Inter-firm Relationships and Asset Prices*

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Job Market Paper

This version: November 14, 2015

ABSTRACT

I study the asset pricing properties that stem from the propagation of shocks within a network economy and the extent to which such a propagation mechanism quantitatively explains asset market phenomena. I show that changes in the propagation of shocks within a network economy are important to understanding variations in asset prices and returns, both in the aggregate and in the cross section. A calibrated model that matches features of customer-supplier networks in the U.S. generates a persistent component in expected consumption growth and stochastic consumption volatility similar to the Long-Run Risks Model of Bansal and Yaron (2004). In the cross section, firms that are more central in the network command higher risk premium than firms that are less central. In the time series, firm-level return volatilities exhibit a high degree of comovement. These two features are consistent with recent empirical evidence.

Keywords: Inter-firm Relationships, Networks, Contagion, Equilibrium Asset Prices.

JEL classification: G12, E32, L10.

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Inter-firm relationships, such as strategic alliances, joint ventures, R&D partnerships, and customer-supplier relationships, are prevalent in modern economies. A growing body of empirical work highlights the importance of these relationships in the case of firms' distress and shows that they may serve as propagation mechanisms of negative shocks to individual firms.[1] For instance, consider South Africa's platinum miners' strike in 2014, which affected the world's top platinum producers, Anglo American Platinum, Impala Platinum, and Lonmin. First, platinum production decreased. Because platinum is used in many industrial applications such as oil cracking, some manufacturing firms may have faced higher production costs, as they needed to restructure their production given the lack of platinum. This, in turn, may have increased costs for some wholesale firms which, in turn, may have decreased some retailers' profits. Namely, a negative shock to a firm (or group of them) may spread to others via inter-firm relationships, and in doing so, potentially alter aggregate economic growth and volatility as well as asset prices and risk premia.

In this paper, I study the asset pricing properties that stem from the propagation of shocks within a network economy and the extent to which such a propagation mechanism quantitatively explains asset market phenomena. To do so, I develop a dynamic, network-based equilibrium model in which the propagation of shocks determines, in large part, firms' cash-flow growth rates. To get a sense of the quantitative impact in asset prices of such a propagation mechanism, I calibrate the model to match features of customer-supplier networks in the U.S. To the best of my knowledge, this study is among the first to explore the extent to which the propagation of shocks within a network economy quantitatively explains asset market phenomena.

The main finding of this paper is that changes in the propagation of shocks within a network economy are important to understand variations in asset prices and returns, both in the aggregate and in the cross section. In the aggregate, a calibrated model generates a persistent component in expected consumption growth and stochastic consumption volatility similar to those in Bansal and Yaron (2004). As in Bansal and Yaron (2004), these two features, together with Epstein-Zin-Weil preferences, help explain characteristics of aggregate asset market data such as the equity premium and the low risk-free rate. The calibrated model also helps in understanding the cross section of expected returns, because it provides a mapping between firms' quantities of risk and firms' location

[1]See Lang and Stulz (1992), Cohen and Frazzini (2008), Hertzel et al. (2008), Menzly and Ozbas (2010), Boone and Ivanov (2012), Kelly, Lustig, and Nieuwerburgh (2013) and Barrot and Sauvagnat (2014).

in the network. For instance, firms that are more central in the network command a higher risk premium than firms that are less central. On average, firms in the highest quintile of centrality yield an annual excess return of 1% over those firms in the lowest quintile. This prediction is aligned with the 3% excess return documented by Ahern (2013) in the network of intersectoral trade. In the time series, firm-level return volatilities exhibit a high degree of comovement—which is consistent with evidence documented by Herskovic et al. (2014) and Duarte et al. (2014).

The main features of the model are as follows. The economy is composed of n firms whose cash-flow growth rates vary stochastically over time. In an otherwise standard dynamic endowment economy, firms' cash-flow growth rates are related via a network of inter-firm relationships, such as a supply chain, which is exogenous and fixed.[2] Each relationship generates benefits that increase a firm's cash-flow growth rate. However, relationships also increase a firm's exposure to negative shocks that affect other firms. In other words, the more relationships a firm is engaged in, the more benefits a firm receives and the higher its exposure to negative shocks that affect other firms in the network. To be more concrete, each firm faces a negative shock to its cash-flow growth rate, independently of others, with probability q—which is time-invariant and equal across firms—at very beginning of each period. Then, these negative shocks spread from one firm to another via inter-firm relationships in a probabilistic manner. In particular, a negative shock to firm i at period t propagates to firm j at t if there exists a sequence of relationships that connects firms i and j in which each relationship in the sequence transmits shocks at period t. For simplicity, each relationship potentially transmits shocks, independently of all other relationships, with probability p_t at period t.[3] The value of p_t captures the relative importance of relationship-specific investments made by the average firm in a network economy. The higher the value of p_t, the more important relationships are on average, and the higher the likelihood that shocks propagate through the economy at period t. To allow changes in the propagation of shocks within the network, the propensity of inter-firm relationships to transmit shocks, p_t, is allowed to vary

[2] The network of inter-firm relationships is assumed to be fixed for two reasons: (a) tractability, and (b) to capture the long-term nature of some customer-supplier relationships that allow connected firms to circumvent difficulties in contracting due to unforeseen contingencies, asymmetries of information, and specificity on firms' investments, e.g. Williamson (1979, 1983).

[3] In the baseline model, I only allow negative shocks to propagate in a probabilistic manner. However, the model can be augmented so that positive and negative shocks propagate over the network using the above probabilistic process. The main results continue to hold as long as the decrease in firms' cash-flows due to negative shocks is larger than the increase in firms' cash-flows due to positive shocks.

over time. As a consequence, the volatility of aggregate cash-flows and the correlation among firms' cash-flows are time-varying. Temporal changes in p_t capture changes in production technologies and complementarities among firms' activities. The pricing is done by a representative agent with Epstein-Zin-Weil preferences to embed the time-varying cash-flow correlation structure—which is endogeneously generated by the network—in a standard asset pricing model.

The above framework has two important properties. First, cash-flow growth rates are independent across firms in the absence of relationships. Second, if only one sequence of relationships connects two firms, the longer the sequence, the smaller the correlation between their cash-flow growth rates. Namely, the more distant two firms are in the network economy, the less related their cash-flows.

The distribution of consumption growth is shaped by two characteristics within the model: (a) the topology of the network of relationships and (b) the propensity of relationships to transmit shocks. Since the network is fixed, the calibrated model is able to generate a persistent component in expected consumption growth and stochastic consumption volatility as long as the propensity of relationships to transmit shocks exhibits persistent time variation. The persistent time variation in the propensity of relationships to transmit shocks within the calibrated model is motivated by the high persistence exhibited by macroeconomic variables that proxy for the level of input specificity faced by the average firm within the U.S. economy. As Barrot and Sauvagnat (2014) show, input specificity is an important driver of the propagation of shocks within customer-supplier networks. Suppliers of specific inputs are more difficult to replace in case of distress, and, thus, shocks may propagate more easily from one firm to another.[4]

In the cross section, shocks to central firms have a higher likelihood of affecting more firms than do shocks to less central firms. As a consequence, central firms are procyclical, whereas less central firms serve as a hedge against aggregate risk and command lower risk premium. Changes in the propensity of relationships to transmit shocks drive fluctuations in growth opportunities and uncertainty across firms. These fluctuations translate into changes in stock prices and returns at equilibrium, which produces a factor structure in returns and returns volatilities at the firm level.

[4]To calibrate the model, I use the time series of R&D/GDP and the number of patents created in the U.S. as measures of the degree of input specificity faced by the average firm in the U.S. The ratio R&D/GDP aims to proxy for the intensity of relationship-specific investments faced by the average firm, whereas the number of patents proxies for how easily the average firm can substitute its inputs whenever a supplier is under distress.

This paper contributes to several strands of the literature. First, it develops a new theoretical framework that relates to a growing body of work focused on understanding the effects of economic linkages in asset pricing properties. Buraschi and Porchia (2012) show that firms more central in a market-based network have lower price dividend ratios and higher expected returns. Using the network of intersectoral trade, Ahern (2013) shows that firms in more central industries have greater exposure to systematic risk. Unlike these papers, my study uses relationships at the firm level to explore the asset pricing properties that stem from the propagation of shocks within a network between firms' cash-flow and the extent to which changes in such a propagation mechanism quantitatively explain asset market phenomena. Using customer-supplier networks, Kelly, Lustig, and Nieuwerburgh (2013) propose that the size distribution and firm volatility distribution are intimately linked. However, they do not explore the equilibrium asset pricing implications of such networks. In a contemporaneous paper, Herskovic (2015) focuses on efficiency gains that come from changes in the input-output network and how those changes are priced in equilibrium. This paper, on the other hand, focuses on how changes in the propagation of shocks within a fixed network alter equilibrium asset prices, risk premia, and stock return volatilities across firms.

I also add to a body of work that explores how granular shocks may lead to aggregate fluctuations in the presence of linkages among different sectors of the economy, e.g. Carvalho (2010), Gabaix (2011), Acemoglu et al. (2012); Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015), Carvalho and Gabaix (2013), among others. This literature focuses mostly on analyzing changes in aggregate economic variables due to changes in the input-output network rather than exploring the asset pricing implications of linkages among firms. This paper expands this literature by exploring the asset pricing implications of linkages at the firm level and studying how changes in the propagation of shocks within a network affect aggregate variables as well as asset pricing, both in the aggregate and in the cross-section.

The rest of the paper is organized as follows: Section I explains the baseline model. Section II describes aggregate output and consumption growth within the baseline model. Section III derives expressions for the market return, the risk-free rate, the price of risk, firms' stock prices, and firms' quantity of risk in large network economies. Section IV uses data on customer-supplier networks in the U.S. as well as macroeconomic variables related to the propagation of shocks within these networks to calibrate the baseline model. Section V shows that changes in the propagation of shocks within large network economies are quantitatively important to understand variations in asset prices and returns, both in the aggregate and in the cross section. Section VI concludes. All proofs, unless otherwise stated, appear in the Appendix.

I. Baseline Model

A. *The Environment*

Consider an economy with one perishable good and an infinite time horizon. Time is discrete and indexed by $t \in \{0, 1, 2, \cdots\}$. The economy is populated by a large number of identical infinitely-lived individuals who are aggregated into a representative infinitely-lived investor with Epstein-Zin-Weil preferences who owns all assets in the economy. In each period, the single good is produced by n infinitely-lived Lucas (1978) trees, henceforth firms, with n being potentially large. In an otherwise standard dynamic endowment economy, firms' cash-flows are related via a network of inter-firm relationships. Interdependencies among firms' cash-flows can be conveniently described by a graph consisting of a set of nodes—which represent firms—together with lines or edges joining certain pairs of nodes—which represent inter-firm relationships. To fix notation, let $\mathcal{G}_n = (\mathcal{F}_n, \mathcal{R}_n)$ denote the network of inter-firm relationships among n firms, where $\mathcal{F}_n$ denotes the set of firms and $\mathcal{R}_n$ denotes the set of inter-firm relationships among them. Because I focus on the effect of $\mathcal{G}_n$ on asset prices rather than on strategic network formation, inter-firm relationships are exogenously determined and fixed before $t = 0$.[5]

B. *The network of inter-firm relationships $\mathcal{G}_n$ and firms' cash-flows*

Firms' cash-flows vary stochastically over time and depend in large part on the network of inter-firm relationships, $\mathcal{G}_n$. The following reduced form formulation of firms' cash-flows captures a simple trade-off in a parsimonious manner. The more relationships a firm is engaged in, the more benefits a firm receives and the higher its exposure to negative shocks that propagate through the network. Let $y_{i,t+1}$ denote firm i's cash-flow at $t+1$, and $Y_t \equiv \prod_{i=1}^{n} y_{i,t}^{1/n}$ denote the aggregate

[5] See Demange and Wooders (2005), Goyal (2007) and Jackson (2008) for a detailed description of network formation models.

output of the economy at t.[6] I assume that $y_{i,t+1}$ follows

$$\log\left(\frac{y_{i,t+1}}{Y_t}\right) \equiv \alpha_0 + \alpha_1 d_i - \alpha_2 \sqrt{n}\widetilde{\varepsilon}_{i,t+1} \ , \quad i \in \{1, \cdots, n\} \tag{1}$$

where parameters α_0, α_1 and α_2 are non-negative and equal across firms. Parameter d_i represents the number of relationships of firm i in $\mathcal{G}_n$, i.e. firm i's degree in $\mathcal{G}_n$. This parameter may differ across firms. The term $\sqrt{n}$ is included as a normalization factor in equation (1), which helps to characterize the equilibrium distribution of aggregate consumption growth later on. Uncertainty in $y_{i,t+1}$ is introduced by a Bernoulli random variable $\widetilde{\varepsilon}_{i,t+1}$, which equals one if firm i faces a negative shock at $t+1$ and zero otherwise. Given that

$$\log\left(\frac{y_{i,t+1}}{Y_t}\right) = \log\left(\frac{y_{i,t+1}}{y_{i,t}}\right) + \log\left(\frac{y_{i,t}}{Y_t}\right) \tag{2}$$

parameter α_2 in equation (1) measures the instantaneous decrease in a firm's cash-flow growth when a firm faces a negative shock, whereas parameter α_1 captures the benefits a firm receives from each relationship it engages in. Parameter α_0 in equation (1) captures the parts of firms' cash-flow growth that are unrelated to benefits or costs associated to inter-firm relationships.[7]

To complete the description of $y_{i,t+1}$, it is necessary to understand how inter-firm relationships affect the distribution of $\widetilde{\varepsilon}_{i,t+1}$ at $t+1$. Such a distribution is determined by the following random-network model. First, each firm faces a negative shock to its cash-flow growth, independently of other firms, with probability q at the very beginning of each period. A negative shock to firm i at $t+1$ propagates to firm j at $t+1$ if there exists a path of relationships in $\mathcal{G}_n$ that connects firms i and j in which each relationship in the path transmits shocks at $t+1$. A path is a sequence of

[6] The definition of Y_t implies that positive aggregate production requires positive production by each firm. To assume that $Y_t \equiv \prod_{i=1}^n y_{i,t}^{1/n}$ is similar to assuming that Y_t is proportional to $\sum_{i=1}^n y_{i,t}$ if n is sufficiently large and all $y_{i,t} \neq 0$. The argument follows from applying a first order Taylor series expansion to $\log(Y_t)$ in which aggregate output, $Y_t \equiv \sum_{i=1}^n y_{i,t}$. A different way of justifying that $Y_t \equiv \prod_{i=1}^n y_{i,t}^{1/n}$ is to consider that every firm produces a different perishable good and each good is necessary to produce other goods in the economy. In such an environment, one obtains asset pricing properties similar to the ones obtained in this paper if the representative investor has preferences over a Cobb-Douglas consumption aggregator of the form $C_t \equiv \prod_{i=1}^n c_{i,t}^{1/n}$, where $c_{i,t}$ represents consumption of the good produced by firm i at time t.

[7] Blume et al. (2013) analyze a similar trade-off in a static environment. They focus, however, on the strategic network formation features of economies in which agents receive benefits from the set of direct links they form, but these links expose them to the risk of being affected by cascades of failures. They provide asymptotic bounds on the welfare of both optimal and stable networks and show that very small amounts of "over-linking" may impose large losses in welfare to networks' participants.

inter-firm relationships that connects a sequence of firms that are each distinct from one another. Each relationship transmits shocks, independently of all other relationships, with probability $\widetilde{p}_{t+1}$ at $t + 1$, which may vary over time.[8] I only allow negative shocks to propagate in a probabilistic manner throughout the network to focus on the propagation of shocks in the case of firms' distress. However, equation (1) can be modified so that positive and negative shocks propagate over the economy. The main results continue to hold as long as the decrease in firms' cash-flows due to negative shocks is larger than the increase in firms' cash-flow due to positive shocks.

The value of $\widetilde{p}_{t+1}$ captures the importance of restrictions on alternative sources of substitutable inputs for the average firm as well as the importance of relationship-specific investments made by the average firm at $t + 1$. The higher the value of $\widetilde{p}_{t+1}$, the more important relationships are on average, and the higher the likelihood that shocks are transmitted via relationships at $t + 1$. For example, in the context of supply chains, Barrot and Sauvagnat (2014) show that input's specificity, switching costs, and complementaries among firms' activities may allow negative shocks to individual firms to propagate and affect other firms in a production chain. The existence of switching costs may prevent firms from restructuring their production sufficiently fast when they need to replace a supplier who is under distress, so negative shocks tend to spread from one firm to another.

To sum up, equation (1) captures the potential consequences of some inter-firm relationships in a simple manner. Despite the fact that firms may use relationships to increase their growth opportunities via efficiency gains, these relationships may have additional consequences because they may also increase a firm's exposure to negative shocks that affect a broader set of firms in the economy. In fact, for a given set of parameters, it follows from equation (1) that firms' expected cash-flow growth rates initially increase in the number of relationships of a firm, but then fall since the benefits associated with relationships are eventually overcompensated by the increase in exposure to negative shocks. Despite the fact that equation (1) is a reduced form formulation, this feature

[8] This random-network model can be thought of as a variation of either a reliability network or a bond percolation model in each period. In a typical reliability network model, the edges of a given network are independently removed with some probability. Remaining edges are assumed to transmit a message. A message from node i to j is transmitted as long as there is at least one path from i to j after edges removal (see Colbourn (1987) for more details). Similarly, in a bond percolation model, edges of a given network are removed at random with some probability. Those edges that are not removed are assumed to percolate a liquid. The question in percolation is whether or not the liquid percolates from one node to another in the network—which is similar to the problem of transmitting a message in a reliability context. For more details see Grimmett (1989), Stauffer and Aharony (1994) and Newman (2010, Chapter 16.1).

of firms' expected cash-flow growth can also be obtained within an equilibrium context. Goyal and Moraga-González (2001) obtain the same feature in firms' profits within strategic environments where firms collaborate in a (regular) R&D network to decrease their production costs but they also compete with their collaborators within the same homogeneous good market.

Given the topology of $\mathcal{G}_n$, the joint distribution of the sequence $\{\widetilde{\varepsilon}_{i,t+1}\}_{i=1}^{n}$ at $t+1$ is determined by two parameters: q, $\widetilde{p}_{t+1}$. The marginal distribution of $\widetilde{\varepsilon}_{i,t+1}$ at $t+1$, conditional on $\widetilde{p}_{t+1}$, depends on q as well as the topology of $\mathcal{G}_n$ and the location of firm i in $\mathcal{G}_n$. In other words,

$$\mathbb{P}\left(\widetilde{\varepsilon}_{i,t+1} = 1 \middle| \widetilde{p}_{t+1}\right) \quad = \quad f\left(q,\ \text{topology of } \mathcal{G}_n, \text{location of firm } i \text{ in } \mathcal{G}_n\right) \tag{3}$$

where $\mathbb{P}\left(\widetilde{\varepsilon}_{i,t+1} = 0 \middle| \widetilde{p}_{t+1}\right) = 1 - \mathbb{P}\left(\widetilde{\varepsilon}_{i,t+1} = 1 \middle| \widetilde{p}_{t+1}\right)$, and $f(\cdot)$ is a mapping characterized by the random-network model described above—which may be hard to characterize in closed-form for general network topologies as n increases.

The above mapping has two important properties. First, in the absence of relationships, $\mathbb{P}\left(\widetilde{\varepsilon}_{i,t+1} = 1 \middle| \widetilde{p}_{t+1}\right) = \mathbb{P}\left(\widetilde{\varepsilon}_{i,t+1} = 1\right) = q$, $\forall\ i$ and $t+1$, so cash-flow growth rates are independent and identically distributed across firms over time. Second, if only one path of relationships exists between two firms, the longer the path, the smaller the correlation between their cash-flows growth rates. Thus, the more distant two firms are in a network in which there is at most one path between two firms, the less related their cash-flows are. Having this feature—which is sometimes called correlation decay, e.g. Gamarnik (2013)—helps a great deal to obtain numerical solutions of the model in large network economies relatively fast.

C. Changes in shock propagation within $\mathcal{G}_n$

Given a network $\mathcal{G}_n$, the correlation structure among firms' cash-flows depends, in large part, on the propensity of relationships to transmit shocks, $\widetilde{p}_t$. Sufficiently small values of $\widetilde{p}_t$ imply that shocks tend to remain locally confined and affect only negligible fractions of the economy, whereas sufficiently large values of $\widetilde{p}_t$ imply that shocks may affect a large fraction of the economy for some network topologies and, thus, alter the distribution of the pricing kernel.

To capture temporal changes in production technologies and complementaries among firms' activities, the propensity $\widetilde{p}_t$ is time-varying and follows a two state ergodic Markov process, taking on either the value p_L or p_H, with $0 \leq p_L < p_H < 1$. The transition probability matrix of $\widetilde{p}_t$, Ω_p, is defined by

$$\begin{aligned} \mathbb{P}(\widetilde{p}_{t+1} = p_H | \widetilde{p}_t = p_H) &= \psi(1-\phi) + \phi \ , \\ \mathbb{P}(\widetilde{p}_{t+1} = p_H | \widetilde{p}_t = p_L) &= \psi(1-\phi) \end{aligned} \tag{4}$$

where ψ is the unconditional probability that $\widetilde{p}_t = p_H$. Parameter ϕ, which measures the persistence in $\widetilde{p}_t$, satisfies $0 \leq \phi < 1$, so $\widetilde{p}_t$ is positively autocorrelated. If $\phi = 0$, then $\widetilde{p}_t$'s are i.i.d. over time. As ϕ tends to 1, $\widetilde{p}_t$'s become perfectly positively correlated over time.

II. Distribution of Consumption Growth

Two components of the model are important to understand equilibrium asset prices: (a) the topology of the network $\mathcal{G}_n$, and (b) the propensity of relationships to transmit shocks. Before I discuss the cross-sectional asset pricing properties that stem from the propagation of shocks within $\mathcal{G}_n$, I study how changes in these two components affect the distribution of aggregate consumption growth and, thus, alter the distribution of the pricing kernel. Let $\Delta\widetilde{c}_{t+1} \equiv \log\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)$ and $\widetilde{x}_{t+1} \equiv \log\left(\frac{Y_{t+1}}{Y_t}\right)$ be the log consumption and output growth at $t+1$, respectively. In equilibrium, $\Delta\widetilde{c}_{t+1} = \widetilde{x}_{t+1}$. From the definition of aggregate output and equation (1) it follows that,

$$\begin{aligned} \Delta\widetilde{c}_{t+1} = \widetilde{x}_{t+1} &= \log\left(\prod_{i=1}^{n}\left(\frac{y_{i,t+1}}{Y_t}\right)^{1/n}\right) \\ &= \sum_{i=1}^{n}\frac{1}{n}\log\left(\frac{y_{i,t+1}}{Y_t}\right) \\ &= \alpha_0 + \alpha_1 \underbrace{\left(\frac{1}{n}\sum_{i=1}^{n} d_i\right)}_{\bar{d}} - \alpha_2\sqrt{n}\underbrace{\left(\frac{1}{n}\sum_{i=1}^{n}\widetilde{\varepsilon}_{i,t+1}\right)}_{\widetilde{W}_{n,t+1}} \\ &= \alpha_0 + \alpha_1 \bar{d} - \alpha_2\sqrt{n}\widetilde{W}_{n,t+1} \ , \end{aligned} \tag{5}$$

where $\bar{d}$ denotes the average number of relationships per firm in the economy, whereas $\widetilde{W}_{n,t+1}$ denotes the average number of firms affected by negative shocks at $t+1$. It follows from equation (5) that the distribution of $\Delta\widetilde{c}_{t+1}$ is determined by the distribution of $\sqrt{n}\widetilde{W}_{n,t+1}$. Because the distribution of $\sqrt{n}\widetilde{W}_{n,t+1}$ is affected by $\widetilde{p}_{t+1}$ and the topology of $\mathcal{G}_n$, these two components also affect the distribution of $\Delta\widetilde{c}_{t+1}$.

To appreciate the importance of $\widetilde{p}_{t+1}$ and the topology of $\mathcal{G}_n$ in shaping the distribution of $\Delta\widetilde{c}_{t+1}$, consider the simple case in which there are no relationships. In this case, $\{\widetilde{\varepsilon}_{i,t+1}\}_{i=1}^n$ is a sequence of i.i.d. Bernoulli random variables, so $n\widetilde{W}_{n,t+1}$ follows a Binomial distribution. By the Lindeberg-Lévy Central Limit Theorem, $\sqrt{n}\widetilde{W}_{n,t+1}$ is normally distributed as n grows large. Provided the absence of relationships, the realization of $\widetilde{p}_{t+1}$ is irrelevant to determining the distribution of $\Delta\widetilde{c}_{t+1}$. From equation (5) it follows that the unconditional mean and variance of $\Delta\widetilde{c}_{t+1}$ are $(\alpha_0 - \alpha_2 q)$ and $q(1-q)\alpha_2^2$, respectively.

In the presence of relationships, however, $\widetilde{p}_{t+1}$ and the topology of $\mathcal{G}_n$ affect the distribution of consumption growth in two important ways. First, all moments of the distribution of $\Delta\widetilde{c}_{t+1}$ at $t+1$ potentially depend on the realization of $\widetilde{p}_{t+1}$ and the topology of $\mathcal{G}_n$. Second, the sequence $\{\widetilde{\varepsilon}_{i,t+1}\}_{i=1}^n$ at $t+1$ is a sequence of dependent random variables, so the conditions under which a Central Limit Theorem (CLT) holds may not be satisfied. In fact, relationships may generate convoluted interdependencies among firms' cash-flows, which it makes difficult to characterize the distribution of $\Delta\widetilde{c}_{t+1}$ for general network topologies.

In general, there is no guarantee that $\Delta\widetilde{c}_{t+1}$ is normally distributed, despite the fact that $\Delta\widetilde{c}_{t+1}$ comes from aggregating shocks to individual firms, as in Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015). In fact, for a large variety of network topologies, simulation shows that the distribution of $\Delta\widetilde{c}_{t+1}$ may differ from a normal distribution. In particular, if $\widetilde{p}_{t+1}$ is sufficiently close to 1 and $\mathcal{G}_n$ is locally connected—i.e., there is at least one path between any two firms in an arbitrarily large neighborhood around any given firm—then a non-negligible fraction of the economy is almost surely affected by shocks to individual firms that propagate over the economy. Therefore, the distribution of $\Delta\widetilde{c}_{t+1}$ may exhibit thicker tails than a normal distribution would. Figure 1 illustrates the previous point. Figure 1(a) depicts an economy with $n = 5$ firms, whereas figure 1(b) depicts the empirical probability density function of $\sqrt{n}\widetilde{W}_{n,t+1}$ for different values of $\widetilde{p}_{t+1}$. As figure 1(b) shows, the distribution of $\sqrt{n}\widetilde{W}_{n,t+1}$ may differ from a normal distribution for large values of $\widetilde{p}_{t+1}$. In particular, as $\widetilde{p}_{t+1}$ tends to one, the distribution of $\sqrt{n}\widetilde{W}_{n,t+1}$ tends to be bimodal.

Despite the existence of relationships and the convoluted dependencies they may generate among firms' cash-flows, the topology of $\mathcal{G}_n$ and $\widetilde{p}_{t+1}$ can be restricted so that $\Delta\widetilde{c}_{t+1}$ is normally distributed as n grows large. In such a case, keeping track of temporal changes of the whole distribution of $\Delta\widetilde{c}_{t+1}$ is equivalent to keeping track of temporal changes in only averages and standard deviations. In particular, if shocks tend to remain locally confined—i.e., shocks only propagate over fractions of the economy that become negligible as n grows large—the sequence $\{\widetilde{\varepsilon}_{i,t+1}\}_{i=1}^{n}$ at $t+1$ becomes a sequence of weakly dependent random variables to which a CLT can be applied. Then, the dynamics of consumption growth can be recast as a version of Hamilton (1989)'s Markov-switching model.

To fix the notation, let $\mathcal{G}_{n+1}$ denote the network $\mathcal{G}_n$, to which I add one new firm and all the relationships the new firm may have with existing firms within $\mathcal{G}_n$. The following proposition imposes sufficient conditions on: (a) the limiting topology of the sequence of networks $\{\mathcal{G}_n\}_{n=1}^{\infty}$, $\mathcal{G}_\infty \equiv \lim_{n\to\infty} \mathcal{G}_n$, and (b) the propensity of relationships to transmit shocks, $\widetilde{p}_{t+1}$, so that $\Delta\widetilde{c}_{t+1}$ is normally distributed as n grows large.

PROPOSITION 1 (Asymptotic Normality of $\Delta\widetilde{c}_{t+1}$): *Given $q > 0$ and a sequence of networks of inter-firm relationships, $\{\mathcal{G}_n\}_{n\geq 1}$, with limiting topology $\mathcal{G}_\infty$, define p_c as*

$$p_c(\mathcal{G}_\infty) \;=\; \sup_{p\in(0,1)} \left\{p : \lim_{n\to\infty} P_q(n) = 0\right\} \tag{6}$$

where $P_q(n)$ denotes the probability that a shock to any given firm within $\mathcal{G}_n$ also affects αn firms via shock propagation, with $\alpha > 0$. If $\widetilde{p}_{t+1} < p_c(\mathcal{G}_\infty)$, then $\sqrt{n}\widetilde{W}_{n,t+1}$ and $\Delta\widetilde{c}_{t+1}$ are normally distributed at $t+1$ as n grows large.

Let $\mu_{c,t+1}$ and $\sigma_{c,t+1}$ denote the mean and volatility of $\Delta\widetilde{c}_{t+1}$, conditional on knowing $\widetilde{p}_{t+1}$ at $t+1$. Under the conditions of proposition 1, the distribution of $\Delta\widetilde{c}_{t+1}$ can be characterized in terms of the pair $(\mu_{c,t+1}, \sigma_{c,t+1})$. Since the network is fixed, the dynamics of $(\mu_{c,t+1}, \sigma_{c,t+1})$ is fully determined by the dynamics of $\widetilde{p}_{t+1}$. Thus, the economy follows a Markov process with a continuum of values for aggregate consumption and its growth rate, $\Delta\widetilde{c}_{t+1}$, but only two values for the first two moments of the distribution of $\Delta\widetilde{c}_{t+1}$, as in Kandel and Stambaugh (1991).

The following corollaries provide a more detailed characterization of those large network economies in which $\Delta\widetilde{c}_{t+1}$ is normally distributed. In particular, they report the limiting topology of the sequence of networks $\{\mathcal{G}_n\}_{n=1}^{\infty}$, $\mathcal{G}_\infty$, and the value of the critical probability p_c. Corollary 1 focuses on large networks in which all firms have the same number of relationships.

COROLLARY 1 (Symmetric Networks): *Given a sequence of networks of inter-firm relationships, $\{\mathcal{G}_n\}_{n\geq 1}$, with limiting topology $\mathcal{G}_\infty$,*[9]

- $p_c = 1 - 2\sin\left(\frac{\pi}{18}\right) \approx 0.65$ *if $\mathcal{G}_\infty$ is the two dimensional honeycomb lattice.*
- $p_c = \frac{1}{2}$ *if $\mathcal{G}_\infty$ is the two dimensional square lattice.*
- $p_c = 2\sin\left(\frac{\pi}{18}\right) \approx 0.34$ *if $\mathcal{G}_\infty$ is the two dimensional triangular lattice.*
- $p_c = \frac{1}{z-1}$ *if $\mathcal{G}_\infty$ is the Bethe lattice with z neighbors per each firm.*

Figure 2 illustrates each of the network economies considered in corollary 1. Corollary 2 focuses on large networks in which the number of relationships may differ across firms—whose topologies more closely resemble real economies.

COROLLARY 2 (Asymmetric Networks): *Given a sequence of networks of inter-firm relationships, $\{\mathcal{G}_n\}_{n\geq 1}$,*

- $p_c = \frac{1}{\textit{branching number of } \mathcal{G}_\infty}$ *if $\mathcal{G}_\infty$ is a tree. The branching number of a tree is the average number of relationships per firm in a tree.*[10]
- $p_c = \frac{1}{e_M}$ *if $\mathcal{G}_n$ is sparse and locally treelike. $\mathcal{G}_n$ is said to be sparse if the number of relationships in $\mathcal{G}_n$ increases linearly with n, as n increases. $\mathcal{G}_n$ is said to be locally treelike if an arbitrarily large neighborhood around any given firm takes the form of a tree. Parameter e_M is the leading eigenvalue of the matrix*

$$M_n = \begin{pmatrix} A_n & \mathbb{I}_n - D_n \\ \mathbb{I}_n & 0 \end{pmatrix} \tag{7}$$

where A_n is the adjacency matrix of $\mathcal{G}_n$, i.e. the $n \times n$ matrix in which $A_{ij} = 1$ if there is a relationship between firms i and j and zero otherwise. $\mathbb{I}_n$ is the $n \times n$ identity matrix, and D_n is the diagonal matrix that contains the number of relationships per firm along the diagonal.

[9] A lattice is a graph whose drawing can be embedded in $\mathbb{R}^n$. The two dimensional honeycomb lattice is a graph in 2D that resembles a honeycomb. The two dimensional square lattice is a graph that resembles the $\mathbb{Z}^2$ grid. The two dimensional triangular lattice is a graph in 2D in which each node has 6 neighbors.

[10] A tree is a network in which any two firms are connected by exactly one path. A forest is a network whose components are trees.

III. Equilibrium Asset Prices

To see what the network $\mathcal{G}_n$ and $\widetilde{p}_{t+1}$ imply for equilibrium asset prices, both in the aggregate and in the cross-section, I embed the cash-flows correlation structure that is endogenously generated by the network in a standard asset pricing model. The representative investor has Epstein-Zin-Weil recursive preferences to account for asset pricing phenomena that are challenging to address with power utility preferences. The asset pricing restrictions on the gross return of firm i, $\widetilde{R}_{i,t+1}$, are

$$\mathbb{E}_t\left(\widetilde{M}_{t+1}\widetilde{R}_{i,t+1}\right) \;=\; 1 \tag{8}$$

where $\widetilde{M}_{t+1} \equiv \left[\beta\left(e^{\Delta\widetilde{c}_{t+1}}\right)^{-\rho}\right]^{\frac{1-\gamma}{1-\rho}}\left[\widetilde{R}_{a,t+1}\right]^{\frac{1-\gamma}{1-\rho}-1}$ represents the pricing kernel at $t+1$ and $\widetilde{R}_{a,t+1}$ is the gross return on aggregate wealth—an asset that delivers aggregate consumption as its dividend each period. Parameter $\rho > 0$, $\rho \neq 1$, represents the inverse of the inter-temporal elasticity of substitution, $\gamma > 0$ is the coefficient of relative risk aversion for static gambles, and $\beta > 0$ measures the subjective discount factor under certainty.[11]

To solve the model, I look for equilibrium asset prices so that price-dividend ratios are stationary, as in Mehra and Prescott (1985), Weil (1989), and Kandel and Stambaugh (1991), among many others. Because equilibrium values are time invariant functions of the state of the economy—which is determined by the state of the propensity of relationships to transmit shocks—the index t can be eliminated. Hereinafter, c denotes the current level of aggregate consumption, y denotes the current level of aggregate output, and s denotes the current state of the propensity of relationships to transmit shocks.

I first solve for the price of aggregate wealth and the risk-free rate. These expressions are then used to solve for equilibrium asset prices and expected excess returns in the cross-section. The conditions under which proposition 1 and corollaries 1 or 2 hold are not needed to be satisfied in what follows. If those conditions are satisfied, however, the conditional expectations that appear in the following propositions can be computed in closed-form. Otherwise, I use simulation to compute those conditional expectations.

[11] If $\gamma = \rho$, these recursive preferences collapse to the standard case of VNM time-additive expected utility. The functional form of the Euler equation when $\rho = 1$ is different from the one shown in equation (8). See Weil (1989, Appendix A) for details. I use the standard terminology to describe γ and ρ. However, Garcia, Renault, and Semenov (2006) and Hansen et al. (2007) indicate that this interpretation may not be correct if $\rho \neq \gamma$.

The following proposition determines the current price of aggregate wealth.

PROPOSITION 2 (Price of Aggregate Wealth): *Let $P_a(c,s)$ denote the current price of aggregate wealth. $P_a(c,s) = w_s^a c$, where w_s^a is the solution of the following non-linear system of equations,*

$$w_s^a \;=\; \beta\left(\sum_{s'\in\{H,L\}}\omega_{s,s'}\mathbb{E}\left(e^{(1-\gamma)\Delta\widetilde{c}_{t+1}}\middle|p_{s'}\right)\left(w_{s'}^a+1\right)^{\frac{1-\gamma}{1-\rho}}\right)^{\frac{1-\rho}{1-\gamma}}, \qquad s=\{H,L\} \tag{9}$$

where $\mathbb{E}\left(\cdot\middle|p_{s'}\right)$ denotes the conditional expectation operator if the propensity of relationships to transmit shocks during the next period is $p_{s'}$, and $\omega_{s,s'}$ represents the (s,s') element of Ω_p.

I restrict my analysis to the set of model primitives in which the existence of a non negative solution of (9) is ensured.[12] The expected period gross return of aggregate wealth in the current state is then

$$\mathbb{E}\left(R_a|s\right) \;=\; \sum_{s'\in\{H,L\}}\omega_{s,s'}\frac{w_{s'}^a+1}{w_s^a}\mathbb{E}\left(e^{\Delta\widetilde{c}_{t+1}}\middle|p_{s'}\right), \qquad s=\{H,L\}\,. \tag{10}$$

It follows from equations (9) and (10) that the price and expected period return of aggregate wealth are driven by the dynamics of $\widetilde{p}_t$. In particular, temporal changes in $\widetilde{p}_t$ convey temporal changes in the distribution of aggregate consumption growth, which, in turn, manifest in the price and the expected period return of aggregate wealth. The dynamics of $\widetilde{p}_t$, parameterized by ψ and ϕ, also impact the price and the expected period return of aggregate wealth via $\omega_{s,s'}$, because these two parameters determine: (a) how frequently the economy is in a state in which relationships transmit shocks more often, and (b) how frequently changes in the propensity of relationships to transmit shocks occur.

I next consider the risk-free asset, which pays one unit of the consumption good during the next period with certainty.

[12] Provided that $e^{\Delta\widetilde{c}_t}$ is positive for all t, parameters ρ and γ need to be restricted so that the function $h(\cdot)$ defined as

$$h\left(w_i^a\right) \;\equiv\; \beta\left(\sum_{j\in\{H,L\}}\omega_{i,j}\mathbb{E}\left(e^{(1-\gamma)\Delta\widetilde{c}_{t+1}}\middle|p_j\right)\left(w_j^a+1\right)^{\frac{1-\gamma}{1-\rho}}\right)^{\frac{1-\rho}{1-\gamma}}$$

is continuous. If $h(\cdot)$ is continuous, the system of equations (9) has a solution by Brouwer's Fixed Point Theorem. Further restrictions in the set of parameter values can be imposed such that the solution of the system of equations is unique.

PROPOSITION 3 (Risk-free Rate): *Let $R_f(s)$ denote the period gross return of the risk-free asset in the current state. $R_f(s)$ solves*

$$\frac{1}{R_f(s)} = \beta^{\frac{1-\gamma}{1-\rho}} \left(\sum_{s' \in \{H,L\}} \omega_{s,s'} \mathbb{E}\left(e^{-\gamma \Delta \widetilde{c}_{t+1}} \middle| p_{s'} \right) \left(\frac{w^a_{s'}+1}{w^a_s} \right)^{\frac{\rho-\gamma}{1-\rho}} \right) \qquad s = \{H, L\} \tag{11}$$

where w^a_s are the solutions of the system of equations (9).

It follows from equation (11) that the equilibrium risk-free rate is also driven by the dynamics of $\widetilde{p}_t$, because changes in $\widetilde{p}_t$ drive changes in the distribution of consumption growth and prices of aggregate wealth.

Using the previous expressions, I now study what the network $\mathcal{G}_n$ and $\widetilde{p}_t$ imply for the cross-section of asset prices and risk premia. The following proposition determines the (ex-dividend) stock price of firm i and its expected period return.

PROPOSITION 4 (Firms' Stock Prices and Expected Period Returns): *Let $P_i(y,s)$ denote the current (ex-dividend) stock price of an asset that delivers firms i's cash-flows as its dividend each period. For large n, $P_i(y,s) = v_i(s)y$, where $v_i(s)$ is the solution of the following linear system of equations*

$$\begin{aligned} v_i(s) &= \beta^{\frac{1-\gamma}{1-\rho}} e^{\bar{x}+\frac{\sigma_x^2}{2}} \left(\sum_{s' \in \{H,L\}} \omega_{s,s'} \left(\frac{w^a_{s'}+1}{w^a_s} \right)^{\frac{\rho-\gamma}{1-\rho}} \mathbb{E}\left(e^{(\tau-\gamma)\Delta \widetilde{c}_{t+1}} \middle| p_{s'} \right) v_i(s') \right) \\ &+ \beta^{\frac{1-\gamma}{1-\rho}} e^{\alpha_0+\alpha_1 d_i} \left(\sum_{s' \in \{H,L\}} \omega_{s,s'} \left(\frac{w^a_{s'}+1}{w^a_s} \right)^{\frac{\rho-\gamma}{1-\rho}} \mathbb{E}\left(e^{-\gamma \Delta \widetilde{c}_{t+1}} \middle| p_{s'} \right) [1-\pi_i(p_{s'})] \right) \end{aligned} \tag{12}$$

where $\pi_i(p_{s'}) \equiv \mathbb{E}\left(\widetilde{\varepsilon}_{i,t+1} \middle| \widetilde{p}_{t+1} = p_{s'} \right)$ and $s = \{H, L\}$. Moreover, the expected one period gross return of firm i is given by

$$\mathbb{E}\left(\widetilde{R}_{i,t+1} \middle| s \right) = \frac{1}{v_i(s)} \left(\sum_{s' \in \{H,L\}} \omega_{s,s'} \left\{ v_i(s') \mathbb{E}\left(e^{\widetilde{x}_{t+1}} \middle| p_{s'} \right) + e^{\alpha_0+\alpha_1 d_i} \left(1-\pi_i(p_{s'}) \right) \right\} \right). \tag{13}$$

To appreciate the importance of a firm's location in $\mathcal{G}_n$ in asset prices and returns, suppose $\mathcal{G}_n$ is symmetric. Then, $d_i = \bar{d}$ and $\pi_i = \bar{\pi} \geq q$ for all i. It then follows from the second term in the right hand side of (12) that all firms have the same price in a given period. As equation (12) shows, differences in prices across firms arise solely from differences in firms' locations in $\mathcal{G}_n$. Differences in prices across firms are driven not only by the number of relationships of a firm, captured by d_i, but also by the set of firms to which a firm is connected, captured by π_i. The same applies for the cross-section of expected excess returns. Differences in expected excess returns across firms arise solely from differences across firms' locations in $\mathcal{G}_n$. To understand the cross-section of firms' risk premia, equation (8) can be rewritten as a beta pricing model,

$$\mathbb{E}\left(\widetilde{R}_{i,t+1}\middle|s\right) - R_f(s) \quad = \quad \underbrace{\left(\frac{\operatorname{Cov}\left(\widetilde{R}_{i,t+1}, \widetilde{M}_{t+1}\middle|s\right)}{\operatorname{Var}\left(\widetilde{M}_{t+1}\middle|s\right)}\right)}_{\beta_{i,\widetilde{M}}(s)} \underbrace{\left(\frac{-\operatorname{Var}\left(\widetilde{M}_{t+1}\middle|s\right)}{\mathbb{E}\left(\widetilde{M}_{t+1}\middle|s\right)}\right)}_{\lambda_{\widetilde{M}}(s)} \tag{14}$$

where $\beta_{i,\widetilde{M}}(s)$ and $\lambda_{\widetilde{M}}(s)$ denote the quantity of risk in firm i and the price of risk in state s, respectively. The following proposition determines $\lambda_{\widetilde{M}}(s)$.

PROPOSITION 5 (Conditional Price of Risk: $\lambda_{\widetilde{M}}(s)$): *The conditional price of risk in state s, $\lambda_{\widetilde{M}}(s)$, equals*

$$\lambda_{\widetilde{M}}(s) \quad = \quad \frac{1}{R_f(s)} - R_f(s)\left(\beta^{2\left(\frac{1-\gamma}{1-\rho}\right)} \sum_{s' \in \{H,L\}} \omega_{s,s'} \left(\frac{w^a_{s'}+1}{w^a_s}\right)^{2\left(\frac{\rho-\gamma}{1-\rho}\right)} \mathbb{E}\left(e^{-2\gamma\Delta\widetilde{c}_{t+1}}\middle|p_{s'}\right)\right) \tag{15}$$

where $R_f(s)$ denotes the period gross return of the risk-free asset in state s.

As equation (15) shows, the price of risk is time-varying, because the propensity of relationships to transmit shocks varies over time. Changes in $\widetilde{p}_t$ introduce changes in the distribution of aggregate consumption growth, in the price of aggregate wealth, and in the risk-free rate, which, in turn, manifest in changes of the price of risk. To compute firms' quantities of risk, one can rearrange equation (14), which yields

$$\beta_{i,\widetilde{M}}(s) = \frac{\mathbb{E}\left(\widetilde{R}_{i,t+1}\middle|s\right) - R_f(s)}{\lambda_{\widetilde{M}}(s)} \tag{16}$$

so that firms' conditional quantities of risk can be computed from equations (11), (13), and (15). As a consequence, firm i's quantity of risk is driven by (a) firm i's location in $\mathcal{G}_n$, which alters

$\mathbb{E}\left(\widetilde{R}_{i,t+1}|s\right)$, and (b) the dynamics of $\widetilde{p}_t$ and topology of $\mathcal{G}_n$, which alter $R_f(s)$, $\lambda_{\widetilde{M}}(s)$ and $\mathbb{E}\left(\widetilde{R}_{i,t+1}|s\right)$.[13]

IV. Calibration

So far, the model illustrates how the propagation of shocks within a network economy alters equilibrium asset prices. I now calibrate the model to get a sense of the extent to which such a propagation mechanism quantitatively explains asset market phenomena. Section IV.A describes the data and the strategy I use to calibrate the network $\mathcal{G}_n$. Section IV.B describes the selection of the rest of parameters in the model.

A. Description of Data and Customer-Supplier Networks

I use annual data on customer-supplier relationships among U.S. firms to pin down the topology of $\mathcal{G}_n$. Statement of Financial Accounting Standards (SFAS) No.131 requires firms to report the existence of customers who represent more than 10% of their annual sales. This information is available on COMPUSTAT files. However, these files tend to list only abbreviations of customers' names. I then resort to the Cohen and Frazzini (2008) dataset on customer-supplier relationships—a subset of the COMPUSTAT database—in which firms' principal customers are uniquely identified.[14] Their dataset consists of 6,425 different firms, considers common stocks, and represents 26,781 unique annual customer-supplier relationships from 1980 to 2005. Customer-supplier relationships last about 3 years on average, and the distribution of firms' size resembles the size distribution of the CRSP universe over the sample period. The size distribution of firms' principal customers, however, is tilted toward large companies. The average customer size is above the 90th size percentile of CRSP firms.

To proxy for those relationships that relate firms' cash-flow growth rates, I consider customer-supplier relationships in which a customer represents at least 20% of a firm's annual sales. My results, however, do not qualitatively change if I decrease that threshold from 20% to 10%. I construct undirected and non-weighted customer-supplier networks at the annual frequency over the sample period. Then, two firms are connected in a given year if one firm represents at least 20% of

[13]In an unreported proposition I also compute firms' quantities of risk as a function of the primitives of the model.
[14]Data available at: http://www.econ.yale.edu/∼ af227/

another firm's sales during that year. Figures 3(a) and 3(b) depict the customer-supplier networks in 1980 and 1986 respectively, in which nodes represent firms and the size of each node is proportional to the number of customer-supplier relationships a firm takes part in. Table II illustrates some of the characteristics of the time series of customer-supplier networks. The average number of firms per network is 388, whereas, on average, there are 281 customer-supplier relationships per network. As in many economic and social networks, the number of relationships varies dramatically across firms.

To select the benchmark topology of $\mathcal{G}_n$, I generate a large network with $n = 400$ firms so that such a network simultaneously matches some of the characteristics of the time series of customer-supplier networks reported in Table II. In particular, the selected topology for $\mathcal{G}_n$ matches the average size of each of the five largest components and the average empirical degree distribution of customer-supplier networks. I restrict the topology of $\mathcal{G}_n$ to be one with no cycles so that firms' probabilities of facing negative shocks in each state of the economy—$\{\pi_i(p_s)\}_{i=1}^n$ with $p_s \in \{p_L, p_H\}$ in equations (12) and (13)—are easy to compute.[15] Such restriction seems to be innocuous, because cycles are not frequent in the customer-supplier dataset. Figure 4(a) depicts the topology of the benchmark economy, whereas figure 4(b) depicts its degree distribution.

Selecting the network topology using this data has one important caveat. Because many firms in the economy, as well as their relationships, are not included in this dataset, one may be able to construct, in the most favorable case, a network that closely resembles only a small fraction of the aggregate economy. This is because firms need to be sufficiently large to be publicly traded and to represent at least 20% of the annual sales of a publicly traded company. To partially ensure that the topology selected in the benchmark economy provides a fair representation of the network that underlies the aggregate U.S. economy, I compare the benchmark network with networks that are uncovered using BEA input-output tables. As table VI shows, the network in the benchmark economy does a good job at representing some features of the U.S. input-output network, and in doing so, potentially provides a reasonable representation of the aggregate U.S. economy.[16]

[15] A cycle consists of a sequence of firms starting and ending at the same firm, with each two consecutive firms in the sequence directly connected to each other in the network.

[16] It is an empirical issue whether a network uncovered using BEA input-output tables provides a sensible representation of the network structure that underlies the U.S. economy—I leave this for future research. Another way to uncover the underlying network using the framework in this paper is to use probabilistic graphical models, which are commonly used to represent statistical relationships in large and complex systems, since my baseline model predicts certain behavior of returns covariances across stocks. For instance, one may calibrate the network using a graphical

B. *Selecting the rest of parameter values*

Given the network topology uncovered in section IV.A, I calibrate the rest of the parameters in the model at the monthly frequency to be consistent with the empirical literature. Table III reports the key parameter values in the calibrated model.

For the sake of illustration, these parameters can be separated into four groups. Parameters in the first group define the preferences of the representative investor, which I select in line with Bansal and Yaron (2004). Thus, I set $\beta = 0.997$, $\gamma = 10$ and $\rho = 0.65$ (IES ≈ 1.5).

Parameters in the second group define the dynamics of firms' cash-flows. I use annual data on earnings per share from COMPUSTAT to proxy for firms' cash-flows. I restrict my focus to firms mentioned in the customer-supplier dataset, because the value of parameters $\{d_i\}_{i=1}^n$ in equation (1)—which correspond to the number of relationships of firms—is available only for those firms. To estimate parameters α_0, α_1 and α_2 in equation (1), I run cross-sectional OLS regressions at the annual frequency and then compute their equivalents at the monthly frequency. To run such cross-sectional regressions, I need to determine whether firm i faces a negative shock in a given year. To do so, I explore the temporal variation of firms' cash-flows and run time series regressions at the firm level, correcting for the existence of linear time trends.[17] By doing so, I identify the years in which each firm faces a negative shock. This allows me to compute annual estimates for α_0, α_1 and α_2 from 1980 to 2004, which are depicted in figure 5.[18] I then set parameters α_0, α_1 and α_2 to be equal to the time series average estimates. Thus, $\alpha_0 = 0.3$, $\alpha_1 = 0.1$ and $\alpha_2 = 0.07$.[19]

lasso estimator (GLASSO) to match observed returns covariances. In doing so, one estimates an undirected and temporally invariant network by estimating a sparse inverse covariance matrix using a lasso (L1) penalty as in Friedman, Hastie, and Tibshirani (2008). The basic estimation strategy assumes that observations have a multivariate Gaussian distribution with mean μ and covariance matrix Σ. If the ij^{th} component of Σ^{-1} is zero, then variables i and j are conditionally independent, given the rest of the variables, which is graphically represented as the lack of an edge between variables i and j in $\mathcal{G}_n$. The normality assumption can be relaxed as in Liu et al. (2012).

[17] Namely, I run the following time series regression at the firm level,

$$\log\left(\frac{y_{i,t}}{Y_{t-1}}\right) = \beta_0 + \beta_1 * t + \epsilon_t. \tag{17}$$

I consider that firm i faces a negative shock at year t if $\log\left(\frac{y_{i,t}}{Y_{t-1}}\right)$ is below the value predicted by regression (17) for more than one standard deviation of the residuals computed from (17).

[18] Estimates of α_0, α_1, and α_2 are statistically significant for most of the years within the sample. In particular, out of 25 years in the sample, α_0, α_1, and α_2 are statistically significant at the 95%, 12, 8, and 20 years, respectively.

[19] To determine the benchmark values of α_0, α_1, and α_2 at the monthly frequency, I assume that $y_{i,\text{year}} = 12 \times y_{i,\text{month}}$, with $i \in \{1, \cdots, n\}$. Provided that data on firms' cash-flows is at the annual frequency, this assumption facilitates the computation of parameters α_0, α_1, and α_2 at the monthly frequency because $Y_{\text{year}} = 12 \times Y_{\text{month}}$ so that $\log\left(\frac{y_{i,\text{year}+1}}{Y_{\text{year}}}\right) = \log\left(\frac{y_{i,\text{month}+1}}{Y_{\text{month}}}\right)$.

Parameters in the third group define the process followed by the propagation of shock within a network economy. The calibration of these parameters has only limited guidance from prior studies. There are five parameters in this group: (i) the coefficient that measures how frequently firms face negative idiosyncratic shocks, q; the values that $\widetilde{p}_t$ may take in each period, p_L and p_H; the coefficient that measures how frequently relationships exhibit high propensity to transmit shocks, ψ; and the coefficient that measures the persistence of the stochastic process followed by $\widetilde{p}_t$, ϕ. I choose the benchmark values in this group by either using available studies or matching important moments in data.[20]

To select parameter ϕ, I explore the time variation of macroeconomic variables that proxy for the degree of input specificity in the U.S., motivated by evidence in Barrot and Sauvagnat (2014). Barrot and Sauvagnat (2014) posit that input specificity is a key driver of the propagation of shocks within supply chains. Their idea is simple. The more specific the inputs a firm uses, the more difficult it is to restructure its production if it needs to replace a supplier who is under distress, and, thus, the more likely it is that such a firm is affected by shocks to its suppliers. It is, then, natural to think that the higher the degree of input specificity faced by the average firm in the economy, the higher the likelihood that negative shocks spread from one firm to another within a customer-supplier network. To proxy for the degree of input specificity faced by the average firm in the network economy, I use the ratio of non federally funded R&D/GDP and the number of patents created in the U.S. These two measures aim to proxy for (i) the relative importance of relationship-specific investments made by the average firm, and (ii) how easily the average firm can substitute suppliers who are under distress.[21] Figure 6 depicts the time series for R&D/GDP from 1953 to 2002 in the U.S., as well as the number of patents created in the U.S. from 1963 to 2009. I then set $\phi = 0.925$ so that the time series, followed by the propensity of relationships to transmit shocks, is as persistent as the time series of either R&D/GDP or the number of patents created in the U.S.[22] I select the rest of the parameters in this group by matching important moments in

[20] A similar strategy is used in Zhang (2005) to pin down parameters to which there is only limited guidance from prior studies.

[21] Barrot and Sauvagnat (2014) construct three measures of suppliers' specificity in their study. The first measure uses information that classifies inputs as differentiated or homogeneous, depending on whether they are sold on an organized exchange or not. The second measure uses suppliers' R&D expenses to capture the importance of relationship-specific investments, whereas the third measure uses the number of patents issued by suppliers to capture restrictions on alternative sources of substitutable inputs.

[22] To pin down the persistence of these time series, I fit autoregressive processes to the time series of non-federal R&D/GDP and the time series of the number of patents created in the U.S. by selecting the complexity of the model

data. In particular, parameters q, p_L, p_H, and ψ are chosen so that the first two moments of the time-aggregated annual growth rates of consumption and dividends generated by the calibrated model are similar to those of observed annual data. I then set $q = 0.2$, $p_L = 0.38$, $p_H = 0.45$, and $\psi = 0.5$.

Parameters in the fourth group define the difference between aggregate output and consumption growth. Within the baseline model, output growth equals consumption growth at equilibrium. To provide a more realistic description of dividends and improve the fit of the calibrated model to data, I augment the baseline model so that consumption and dividends are two different processes within the benchmark economy. Similar to many others, including Cecchetti, Lam, and Mark (1993), Abel (1999), Campbell (1999, 2003), and Bansal and Yaron (2004), I assume that dividend and consumption growth jointly satisfy (within the augmented model),

$$(\widetilde{x}_{t+1} - \bar{x}) = \tau \left(\Delta \widetilde{c}_{t+1} - \bar{c}\right) + \sigma_x \widetilde{\xi}_{t+1} \ , \tag{18}$$

where $\bar{x}$ and $\bar{c}$ are constant and represent the unconditional means of log output and consumption growth, respectively. Parameter $\tau > 0$ and $\widetilde{\xi}_{t+1}$ is i.i.d. normal with mean zero and unit variance. Thus, the representative investor is implicitly assumed to have access to labor income in the augmented model. For simplicity, $\widetilde{\xi}_{t+1}$ is independent of both $\Delta\widetilde{c}_{t+1}$ and variables $\{\widetilde{\varepsilon}_{i,t+1}\}_{i=1}^{n}$. As in Abel (1999), parameter τ represents the leverage ratio on equity. If $\bar{x} = \bar{c} = \sigma_x = 0$, then aggregate consumption and dividend growth are specified as in Abel (1999). If $\bar{x} = \bar{c} = \sigma_x = 0$ and $\tau = 1$, then the market portfolio is a claim to total wealth and I recover the baseline model. I set $\bar{c} = 0.019/12$ and $\bar{x} = 0.038/12$ so that the unconditional means of consumption and dividend growth generated by the benchmark economy are similar to the ones found in data. I follow Bansal and Yaron (2004) and set $\tau = 3$. I set $\sigma_x = 0.0262$ so that the volatility of dividends generated by the benchmark economy is similar to the one found in data.

Despite the fact that aggregate output and consumption are two different processes within the augmented model, both of these processes are still determined, in large part, by the propagation of shocks within the network. In particular, the distribution of $\widetilde{x}_{t+1}$ is fully determined by the

using the Akaike information criterion. The fitted AR models are both highly persistent. In particular, the fitted AR model for the number of patents in the U.S has a persistence parameter equal to 0.93, whereas the fitted AR model for R&D/GDP has a persistence parameter equal to 0.85.

propagation of shocks as equation (5) shows, whereas the distribution of $\Delta\widetilde{c}_{t+1}$ is also determined by the propagation of shocks as equation (18) states.

V. Implications of the Calibrated Model

This section studies the asset market implications of the calibrated model. It shows that changes in the propagation of shocks, within networks of inter-firm relationships that resemble real economies, are quantitatively important to understand variations in asset prices and returns, both in the aggregate and in the cross section.

A. *Asset Market Phenomena, Network Economies, and Long-Run Risks*

Table IV exhibits moments generated under the benchmark parameterization. Table IV suggests that the model does a reasonable job at matching important asset pricing moments as well as moments of consumption and dividend growth. The benchmark parameterization delivers an average annual log consumption growth of 1.8%, an annual volatility of log consumption growth of 4.7%, an average annual log dividend growth of 3.8%, and an annual volatility of log dividend growth of 14.9%, all values similar to those found in data. It also delivers an average market return of 12%, an annual volatility of the market return of 18.92%, an average risk-free rate of 2.16%, an annual volatility of the risk-free rate of 0.7%, an annual equity premium of 10%, and a Sharpe ratio of 0.52. With the exception of the volatility of the risk-free rate and Sharpe ratio, all values are aligned with those found in data.

Besides matching the above moments, the calibrated model generates a persistent component in expected consumption growth and stochastic consumption volatility similar to those assumed by the Long-Run Risks Model (LRR) of Bansal and Yaron (2004). As Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2012) show, these two features, together with Epstein-Zin-Weil preferences, help to quantitatively explain an array of important asset market phenomena.[23] Table V reports means and volatilities based on 300 simulated economies over 620 monthly observations

[23] Since Bansal and Yaron (2004), several authors have used the long-run risk framework to explain an array of market phenomena. For instance, Kiku (2006) provides an explanation of the value premium within the long-run risks framework. Drechsler and Yaron (2011) show that a calibrated long-run risks model generates a variance premium with time variation and return predictability that is consistent with data. Bansal and Shaliastovich (2013) develop a long-run risks model that accounts for bond return predictability and violations of uncovered interest parity in currency markets.

of several similarity measures between time series generated with either the calibrated model or the LRR model. As table V suggests, both models generate similar time series for expected consumption growth and stochastic consumption volatility.

It is important to appreciate that these two features are endogenously generated within my model rather than exogenously imposed, as in many asset pricing models. The calibrated model generates these two features because the propensity of relationships to transmit shocks follows a persistent process, that is consistent with data, and inter-firm relationships are long-term. Despite the fact that these two features are endogenously generated, I do not claim that my model provides a complete micro-foundation of long-run risks. The reason is that inter-firm relationships are exogenous and fixed within my model. Nonetheless, this model provides a novel link between equilibrium asset prices and the propagation of firm level shocks, within networks that resemble real economies, that is consistent with the existence of long-run risks. The model suggests that changes in technologies and complementaries among firms activities within network economies are quantitatively relevant to understand variations in asset prices and returns. This is particularly important in modern economies provided the high degree of interconnectedness among firms. In doing so, the model provides a new perspective on the potential sources of long-run risks. The framework presented in this paper is also able to nest long-run risk models under suitable assumptions as Appendix C demonstrates.

B. *Firms' Centrality and the Cross-Section of Risk Premia*

Besides helping to explain aggregate asset market phenomena, the model helps to understand the cross-section of expected returns because it provides a mapping between firms' quantities of priced risk and firms' importance in the network. To measure the importance of a firm in the inter-firm relationships network, I define the centrality of firm i at time t as the expected number of firms that can be affected by a shock to firm i at time t. This measure captures the relative importance of firm i in transmitting shocks over the economy. Shocks to firm i may alter aggregate output and consumption growth to the extent to which they propagate over a non-negligible fraction of the economy and, thus, alter firm i's risk premium. If the economy contains no cycles, the centrality of firm i at period t, $\chi_{i,t}$, equals to

$$\chi_{i,t} = \sum_{d=1}^{L_i} n_i^j \widetilde{p}_t^j \tag{19}$$

where n_i^j denotes the number of firms that are at a distance j from firm i in $\mathcal{G}_n$; and $\widetilde{p}_t$ denotes the realization of $\widetilde{p}_t$ at time t. Firms i and k are said to be at a distance j if the shortest path between i and k has length j. L_i denotes the largest distance between any given firm within $\mathcal{G}_n$ and firm i.[24]

Figure 7(a) shows firms' conditional risk premia as a function of firms' centrality. It follows from figure 7(a) that firms that are more central in the network command higher risk premium than firms that are less central. Figure 7(b) shows firms' conditional quantity of risk, $\beta_{i,\widetilde{M}}$, as a function of firms' centrality. It follows from figure 7(b) that firms that are more central have higher quantity of risk than firms that are less central. Shocks to central firms have higher likelihood of affecting more firms on average than do shocks to less central firms. As a consequence, central firms tend to be procyclical, whereas less central firms serve as a hedge against aggregate risk and, thus, command lower risk premium. On average, firms in the highest quintile of centrality yield an annual excess return of 1% over those firms in the lowest quintile—which is aligned with the 3% excess return documented by Ahern (2013) within the network of intersectoral trade.[25]

[24] If the network contains no cycles, the probability that k firms that are at a distance j from firm i are also affected by shocks to firm i at period t, $\mathbb{P}_i^j(k)$, is given by

$$\mathbb{P}_i^j(k) \quad = \quad \binom{n_i^j}{k} \left(\widetilde{p}_t^j\right)^k \left(1 - \widetilde{p}_t^j\right)^{n_i^j - k}$$

The expected number of firms that are at a distance j from firm i and are also affected by a shock to firm i at period t is $n_i^j \widetilde{p}_t^j$. As a consequence, the expected number of firms that can be affected by shocks to firm i is given by (19). If there is no path between firm i and other firms within $\mathcal{G}_n$, define $L_i = \infty$.

[25] The 1% excess return comes from 200 simulated economies over 1100 monthly observations. I disregard the first 100 observations in each simulation to eliminate the potential bias coming from the initial condition. At the beginning of each year, I sort firms into five quintiles based on centrality and form five equally weighted portfolios, which I keep over the next twelve months. The 1% excess return corresponds to the average annual return of a strategy that goes long in the portfolio with those firms with the highest centrality and short in the portfolio with those firms with the lowest centrality. Despite that Ahern (2013) uses a different network to compute his results, Table VI shows that the network topologies used by Ahern (2013) are similar to the network topology used in this paper.

C. *Factor Structure on Firm-Level Return Volatility*

The calibrated model also generates a high degree of common time variation in return volatilities at the firm level, which is aligned with recent empirical evidence, e.g. Herskovic et al. (2014), Duarte et al. (2014). To facilitate comparison with evidence documented by Herskovic et al. (2014), figure 8 illustrates annual total return volatility at the firm level averaged within start-of-year size quintiles. As figure 8 shows, firms of all size exhibit similar time series volatility patterns. On average, the first principal component of the cross-section of annual return volatility accounts for 99% of the variance. Within the model, the existence of this factor structure is not surprising, because fluctuations in the propensity of relationships to transmit shocks drive changes in growth opportunities and uncertainty across firms, which translate into changes in prices and returns at the firm level. Provided that returns respond to a common factor—given by the propensity $\widetilde{p}_t$—firm level return volatilities inherit a factor structure.[26]

VI. Conclusion

This paper suggests that the propagation of firm level shocks within network economies are quantitatively important to understanding asset prices and returns, both in the aggregate and in the cross-section. Changes in either the network that underlies the aggregate economy or the propensity of relationships to transmit shocks within a fixed network may alter aggregate variables, such as output and consumption, which, in turn, alter equilibrium asset prices and returns.

I show that a calibrated model that matches features of customer-supplier networks in the U.S. as well as features of macroeconomic variables that aim to proxy for the propagation of firm level shocks within these networks, generates a persistent component in expected consumption growth and stochastic consumption volatility similar to that of the Long-Run Risk Model of Bansal and Yaron (2004). As Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2012) show, these two features, together with Epstein-Zin-Weil preferences, help to explain characteristics of aggregate asset market

[26] Recent empirical evidence also suggests the existence of common time variation in firm level idiosyncratic volatilities, e.g. Herskovic et al. (2014), Duarte et al. (2014). In unreported results, I explore the extent to which firm level idiosyncratic volatilities exhibit a factor structure within the calibrated model. After removing the market as a common factor of return volatilities, the high degree of common time variation in firm level return volatilities tends to disappear. On average, the first principal component of the cross-section of annual idiosyncratic volatility accounts only for 3% of the variance (see figure 8(b)).

data such as the equity premium and low risk-free rate. The model also helps in understanding the cross-section of expected returns, as it provides a mapping between firms' quantities of priced risk and firms' importance in the network. In the cross section, firms that are more central in the network command higher risk premium than firms that are less central. Shocks to central firms have higher likelihood of affecting more firms on average than do shocks to less central firms. As a consequence, central firms tend to be procyclical, whereas less central firms serve as a hedge against aggregate risk and, thus, command lower risk premium. In the time series, firm-level return volatilities exhibit a high degree of comovement. These two features are consistent with recent empirical evidence.

Future research in this area is needed. The data currently available allows us to uncover only a partial representation of the network that underlies the aggregate U.S. economy. More detailed data may improve the calibration of the model as well as the precision of its estimates. In addition, more empirical evidence along the lines of Barrot and Sauvagnat (2014) may help to uncover a better representation of the process that determines the propagation of shock via inter-firm relationships. Finally, several questions remain open and should be studied further. For instance, how does the multiplicity of relationships among firms quantitatively affect asset prices? To what extent does endogenous network formation affect asset prices? I believe that the framework developed in this paper may be of great use in starting to answer these challenging questions.

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Appendix A: Proofs

This section contains the proofs of propositions and corollaries in the paper. The following computations consider two assumptions:

ASSUMPTION 1: *Firm i's output at time* $t+1$, $y_{i,t+1}$, *follows*

$$\log\left(\frac{y_{i,t+1}}{Y_t}\right) \quad \equiv \quad \alpha_0 + \alpha_1 d_i - \alpha_2\sqrt{n}\widetilde{\varepsilon}_{i,t+1} \tag{1}$$

where $\widetilde{\varepsilon}_{i,t+1}$ *denotes a Bernoulli random variable which equals one if firm i faces a negative shock at* $t+1$ *and zero otherwise. For a given* $\mathcal{G}_n$, *parameter* d_i *denotes firm i's degree. Parameters* α_0, α_1 *and* α_2 *are non-negative real numbers.*

ASSUMPTION 2: *Let* $\widetilde{x}_{t+1} \equiv \log\left(\frac{Y_{t+1}}{Y_t}\right)$ *be the log output growth rate of the economy at time* $t+1$, *and let* $\Delta\widetilde{c}_{t+1} \equiv \log\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)$, *be the log aggregate consumption growth rate. The processes for* $\widetilde{x}_{t+1}$ *and* $\Delta\widetilde{c}_{t+1}$ *satisfy*

$$\widetilde{x}_{t+1} - x^* = \tau(\Delta\widetilde{c}_{t+1} - c^*) + \sigma_x\widetilde{\xi}_{t+1} \tag{2}$$

where $\bar{x}$ *and* $\bar{c}$ *are real numbers,* $\tau > 0$, $\sigma_x > 0$; *and* $\widetilde{\xi}_{t+1} \xrightarrow{d}$ *i.i.d.* $\mathcal{N}(0,1)$. *Variable* $\widetilde{\xi}_{t+1}$ *is independent of* $\Delta\widetilde{c}_{t+1}$ *and* $\{\widetilde{\varepsilon}_{i,t+1}\}_{i=1}^n$ *at* $t+1$.

To simplicity notation, define $\bar{x} \equiv x^* - \tau c^*$. Let s_t denote the state of $\widetilde{p}_t$ at period t. Given $\mathcal{G}_n$, s_t determines the distributions of aggregate output and consumption growth at period t. Provided that $\widetilde{p}_t$ varies over time, the distributions of aggregate output and consumption growth vary over time as well, and the dynamics of the moments of these distributions satisfy the Markov property.

Sketch of proof of Proposition 1 and Corollaries 1 and 2. Given a sequence of network topologies $\{\mathcal{G}_n\}_{n=1}^{\infty}$, with limiting topology $\mathcal{G}_\infty$, and the realization of $\widetilde{p}_t$ at time t, the goal is to find the conditions under which $\sqrt{n}\widetilde{W}_{n,t}$ is normally distributed as n grows large.

Without loss of generality, fix t so that subscript t on the sequence $\{\widetilde{\varepsilon}_{i,t}\}_{i=1}^n$ can be eliminated. If the sequence of Bernoulli random variables $\{\widetilde{\varepsilon}_i\}_{i=1}^n$ is independent, the Lindeberg-Lévy central limit theorem implies that $\sqrt{n}\widetilde{W}_n$ is normally distributed as n grows large. Consequently, if $\widetilde{p}_t = 0$ firms' cash-flows are independent and $\sqrt{n}\widetilde{W}_n$ is asymptotically normally distributed.

In the presence of inter-firm relationships, however, cash-flows of connected firms are correlated if $\widetilde{p}_t > 0$. Despite that the sequence $\{\widetilde{\varepsilon}_i\}_{i=1}^n$ may be dependent, $\sqrt{n}\widetilde{W}_n$ may still be asymptotically normally distributed if the dependence among variables $\{\widetilde{\varepsilon}_i\}_{i=1}^n$ is sufficiently weak in a sense to be defined.

To better understand the main idea behind the proof, it is illustrative to review statistical concepts such as α-mixing, stationary processes and m-dependent sequences. I do so in what follows. For the sequence $\{\widetilde{\varepsilon}_i\}_{i=1}^n$, let α_n be a non-negative number such that

$$\left|\mathbb{P}(A\cap B) - \mathbb{P}(A)\mathbb{P}(B)\right| \quad \leq \quad \alpha_n \tag{3}$$

with $A \in \sigma(\widetilde{\varepsilon}_1, \cdots, \widetilde{\varepsilon}_k)$, $B \in \sigma(\widetilde{\varepsilon}_{k+n}, \widetilde{\varepsilon}_{k+n+1}, \cdots)$, $k \geq 1$ and $n \geq 1$; where $\sigma(\cdot)$ denotes the σ-algebra defined on the power set of $\{0,1\}^n \equiv \{0,1\} \times \cdots \times \{0,1\}$. The sequence $\{\widetilde{\varepsilon}_i\}_{i=1}^n$ is said to be α-mixing if $\alpha_n \to 0$ as n grows large. In other words, $\widetilde{\varepsilon}_k$ and $\widetilde{\varepsilon}_{k+n}$ are approximately independent for large n. The sequence is said to be stationary if the distribution of $(\widetilde{\varepsilon}_l, \widetilde{\varepsilon}_{l+1}, \cdots, \widetilde{\varepsilon}_{l+j})$ does not depend on l. If $\{\widetilde{\varepsilon}_i\}_{i=1}^n$ is α-mixing and stationary, $\sqrt{n}\widetilde{W}_n$ follows a normal distribution as n grows large—see Billingsley (1995, Theorem 27.4). A special case of the above result occurs if there exists an ordering of the sequence $\{\widetilde{\varepsilon}_i\}_{i=1}^n$ such that the dependence between variables $\widetilde{\varepsilon}_k$ and $\widetilde{\varepsilon}_j$ decreases as the distance between them increases in such an ordering. In particular, if there exists such an ordering and a positive $m \geq 0$ such that $(\widetilde{\varepsilon}_1, \cdots, \widetilde{\varepsilon}_k)$ and $(\widetilde{\varepsilon}_{1,k+s}, \cdots, \widetilde{\varepsilon}_{k+s+l})$ are independent whenever $s > m$, the sequence $\{\widetilde{\varepsilon}_i\}_{i=1}^n$ is said to be m-dependent in which case $\sqrt{n}\widetilde{W}_n$ follows a normal distribution for large n. An independent sequence is 0-dependent using this terminology.

In what follows, I apply the same idea behind a m-dependent sequence. In particular, I impose that negative shocks tend to remain locally confined as n grows large so that there always exist an index ordering $\mathcal{I}$ that makes the sequence $\{\widetilde{\varepsilon}_i\}_{i\in\mathcal{I}}$ to be m-dependent, in the sense described above.

For a given network topology, let $0 < p_c \leq 1$ be a real number such that for all $\widetilde{p}_t < p_c$, negative shocks only spread over clusters of firms of finite size. Provided that the size of such clusters becomes negligible compared to the size of the economy as n grows large, and there is an infinite number of small clusters, all independent among each other, $\sqrt{n}\widetilde{W}_n$ is normally distributed as n grows large. For instance, define m as the largest expected diameter of such clusters and the corresponding index ordering $\mathcal{I}$ such that whenever $s > m$, $(\widetilde{\varepsilon}_1, \cdots, \widetilde{\varepsilon}_k)$ and $(\widetilde{\varepsilon}_{1,k+s}, \cdots, \widetilde{\varepsilon}_{k+s+l})$ are independent in $\{\widetilde{\varepsilon}_i\}_{i\in\mathcal{I}}$.

To find the threshold p_c, it is illustrative to compute the probability that at least one negative shock spreads over $n-1$ different firms. Let P_n denote such a probability. Given how shocks spread from one firm to another, P_n equals

$$
\begin{aligned}
P_n &= (1-(1-q)^n)\,\mathbb{P}\,[\text{there is at least one open walk connecting } n \text{ firms }] \qquad (4)\\
&\approx (1-e^{-nq})\mathbb{P}\,[\text{there is at least one open walk connecting } n \text{ firms }] \quad (\text{for large } n)
\end{aligned}
$$

where q is the probability that a firm faces a negative idiosyncratic shock. A walk is a sequence of relationships which connect a sequence of firms that may not be all distinct from one another. A walk is considered to be open at t if all the relationships that compose the walk transmit negative shocks at t.

I focus on the limit of P_n as n grows large. Provided that $0 < q < 1$, the first term in the right-hand side of (4) tends to 1 as $n \to \infty$ at an exponential rate. As a consequence, if the second term in the right-hand side of (4) tends to 0 as n grows large, negative idiosyncratic shocks tend to remain locally confined since, almost surely, no firm belongs to an infinite open walk. Then, to determine the conditions under which a CLT-type of result applies is related to determine the probability, as $n \to \infty$, that a given firm belongs to an infinite open walk. Given a sequence $\{\mathcal{G}_n\}_n$, with limiting distribution $\mathcal{G}_\infty$, define p_c as

$$
p_c(\mathcal{G}_\infty) = \sup_{p\in(0,1)} \left\{ p : \lim_{n\to\infty} P_n = 0 \right\} \qquad (5)
$$

I write $p_c = p_c(\mathcal{G}_\infty)$ since p_c may depend on the network topology in the limit. Therefore, if $\widetilde{p}_t < p_c$ then $\sqrt{n}\widetilde{W}_n$ follows a normal distribution as n grows large since all open walks are almost surely finite and their size distribution has a tail which tend to decrease with n sufficiently fast.[27]

To prove normality, condition $\widetilde{p}_t < p_c$ may be stronger than necessary. Imposing such a condition, however, greatly facilitates the proof since the determination of p_c has been extensively studied in percolation theory, e.g. Grimmett (1989) and Stauffer and Aharony (1994). In percolation, p_c is sometimes called the critical probability or critical phenomenon of the model, because it indicates the arrival of an infinite connected component as $n \to \infty$ within a particular model.

To illustrate how p_c can be determined, consider the following two simple examples:

- Imagine n firms are arranged in a straight line and each relationship may transmit shocks with probability p. The probability that the line is open is p^n, which tends to zero as $n \to \infty$, so that $p_c = 1$.
- Suppose n firms are arranged in a circle. The probability that the circle is open tends to zero as $n \to \infty$. Think about putting the endpoints of an infinitely line together. Thus, $p_c = 1$.

Taking results from bond percolation, Table I reports critical probabilities for several symmetric network topologies. As Table I shows, p_c varies across networks. For instance, if $\mathcal{G}_\infty$ is the two dimensional honeycomb lattice then $p_c = 1 - 2\sin\left(\frac{\pi}{18}\right) \approx 0.65$ whereas if $\mathcal{G}_\infty$ is the two dimensional square lattice then $p_c = \frac{1}{2}$.

The previous analysis determines conditions under which $\sqrt{n}\widetilde{W}_n$ is normally distributed for some large symmetric networks. But what happens in other network topologies? In particular, under what conditions is $\sqrt{n}\widetilde{W}_n$ asymptotically normally distributed in large asymmetric networks? Using random walks on trees, Lyons (1990) shows that if $\mathcal{G}_\infty$ is a tree then

$$p_c = \frac{1}{\text{branching number of } \mathcal{G}_\infty} \tag{6}$$

where the branching number of a tree is the average number of branches per node in a tree.[28] A tree is a connected graph in which two given nodes are connected by exactly one path. A tree is said to be z-regular if each node has degree z. If $\mathcal{G}_\infty$ is an z-regular tree, the average number of branches per node is $z-1$ so $p_c = \frac{1}{z-1}$; which is consistent with Table I.[29]

One can generalize the previous result for topologies where $\mathcal{G}_\infty$ is sparse and locally treelike. G_n is said to be sparse if G_n has m edges and $m = O(n)$. Notation $m = O(n)$ indicates that m grows, at most, linearly with n so there exists a positive number c such that $\left|\frac{m}{n}\right| < c$ for all n. Namely, G_n is sparse if only a small fraction of the possible $\frac{n(n-1)}{2}$ edges are present. G_∞ is said to be locally treelike if in the limit an arbitrarily large neighborhood

[27] For instance, if $\mathcal{G}_n = \mathbb{L}^d$, where $\mathbb{L}^d$ represents the d-dimensional lattice, the probability that an open walk has size n is proportional to $\exp(-\zeta(p)n)$—see Grimmett (1989, Chapters 5 and 7).

[28] For a concrete definition of the branching number see Lyons (1990, page 935).

[29] To motivate the previous result, it is informative to compute the percolation threshold in the Bethe lattice with z neighbors per every node. Start at the root and check whether there is a chance of finding an infinite open path from the root. Starting from the root, one has $(z-1)$ new edges emanating from each new node in each layer of the lattice. Each of these $(z-1)$ new edges leads to one new node, which is affected with probability p. On average, $(z-1)p$ nodes are affected at each layer of the lattice. If $(z-1)p < 1$ then the average number of affected nodes decreases in each layer by a factor of $(z-1)p$. As a consequence, if $(z-1)p < 1$ the probability of finding an infinite open path goes to zero exponentially in the path length. Thus, $p_c = \frac{1}{z-1}$ for the Bethe lattice with z neighbors for every node.

around any node takes the form of a tree. Using the previous idea and reformulating percolation in trees as a message passing process, Karrer, Newman, and Zdeborová (2014) shows that if $\mathcal{G}_\infty$ is sparse and locally treelike then

$$p_c \quad = \quad \frac{1}{\epsilon_H} \tag{7}$$

where ϵ_H is the leading eigenvalue of the $2n \times 2n$ matrix

$$M \quad = \quad \begin{pmatrix} A & \mathbb{I} - D \\ \mathbb{I} & 0 \end{pmatrix} \tag{8}$$

where A is the adjacency matrix that represents $\mathcal{G}_n$, $\mathbb{I}$ is the $n \times n$ identity matrix, and D is the diagonal matrix with the number of relationships per firm along the diagonal, e.g. Karrer, Newman, and Zdeborová (2014). Parameter e_H is always real. For a sparse network this matrix is also sparse, with only $2m + 2n$ nonzero elements, which permits rapid numerical calculation of the leading eigenvalue. For the network that characterize the benchmark economy one obtains

$$e_H = 1 \quad \rightarrow \quad p_c \approx 1 \tag{9}$$

$$\text{branching number} = 1.185 \quad \rightarrow \quad p_c \approx 0.85 \tag{10}$$

$\square$

Proof of Proposition 2. I look for an equilibrium such that the price dividend ratio is stationary. I conjecture that if c is the current aggregate consumption and s the current state of $\widetilde{p}_t$, then $P_a(c, s) = w_s^a c$, in which P_a is the price of aggregate wealth and w_s^a a number that depends on state s. If $s_t = s$ and $s_{t+1} = s'$, the realized gross return at time $t + 1$ of the asset that delivers aggregate consumption as its dividend each period, $\widetilde{R}_{a,t+1}$, equals

$$\widetilde{R}_{a,t+1} \quad = \quad \frac{\widetilde{P}_{a,t+1} + \widetilde{C}_{t+1}}{P_{a,t}} \quad = \quad \frac{w_{s'}^a + 1}{w_s^a} \frac{\widetilde{C}_{t+1}}{C_t} \tag{11}$$

Setting $\widetilde{R}_{i,t+1} = \widetilde{R}_{a,t+1}$ in equation (8) yields,

$$\begin{aligned} \mathbb{E}_t \left(\left[\beta \left(\frac{\widetilde{C}_{t+1}}{C_t} \right)^{-\rho} \right]^{\frac{1-\gamma}{1-\rho}} \left[\widetilde{R}_{a,t+1} \right]^{\frac{1-\gamma}{1-\rho}} \right) \quad &= \quad 1 \\ \Rightarrow \mathbb{E} \left(\left[\beta \left(\frac{\widetilde{C}_{t+1}}{C_t} \right)^{-\rho} \right]^{\frac{1-\gamma}{1-\rho}} \left[\frac{w_{s'}^a + 1}{w_s^a} \frac{\widetilde{C}_{t+1}}{C_t} \right]^{\frac{1-\gamma}{1-\rho}} \Big| p_s \right) \quad &= \quad 1 \end{aligned} \tag{12}$$

Provided that s_t follows a Markov process, equation (12) can be rewritten as

$$\beta^{\frac{1-\gamma}{1-\rho}} \left(\sum_{s'=H,L} \omega_{s,s'} \mathbb{E} \left(\left(\frac{\widetilde{C}_{t+1}}{C_t} \right)^{1-\gamma} \Big| p_{s'} \right) \left(\frac{w_{s'}^a + 1}{w_s^a} \right)^{\frac{1-\gamma}{1-\rho}} \right) \quad = \quad 1 \tag{13}$$

Reordering equation (13) yields,

$$w_s^a \quad = \quad \beta\left(\sum_{s'=H,L}\omega_{s,s'}\mathbb{E}\left(e^{(1-\gamma)\Delta\widetilde{c}_{t+1}}\middle|p_{s'}\right)(w_{s'}^a+1)^{\frac{1-\gamma}{1-\rho}}\right)^{\frac{1-\rho}{1-\gamma}} \qquad s=H,L \tag{14}$$

which completes the proof.

REMARK 1: *If $\sqrt{n}\widetilde{W}_{n,t+1}$ is normally distributed, then*

$$\mathbb{E}\left(e^{(1-\gamma)\Delta\widetilde{c}_{t+1}}\middle|s\right) \quad = \quad \exp\left(\frac{(1-\gamma)(\alpha_0+\alpha_1\bar{d}-\alpha_2\mu_s-\bar{x})}{\tau}+\frac{(1-\gamma)^2}{2}\left(\frac{\alpha_2^2\sigma_s^2-\sigma_x^2}{\tau^2}\right)\right) \qquad s=H,L \tag{15}$$

where

$$\mu_H \equiv \lim_{n\to\infty}\mathbb{E}\left(\sum_{i=1}^{n}\frac{\widetilde{\varepsilon}_{i,t+1}}{\sqrt{n}}\middle|\widetilde{p}_{t+1}=p_H\right) \quad \textit{and} \quad \sigma_H^2 \equiv \lim_{n\to\infty} Var\left(\sum_{i=1}^{n}\frac{\widetilde{\varepsilon}_{i,t+1}}{\sqrt{n}}\middle|\widetilde{p}_{t+1}=p_H\right)$$
$$\mu_L \equiv \lim_{n\to\infty}\mathbb{E}\left(\sum_{i=1}^{n}\frac{\widetilde{\varepsilon}_{i,t+1}}{\sqrt{n}}\middle|\widetilde{p}_{t+1}=p_L\right) \quad \textit{and} \quad \sigma_L^2 \equiv \lim_{n\to\infty} Var\left(\sum_{i=1}^{n}\frac{\widetilde{\varepsilon}_{i,t+1}}{\sqrt{n}}\middle|\widetilde{p}_{t+1}=p_L\right)$$

and the above constants are assumed to be finite so that equation (15) is well-defined.

REMARK 2 (Price of Market Return): *If consumption and output growth differ I compute the price of the market return as follows. I conjecture that if y is the current aggregate output and s the current state of $\widetilde{p}_t$, then $P_m(c,s)=w_s^m y$, where P_m is the price of the market portfolio and w_s^m a number that depends on state s. If $s_t=s$ and $s_{t+1}=s'$, then the realized gross return at time $t+1$ of the asset that delivers aggregate output as its dividend each period, $\widetilde{R}_{m,t+1}$, equals*

$$\widetilde{R}_{m,t+1} \quad = \quad \frac{\widetilde{P}_{m,t+1}+Y_{t+1}}{P_{m,t}} \quad = \quad \frac{w_{s'}^m+1}{w_s^m}\frac{Y_{t+1}}{Y_t} \tag{16}$$

Setting $\widetilde{R}_{i,t+1}=\widetilde{R}_{m,t+1}$ in equation (8) yields,

$$\mathbb{E}_t\left(\left[\beta\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\rho}\right]^{\frac{1-\gamma}{1-\rho}}\left[\widetilde{R}_{a,t+1}\right]^{\frac{1-\gamma}{1-\rho}-1}\widetilde{R}_{m,t+1}\right) \quad = \quad 1$$
$$\Rightarrow \mathbb{E}\left(\left[\beta\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\rho}\right]^{\frac{1-\gamma}{1-\rho}}\left[\frac{w_{s'}^a+1}{w_s^a}\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)\right]^{\frac{1-\gamma}{1-\rho}-1}\left(\frac{w_{s'}^m+1}{w_s^m}\widetilde{X}_{t+1}\right)\middle|p_s\right) \quad = \quad 1 \tag{17}$$

where $\widetilde{X}_{t+1}=\frac{Y_{t+1}}{Y_t}$. Provided that s_t follows a Markov process, equation (17) can be rewritten as

$$\beta^{\frac{1-\gamma}{1-\rho}}\left(\sum_{s'=\{H,L\}}\omega_{s,s'}\mathbb{E}\left(\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\gamma}\widetilde{X}_{t+1}\middle|p_{s'}\right)\left(\frac{w_{s'}^a+1}{w_s^a}\right)^{\frac{1-\gamma}{1-\rho}-1}\left(\frac{w_{s'}^m+1}{w_s^m}\right)\right) \quad = \quad 1 \tag{18}$$

Reordering equation (18) yields,

$$w_s^m = \beta^{\frac{1-\gamma}{1-\rho}} \left(\sum_{s' \in \{H,L\}} \omega_{s,s'} \mathbb{E}\left(e^{-\gamma \Delta \widetilde{c}_{t+1} + \widetilde{x}_{t+1}} \middle| p_{s'} \right) \left(\frac{w_{s'}^a + 1}{w_s^a} \right)^{\frac{1-\gamma}{1-\rho}-1} (w_{s'}^m + 1) \right) \qquad s = \{H, L\} \tag{19}$$

It follows from (2) that $-\gamma \Delta \widetilde{c}_{t+1} + \widetilde{x}_{t+1} = \bar{x} + (\tau - \gamma)\Delta \widetilde{c}_{t+1} + \sigma_x \widetilde{\xi}_{t+1}$. *Therefore, (19) equals to*

$$w_s^m = \beta^{\frac{1-\gamma}{1-\rho}} e^{\bar{x} + \frac{\sigma_x^2}{2}} \left(\sum_{s' \in \{H,L\}} \omega_{s,s'} \mathbb{E}\left(e^{(\tau-\gamma)\Delta \widetilde{c}_{t+1}} \middle| p_{s'} \right) \left(\frac{w_{s'}^a + 1}{w_s^a} \right)^{\frac{1-\gamma}{1-\rho}-1} (w_{s'}^m + 1) \right) \qquad s = \{H, L\} \tag{20}$$

□

Proof of Proposition 3. Setting $\widetilde{R}_{i,t+1} = R_f$ in equation (8) yields,

$$\mathbb{E}\left(\left[\beta \left(\frac{\widetilde{C}_{t+1}}{C_t} \right)^{-\rho} \right]^{\frac{1-\gamma}{1-\rho}} \left[\widetilde{R}_{a,t+1} \right]^{\frac{1-\gamma}{1-\rho}-1} \middle| p_s \right) = \frac{1}{R_f(s)}, \qquad s = \{H, L\}. \tag{21}$$

Provided that s_t follows a Markov process and $P_a(c,s) = w_s^a c$, the left hand side of equation (21) can be rewritten as the following sum

$$\beta^{\frac{1-\gamma}{1-\rho}} \left(\sum_{s'=H,L} \omega_{s,s'} \mathbb{E}\left(\left(\frac{\widetilde{C}_{t+1}}{C_t} \right)^{-\gamma} \middle| p_{s'} \right) \left(\frac{w_{s'}^a + 1}{w_s^a} \right)^{\frac{\rho-\gamma}{1-\rho}} \right)$$

Therefore,

$$\frac{1}{R_f(s)} = \beta^{\frac{1-\gamma}{1-\rho}} \left(\sum_{s'=H,L} \omega_{s,s'} \mathbb{E}\left(e^{-\gamma \Delta \widetilde{c}_{t+1}} \middle| p_{s'} \right) \left(\frac{w_{s'}^a + 1}{w_s^a} \right)^{\frac{\rho-\gamma}{1-\rho}} \right), \qquad s = \{H, L\}$$

which completes the proof □

Proof of Proposition 4. Consider $s_t = s$ and $s_{t+1} = s'$. Equation (8) can be rewritten as,

$$P_{i,t} = \mathbb{E}_t \left(\widetilde{M}_{t+1} \left(\widetilde{P}_{i,t+1} + y_{i,t+1} \right) \right) \qquad i = 1, \cdots, n \tag{22}$$

where

$$\widetilde{M}_{t+1} \equiv \left[\beta \left(\frac{\widetilde{C}_{t+1}}{C_t} \right)^{-\rho} \right]^{\frac{1-\gamma}{1-\rho}} \left[\widetilde{R}_{a,t+1} \right]^{\frac{1-\gamma}{1-\rho}-1}$$

represents the pricing kernel. Dividing equation (22) by Y_t yields

$$\frac{P_{i,t}}{Y_t} = \mathbb{E}_t \left(\widetilde{M}_{t+1} \widetilde{X}_{t+1} \frac{\widetilde{P}_{i,t+1}}{Y_{t+1}} \right) + \mathbb{E}_t \left(\widetilde{M}_{t+1} \frac{y_{i,t+1}}{Y_t} \right) \qquad i = 1, \cdots, n \tag{23}$$

which can be rewritten as

$$v_{i,t} \quad = \quad \mathbb{E}_t\left(\widetilde{M}_{t+1}\widetilde{X}_{t+1}v_{i,t+1}\right) + \mathbb{E}_t\left(\widetilde{M}_{t+1}\frac{y_{i,t+1}}{Y_t}\right) \qquad i = 1, \cdots, n \tag{24}$$

with $v_{i,t} \equiv v_i(s) \equiv \frac{P_{i,t}}{Y_t}$. Provided that s_t follows a Markov process and $P_a(c, s) = w_s^a c$, the first term in the right hand side of equation (24) can be rewritten as

$$\mathbb{E}_t\left(\widetilde{M}_{t+1}\widetilde{X}_{t+1}v_{i,t+1}\right) \quad = \quad \beta^{\frac{1-\gamma}{1-\rho}}e^{\bar{x}+\frac{\sigma_x^2}{2}}\left(\sum_{s'=H,L}\omega_{s,s'}\left(\frac{w_{s'}^a+1}{w_s^a}\right)^{\frac{\rho-\gamma}{1-\rho}}\mathbb{E}\left(e^{(\tau-\gamma)\Delta\widetilde{c}_{t+1}}\big|p_{s'}\right)v_i(s')\right) \tag{25}$$

whereas the second term in the right hand side of equation (24) can be rewritten as

$$\mathbb{E}_t\left(\widetilde{M}_{t+1}\frac{y_{i,t+1}}{Y_t}\right) \quad = \quad e^{\alpha_0+\alpha_1 d_i}\mathbb{E}_t\left(\widetilde{M}_{t+1}e^{-\alpha_2\sqrt{n}\widetilde{\varepsilon}_{i,t+1}}\right) \tag{26}$$

The expectation term in the right hand side of equation (26) can be written as

$$\begin{aligned}\mathbb{E}_t\left(\widetilde{M}_{t+1}e^{-\alpha_2\sqrt{n}\widetilde{\varepsilon}_{i,t+1}}\right) \quad &= \quad \beta^{\frac{1-\gamma}{1-\rho}}\left(\sum_{s'=H,L}\omega_{s,s'}\mathbb{E}\left(\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\gamma}e^{-\alpha_2\sqrt{n}\widetilde{\varepsilon}_{i,t+1}}\Bigg|p_{s'}\right)\left(\frac{w_{s'}^a+1}{w_s^a}\right)^{\frac{\rho-\gamma}{1-\rho}}\right)\\ &= \quad \beta^{\frac{1-\gamma}{1-\rho}}\left(\sum_{s'=H,L}\omega_{s,s'}\mathbb{E}\left(e^{-\gamma\Delta\widetilde{c}_{t+1}-\alpha_2\sqrt{n}\widetilde{\varepsilon}_{i,t+1}}\Big|p_{s'}\right)\left(\frac{w_{s'}^a+1}{w_s^a}\right)^{\frac{\rho-\gamma}{1-\rho}}\right)\end{aligned}$$

As a consequence,

$$\begin{aligned}v_i(s) \quad &= \quad \beta^{\frac{1-\gamma}{1-\rho}}e^{\bar{x}+\frac{\sigma_x^2}{2}}\left(\sum_{s'=H,L}\omega_{s,s'}\left(\frac{w_{s'}^a+1}{w_s^a}\right)^{\frac{\rho-\gamma}{1-\rho}}\mathbb{E}\left(e^{(\tau-\gamma)\Delta\widetilde{c}_{t+1}}\big|p_{s'}\right)v_i(s')\right)\\ &+ \quad \beta^{\frac{1-\gamma}{1-\rho}}e^{\alpha_0+\alpha_1 d_i}\left(\sum_{s'=H,L}\omega_{s,s'}\mathbb{E}\left(e^{-\gamma\Delta\widetilde{c}_{t+1}-\alpha_2\sqrt{n}\widetilde{\varepsilon}_{i,t+1}}\Big|p_{s'}\right)\left(\frac{w_{s'}^a+1}{w_s^a}\right)^{\frac{\rho-\gamma}{1-\rho}}\right) \qquad i = 1, \cdots, n\end{aligned}$$

Define $\pi_i(s') \equiv \mathbb{E}\left[\widetilde{\varepsilon}_{i,t+1}|s_{t+1} = s'\right]$. It is worth noting that

$$\begin{aligned}-\gamma\Delta\widetilde{c}_{t+1} - \alpha_2\sqrt{n}\widetilde{\varepsilon}_{i,t+1} \quad &= \quad -\gamma\underbrace{\left(\frac{1}{\tau}\left\{\alpha_0+\alpha_1\bar{d}-\alpha_2\left(\sum_{j\neq i}\frac{\widetilde{\varepsilon}_{j,t+1}}{\sqrt{n}}\right)-\sigma_x\widetilde{\xi}_{t+1}-\bar{x}\right\}\right)}_{\Delta\widetilde{c}_{-i,t+1}}-\alpha_2\sqrt{n}\left(1-\frac{\gamma}{\tau n}\right)\widetilde{\varepsilon}_{i,t+1}\\ &= \quad -\gamma\Delta\widetilde{c}_{-i,t+1} - \alpha_2\sqrt{n}\left(1-\frac{\gamma}{\tau n}\right)\widetilde{\varepsilon}_{i,t+1}\end{aligned} \tag{27}$$

Since $\Delta\widetilde{c}_{-i,t+1}$ and $\widetilde{\varepsilon}_{i,t+1}$ are independent

$$\begin{aligned}\mathbb{E}\left(e^{-\gamma\Delta\widetilde{c}_{-i,t+1}-\alpha_2\sqrt{n}\left(1-\frac{\gamma}{\tau n}\right)\widetilde{\varepsilon}_{i,t+1}}\Big|s\right) \quad &= \quad \mathbb{E}\left(e^{-\gamma\Delta\widetilde{c}_{-i,t+1}}\big|p_s\right)\left\{\pi_i(s)e^{-\alpha_2\sqrt{n}\left(1-\frac{\gamma}{\tau n}\right)}+(1-\pi_i(s))\right\}\\ &\approx \quad \mathbb{E}\left(e^{-\gamma\Delta\widetilde{c}_{t+1}}\big|p_s\right)(1-\pi_i(s))\end{aligned} \tag{28}$$

where the last approximation is accurate for large n. If the distribution of $\Delta\widetilde{c}_{t+1}$ is known, the expectation of

$-\gamma\Delta\widetilde{c}_{t+1}-\alpha_2\sqrt{n}\widetilde{\varepsilon}_{i,t+1}$ can be approximated using equation (28). Therefore,

$$\begin{aligned} v_i(s) &= \beta^{\frac{1-\gamma}{1-\rho}} e^{\bar{x}+\frac{\sigma_x^2}{2}} \left(\sum_{s'=H,L} \omega_{s,s'} \left(\frac{w_{s'}^a+1}{w_s^a} \right)^{\frac{\rho-\gamma}{1-\rho}} \mathbb{E}\left(e^{(\tau-\gamma)\Delta\widetilde{c}_{t+1}} \big| p_{s'} \right) v_i(s') \right) \\ &+ \beta^{\frac{1-\gamma}{1-\rho}} e^{\alpha_0+\alpha_1 d_i} \left(\sum_{s'=H,L} \omega_{s,s'} \mathbb{E}\left(e^{-\gamma\Delta\widetilde{c}_{t+1}} \big| p_s \right) (1-\pi_i(s)) \left(\frac{w_{s'}^a+1}{w_s^a} \right)^{\frac{\rho-\gamma}{1-\rho}} \right) \qquad i=1,\cdots,n \end{aligned}$$

which completes the proof $\square$

Proof of Proposition 5. Recall

$$\mathrm{Var}\left(\widetilde{M}_{t+1}\big|s\right) = \mathbb{E}\left(\widetilde{M}_{t+1}^2\big|s\right) - \mathbb{E}^2\left(\widetilde{M}_{t+1}\big|s\right) \tag{29}$$

The first term in the right hand side of equation (29) can be rewritten as

$$\mathbb{E}\left(\widetilde{M}_{t+1}^2\big|s\right) = \beta^{2\left(\frac{1-\gamma}{1-\rho}\right)} \left(\sum_{s'=H,L} \omega_{s,s'} \mathbb{E}\left(\left(\frac{\widetilde{C}_{t+1}}{C_t} \right)^{-2\gamma} \Big| p_{s'} \right) \left(\frac{w_{s'}^a+1}{w_s^a} \right)^{2\left(\frac{\rho-\gamma}{1-\rho}\right)} \right) \tag{30}$$

Provided that $\lambda_m(s) \equiv -\frac{\mathrm{Var}\left(\widetilde{M}_{t+1}|s\right)}{\mathbb{E}\left(\widetilde{M}_{t+1}|s\right)}$ and $\mathbb{E}\left(\widetilde{M}_{t+1}\big|s\right) = \frac{1}{R_f(s)}$, it then follows from equation (29) that

$$\lambda_m(s) = \frac{1}{R_f(s)} - R_f(s) \left(\beta^{2\left(\frac{1-\gamma}{1-\rho}\right)} \sum_{s'=H,L} \omega_{s,s'} \left(\frac{w_{s'}^a+1}{w_s^a} \right)^{2\left(\frac{\rho-\gamma}{1-\rho}\right)} \mathbb{E}\left(e^{-2\gamma\Delta\widetilde{c}_{t+1}} \big| p_{s'} \right) \right), \qquad s=\{H,L\}$$

which completes the proof $\square$

Appendix B: Simulation of the Model

This section describes the algorithm I use to compute firms' probabilities of facing negative shocks in each state of nature so one can compute asset prices and returns at the firm level using proposition 4. Let s_t denote the state of $\widetilde{p}_t$ at period t. To simplify the computation of probabilities $\{\pi_i(s_t)\}_{i=1}^n$, I restrict the topology of $\mathcal{G}_n$. In general topologies, computing $\{\pi_i(s_t)\}_{i=1}^n$ is hard, because the number of states that need to be considered increases exponentially with n. In economies with no cycles, however, computing $\{\pi_i(s_t)\}_{i=1}^n$ is easier. In those economies, computing $\{\pi_i(s_t)\}_{i=1}^n$ can be framed as a recursive problem as the following algorithm describes.

Algorithm *Firms Probabilities* (G_n, $\widetilde{p}_t, q$)

($*$ Description: Algorithm that computes firms' probabilities of facing negative shocks if G_n is a forest $*$)

Input: G_n (a forest), $\widetilde{p}_t$, q.

Output: The set of probabilities of firms facing a negative shock at time t, $\{\pi_i(s_t)\}_{i=1}^n$

1. **for** each firm $i \in G_n$

2. Determine the subgraph of G_n wherein firm i participates. Denote such a graph as T_i and label firm i

as its root.[30]

3. **if** firm i has a no connections

4. **return** $\pi_i(s_t) = q$

5. **else** **return** Prob(i,T_i,$\widetilde{p}_t$,q)

where Prob(i,T_i,$\widetilde{p}_t$,q) corresponds to the following recursive program,

Algorithm *Prob(i,T_i,$\widetilde{p}_t$,q)*

($*$ Description: Recursive algorithm that computes firm i's probability of facing a negative shock $*$)

Input: A node i in $\mathcal{G}_n$, the tree T_i wherein node i is the root, $\widetilde{p}_t$ and q.

Output: $\pi_i(s_t)$

1. Determine the set of children of node i in T_i, say $\mathcal{C}_i$.[31]

2. **if** $\mathcal{C}_i = \emptyset$

3. **return** $\pi_i(s_t) = q$

4. **else** **if** every node in $\mathcal{C}_i$ has no children

5. **return** $\pi_i(s_t) = q + (1-q)\left(1 - (1-\widetilde{p}_t q)^{|\mathcal{C}_i|}\right)$

6. **else** **return** $\pi_i(s_t) = q + (1-q)\left(1 - \prod_{k \in \mathcal{C}_i}\left(1 - \widetilde{p}_t \text{Prob}(k, T_{i,k}, \widetilde{p}_t, q)\right)\right)$ [32]

where $\mathcal{C}_i$ denotes the cardinality of set $\mathcal{C}_i$.

In economies with no cycles, it is also simple to compute the first two moments of the distribution of $\sqrt{n}\widetilde{W}_{n,t+1}$ at $t+1$. Let μ_s, σ_s^2 denote the mean and variance of $\sqrt{n}\widetilde{W}_{n,t+1}$ if $s_{t+1} = s$, respectively. In other words,

$$\mu_s = \lim_{n\to\infty} \mathbb{E}\left(\sum_{i=1}^{n} \frac{\widetilde{\varepsilon}_{i,t+1}}{\sqrt{n}}\middle| s\right) = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi_i(s)}{\sqrt{n}} \qquad s = L, H \tag{1}$$

and

$$\begin{aligned} \sigma_s^2 &= \lim_{n\to\infty} \text{Var}\left(\sum_{i=1}^{n} \frac{\widetilde{\varepsilon}_{i,t+1}}{\sqrt{n}}\middle| s\right) \\ &= \lim_{n\to\infty}\left\{\frac{1}{n}\sum_{i=1}^{n}\pi_i(s)\left(1-\pi_i(s)\right) + \frac{1}{n}\sum_{(i,j)\in\mathcal{R}_n} \text{Cov}\left(\widetilde{\varepsilon}_{i,t+1}, \widetilde{\varepsilon}_{j,t+1}\middle| s_t = s\right)\right\} \qquad s = L, H \end{aligned} \tag{2}$$

The second equation can be simplified further. If there exists a path between firm i and j after edges are removed at time $t+1$ then $\widetilde{\varepsilon}_{i,t+1} = \widetilde{\varepsilon}_{j,t+1}$. If there is no path between firm i and j in $\mathcal{G}_n$, variables $\widetilde{\varepsilon}_{i,t+1}$ and $\widetilde{\varepsilon}_{j,t+1}$ are independent. It then follows,

$$\begin{aligned} \mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t}\widetilde{\varepsilon}_{j,t}\middle| \text{ there is a path between } i \text{ and } j\right] &= \text{Var}_t\left[\widetilde{\varepsilon}_{i,t}\right] + \mathbb{E}_t^2\left[\widetilde{\varepsilon}_{i,t}\right] = \pi_i(s)(1-\pi_i(s)) + \pi_i^2(s) \\ \mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t}\widetilde{\varepsilon}_{j,t}\middle| \text{ there is no path between } i \text{ and } j\right] &= \mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t}\right]\mathbb{E}_t\left[\widetilde{\varepsilon}_{j,t}\right] = \pi_i(s)\pi_j(s) \end{aligned}$$

[30] Note that such a graph is a tree provided that G_n is a forest.

[31] In a rooted tree, the parent of a node is the node connected to it on the path to the root. Every node except the root has a unique parent. A child of a node v is a node of which v is the parent.

[32] Tree $T_{i,k}$ denotes the branch of tree T_i that starts at node k.

Hence,

$$\begin{aligned}\mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t}\widetilde{\varepsilon}_{j,t}\right] &= \mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t}\widetilde{\varepsilon}_{j,t}\middle| \text{ there is a path between } i \text{ and } j\right]\mathbb{P}\left[\text{there is a path between } i \text{ and } j \text{ at } t\right] \\ &+ \mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t}\widetilde{\varepsilon}_{j,t}\middle| \text{ there is no path between } i \text{ and } j\right]\mathbb{P}\left[\text{there is no path between } i \text{ and } j \text{ at } t\right] \\ &= \left(\pi_i(s)(1-\pi_i(s))+\pi_i^2(s)\right)\mathbf{P}_{ij}(s)+\pi_i(s)\pi_j(s)\left(1-\mathbf{P}_{ij}(s)\right)\end{aligned}$$

where $\mathbf{P}_{ij}(s) \equiv \mathbb{P}\left[\text{there is a path between } i \text{ and } j \text{ if } s_t = s\right]$. Thus,

$$\begin{aligned}\mathrm{Cov}_t\left[\widetilde{\varepsilon}_{i,t},\widetilde{\varepsilon}_{j,t}\right] &= \mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t}\widetilde{\varepsilon}_{j,t}\right]-\mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t}\right]\mathbb{E}_t\left[\widetilde{\varepsilon}_{j,t}\right] \\ &= \left(\pi_i(s)(1-\pi_i(s))+\pi_i^2(s)\right)\mathbf{P}_{ij}(s)+\pi_i(s)\pi_j(s)\left(1-\mathbf{P}_{ij}(s)\right)-\pi_i(s)\pi_j(s) \\ &= \pi_i(s)\left(1-\pi_j(s)\right)\mathbf{P}_{ij}(s)\end{aligned}$$

Therefore,

$$\sigma_s^2 = \lim_{n\to\infty}\left\{\frac{1}{n}\sum_{i=1}^{n}\pi_i(s)\left(1-\pi_i(s)\right)+\frac{1}{n}\sum_{(i,j)\in\mathcal{R}_n}\pi_i(s)\left(1-\pi_j(s)\right)\mathbf{P}_{ij}(s)\right\} \qquad s=L,H$$

To compute $\mathbf{P}_{ij}(s)$ I need to determine the set of paths that connect firms i and j on $\mathcal{G}_n$. If there is more than one path connecting firm i and j, computing $\mathbf{P}_{ij}(s)$ is difficult, because shocks can be transmitted by any of those paths which may be of different length. On the other hand, if there is only one path connecting any two given firms, say firm i and j, $\mathbf{P}_{ij}(s)$ is a function of the length of the unique path connecting firms i and j. It then becomes handy to restrict the topology of $\mathcal{G}_n$ so that it does not have cycles. The following remark describes $\mathbf{P}_{ij}(s)$ when $\mathcal{G}_n$ is a forest.

REMARK 3: *Suppose $\mathcal{G}_n$ is a forest, namely there are no cycles. Then, every component of $\mathcal{G}_n$ is a tree. Provided that any two given firms are jointed by a unique path (in case such a path exists),*

$$\boldsymbol{P}_{ij}(s) = \begin{cases}\widetilde{p}_t^{\,l_{i,j}} & \textit{where } l_{i,j} \textit{ is the length of the (unique) path between } i \textit{ and } j \textit{ in } \mathcal{G}_n \\ 0 & \textit{there is no path between } i \textit{ and } j\end{cases} \tag{3}$$

Appendix C: Network Economies and Long-Run Risks

This section shows how the baseline model can be recast so that it generates dynamics that are consistent with long-run risks models. In what follows, both the mean and volatility of firms' growth rate of cash-flows have a persistent component. I use approximations similar to those used by Campbell and Shiller (1989) and Bansal and Yaron (2004) to derive approximated solutions for equilibrium asset prices.

Recall that $P_{i,t+1}$ is the share price of firm i at $t+1$. For simplicity assume $\bar{x} = \sigma_x = 0$ and $\tau = 1$ so that the following two conditions hold at equilibrium

$$P_{a,t+1} = \sum_{i=1}^{n} P_{i,t+1} \tag{1}$$

$$\widetilde{c}_{t+1} = \prod_{i=1}^{n} y_{i,t+1}^{1/n} \tag{2}$$

Define

$$g_{i,t+1} \equiv \log\left(\frac{y_{i,t+1}}{c_t}\right) \quad , \quad g_{t+1} \equiv \log\left(\frac{\widetilde{c}_{t+1}}{c_t}\right) \tag{3}$$

$$z_{i,t+1} \equiv \log\left(\frac{P_{i,t+1}}{\widetilde{c}_{t+1}}\right) \quad , \quad z_{t+1} \equiv \log\left(\frac{P_{a,t+1}}{\widetilde{c}_{t+1}}\right) \tag{4}$$

Provided Y_{t+1} definition, it follows

$$g_{t+1} = \sum_{i=1}^{n} \frac{1}{n} g_{i,t+1} \tag{5}$$

Using first order Taylor approximations yields

$$z_{t+1} \approx w_0 + \sum_{i=1}^{n} w_i z_{i,t+1} \tag{6}$$

where $w_i \approx \mathbb{E}\left(\frac{z_{i,t}}{\sum_{j=1}^{n} z_{j,t}}\right)$, and $\sum_{i=1}^{n} w_i = 1$. The term w_0 is selected to ensure that first order approximations hold in levels as well. Define the continuous return of firm i at $t+1$ as

$$r_{i,t+1} \equiv \log\left(\frac{P_{i,t+1} + y_{i,t+1}}{P_{i,t}}\right) \tag{7}$$

and the continuous return on aggregate wealth at $t+1$ as:

$$r_{a,t+1} \equiv \log\left(\frac{P_{a,t+1} + \widetilde{c}_{t+1}}{P_{a,t}}\right) \tag{8}$$

Using first order Taylor approximations yields[33]

$$r_{i,t+1} \approx k_i + \rho_i z_{i,t+1} - z_{i,t} + \rho_i g_{t+1} + (1-\rho_i) g_{i,t+1} \tag{9}$$

$$r_{a,t+1} \approx k_m - z_t + \rho_m z_{t+1} + g_{t+1} \tag{10}$$

[33] Approximation (10) follows directly from Bansal and Yaron (2004) which in turns follows from the dividend-ratio model of Campbell and Shiller (1989). Approximation (9) follows from Campbell and Shiller (1989) once noting that

$$\begin{aligned} r_{i,t+1} &\approx k_i + \log\left(\frac{y_{i,t}}{P_{i,t}}\right) - \rho_i \log\left(\frac{y_{i,t+1}}{P_{i,t+1}}\right) + \log\left(\frac{y_{i,t+1}}{y_{i,t}}\right) \\ &= k_i + \log\left(\frac{y_{i,t}}{c_{t-1}} \frac{c_t}{P_{i,t}} \frac{c_{t-1}}{c_t}\right) - \rho_i \log\left(\frac{y_{i,t+1}}{c_t} \frac{c_{t+1}}{P_{i,t+1}} \frac{c_t}{c_{t+1}}\right) + \log\left(\frac{y_{i,t+1}}{c_t} \frac{c_t}{c_{t-1}} \frac{c_{t-1}}{y_{i,t}}\right) \\ &= k_i + \rho_i z_{i,t+1} - z_{i,t} + \rho_i g_{t+1} + (1-\rho_i) g_{i,t+1} \end{aligned}$$

where $\{k_i\}_{i=1}^{n}$ and k_m ensure that first order approximations hold in levels as well. Provided $g_{i,t+1}$ definition, $g_{i,t+1}$ can be approximated by

$$g_{i,t+1} \approx x_{i,t} + \sigma_{i,t}\eta_{i,t+1} \tag{11}$$

where

$$x_{i,t} \equiv \alpha_0 + \alpha_1 d_i - \alpha_2 \mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t+1}\right] \tag{12}$$

$$\sigma^2_{i,t} \equiv \alpha_2^2 \mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t+1}\right]\left(1 - \mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t+1}\right]\right) \tag{13}$$

Note that $x_{i,t}$ determines $\mathbb{E}_t[g_{i,t+1}]$ and $\sigma_{i,t}$ determines the conditional volatility of $g_{i,t+1}$, given the information at time t. Provided that $\mathcal{G}_n$ does not vary over time and $\widetilde{p}_t$ follows a two state ergodic Markov process, the processes that $x_{i,t}$ and $\sigma^2_{i,t}$ follow can be approximated by:

$$x_{i,t+1} \approx \mu_0 + \mu_1 x_{i,t} + \mu_2 \sigma_{i,t}\zeta_{p,t+1} \tag{14}$$

$$\sigma^2_{i,t+1} \approx \nu_0 + \nu_1 \sigma^2_{i,t} + \nu_2 \sigma_p \zeta_{p,t+1} \tag{15}$$

where $0 < \mu_1 < 1$, $\mu_2 > 0$, $0 < \nu_1 < 1$ and $\nu_2 > 0$. Variable $\zeta_{p,t+1} \xrightarrow{d} \mathcal{N}(0,1)$ represents the uncertainty coming from unexpected changes in $\widetilde{p}_{t+1}$. Variables $\eta_{i,t+1} \xrightarrow{d} \mathcal{N}(0,1)$ represents the uncertainty coming from idiosyncratic productivity shocks at the firm level, with $\eta_{i,t+1} \perp\!\!\!\perp \eta_{j,t+1}$, $\forall\ j \neq i$. In the baseline model, parameter q is related to variables $\eta_{i,t+1}$ in the approximated solution. Provided how negative shocks are propagated, $\eta_{i,t+1} \perp\!\!\!\perp \zeta_{p,t+1}$, $\forall\ i$.[34]

With the above definitions and approximations at hand, I now study the asset pricing implication of inter-firm relationships. The pricing kernel equals

$$m_{t+1} \equiv \theta \ln(\delta) - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{a,t+1} \tag{16}$$

I derive firm i's price and return using the pricing kernel and the standard first order condition

$$\mathbb{E}_t\left[\exp\left(m_{t+1} + r_{i,t+1}\right)\right] = 1 \tag{17}$$

I first solve for the return of the market portfolio $r_{a,t+1}$ substituting $r_{i,t+1}$ by $r_{a,t+1}$. Then I solve for the risk-free rate. Finally I solve for the risk premium of firm i, $\forall\ i \in \{1, \cdots, n\}$.

Return of the Market Portfolio: Following Bansal and Yaron (2004) I conjecture that firm i's logarithm of the price-consumption ratio follows:

$$z_{i,t} = a_0 + a_1 x_{i,t} + a_2 \sigma^2_{i,t} \tag{18}$$

To solve for constants a_0, a_1 and a_2 I use equations (5), (6) and (10) into the Euler equation (17). Since $\eta_{i,t+1}$, $\zeta_{p,t+1}$

[34] Let $\widetilde{x}$ and $\widetilde{y}$ be two random variables. I write $\widetilde{x} \perp\!\!\!\perp \widetilde{y}$ to denote that $\widetilde{x}$ is independent of $\widetilde{y}$.

are conditionally normal, $\forall\ i \in \{1, \cdots, n\}$, $r_{a,t+1}$ and m_{t+1} are also normal. Exploiting this normality, I write down the Euler equation in terms of the state variables $\{x_{i,t}, \sigma_{i,t}\}_{i=1}^{n}$. As the Euler equation must hold for all values of the states variables, the terms involving $x_{i,t}$ must satisfy:

$$\frac{1}{n}\left(1-\frac{1}{\psi}\right) - w_i a_1 + \rho_m a_1 \mu_1 w_i \quad = \quad 0 \tag{19}$$

ASSUMPTION 3: *Consider that* $\sum_{i=1}^{n} w_i x_{i,t} \approx \frac{1}{n}\sum_{i=1}^{n} x_{i,t}$.

It is worth noting that if most firms in $\mathcal{G}_n$ have a similar number of connections, then assumption 3 is satisfied. For instance, if $\mathcal{G}_n$ is regular, i.e. all firms have the same degree, then $w_i \approx \frac{1}{n}$ for most firms in $\mathcal{G}_n$. If assumption 3 is satisfied, I then can rewrite equation (19) as

$$w_i\left(1-\frac{1}{\psi}\right) - w_i a_1 + \rho_m a_1 \mu_1 w_i \quad \approx \quad 0 \tag{20}$$

as a consequence,

$$a_1 \quad \approx \quad \frac{\left(1-\frac{1}{\psi}\right)}{1-\mu_1\rho_m} \tag{21}$$

ASSUMPTION 4: *Assume that for most firms in* $\mathcal{G}_n$, $\sigma_{i,t} \approx \sigma_{i,t}^2$.

Using assumption 4 and collecting all the terms that involve $\sigma_{i,t}^2$ yields

$$-w_i a_2 + \rho_m w_i a_2 \nu_1 + \frac{\theta}{2}\left(\frac{1}{n}\right)^2\left(1-\frac{1}{\psi}\right)^2 + \frac{\theta}{2}\rho_m^2 w_i^2\left(a_1^2\mu_2^2 + a_1\mu_2 a_2\nu_2\sigma_p\right) \quad \approx \quad 0 \tag{22}$$

If assumption 3 is satisfied and n is sufficiently large, then $w_i \approx w_i^2$ for most firms. Then equation (22) can be rewritten as

$$-a_2 + \rho_m a_2 \nu_1 + \frac{\theta}{2}\left(1-\frac{1}{\psi}\right)^2 + \frac{\theta}{2}\rho_m^2\left(a_1^2\mu_2^2 + a_1\mu_2 a_2\nu_2\sigma_p\right) \quad \approx \quad 0 \tag{23}$$

It then follows,

$$a_2 \quad \approx \quad \frac{\frac{\theta}{2}\left(\left(1-\frac{1}{\psi}\right)^2 + \rho_m^2 a_1^2\mu_2^2\right)}{1-\nu_1\rho_m + \frac{\theta}{2}\rho_m^2 a_1\mu_2\nu_2\sigma_p} \tag{24}$$

Given the solution for $z_{i,t}$ the innovation to the return of aggregate wealth is given by

$$\begin{aligned}
r_{a,t+1} - \mathbb{E}_t[r_{a,t+1}] \quad &\approx \quad \rho_m\left(a_1\mu_2\left(\sum_{i=1}^{n} w_i\sigma_{i,t}\right) + a_2\nu_2\sigma_p\right)\zeta_{p,t+1} + \frac{1}{n}\sum_{i=1}^{n}\sigma_{i,t}\eta_{i,t+1} \\
&\approx \quad \rho_m\left(a_1\mu_2\left(\sum_{i=1}^{n} w_i\sigma_{i,t}\right) + a_2\nu_2\sigma_p\right)\zeta_{p,t+1} + \sum_{i=1}^{n} w_i\sigma_{i,t}\eta_{i,t+1} \\
&= \quad \rho_m\Delta_{p,t}\zeta_{p,t+1} + \sum_{i=1}^{n} w_i\sigma_{i,t}\eta_{i,t+1} \qquad (25)
\end{aligned}$$

where $\Delta_{p,t} \equiv a_1\mu_2\left(\sum_{i=1}^n w_i\sigma_{i,t}\right) + a_2\nu_2\sigma_p$. The conditional variance of aggregate wealth is given by

$$\mathrm{Var}_t[r_{a,t+1}] \quad \approx \quad \rho_m^2\Delta_{p,t}^2 + \sum_{i=1}^n w_i^2\sigma_{i,t}^2 \tag{26}$$

Hereinafter, I assume that assumptions 3 and 4 are satisfied.

Pricing Kernel: Using equations (5) and (10), I rewrite the pricing kernel in terms of the state variables,

$$\begin{aligned} m_{t+1} \quad &\equiv \quad \theta\ln(\delta) - \frac{\theta}{\psi}g_{t+1} + (\theta-1)r_{a,t+1} \qquad (27) \\ &\approx \quad \theta\ln(\delta) - \frac{\theta}{\psi}\left(\sum_{i=1}^n w_i(x_{i,t}+\sigma_{i,t}\eta_{i,t+1})\right) \\ &+ \quad (\theta-1)\left(k_m - w_0 - \sum_{i=1}^n w_i\left(a_0 + a_1x_{i,t} + a_2\sigma_{i,t}^2\right)\right) \\ &+ \quad (\theta-1)\rho_m\left(w_0 + \sum_{i=1}^n w_i\left(a_0 + a_1\mu_0 + a_1\mu_2x_{i,t} + a_1\mu_2\sigma_{i,t}\zeta_{p,t+1}\right)\right) \\ &+ \quad (\theta-1)\rho_m\left(\sum_{i=1}^n w_i\left(a_2\nu_0 + a_2\nu_1\sigma_{i,t}^2 + a_2\nu_2\sigma_p\zeta_{p,t+1}\right)\right) \\ &+ \quad (\theta-1)\left(\sum_{i=1}^n w_i(x_{i,t}+\sigma_{i,t}\sigma_{i,t+1})\right) \end{aligned}$$

Innovations to the pricing kernel are then given by

$$m_{t+1} - \mathbb{E}_t[m_{t+1}] \quad \approx \quad \lambda_{m,q}\left(\sum_{i=1}^n w_i\sigma_{i,t}\eta_{i,t+1}\right) + \lambda_{m,p}\Delta_{p,t}\zeta_{p,t+1} \tag{28}$$

where λ's represent the aggregate market prices of risk for each source of risk, namely $\{\eta_{i,t+1}\}_{i=1}^n$ and $\zeta_{p,t+1}$, which are defined as

$$\begin{aligned} \lambda_{m,q} \quad &\equiv \quad \theta\left(1-\frac{1}{\psi}\right) - 1 \\ \lambda_{m,p} \quad &\equiv \quad (\theta-1)\rho_m \end{aligned}$$

It follows from equation (28) that the conditional variance of the pricing kernel is given by

$$\mathrm{Var}_t[m_{t+1}] \quad \approx \quad \lambda_{m,q}^2\left(\sum_{i=1}^n w_i^2\sigma_{i,t}^2\right) + \lambda_{m,p}^2\Delta_{p,t}^2 \tag{29}$$

Equity Premium: The risk premium of the market return (aggregate wealth) is determined by the conditional covariance between the market portfolio and the pricing kernel. It then follows

$$\mathbb{E}_t[r_{a,t+1} - r_{f,t}] \quad = \quad -\mathrm{Cov}_t\left(m_{t+1} - \mathbb{E}_t[m_{t+1}], r_{a,t+1} - \mathbb{E}_t[r_{a,t+1}]\right) - \frac{1}{2}\mathrm{Var}_t(r_{a,t+1}) \tag{30}$$

Using equations (25) and (28) into the above equation yields

$$\mathbb{E}_t[r_{a,t+1} - r_{f,t}] \quad \approx \quad -\left(\lambda_{m,q} + \frac{1}{2}\right)\left(\sum_{i=1}^n w_i^2\sigma_{i,t}^2\right) - \rho_m\left(\lambda_{m,p} + \frac{\rho_m}{2}\right)\Delta_{p,t}^2 \tag{31}$$

Risk-free Rate: As in Bansal and Yaron (2004) the risk-free rate satisfies

$$r_{f,t} \quad = \quad -\ln(\delta) + \frac{1}{\psi}\mathbb{E}_t[g_{t+1}] + \frac{1-\theta}{\theta}\mathbb{E}_t[r_{a,t+1} - r_{f,t}] - \frac{1}{2\theta}\mathrm{Var}_t[m_{t+1}] \tag{32}$$

Using equations (29) and (31) into the above equation yields,

$$\begin{aligned} r_{f,t} \quad &\approx \quad -\ln(\delta) + \frac{1}{\psi}\left(\sum_{i=1}^{n} w_i x_{i,t}\right) \\ &- \quad \frac{1-\theta}{\theta}\left(\left(\lambda_{m,q} + \frac{1}{2}\right)\left(\sum_{i=1}^{n} w_i^2\sigma_{i,t}^2\right) + \rho_m\left(\lambda_{m,p} + \frac{\rho_m}{2}\right)\Delta_{p,t}^2\right) \\ &- \quad \frac{1}{2\theta}\left(\lambda_{m,q}^2\left(\sum_{i=1}^{n} w_i^2\sigma_{i,t}^2\right) + \lambda_{m,p}^2\Delta_{p,t}^2\right) \end{aligned} \tag{33}$$

Risk Premium in the Cross-Section: As with the market portfolio, the risk premium of firm i is determined by the conditional covariance between firm i's return and the pricing kernel. It then follows

$$\mathbb{E}_t[r_{i,t+1} - r_{f,t}] \quad = \quad -\mathrm{Cov}_t\left(m_{t+1} - \mathbb{E}_t[m_{t+1}], r_{i,t+1} - \mathbb{E}_t[r_{i,t+1}]\right) - \frac{1}{2}\mathrm{Var}_t(r_{i,t+1}) \tag{34}$$

It becomes handy to compute the innovations on firm i's return. Using equation (9) it can be shown

$$\begin{aligned} r_{i,t+1} - \mathbb{E}_t[r_{i,t+1}] \quad &\approx \quad \rho_i\left(a_1\mu_2\sigma_{i,t} + a_2\nu_2\sigma_p\right)\zeta_{p,t+1} \\ &+ \quad \rho_i\left(\sum_{j\neq i}^{n} w_j\sigma_{j,t}\eta_{j,t+1}\right) + (1-\rho_i(1-w_i))\sigma_{i,t}\eta_{i,t+1} \\ &= \quad \rho_i\nabla_{p,t}\zeta_{p,t+1} + \rho_i\left(\sum_{j\neq i}^{n} w_j\sigma_{j,t}\eta_{j,t+1}\right) + (1-\rho_i(1-w_i))\sigma_{i,t}\eta_{i,t+1} \end{aligned} \tag{35}$$

where $\nabla_{p,t} \equiv a_1\mu_2\sigma_{i,t} + a_2\nu_2\sigma_p$. It then follows from equation (35)

$$\mathrm{Var}_t(r_{i,t+1}) \quad \approx \quad \rho_i^2\nabla_{p,t}^2 + \rho_i^2\sum_{j\neq i}^{n} w_j^2\sigma_{j,t}^2 + (1-\rho_i(1-w_i))^2\sigma_{i,t}^2 \tag{36}$$

Using equations (28), (35) and (36) into (34) yields

$$\begin{aligned} \mathbb{E}_t[r_{i,t+1} - r_{f,t}] \quad &\approx \quad -\rho_i\left(\sum_{j\neq i}^{n} w_j^2\sigma_{j,t}^2\right)\left(\lambda_{m,q} + \frac{\rho_i}{2}\right) \\ &- \quad \left(\lambda_{m,q}w_i + \frac{1}{2}(1-\rho_i(1-w_i))\right)(1-\rho_i(1-w_i))\,\sigma_{i,t}^2 \\ &- \quad \rho_i\nabla_{p,t}\left(\lambda_{m,p}\Delta_{p,t} + \frac{\rho_i}{2}\nabla_{p,t}\right) \end{aligned} \tag{37}$$

Topology of $\mathcal{G}_n$ and the Cross-Section of Risk Premia: Let e_i denote a measure of centrality of firm i in $\mathcal{G}_n$. For example, e_i may represent a firm degree, closeness, betweenness or eigenvector centrality. Differentiating equation (37) with respect to e_i yields

$$\begin{aligned}\frac{\partial \mathbb{E}_t[r_{i,t+1}-r_{f,t}]}{\partial e_i} &\approx -2\rho_i\left(\sum_{j\neq i}^{n} w_j\sigma_{j,t}\frac{\partial \sigma_{j,t}}{\partial e_i}\right)\left(\lambda_{m,q}+\frac{\rho_i}{2}\right)\\ &- 2\left(\lambda_{m,q}w_i+\frac{1}{2}(1-\rho_i(1-w_i))\right)(1-\rho_i(1-w_i))\,\sigma_{i,t}\frac{\partial \sigma_{i,t}}{\partial e_i}\\ &- \rho_i\lambda_{m,p}\nabla_{p,t}\frac{\partial \Delta_{p,t}}{\partial e_i}-\rho_i\left(\lambda_{m,p}\Delta_{p,t}+\rho_i\nabla_{p,t}\right)\frac{\partial \nabla_{p,t}}{\partial e_i} \qquad (38)\end{aligned}$$

where $\frac{\partial \nabla_{p,t}}{\partial e_i}=a_1\mu_2\frac{\partial \sigma_{i,t}}{\partial e_i}$ and $\frac{\partial \Delta_{p,t}}{\partial e_i}=a_1\mu_2\left(\sum_{k=1}^{n} w_k\frac{\partial \sigma_{k,t}}{\partial e_i}\right)$.

As Bansal and Yaron (2004), consider $\gamma=10$ and $\psi=1.5$. Thus, $a_1>0$ and $\theta<0$. As a consequence,

- $\lambda_{m,p}<0$
- $\lambda_{m,q}<0$
- $\lambda_{m,q}+\frac{\rho_i}{2}<0$
- $\lambda_{m,q}w_i+\frac{1}{2}(1-\rho_i(1-w_i))<0$

If either μ_2, ν_2 or σ_p are sufficiently large such that $a_2>0$ then

- $\nabla_{p,t}>0$
- $\lambda_{m,p}\Delta_{p,t}+\rho_i\nabla_{p,t}<0$

Consider further that $\sigma_{k,t}$ are weakly increasing functions of e_i, $\forall\ k\in\{1,\cdots,n\}$. If the following sum

$$- \quad 2\rho_i\left(\sum_{j\neq i}^{n} w_j\sigma_{j,t}\frac{\partial \sigma_{j,t}}{\partial e_i}\right)\left(\lambda_{m,q}+\frac{\rho_i}{2}\right)-\rho_i\lambda_{m,p}\nabla_{p,t}a_1\mu_2\left(\sum_{j\neq i}^{n} w_j\frac{\partial \sigma_{j,t}}{\partial e_i}\right)$$

is greater than

$$\begin{aligned}&- \quad 2\left(\lambda_{m,q}w_i+\frac{1}{2}(1-\rho_i(1-w_i))\right)(1-\rho_i(1-w_i))\,\sigma_{i,t}\frac{\partial \sigma_{i,t}}{\partial e_i}-\rho_i\lambda_{m,p}\nabla_{p,t}a_1\mu_2 w_i\frac{\partial \sigma_{i,t}}{\partial e_i}\\ &- \quad \rho_i a_1\mu_2\frac{\partial \sigma_{i,t}}{\partial e_i}\left(\lambda_{m,p}\Delta_{p,t}+\rho_i\nabla_{p,t}\right)\end{aligned}$$

Then $\frac{\partial \mathbb{E}_t[r_{i,t+1}-r_{f,t}]}{\partial e_i}\geq 0$. If the above inequality holds, then firm i is more procyclical than firms with centrality scores smaller than e_i, because shocks to firm i tend to affect a higher number of firms in the economy than do shocks to firms with scores smaller than e_i. In such an environment, an increase on firm i's centrality increases the effect that firm i plays on aggregate volatility, which is measured by terms $\left(\sum_{j\neq i}^{n} w_j\sigma_{j,t}\frac{\partial \sigma_{j,t}}{\partial e_i}\right)$ and $\left(\sum_{j\neq i}^{n} w_j\frac{\partial \sigma_{j,t}}{\partial e_i}\right)$. The increase in risk tends to overcompensate the increase in firm i's growth opportunities. On the other hand, firms with small e_i tend to be less procyclical than firms with large e_i, and thus they serve as a hedge to aggregate risk.

Appendix D: Tables and Figures

This section contains the tables and figures mentioned in the paper and in the appendix.

Table I
Critical probability for different symmetric network topologies

The table reports critical probabilities for different symmetric network topologies. Besides reporting the two examples described in Appendix A, the table reproduces a subset of the values reported in Stauffer and Aharony (1994, Table 1). The first column reports the topology of $\mathcal{G}_\infty$. The second column reports the number of neighbors of any given node in $\mathcal{G}_\infty$. The third column reports the critical probability, $p_c(\mathcal{G}_\infty)$. Despite that $\mathcal{G}_\infty$ may be highly connected, if $\widetilde{p}_t < p_c(\mathcal{G}_\infty)$ then no infinite component emerges as $n \to \infty$, and thus $\sqrt{n}\widetilde{W}_n$ is asymptotically normally distributed. For illustrative purposes, figure 2(a) depicts a 2D Honeycomb lattice, figure 2(b) depicts a 2D Squared lattice; figure 2(c) depicts a 2D Triangular lattice and figure 2(d) depitcs a Bethe lattice with $z = 3$. The Bethe lattice of degree z is defined as an infinite tree in which any node has degree z. For n finite such topologies are called Cayley Trees.

Topology of $\mathcal{G}_\infty$	**Number of neighbors**	$p_c(\mathcal{G}_\infty)$
Infinite Line (1D lattice)	2	1
Infinite Circle	2	1
2D Honeycomb lattice	3	$1 - 2\sin\left(\frac{\pi}{18}\right)$
2D Squared lattice	4	$\frac{1}{2}$
2D Triangular lattice	6	$2\sin\left(\frac{\pi}{18}\right)$
Bethe lattice	z	$\frac{1}{z-1}$

Table II
Characteristics of Customer-Supplier Networks

The table reports characteristics of customer-supplier networks generated at the annual frequency using the Cohen and Frazzini (2008) dataset from 1980 to 2004. Two firms are connected in the network of year t if one of them represents at least 20% of the other firm's sales during year t. The number of components (clusters) in each network is computed via two consecutive depth-first searches. Provided that degree distributions exhibit fat tails, one can approximate them via power law distributions at least in the upper tail. Namely, the probability of a given degree d in the network of year t, $\mathbb{P}^t(d)$, can be expressed as $\mathbb{P}^t(d) = a_t d^{-\xi_t}$, where $a_t > 0$ and $\xi_t > 1$ are parameters to be estimated. The last row shows the average and standard deviation of the MLE estimators for ξ_t, over the sample period.

Characteristic	**Mean**	**Standard Deviation**
Number of firms per customer-supplier network	388	178
Number of relationships per customer-supplier network	281	154
Number of components per network	122	47
Size of the largest component	30	13
Size of the second largest component	24	11
Size of the third largest component	12	8
Size of the fourth largest component	9	7
Size of the fifth largest component	6	4
Exponent of fitted power law to the degree distribution	3.06	0.28

Table III
Benchmark Parameterization

The table reports the list of parameter values in the benchmark parametrization. I set $\bar{c} = 0.019/12$ and $\bar{x} = 0.038/12$ so that the unconditional means of consumption and dividend growth generated by the benchmark economy are similar to the ones found in data. I follow Bansal and Yaron (2004) and I set $\tau = 3$. I set $\sigma_x = 0.0262$ to match the volatility of dividends. I divide the rest of parameter values into three groups. Parameters in the first group define the preferences of the representative investor: β represents the time discount factor; γ represents the coefficient of relative risk aversion for static gambles; and ρ represents the inverse of the inter-temporal elasticity of substitution. Parameters in the second group describe firms' cash-flows: α_0 measures the part of firms' cash-flows unrelated to inter-firm relationships; α_1 measures the marginal benefit a firm receives from each relationship; and α_2 measures the decrease in a firm's cash-flow if a firm faces a negative shock. Given a network topology, parameters in the third group define the stochastic process that determines the propagation of shocks within the network economy: p_L and p_H are the values that the propensity of relationships to transmit negative shocks; q measures how frequently firms face negative idiosyncratic shocks; ψ measures how frequently relationships exhibit high propensity to transmit negative shocks; and ϕ measures the persistence of the stochastic process followed by $\widetilde{p}_t$.

Preferences			**Firms' Cash-flows**			**Propagation of shocks**				
β	γ	ρ	α_0	α_1	α_2	p_L	p_H	q	ψ	ϕ
0.997	10	0.65	0.3	0.1	0.07	0.38	0.45	0.2	0.5	0.925

Table IV
Moments under the Benchmark Parameterization

The table reports the first two moments of consumption and dividend growth as well as a set of key asset pricing moments. Column **Data** reports moments found in data. Column **Model** reports moments generated under the benchmark parametrization described in Table III. Column **BY2004** reports moments generated under the Long-Run Risks Model of Bansal and Yaron (2004). Data on consumption and dividends is obtained from Robert Shiller's website http://www.econ.yale.edu/ shiller/data.htm. Moments on the return on aggregate wealth, risk-free rate, equity premium and Sharpe ratio are based on data from 1928 to 2014 and obtained from Aswath Damodaran's website: http://pages.stern.nyu.edu/∼adamodar/. The annual return on aggregate wealth is approximated by the annual return of the S&P 500 while the yield on three month T-bills is used to proxy for the return on the risk-free asset.

Moments	**Data**	**Model**	**BY2004**
Average annual log of consumption growth rate	1.9%	1.9%	1.8%
Annual volatility of log consumption rate	3.5%	4.7%	2.8%
Average annual log dividend growth rate	3.8%	3.8%	1.8%
Annual volatility of the log dividend growth rate	11.63%	14.9%	12.3%
Average annual market return (S&P 500)	11.53%	12%	7.2%
Annual volatility of the market return	19%	18.92%	19.42%
Average annual risk-free rate (3 month T-Bill)	3.53%	2.16%	0.86%
Annual volatility of risk-free rate	3%	0.7%	0.97%
Average annual equity risk premium	8%	10%	6.33%
Average annual Sharpe ratio	0.4	0.52	0.33

Table V
Similarities between the calibrated model and the LRR model

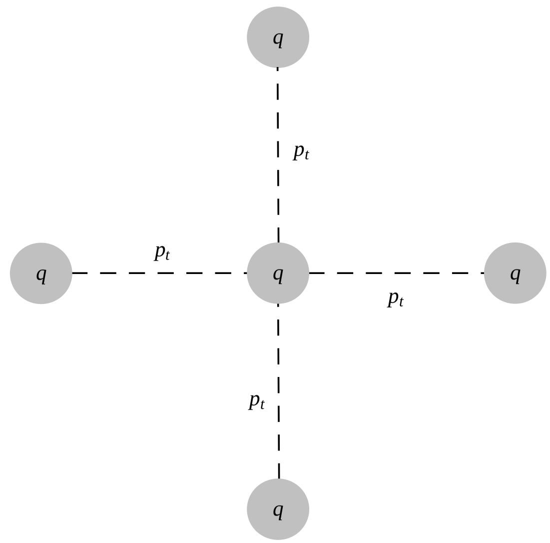
The table reports averages and standard deviations of similarity measures between time series generated with either the calibrated model or the benchmark parameterization in the LRR model of Bansal and Yaron (2004). To compute these measures I assume that the propensity of inter-firm relationships to transmit negative shocks follows: $\widetilde{p}_{t+1} = 0.4 + 0.925(\widetilde{p}_t - 0.4) + 0.006\epsilon_{t+1}$, where ϵ_{t+1} is standard normal and i.i.d over time. Such an AR(1) process can be approximated by the 2-states Markov chain followed by $\widetilde{p}_t$ in the benchmark parameterization. To compute averages and standard deviations, I sample from the calibrated model and the LRR model to construct two finite-sample empirical distributions for each similarity measure: one for expected consumption growth, $\mathbb{E}_t\left[\Delta\widetilde{c}_{t+1}\right]$, and one for the conditional volatility of consumption growth, $\mathrm{Vol}_t\left[\Delta\widetilde{c}_{t+1}\right]$. Reported values are based on 300 simulated samples over 620 periods. The first 500 periods in each sample are disregarded to eliminate bias coming from the initial condition. All similarity measures report scores computed as $\frac{1}{1+distance}$, where *distance* is defined according to each similarity measure. Let $\mathbf{X}_T = (X_1, \cdots, X_T)$ and $\mathbf{Y}_T = (Y_1, \cdots, Y_T)$ denote realizations from two time series, $X = \{X_t\}$ and $Y = \{Y_t\}$. The first and second similarity measures focus on the proximity between X and Y at specific points of time. The euclidean distance (ED) is defined as $\sqrt{\sum_{t=1}^{T}(X_t - Y_t)^2}$, whereas the dynamic time warping (DTW) distance is defined as $\min_r\left(\sum_{i=1}^{m}|X_{a_i} - Y_{b_i}|\right)$, where $r = ((X_{a_1}, Y_{b_1}), \cdots, (X_{a_m}, Y_{b_m}))$ is a sequence of m pairs that preserves the order of observations, i.e. $a_i < a_j$ and $b_i < b_j$ if $j > i$. DTW seeks to find a mapping such that the distance between X and Y is minimized. This way of computing distance allows two time series that are similar but locally out of phase to align in a non-linear manner. The third measure focuses on correlation-based distances. It uses the partial autocorrelation function (PACF) to define distance between time series. In particular, distance is defined as $\sqrt{(\hat{\rho}_{X_t} - \hat{\rho}_{Y_t})'\Omega(\hat{\rho}_{X_t} - \hat{\rho}_{Y_t})}$ where Ω is a matrix of weights, whereas $\hat{\rho}_{X_t}$ and $\hat{\rho}_{Y_t}$ are the estimated partial autocorrelations of X and Y, respectively. The fourth and fifth measures assume that an specific model generates both time series. The idea is to fit the specific model to each time series and then measure the dissimilarity between the fitted models. The fourth measure computes the distance between two time series as the Euclidean distance between the truncated AR operators. In this case, distance is defined as $\sqrt{\sum_{j=1}^{k}(e_{j,X_t} - e_{j,Y_t})^2}$ where $e_{X_t} = (e_{1,X_t}, \cdots, e_{k,X_t})$ and $e_{Y_t} = (e_{1,Y_t}, \cdots, e_{k,Y_t})$ denote the vectors of $AR(k)$ parameter estimators for X and Y, respectively. The fifth measure computes dissimilarity between two time series in terms of their linear predictive coding in ARIMA processes as in Kalpakis, Gada, and Puttagunta (2001). The last measure defines distance based on nonparametric spectral estimators. Let f_{X_T} and f_{Y_T} denote the spectral densities of X_T and Y_T, respectively. In this case, the dissimilarity measure is given by a nonparametric statistic that checks the equality of the log-spectra of the two time series. It defines distance as $\sum_{k=1}^{n}\left[Z_k - \hat{\mu}(\lambda_k) - 2\log(1 + e^{Z_k - \hat{\mu}(\lambda_k)})\right] - \sum_{k=1}^{n}\left[Z_k - 2\log(1 + e^{Z_k})\right]$, where $Z_k = \log(I_{X_T}(\lambda_k)) - \log(I_{Y_T}(\lambda_k))$, and $\hat{\mu}(\lambda_k)$ is the local maximum log-likelihood estimator of $\mu(\lambda_k) = \log(f_{X_T}(\lambda_k)) - \log(f_{Y_T}(\lambda_k))$ computed with local lineal smoothers of the periodograms. All similarity measures are computed using the **R** package TSclust (see Montero and Vilar (2014)).

	$\mathbb{E}_t\left[\Delta\widetilde{c}_{t+1}\right]$		$\mathrm{Vol}_t\left[\Delta\widetilde{c}_{t+1}\right]$	
Similarity measure	**Mean**	**Standard Deviation**	**Mean**	**Standard Deviation**
Euclidean Distance (ED)	0.958	0.012	0.974	0.008
Dynamic Time Warping	0.758	0.091	0.723	0.105
PACF	0.736	0.043	0.743	0.043
ED in AR	0.908	0.100	0.910	0.097
Linear predictive in ARIMA	0.726	0.325	0.729	0.313
Spectral distance	1.0	0.000	1.0	0.000

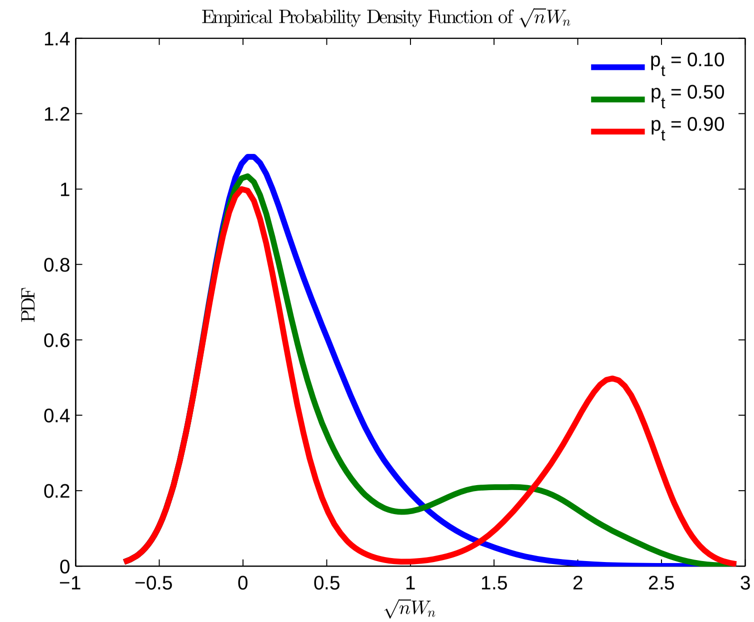
Table VI
Eigenvector Centrality Summary Statistics

The table reports averages of summary statistics for log(eigenvector centrality). To compute averages in the third and fourth columns, I use customer-supplier data on years 1982, 1987, 1992, 1997 and 2002 to be consistent with the years used by Ahern (2013). Using data reported in Ahern (2013, Internet Appendix Table II), the second column present averages of the statistics for log(eigenvector centrality) in inter-sectoral trade networks. The third column presents averages in annual customer supplier networks in which two firms are connected if one firm represents at least 10% of the other firm's annual sales. The fourth column presents averages in annual customer supplier networks in which two firms are connected if one firm represents at least 20% of the other firm's annual sales. The fifth column reports the statistics for log(eigenvector centrality) in the network of the calibrated economy.

Statistic	**Inter-sectoral Networks**	**Customer Supplier Networks** (10%)	**Customer Supplier Networks** (20%)	**Calibrated Network**
Number of sectors/firms	474	750	382	400
Mean	−6.68	−6.74	−6.62	−6.09
Standard Deviation	1.48	1.07	1.31	1.71
Skewness	0.87	4.04	3.28	1.54
Kurtosis	4.45	18.50	12.38	3.70
Minimum	−10.21	−7.01	−7.01	−7.01
1st Percentile	−9.39	−7.01	−7.01	−7.01
25th Percentile	−7.71	−7.01	−7.01	−7.01
Median	−6.85	−7.01	−7.01	−6.09
75th Percentile	−5.90	−7.01	−7.01	−6.42
99th	−2.27	−1.83	−1.67	−2.30
Maximum	−0.17	−0.46	−0.34	−0.74

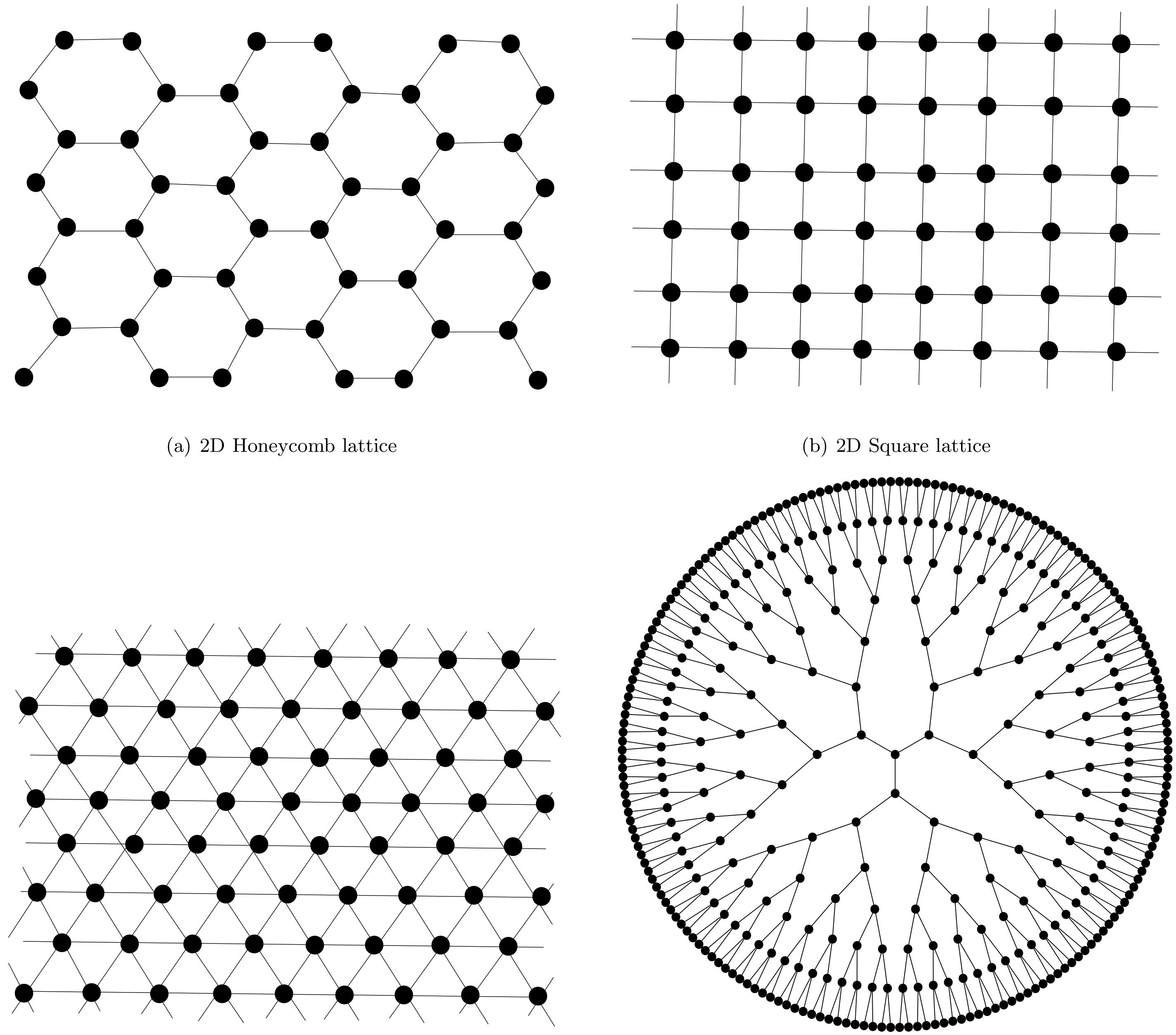


(a) Network Topology



(b) Empirical density function of $\sqrt{n}\widetilde{W}_{n,t}$

Figure 1. The figure illustrates how changes in the propensity of inter-firm relationships to transmit shocks at t, $\widetilde{p}_t$, impact the distribution of $\sqrt{n}\widetilde{W}_{n,t}$. Figure 1(a) depicts an economy with $n = 5$ firms, whereas figure 1(b) depicts estimates of the density function of $\sqrt{n}\widetilde{W}_{n,t}$ for different values of $\widetilde{p}_t$. These estimates are computed via normal kernel smoothing estimators using function ksdensity($\cdot$) in MATLAB.



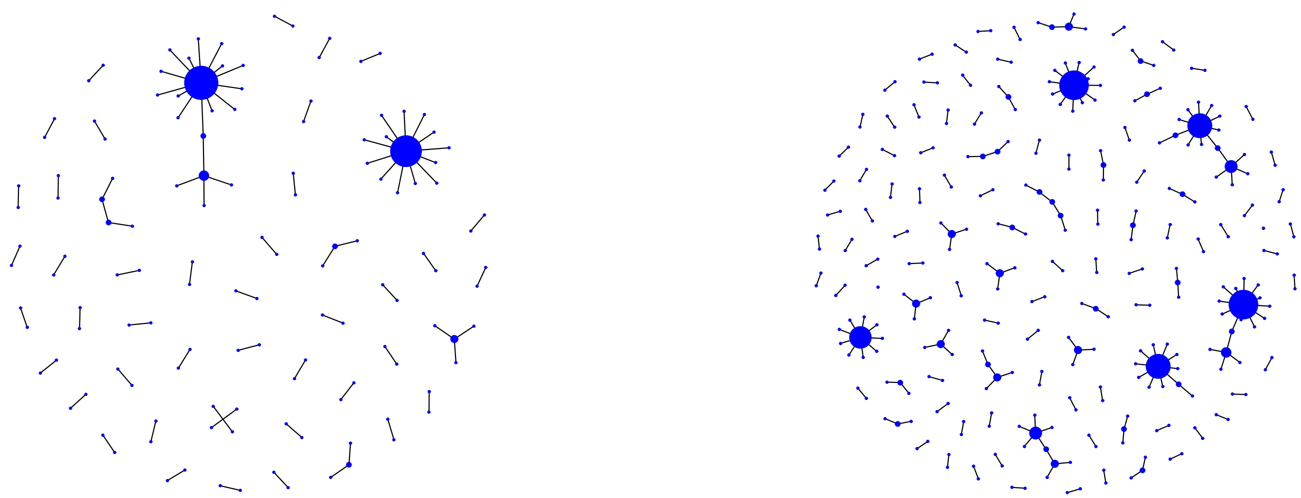
(a) 2D Honeycomb lattice

(b) 2D Square lattice

(c) 2D Triangular lattice

(d) Bethe lattice with degree 3

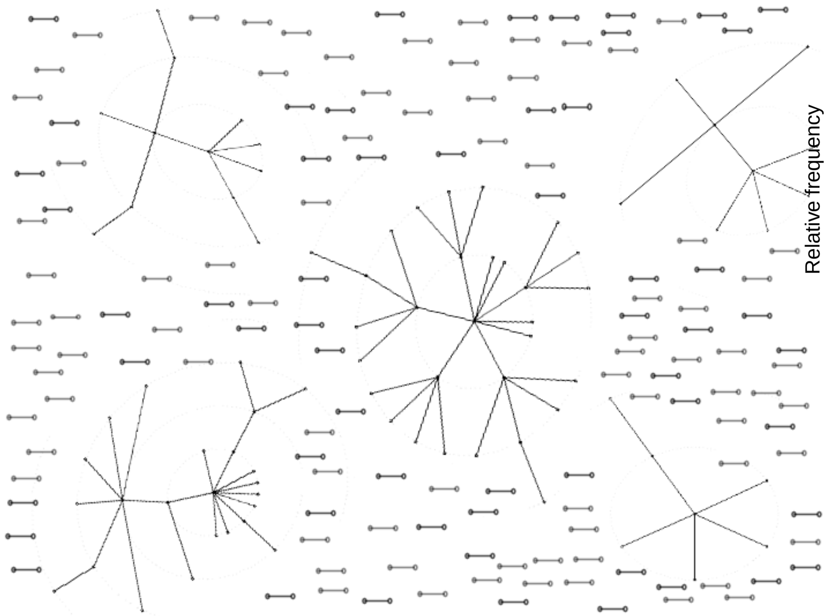
Figure 2. The figure shows the topologies of the symmetric networks considered in corollary 1 and Table I.



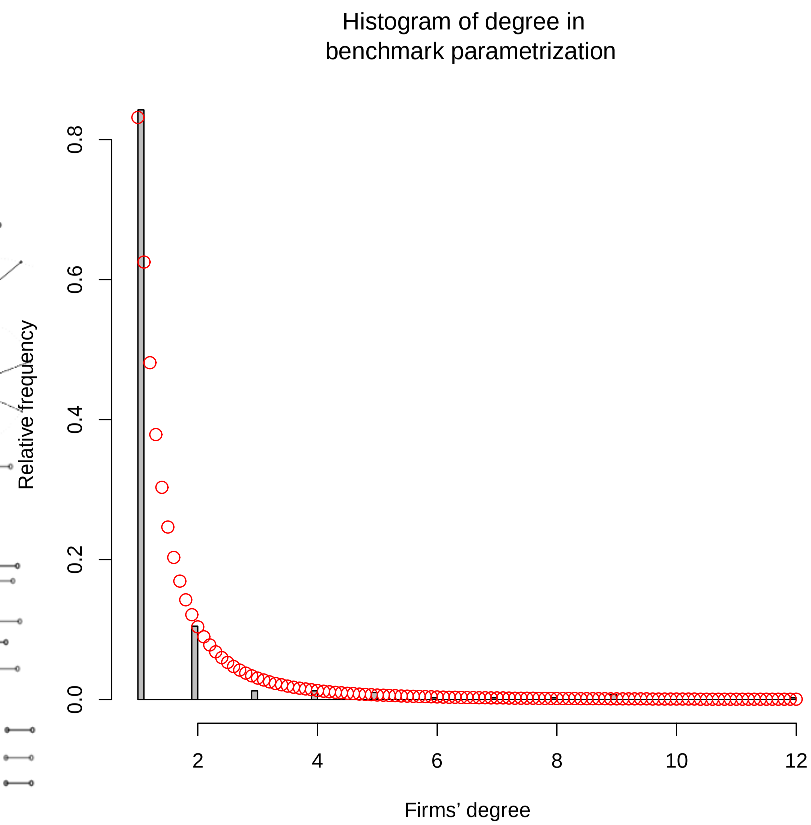
(a) Customer-Supplier Network in 1980

(b) Customer-Supplier Network in 1986

Figure 3. Customer-Supplier Networks in 1980 and 1986. Nodes represent firms while lines represent customer-supplier relationships above the 20% cutoff. The size of each node is proportional to its number of relationships (degree). Figure 3(a) consists of 131 firms and 84 inter-firm relationships. The three firms with the highest degree are: Ford, Sears and K-Mart. Figure 3(b) consists of 316 firms and 213 inter-firm relationships. The three firms with the highest degree are: Medicare-Glaser, Ford and Sears.



(a) Topology of inter-firm relationships network



(b) Degree distribution of inter-firm relationships network

Figure 4. Network of inter-firm relationships under benchmark parametrization. Figure 4(a) plots the topology of the network under the benchmark parameterization. The size of the five largest components match the average size of the five largest components in customer-supplier networks. Figure 4(b) plots the degree distribution under benchmark parametrization (shown with bars). Dots represent a power law distribution with exponent 3.

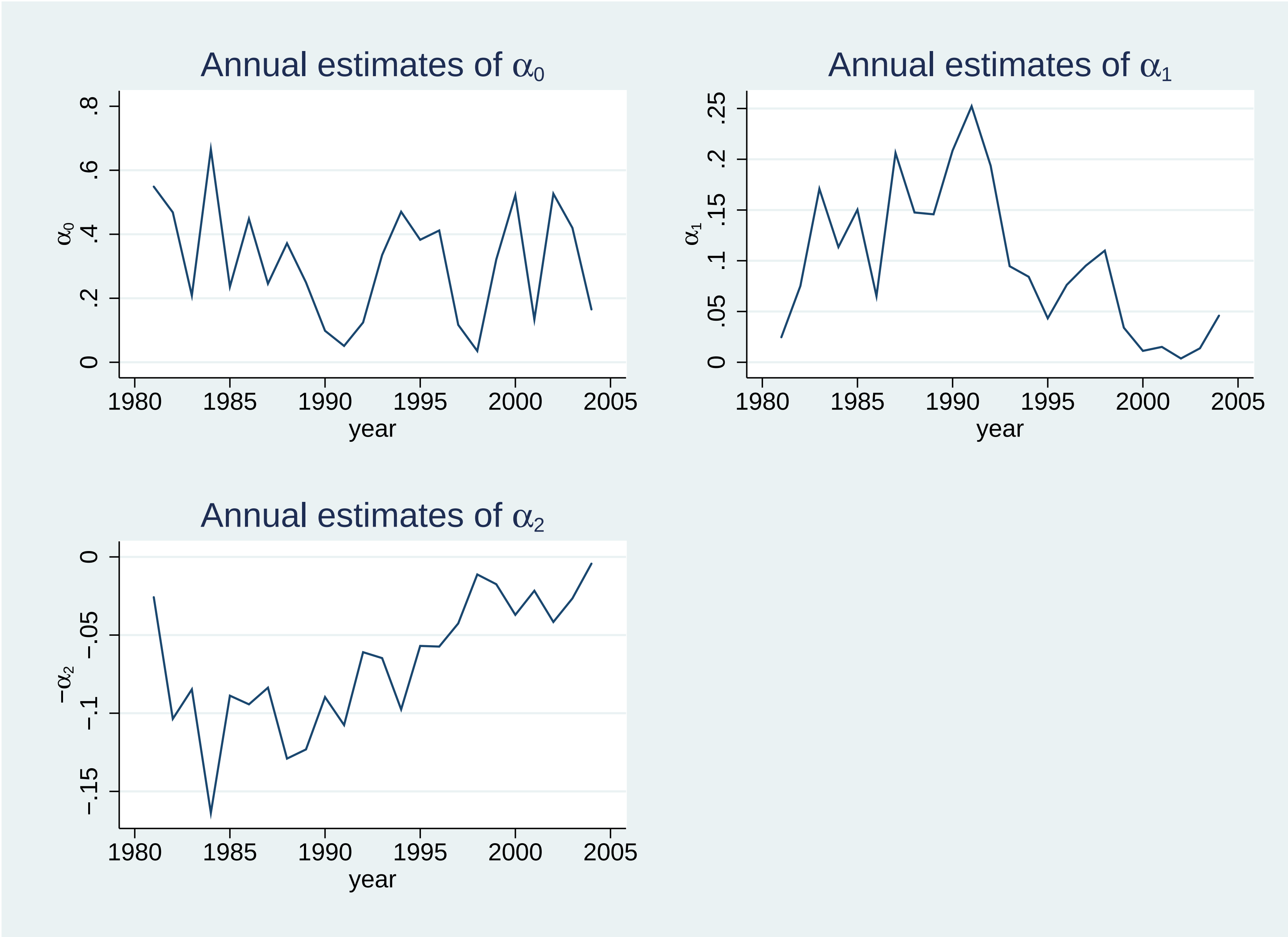
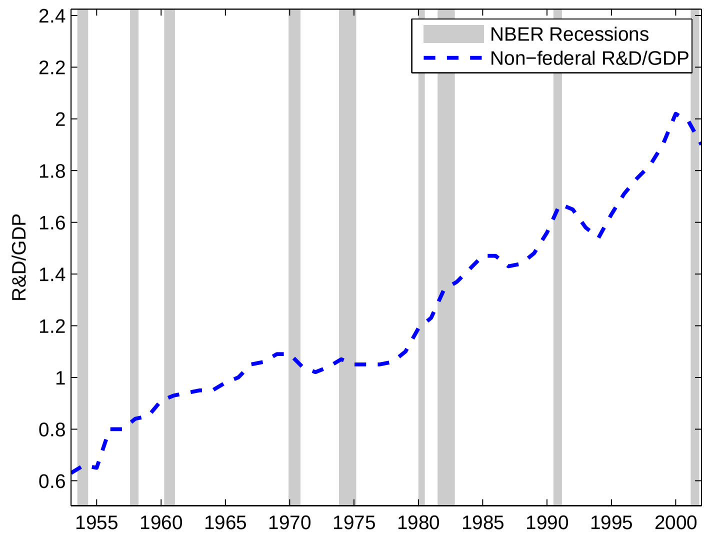
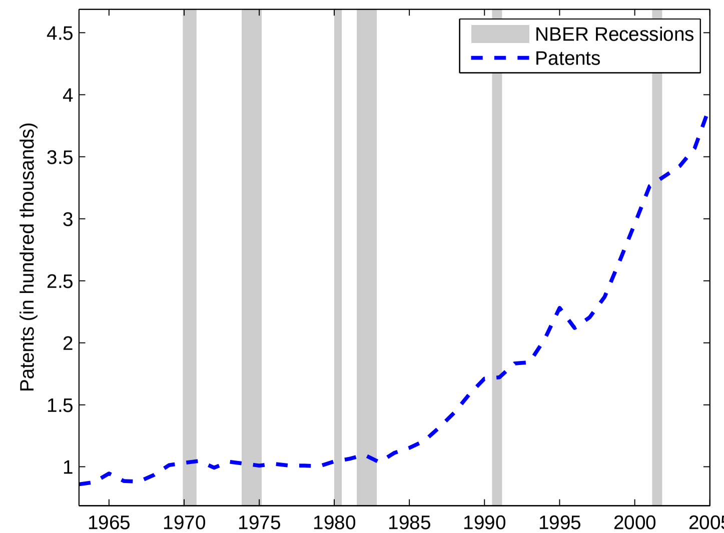


Figure 5. The figure shows annual estimates of parameters α_0, α_1 and α_2 in equation (1). I estimate these parameters using data on earnings per share (EPS) from COMPUSTAT to proxy for $\{y_{i,t}\}_{i=1}^{n}$.

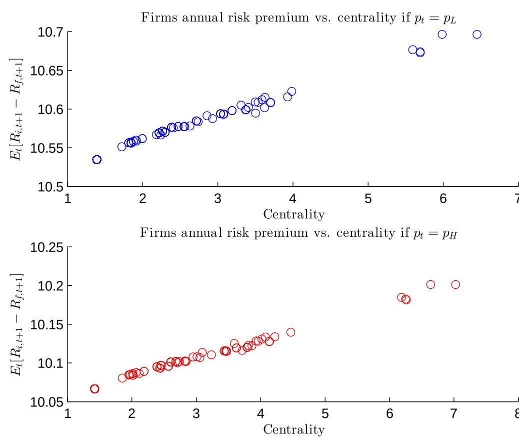


(a) Non federally funded R&D/GDP

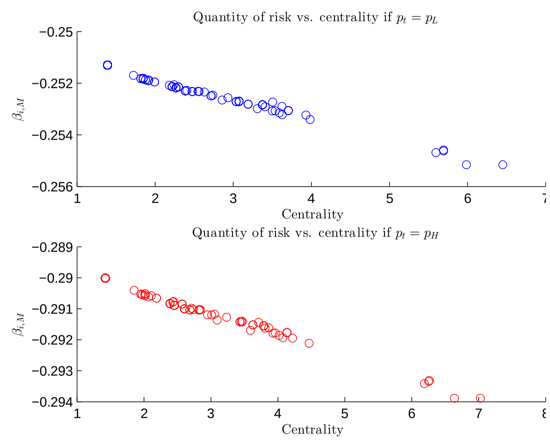


(b) Number of patents created in U.S.

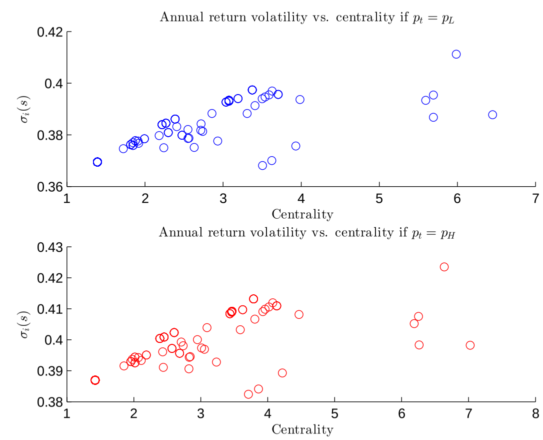
Figure 6. The figure shows the time series of R&D/GDP and the number of patents created in the U.S. Figure 6(a) depicts non-federal ratios of Research and Development (R&D) to GDP from 1953 to 2002 in the U.S. and NBER recessions. Figure 6(b) plots the number of patents created in the U.S. from 1963 to 2009 and NBER recessions. Source: National Science Foundation, http://www.nsf.gov/statistics/.



(a) Firms' risk premium

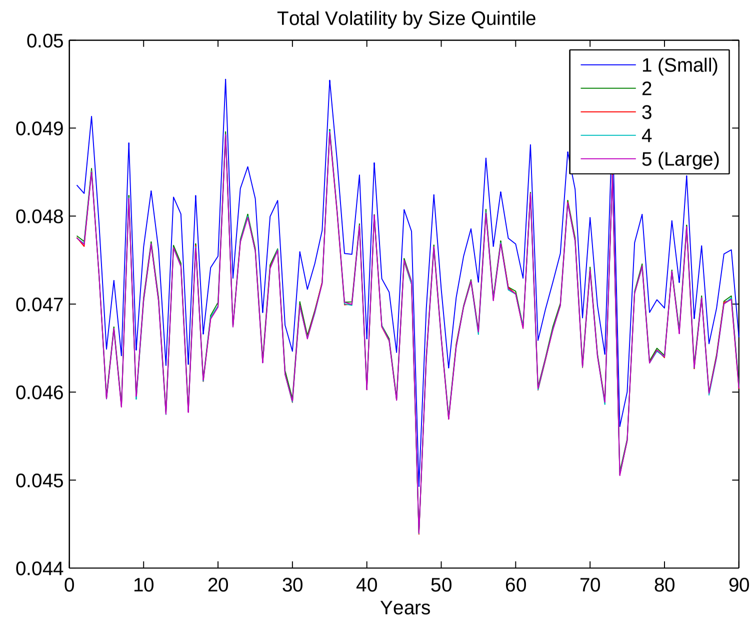


(b) Firms' quantity of risk

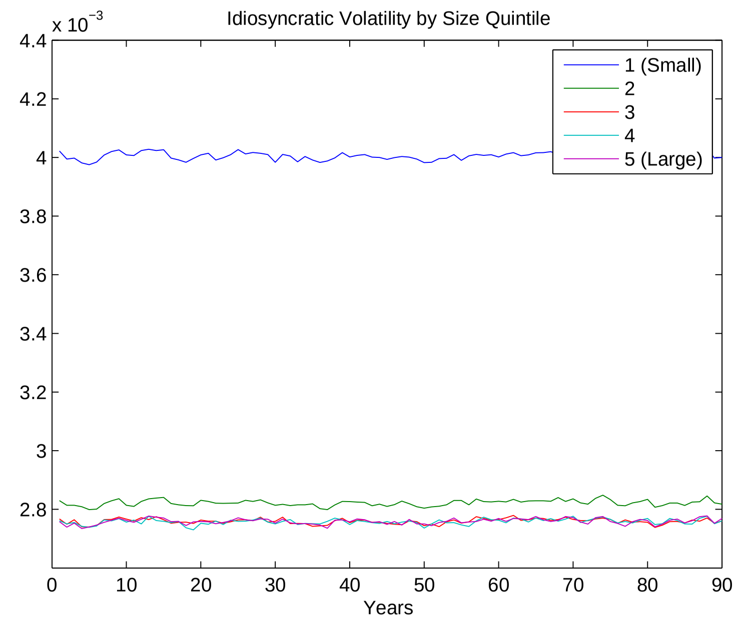


(c) Firms' return volatility

Figure 7. The figure plots firms' conditional risk premium, conditional quantities of risk and conditional annualized return volatilities as a function of firms' centrality in the network. Upper panels show economies in which the propensity $\widetilde{p}_t$ attains p_L whereas bottom panels show economies in which the propensity $\widetilde{p}_t$ attains p_H. Figure 7(a) plots firms' (conditional) annual risk premium as a function of firms centrality. Figure 7(b) plots firms' (conditional) quantity of risk, $\beta_{i,\widetilde{M}}$, as a function of firms' centrality. Figure 7(c) plots firms' (conditional) annual return volatility as a function of firms' centrality.



(a) Total Volatility by Size Quintile



(b) Idiosyncratic Volatility by Size Quintile

Figure 8. Annualized firm level volatilities averaged within size quintiles. I simulate 200 panels with 400 firms over 1,500 periods and disregard the first 500 periods to eliminate biases coming from the initial condition. Within each panel, I compute firm level total volatility as the annualized standard deviation of monthly firm level realized returns. I construct firm level idiosyncratic volatility as the annualized standard deviation of residuals computed from monthly CAPM regressions of firm level excess realized returns on the excess realized return of the market portfolio. This procedure yields panels of firm-year total and idiosyncratic volatilities estimates. Then, at the beginning of each year I sort firms based on size and average annual volatilities within size quintiles. This procedure yields five time series of total and idiosyncratic volatilities per panel. Figure 8(a) plots total volatilities per quintile whereas figure 8(b) plots idiosyncratic volatilities per quintile averaged over the 200 panels.