profits are independent of λ,

$$
\Pi_\lambda(q, \gamma) = \gamma \text{ for all } \lambda \in [0, 1)
\tag{37}
$$

Despite its piecewise structure, the function is continuous everywhere. Specifically, note that the transition between monopolistic and competitive regimes occurs at $\lambda = \frac{2(q-\bar{p})}{\varepsilon} + \frac{4\gamma}{\varepsilon(R-r)}$, which substituted into the competitive profit function yields the same profits as when approaching the threshold from the monopolistic side: $\Pi = \frac{\gamma^2}{(q-\bar{p})(R-r)+2\gamma}$

Taking derivative with respect to λ, one obtains the marginal returns to information for $q > \bar{p} - \frac{\gamma}{R-r}$ as

$$\frac{\partial\Pi_\lambda}{\partial\lambda}(q,\gamma) = \begin{cases} 0 & \text{if } 0 \leq \lambda \leq \frac{2|q-\bar{p}|}{\varepsilon} \\ (R-r)\left(\frac{\varepsilon}{8} - \frac{(\bar{p}-q)^2}{2\lambda^2\varepsilon}\right) & \text{if } \frac{2|q-\bar{p}|}{\varepsilon} < \lambda \leq \frac{2(q-\bar{p})}{\varepsilon} + \frac{4\gamma}{\varepsilon(R-r)} \\ \frac{2\gamma\left(q-\bar{p}+\frac{\gamma}{R-r}\right)}{\lambda^2\varepsilon} & \text{if } \frac{2(q-\bar{p})}{\varepsilon} + \frac{4\gamma}{\varepsilon(R-r)} < \lambda < 1 \end{cases} \tag{38}$$

whereas there are zero marginal returns to information for $q \leq \bar{p} - \frac{\gamma}{R-r}$. Hence we already know that $\lambda_E^* = 0$ for this region, which proves the third statement of the proposition. The first statement of the proposition is equally obvious because profits and marginal returns to information for this case are the same as in the planner's benchmark. It thus only remains to show that marginal returns under threat of competition are lower than those attained under monopoly. Denote the difference in marginal returns to information as

$$\begin{aligned} \Delta(\lambda) &\equiv (R-r)\left(\frac{\varepsilon}{8} - \frac{(\bar{p}-q)^2}{2\lambda^2\varepsilon}\right) - \frac{2\gamma\left(q-\bar{p}+\frac{\gamma}{R-r}\right)}{\lambda^2\varepsilon} \\ &= \frac{(R-r)\varepsilon}{8} - \frac{((q-\bar{p})(R-r)+2\gamma)^2}{2(R-r)\lambda^2\varepsilon} \end{aligned} \tag{39}$$

This term is non-negative if

$$\frac{1}{8}(R-r)\varepsilon \cdot 2(R-r)\lambda^2\varepsilon - ((q-\bar{p})(R-r)+2\gamma)^2 \geq 0$$
$$\Leftrightarrow \left((R-r)\frac{\lambda\varepsilon}{2}\right)^2 - ((q-\bar{p})(R-r)+2\gamma)^2 \geq 0 \tag{40}$$

Substituting into this expression the threshold between monopolistic and competitive regimes, $\lambda = \frac{2(q-\bar{p})}{\varepsilon} + \frac{4\gamma}{\varepsilon(R-r)}$, we see that the left-hand side of this inequality becomes zero. For any larger value of λ it will obviously remain positive. Thus, marginal returns to information are continuous at the threshold, and for higher λ marginal returns to information under competition are strictly less than under monopoly. q.e.d.

Proof of Proposition 7:

Under the conditions stated in the proposition, a reduction of competition has two effects on credit: As one can see by taking the total derivative wrt. γ,

$$\frac{\mathrm{d}}{\mathrm{d}\gamma}\left(m_{\lambda_E^*(\gamma)}^E(q,\gamma)\right) = \frac{\partial m_\lambda^E}{\partial\gamma} + \left.\frac{\partial m_\lambda^E}{\partial\lambda}\right|_{\lambda=\lambda_E^*} \cdot \frac{\partial\lambda_E^*}{\partial\gamma} \tag{41}$$

there is a *direct* effect and an *indirect* one via λ. The two effects have opposite sign: for constant information level, less competition results in less cross-subsidization and thus less

credit. However, less competition also raises information precision which again prompts more cross-subsidization and more credit. I show here that the first effect prevails:

First, note that

$$\frac{\partial m_\lambda^E}{\partial \lambda} = \frac{\partial}{\partial \lambda}\left(1 - \frac{2(q - \bar{p} + \frac{\gamma}{R-r})}{\lambda \varepsilon}\right) = \frac{1 - m_\lambda^E}{\lambda} \tag{42}$$

and, as a consequence of $\Pi_\lambda^E = \gamma m_\lambda^E$,

$$\frac{\partial \Pi_\lambda^E}{\partial \lambda} = \gamma \frac{1 - m_\lambda^E}{\lambda} \tag{43}$$

From here I take derivatives and obtain

$$\frac{\partial^2 \Pi_\lambda^E}{\partial \lambda^2} = -\frac{2\gamma}{\lambda^2}(1 - m_\lambda^E) \text{ and} \tag{44}$$

$$\frac{\partial^2 \Pi_\lambda^E}{\partial \lambda \partial \gamma} = \frac{1 - m_\lambda^E}{\lambda} + \frac{2\gamma}{\lambda^2 \varepsilon (R - r)} \tag{45}$$

I then apply the implicit function theorem on the first-order condition that fixes λ_E^* in order to put bounds on the sensitivity of λ_E^* with respect to γ:

$$\frac{d}{d\gamma}\lambda_E^*(q, \gamma) = -\frac{\frac{\partial^2 \Pi_\lambda^E}{\partial \lambda \partial \gamma}}{\frac{\partial^2 \Pi_\lambda^E}{\partial \lambda^2} - c''(\lambda)} \leq \frac{\frac{\partial^2 \Pi_\lambda^E}{\partial \lambda \partial \gamma}}{-\frac{\partial^2 \Pi_\lambda^E}{\partial \lambda^2}} = \frac{\frac{1 - m_\lambda^E}{\lambda} + \frac{2\gamma}{\lambda^2 \varepsilon (R-r)}}{\frac{2\gamma}{\lambda^2}(1 - m_\lambda^E)} \tag{46}$$

The inequality results from the fact that I assumed a convex cost function, so $c'' \geq 0$. Simplifying further, I obtain

$$\frac{d}{d\gamma}\lambda_E^*(q, \gamma) \leq \frac{\lambda}{2\gamma} + \frac{1}{\varepsilon(R - r)(1 - m_\lambda^E)} \tag{47}$$

and

$$\frac{\partial m_\lambda^E}{\partial \lambda}\frac{d\lambda}{d\gamma} \leq \frac{1 - m_\lambda^E}{2\gamma} + \frac{1}{\varepsilon \lambda (R - r)} \tag{48}$$

The sum of direct and indirect effect then yields

$$\frac{d}{d\gamma}\left(m_{\lambda_E^*(\gamma)}^E(q, \gamma)\right) \leq \frac{1 - m_\lambda^E}{2\gamma} - \frac{1}{\varepsilon \lambda (R - r)} \tag{49}$$

Substituting the definition of m_λ^E, I have

$$\frac{d}{d\gamma}\left(m_{\lambda_E^*(\gamma)}^E(q, \gamma)\right) \leq \frac{q - \bar{p}}{\gamma \lambda \varepsilon} + \frac{1}{\lambda \varepsilon (R - r)} - \frac{1}{\varepsilon \lambda (R - r)} = \frac{q - \bar{p}}{\gamma \lambda \varepsilon} \tag{50}$$

Since I assumed that the average project is credit-worthy, $q < \bar{p}$ holds and the equation is negative. q.e.d.

A.3   Closed-Form Solutions

For the specific cost function $c(\lambda) = c_0 \frac{\lambda}{1-\lambda}$ the model has closed-form solutions for $\lambda^*(q)$: In the monopolistic case, the first-order condition has the form

$$\frac{c_0}{(\lambda - 1)^2} \;=\; b - \frac{a}{\lambda^2}$$

with $a \equiv \frac{(R-r)(\bar{p}-q)^2}{2\varepsilon}$ and $b \equiv \frac{(R-r)\varepsilon}{8}$. Solving for λ leads to a quartic equation for which the solution is known in standard tables. Two roots can be discarded because they describe local minima, so only two solutions λ_1^* and λ_2^* remain. The correct solution is identified as the only one that lies within the $[0, 1]$ interval (depending on q, either the one or the other branch applies). Define

$$
\begin{aligned}
\gamma &\equiv a - b - c_0 \\
\phi &\equiv 108a^2 b - 108ab^2 - 108ab\gamma - 2\gamma^3 \\
\beta &\equiv \frac{\sqrt[3]{2}\gamma^2}{3a\left(\phi + \sqrt{\phi^2 - 4\gamma^6}\right)^{1/3}} + \frac{\left(\phi + \sqrt{\phi^2 - 4\gamma^6}\right)^{1/3}}{3\sqrt[3]{2}a}
\end{aligned}
$$

The two solutions are then given by

$$\lambda_1^* \;=\; \frac{1}{2}\sqrt{2 + \beta - \frac{4\gamma}{3a} - \frac{2(a - 2b - \gamma)}{a\sqrt{1 - \beta - \frac{2\gamma}{3a}}}} - \frac{1}{2}\sqrt{1 - \beta - \frac{2\gamma}{3a}} + \frac{1}{2} \tag{51}$$

$$\lambda_2^* \;=\; \frac{1}{2}\sqrt{2 + \beta - \frac{4\gamma}{3a} + \frac{2(a - 2b - \gamma)}{a\sqrt{1 - \beta - \frac{2\gamma}{3a}}}} + \frac{1}{2}\sqrt{1 - \beta - \frac{2\gamma}{3a}} + \frac{1}{2} \tag{52}$$

In the competitive case, the first-order condition is quadratic in λ and can be solved to yield as the unique non-negative solution

$$\lambda_E^*(q, \gamma) = \frac{1}{1 + \sqrt{\frac{c_0}{\delta}}} \quad \text{where } \delta = 2\gamma\varepsilon^{-1}\left(q - \bar{p} + \frac{\gamma}{R-r}\right) \tag{53}$$