

The Geography of Beliefs*

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Abstract

Empirical evidence shows that beliefs of households deviate from rational expectations and instead may be influenced by characteristics such as place of residence, culture, and socioeconomic status, which can be modeled using network theory. We develop a model where a household's beliefs about stock returns are an *endogenous* outcome of its location in a bipartite network of households and firms. We use this model to establish the relation between households' beliefs about stock returns, where the beliefs are unobservable, and the portfolio weights allocated to these stocks by these households. We use Finnish data for 125 stocks and the portfolio holdings of 405,628 households to estimate our model and find that household-firm distance in the network has a statistically and economically significant effect on household beliefs about firm-level stock returns. Our estimates show that agents are connected to firms within a radius of about 143 miles from where they live, and geography has a strong effect on beliefs: a one standard deviation decrease in an agent's distance to a firm's headquarters predicts an increase in portfolio holdings by 165%. Our work provides a belief-based microfoundation for the bias toward local stocks that has been documented empirically.

Keywords: household finance, behavioral finance, networks.

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1 Introduction and Motivation

Rational expectations is a mathematically convenient fiction beloved of economists. However, “the rational expectation hypothesis is strongly rejected” (Landier, Ma, and Thesmar, 2017). This finding necessitates taking a stance on how to deviate from rational expectations, while still imposing rigor on the manner in which beliefs are formed. In this paper, we develop a theoretical framework to demonstrate how one can deviate from rational expectations in a disciplined fashion, and then establish empirically using data on Finnish households and firms that this framework performs well in explaining belief formation.

In the framework we develop, households’ beliefs are derived *endogenously* based on their location relative to firms within a bipartite network.¹ Networks are ubiquitous in modern economies: “Networks determine our information, influence our opinions, and shape our political attitudes” (Acemoglu and Ozdaglar, 2009). Substantial evidence shows that individuals’ beliefs are biased by location in a broad sense of the term: concrete geographical neighborhood or a more abstract positional descriptor such as culture, economic status, and social standing. For example, Das, Kuhnen, and Nagel (2017) show that a person’s socio-economic status influences their beliefs about macroeconomic variables; Guiso, Sapienza, and Zingales (2006) describe the direct impact of culture on beliefs; Kuchler and Zafar (2018) find that when individuals form beliefs about aggregate house prices they overweight house-price observations from their local area; Shive (2010) and Bailey, Cao, Kuchler, and Stroebel (2018) find that investors’ expectations are influenced by the experiences of other people within their social network.²

In our framework, each household regards its benchmark model as an approximation. Households believe that the data come from an unknown member of a set of models where firm-level expected returns differ from those in its benchmark model; i.e. each alternative model is characterized by an alternative probability measure. The household’s concern

¹Bipartite networks are a particular class of networks, whose nodes are divided into two sets, and only connections between nodes in different sets are allowed. For an introduction to the theory of networks, see the books by Easley and Kleinberg (2010) and Jackson (2010).

²There is also a large complementary literature showing that beliefs are influenced by personal experiences *over time* as opposed to location; see, for example, Vissing-Jorgensen (2003), Kaustia and Knüpfer (2008, 2012), Choi, Laibson, Madrian, and Metrick (2009), Greenwood and Nagel (2009), Chiang, Hirshleifer, Qian, and Sherman (2011), Malmendier and Nagel (2011, 2015), and Knüpfer, Rantapuska, and Sarvimäki (2017).

about model misspecification induces it to prefer decision rules that work over the set of alternative probability measures as opposed to one specific probability measure. At the same time, there is an “information penalty” for deviating from the benchmark probability measure. The household’s information penalty depends on not just the alternative probability measure; it also depends on its network location, in contrast with relative entropy.³ For instance, if the household is located further from a particular firm, this reduces the size of the information penalty incurred for considering a given alternative probability measure. Hence, the further a household’s location from a particular firm, the closer to zero will be the household’s endogenous personal expectation for the firm’s risk premium (i.e., the stock return in excess of the risk-free).

We use the network structure to represent the geographical location—broadly interpreted to include also cultural, social, and linguistic distance—of each household relative to firms and use this to derive the beliefs or probability measure of the household. Once we have derived beliefs, we derive their implications for the portfolio decisions of each household. Because beliefs cannot be observed, in our empirical work we exploit portfolio holdings to infer these beliefs.

We consider a model with H heterogeneous households (each with its own beliefs) and N heterogeneous firms. As in Cox, Ingersoll, and Ross (1985), the physical capital of the N heterogeneous firms is subject to exogenous shocks. But, in contrast with Cox et al., we have heterogeneous households with Epstein and Zin (1989) and Weil (1990) preferences coupled with household-specific beliefs, as described above (and explained in detail below).

We solve this model in closed form and demonstrate how endogenous beliefs impact the portfolio choices of households. In our model, each household evaluates investment opportunities using its own personal beliefs, which depend on the household’s location within the network of firms. Consequently, no household holds the market portfolio; instead,

³The relative entropy of an alternative probability measure with respect to a benchmark measure is the standard way of quantifying the change in information when an alternative probability measure replaces the benchmark measure. As such, relative entropy is based purely on information and ignores network location. If we were to ignore the influence of network location on the size of the information penalty, then our framework would reduce to that in Hansen and Sargent (2007).

each household’s portfolio is biased because of its personal beliefs. We use this model to infer household beliefs, which are unobservable, from observations on portfolio holdings.

Our work is closely related to the seminal work of Huberman (2001) and Grinblatt and Keloharju (2001), who document that households tend to overinvest in firms that are “familiar” to them. Huberman (2001) shows that familiarity depends on geographical distance, while Grinblatt and Keloharju (2001) find that it can depend also on cultural and language distance.⁴ There is a large subsequent “local bias” literature that documents the tendency of households to invest in companies “close” to them. Massa and Simonov (2006) find evidence that households bias their portfolio toward stocks that are geographically or professionally close and argue that this bias is information driven. In contrast, Seasholes and Zhu (2010), Døskeland and Hvide (2011), Baltzer, Stolper, and Walter (2013, 2015) find that while portfolios are indeed biased toward local stocks, the local holdings do not generate abnormal performance. Bodnaruk (2009) provides compelling evidence of the effect of geographical distance by showing that as investors change their place of residence and thereby the distance from the companies they invest in, they also adjust their portfolio weights. In contrast to these studies that focus on individual investors, Coval and Moskowitz (1999b,a) find that professional fund managers also bias their holdings toward local stocks and earn abnormal returns from nearby investments. Pool, Stoffman, and Yonker (2012) also study professional U.S. mutual-fund managers and find that they bias their portfolios toward stocks from their home states; however, home-state stocks do not outperform other holdings, suggesting that home-state investments are not informed.

Our work builds on this literature on local bias in portfolio holdings by providing a *belief-based microfoundation* for the local bias documented in these papers. We find empirical support for our model, by testing novel predictions that arise within our framework. For instance, the main regression tests in Grinblatt and Keloharju (2001) arise naturally in our framework, but their firm fixed effects now have particular economic interpretations. We show that the firm fixed effects in their tests represent distributional properties of stock

⁴Lindblom, Mavruk, and Sjögren (2018) find that, in addition to local bias, individual investors who live in their birthplace invest almost three times more in local firms than other locals. Laudenbach, Malmendier, and Niessen-Ruenzi (2018) demonstrate that even decades after reunification, East Germans still invest significantly less in the stock market and are more likely to hold stocks of companies in communist countries (China, Russia, Vietnam), and less likely to invest in American companies and the financial sector.

returns (excess return per unit of variance risk), and postal-code fixed effects represent household preferences (risk aversion). Our belief-based model for their, and our, results is supported by the positive relation between the estimated excess return per unit of variance risk and out-of-sample firm performance. Novel empirical findings consistent with our model are the strong overlap between portfolio holdings of households within the same postal code and the strong relation between proximity and positive total portfolio holdings of a stock at the postal code level. Our empirical results are robust to various alternative specifications, and our modeling approach is flexible in allowing for belief-based explanations for other types of biases, e.g., cultural and linguistic barriers.

In particular, we test our model of belief formation using Finnish data on the portfolio holdings in 125 stocks of 405,628 households in 2,923 postal code areas. Using the portfolio holdings of Finnish households to infer their beliefs, we find that geographical household-firm distance within the network has a statistically and economically significant effect on the beliefs of households about firms' stock returns. We find that the sensitivity coefficient representing distance is highly statistically significant in all regressions—univariate regressions, regressions including risk-aversion fixed effects, regressions including risk-aversion and stock-characteristic fixed effects, and panel regressions with robust standard errors double clustered at the firm and postal code level. The results are also *economically* significant: a one standard deviation decrease in distance to a firm's headquarters predicts an increase in portfolio holdings by a factor of 2.645.

A possible concern with these results is that they may be driven by households in Helsinki, the main conurbation in Finland, and there may be differences in behavior between rural and urban households. To check if this is indeed the case, we run regressions *excluding* stocks and postal codes in the Helsinki area and find that the results are still both statistically and economically highly significant. Another potential concern is that it may not be geographical distance per se that drives beliefs, but rather employment; i.e. households may tend to invest in the firms they work for, which are also likely to be close to where they live. To investigate if this is the case, we *exclude* observations for which the household and firm headquarters are close to each other (under 8 miles in one specification

and under 24 miles in another specification). We find that the results remain qualitatively similar, and thus, an “employment” effect does not seem to be driving the results.

In addition to the literature on local bias already discussed above, our paper is related to two other literatures. The first is the literature on robust decision making in finance and economics, which is described in Hansen and Sargent (2007). The key idea we take from this literature is that decision makers are uncertain about the benchmark model they use to make decisions. Consequently, they consider a range of models around the benchmark and make decisions that are robust with respect to the worst-case model. At the same time, there is a penalty for deviating from the benchmark model. This penalty is the relative entropy of the probability measure for each model that is considered with respect to the probability measure of the benchmark model. Trojani and Vanini (2004) study the implications of model uncertainty for portfolio choice in continuous-time economies with heterogeneous investors. Gagliardini, Porchia, and Trojani (2008) provide an application of robust decision making in the Cox et al. (1985) model to study the implications for the term structure of interest rates. Bhandari, Borovička, and Ho (2019) develop a model where agents’ subjective beliefs are endogenous consequences of model misspecification.⁵ Our approach extends this literature by using a penalty that incorporates network effects, in particular the location of a household relative to firms.

Our paper is therefore related to the literature on networks in economics and finance. A growing literature explores the implications of network structure for economic variables such as aggregate fluctuations (Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi, 2012, Gabaix, 2011), systemic risk and financial stability (Acemoglu, Ozdaglar, and Tahbaz-Salehi, 2015, Farboodi, 2017), systematic risk and asset pricing (Ahern, 2013), return predictability (Cohen and Frazzini, 2008), mortgage default risk (Stanton, Walden, and Wallace, 2014, 2018), merger waves (Ahern and Harford, 2014), information acquisition (Herskovic and Ramos, 2017), and asset pricing implications of information diffusion through networks (Walden, 2019). Allen and Babus (2009) and Jackson (2010, 2014) provide comprehensive surveys

⁵In contrast, Klibanoff, Marinacci, and Mukerji (2005) develop a model of “smooth” ambiguity in which the investor uses a nonlinear function to evaluate expected utility values. Maccheroni, Marinacci, and Rustichini (2006) show that these preferences are nested in a wider class of preferences called “divergence preferences.” For surveys of this literature, see Hansen and Sargent (2007), Guidolin and Rinaldi (2009), Epstein and Schneider (2010), and Gilboa and Marinacci (2016).

of this literature. Our work contributes to this literature by showing how networks can influence beliefs.

The rest of this paper is organized as follows. We describe the main features of our model in Section 2. The choice problem of an individual household with subjective beliefs is solved in Section 3. We evaluate the predictions of the model empirically in Section 4. We conclude in Section 5. Proofs for all results and additional empirical results are reported in the appendices.

2 The Model

In this section, we develop a model of a finite number of firms and households in a stochastic dynamic equilibrium economy. The location of a household relative to all firms in the economy is described by a network structure. The beliefs of a household are determined by its location within the network and the network itself.

2.1 Firms

There are N firms indexed by $n \in \{1, \dots, N\}$. The value of the capital stock in each firm at date t is denoted by $K_{n,t}$ and the output flow by

$$Y_{n,t} = \alpha_n K_{n,t},$$

for some firm-specific technology level $\alpha_n > 0$. The level of a firm's capital stock can be increased by investment at the rate $I_{n,t}$. We thus have the following capital accumulation equation for an individual firm:

$$dK_{n,t} = I_{n,t} dt + \sigma_n K_{n,t} dZ_{n,t},$$

where σ_n , the volatility of the exogenous shock to a firm's capital stock, is constant. The term $dZ_{n,t}$ is the increment in a standard Brownian motion and is firm-specific; the correlation between $dZ_{n,t}$ and $dZ_{m,t}$ for $n \neq m$ is given by $0 < \rho < 1$, which is also assumed to be constant over time and the same for all pairs $n \neq m$. Firm-specific shocks create heterogeneity across firms. The $N \times N$ variance-covariance matrix of returns on firms' capital

stocks is given by $V = [V_{nm}]$ and Ω denotes the correlation matrix, where the elements of these matrices are

$$V_{nm} = \begin{cases} \sigma_n^2, & n = m, \\ \rho_{nm}\sigma_n\sigma_m, & n \neq m, \end{cases} \quad \text{and} \quad \Omega = \begin{cases} 1, & n = m, \\ \rho_{nm}, & n \neq m. \end{cases}$$

A firm's output flow is divided between its investment flow and dividend flow:

$$Y_{n,t} = I_{n,t} + D_{n,t}.$$

We can therefore rewrite the capital accumulation equation as

$$dK_{n,t} = (\alpha_n K_{n,t} - D_{n,t})dt + \sigma_n K_{n,t} dZ_{n,t}.$$

In the Cox et al. (1985) model, the expected return on a firm's physical capital, α_n , equals the return on its stock. Similarly, the volatility of the return on a firm's capital, σ_n , equals the volatility of the return on its stock.

2.2 The Investment Opportunities of Households

There are H households, indexed by $h \in \{1, \dots, H\}$. Households can invest their wealth in two classes of assets. The first is a risk-free asset, which has an interest rate i that we assume is constant over time. Let $B_{h,t}$ denote the stock of wealth invested by household h in the risk-free asset at date t . Then, the change in $B_{h,t}$ is given by

$$\frac{dB_{h,t}}{B_{h,t}} = i dt.$$

Additionally, households can invest in the N risky firms, or equivalently, in the stocks of these N firms. We denote by $K_{hn,t}$ the stock of household h 's wealth invested in the n 'th risky firm. Given that the household's wealth, $W_{h,t}$, is invested in the risk-free asset and the N risky firms, we have that:

$$W_{h,t} = B_{h,t} + \sum_{n=1}^N K_{hn,t}.$$

The proportion of a household's wealth invested in firm n is denoted by ω_{hn} , and so

$$K_{hn,t} = \omega_{hn} W_{h,t},$$

so that the amount of household h 's wealth invested in the risk-free asset is

$$B_{h,t} = \left(1 - \sum_{h=1}^N \omega_{hn}\right) W_{h,t}.$$

The dividends distributed by firm n are consumed by household h according to share of firm n that household h holds:

$$C_{hn,t} = D_{hn,t} = \frac{K_{hn,t}}{K_{n,t}} D_{n,t},$$

where $C_{hn,t}$ is the consumption rate of household h from the dividend flow of firm n . Hence, the dynamic budget constraint for household h is given by

$$\frac{dW_{h,t}}{W_{h,t}} = \left(1 - \sum_{n=1}^N \omega_{hn,t}\right) idt + \sum_{n=1}^N \omega_{hn,t} \left(\alpha_n dt + \sigma_n dZ_{n,t}\right) - \frac{C_{h,t}}{W_{h,t}} dt,$$

where $C_{h,t}$ is the consumption rate of household h and $C_{h,t} = \sum_{n=1}^N C_{hn,t}$.

2.3 Network Structure

We endogenize household beliefs by linking them to the proximity of a household to each firm in the economy. That is, beliefs are the consequence of differences in locations across households, which we represent via a network structure. First, we explain the underlying network structure. Second, we show how household beliefs are modeled. Third, we show how household beliefs are impacted by the location of a household within a network.

The separation of household h from firm n is denoted by d_{hn} . This could be a geographical distance or a more abstract measure of separation, such as cultural or linguistic distance. For example, an English-speaking household would be farther away from a German company based in Germany, than a German-speaking household that is located on the same street as the English-speaking household. We map the measure of separation, d_{hn} , into a measure of proximity, ϕ_{hn} , which is constrained to lie in the interval $[0, 1]$, by using

the following function

$$\phi_{hn} = \begin{cases} e^{-\kappa d_{hn}} & , d_{hn} \leq \bar{d}, \\ 0 & , d_{hn} > \bar{d}, \end{cases} \quad (1)$$

where $\kappa \geq 0$ is a measure of the sensitivity of ϕ_{hn} to d_{hn} and \bar{d} is a constant denoting some threshold value. Thus, $\phi_{hn} = 1$ when the separation measure for household h relative to firm n is 0 and $\phi_{hn} = 0$ when the separation measure for household h relative to firm n exceeds the threshold \bar{d} .

The exponential decrease in proximity as a function of distance in equation (1) is akin to a setting where multiplicative noise increases the uncertainty of information. Specifically, consider a random signal \tilde{x} that is transmitted from its origin to a recipient through a sequence of d steps (for example word-of-mouth communication in a social network), so that in each of the steps the signal is garbled with multiplicative, independently distributed, noise, $\tilde{x} \mapsto \tilde{x} \times \tilde{S}_1 \times \tilde{S}_2 \cdots \times \tilde{S}_d$, where \tilde{x} and \tilde{S}_i , $i = 1, \dots, d$, are log-normally distributed. The variance of the garbled signal will then increase exponentially with distance, i.e., the precision will decrease exponentially. The threshold in the specification is akin to a setting where background noise makes a signal uninformative beyond a certain distance. We therefore view the functional form for the proximity specification in equation (1) as natural and parsimonious.

In contrast with most existing work in finance, where a network consists of agents of one type, we have a network consisting of two types: households and firms. Such a network is known as a *bipartite* network. In our case, the bipartite network of households and firms is described via the H by N matrix \mathbb{D} , where

$$\mathbb{D} = [d_{hn}]_{hn}, \quad h \in \{1, \dots, H\}, n \in \{1, \dots, N\}.$$

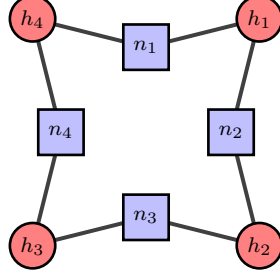
The matrix \mathbb{D} is called the biadjacency matrix of the bipartite network. From the biadjacency matrix of separation measures for household h , we obtain the proximity matrix

$$\Phi = [\phi_{hn}]_{hn}, \quad h \in \{1, \dots, H\}, n \in \{1, \dots, N\}.$$

Below, we provide two examples of bipartite networks. In our first example, illustrated in Figure 1, we assume the number of households equals the number of firms ($H = N$) and

Figure 1: First example of network

In this figure, we assume that the number of firms is equal to the number of households, $N = H = 4$, and that each household has a separation measure of $d \in (0, \bar{d})$ with respect to 2 firms and a separation measure of $\bar{d} + \epsilon$, where $\epsilon > 0$ with respect to all other firms.



that each household has a separation measure of $d \in (0, \bar{d})$ with respect to 2 firms and a separation measure of $\bar{d} + \epsilon$, where $\epsilon > 0$ with respect to all other firms. Let the firms be arranged in a circle, and let each household h be equally distant from the two firms nearest to it on either side. Thus, in this case the biadjacency matrix of separation measures is given by the following N by N matrix

$$\mathbb{D} = \begin{bmatrix} d & d & \bar{d} + \epsilon & \bar{d} + \epsilon & \cdots & \bar{d} + \epsilon \\ \bar{d} + \epsilon & d & d & \bar{d} + \epsilon & \cdots & \bar{d} + \epsilon \\ \vdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ \bar{d} + \epsilon & \bar{d} + \epsilon & \cdots & \bar{d} + \epsilon & d & d \\ d & \bar{d} + \epsilon & \cdots & \bar{d} + \epsilon & \bar{d} + \epsilon & d \end{bmatrix}.$$

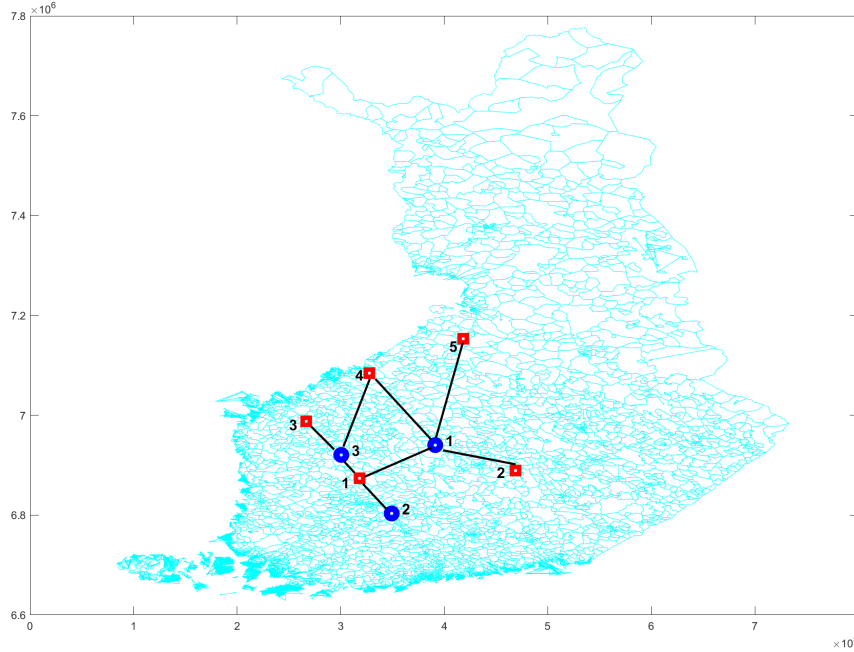
Hence, defining $\phi = e^{-\kappa d}$, the matrix of proximity measures is given by

$$\Phi = \begin{bmatrix} \phi & \phi & 0 & 0 & \cdots & 0 \\ 0 & \phi & \phi & 0 & \cdots & 0 \\ \vdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ 0 & 0 & \cdots & 0 & \phi & \phi \\ \phi & 0 & \cdots & 0 & 0 & \phi \end{bmatrix}.$$

In our second example, consider Figure 2 below, which shows a map of Finland (in cyan) decomposed into 3036 postal code regions. We will use this decomposition in our empirical tests in Section 4. The five red squares represent five households, situated at the points p_1, \dots, p_5 , and the three blue circles represent three firms situated at p^1, p^2 and p^3 .

Figure 2: Second example of network

In this figure, we assume that the number of firms $N = 3$, with these firms represented by the three blue circles and the number of households is $H = 5$, with these households represented by the five red squares. The underlying map (in cyan), shows 3,036 Finnish postal code regions, obtained from the Finnish postal services company, Posti Group Corporation.



Geographical distance is used to define the firm-household network, so that

$$d_{hn} = D((x_h, y_h), (x^n, y^n)) = \sqrt{(x_h - x^n)^2 + (y_h - y^n)^2},$$

where the Euclidean distance function D is normalized so that the two postal codes that are farthest away from each other (postal codes 2 and 3) are at unit distance, and $p_h = (x_h, y_h)$, $p^n = (x^n, y^n)$.

The constant is $\kappa = 0.9$, and the threshold is set to $\bar{d} = 0.5$. This leads to the following matrix of proximity measures

$$\Phi = \begin{bmatrix} 0.771 & 0.852 & 0.905 \\ 0.770 & 0 & 0 \\ 0 & 0 & 0.850 \\ 0.720 & 0 & 0.736 \\ 0.676 & 0 & 0 \end{bmatrix}.$$

The proximity function is the highest for household 1 and firm 3 ($\phi_{13} = 0.905$), because they represent the geographically closest household-firm pair. There are in total eight household-firm connections; the remaining firm-household pairs are farther apart than the threshold $\bar{d} = 0.5$, and therefore, are not connected.

To show the flexibility of our framework, we also consider a variation of this example with a more general distance function. The distance function is now affected not only by geographical distance, but also cultural distance, as suggested in Grinblatt and Keloharju (2001). Culture could, for example, be measured by the main language spoken by the firm's CEO. Assume that the CEOs' main language is represented by $\ell^n \in \{0, 1\}$, where 0 represents a Swedish-speaking CEO and 1 a Finnish-speaking CEO. We shall assume that $\ell^1 = \ell^2 = 1$, and $\ell^3 = 0$. Also, assume similarly that $\ell_h \in \{0, 1\}$ represents the predominant language spoken by households in the different postal codes. We shall assume that $\ell_1 = \ell_4 = 0$, while $\ell_2 = \ell_3 = \ell_5 = 1$.

Then the generalized distance function takes the form $d_{hn} = d(p_h, p^n) + c|\ell_h - \ell^n|$, where the constant $c = 0.25$, leading to the following matrix of proximity measures

$$\Phi = \begin{bmatrix} 0 & 0.680 & 0.905 \\ 0.770 & 0 & 0 \\ 0 & 0 & 0.679 \\ 0 & 0 & 0.736 \\ 0.676 & 0 & 0 \end{bmatrix}.$$

With the generalized distance function, the connections between firm 1 and households 1 and 4 no longer exist, $\phi_{11} = \phi_{41} = 0$, because the effect of their cultural distance is to increase the distance beyond the threshold $\bar{d} = 0.5$. The connections ϕ_{12} and ϕ_{31} also decrease, from 0.852 to 0.680 and from 0.850 to 0.679, respectively, but stay positive, i.e., the connections remain.

2.4 Beliefs of Households

Each household h has its own beliefs, represented by its personal probability measure \mathbb{Q}^h , which differs from the physical (objective) probability measure \mathbb{P} . We define the beliefs of household h below.

Definition 1. *If we consider an event A which can occur at time $T > t$, household h 's personal expectation that event A could occur, conditional on date- t information, is given by*

$$E_t^{\mathbb{Q}^{\nu_h}} [I_A] = E_t^{\mathbb{P}} \left[\frac{M_{h,T}}{M_{h,t}} I_A \right],$$

where I_A is the indicator function associated with event A , and $M_{h,t}$ is given by

$$\frac{dM_{h,t}}{M_{h,t}} = \boldsymbol{\nu}_{h,t}^\top (\Omega \boldsymbol{\sigma})^{-1} d\mathbf{Z}_t,$$

where $\boldsymbol{\sigma} = \text{diag}(\sigma_1, \dots, \sigma_N)$, $\mathbf{Z}_t = (Z_{1,t}, \dots, Z_{N,t})^\top$, and

$$\boldsymbol{\nu}_{h,t} = (\nu_{h1,t}, \dots, \nu_{hN,t})^\top \quad (2)$$

is the vector of divergences of household expectations from rational expectations.

To understand the intuition behind the above definition, observe that the expected rate of return for firm n 's stock under household h 's expectations is $\alpha_n + \nu_{hn,t}$. We can thus see that the divergence in household h 's beliefs about the expected rate of return on firm n (relative to the physical probability measure) is ν_{hn} . Collecting the divergences of household h for all N firms, we obtain its vector of personal divergences, as defined in equation (2). From Girsanov's Theorem, we know that choosing a vector of personal divergences is equivalent to a household changing the objective physical measure to a new measure, denoted by \mathbb{Q}^{ν_h} . When its vector of personal divergences is the zero vector, then household h 's personal beliefs coincide with the objective beliefs represented by \mathbb{P} .

In Definition 1, the only source of different beliefs between households is the vector of divergences, $\boldsymbol{\nu}_{h,t}$, which will be determined by a household's location. As a consequence, households in the same location will be predicted to hold identical portfolios. Moreover, as we shall see, households will neither overinvest nor shortsell stocks. These features are, of course, inconsistent with what is observed in practice, and are easily avoided with an extended specification. Specifically, we may replace $M_{h,t}$ in Definition (1) with the process

$$\frac{dM_{h,t}}{M_{h,t}} = (\boldsymbol{\nu}_{h,t} + \boldsymbol{\xi}_h)^\top (\Omega \boldsymbol{\sigma})^{-1} d\mathbf{Z}_t,$$

where

$$\boldsymbol{\xi}_h = (\xi_{h1}, \dots, \xi_{hN})^\top,$$

and ξ_{hn} are random variables with mean zero, i.i.d. across agents and stocks. The additional variation in individual household's expectations introduced by this extension generates further heterogeneity in individual household beliefs and holdings, but cancels out in aggregate in markets with many households. For simplicity, we use the base specification in the forthcoming derivation; the derivation under the extension is very similar.

We can use household h 's personal beliefs to understand how it forms expectations about the rate of return on its invested wealth. Household h 's personal belief about the expected rate of return on its wealth is $\sum_{n=1}^N \omega_{hn,t}(\alpha_n + \nu_{hn,t})$ instead of $\sum_{n=1}^N \omega_{hn,t}\alpha_n$, a change of

$$\sum_{n=1}^N \omega_{hn,t} \nu_{hn,t}$$

relative to the physical measure \mathbb{P} . This change can be regarded as a personal divergences in household h 's portfolio return expectations, which we can write more succinctly as

$$\boldsymbol{\omega}_{h,t}^\top \boldsymbol{\nu}_{h,t},$$

where $\boldsymbol{\omega}_{h,t} = (\omega_{h1,t}, \dots, \omega_{hN,t})^\top$ is the column vector of portfolio weights.

We now use the vector of household expectations to define the *network-weighted information loss* of an individual household.

Definition 2. *The network-weighted information loss for household h is given by*

$$\widehat{L}_{h,t} = \boldsymbol{\nu}_{h,t}^\top \boldsymbol{\sigma}^{-1} F_h \boldsymbol{\sigma}^{-1} \boldsymbol{\nu}_{h,t}, \quad (3)$$

where $\boldsymbol{\nu}_{h,t}$ is household h 's vector of personal divergences and F_h is the $N \times N$ diagonal matrix

$$F_h = \text{diag} \left(\frac{\phi_{h1}}{1 - \phi_{h1}}, \dots, \frac{\phi_{hN}}{1 - \phi_{hN}} \right), \quad \phi_{hn} \in [0, 1], \quad n \in \{1, \dots, N\}.$$

To understand the motivation underlying the above definition, observe that we can rewrite equation (3) as

$$\widehat{L}_{h,t} = \sum_{n=1}^N \frac{\phi_{hn}}{1 - \phi_{hn}} \frac{\nu_{hn,t}^2}{\sigma_n^2}.$$

In the above expression, we can interpret $\nu_{hn,t}^2/\sigma_n^2$ as a measure of the information about firm n that is discarded by household h when it uses its personal belief \mathbb{Q}^{ν^h} as opposed to the objective belief \mathbb{P} . We then weight each of these information losses by $\frac{\phi_{hn}}{1-\phi_{hn}}$. Doing so ensures that an information loss impacts $\widehat{L}_{h,t}$ only when a household's proximity with respect to a particular firm is not zero and that the impact of the information loss increases with proximity and becomes infinitely large when $\phi_{hn} = 1$, i.e. when the separation measure is 0. In this manner, a household's location within a network structure determines its network-weighted information loss.

Our definition of a household's network-weighted information loss is closely related to the relative entropy per unit time of the objective belief \mathbb{P} with respect to the personal belief \mathbb{Q}^{ν^h} (also known as the Kullback-Leibler divergence from \mathbb{Q}^{ν^h} to \mathbb{P}). Observe that the relative entropy per unit time of the objective belief \mathbb{P} with respect to the personal belief \mathbb{Q}^{ν^h} is given by

$$D^{KL}[\mathbb{P}|\mathbb{Q}^{\nu^h}] = \frac{1}{dt} E_t \left[\left(\frac{dM_{h,t}}{M_{h,t}} \right)^2 \right] = \boldsymbol{\nu}_{h,t}^\top \boldsymbol{\sigma}^{-1} [(\boldsymbol{\Omega})^{-1}]^\top \boldsymbol{\sigma}^{-1} \boldsymbol{\nu}_{h,t}.$$

In the above expression for relative entropy, we can see that when firm-level information losses are summed up, they are weighted by the correlation structure underlying firm-level returns, as encapsulated by the correlation matrix $\boldsymbol{\Omega}$. In contrast, our definition of a household's network-weighted information loss in (3) uses the household's *location* in the bipartite network of firms and households to weight the information losses.

2.5 Intertemporal Aggregator with Endogenous Beliefs

Each household maximizes its date- t utility level, $U_{h,t}$, defined as in Epstein and Zin (1989) by an intertemporal aggregation of date- t consumption flow, $C_{h,t}$, and the date- t personal

certainty-equivalent of date $t + dt$ utility:⁶

$$U_{h,t} = \mathcal{A}_h(C_{h,t}, \mu_{h,t}^\nu[U_{h,t+dt}]),$$

where $\mathcal{A}_h(\cdot, \cdot)$ is the time aggregator, defined by

$$\mathcal{A}_h(x, y) = \left[(1 - e^{-\delta_h dt})x^{1-\frac{1}{\psi_h}} + e^{-\delta_h dt}y^{1-\frac{1}{\psi_h}} \right]^{\frac{1}{1-\frac{1}{\psi_h}}}, \quad (4)$$

in which $\delta_h > 0$ is the rate of time preference, $\psi_h > 0$ is the elasticity of intertemporal substitution, and $\mu_{h,t}^\nu[U_{h,t+dt}]$ is the date- t personal certainty equivalent of $U_{h,t+dt}$. We explain below how this certainty-equivalent value is determined.

2.6 Household-Specific Certainty Equivalent

We now use the concepts of a household's beliefs and its network-weighted information loss to define its personal certainty-equivalent, which is fundamental to how we endogenize belief formation as a function of household location within the network structure.

Definition 3. *The date- t personal certainty-equivalent of date- $t + dt$ household utility is given by*

$$\mu_{h,t}^\nu[U_{h,t+dt}] = \widehat{\mu}_{h,t}^\nu[U_{h,t+dt}] + U_{h,t}L_{h,t}dt, \quad (5)$$

where $\widehat{\mu}_{h,t}^\nu[U_{h,t+dt}]$ is defined by

$$u_{\gamma_h}(\widehat{\mu}_{h,t}^\nu[U_{h,t+dt}]) = E_t^{\mathbb{Q}^{\nu_h}}[u_{\gamma_h}(U_{h,t+dt})], \quad (6)$$

$$u_{\gamma_h}(x) = \frac{x^{1-\gamma_h}}{1-\gamma_h},$$

and

$$L_{h,t} = \frac{1}{2\gamma_h}\widehat{L}_{h,t}.$$

The first part of the definition of the personal certainty-equivalent is just the standard definition of a certainty-equivalent based on power utility with relative risk aversion γ_h , but

⁶The only difference with Epstein and Zin (1989) is that we work in continuous time, whereas they work in discrete time. The continuous-time version of recursive preferences is known as stochastic differential utility (SDU), and is derived formally in Duffie and Epstein (1992). Schroder and Skiadas (1999) provide a proof of existence and uniqueness for the finite horizon case.

using the personal belief \mathbb{Q}^{ν^h} . The second part of the definition hinges on the expression for $L_{h,t}$, which is a multiple of the network-weighted information loss from using the personal belief \mathbb{Q}^{ν^h} instead of the objective belief \mathbb{P} .

When a household chooses its beliefs by choosing a vector of divergences $\boldsymbol{\nu}_{h,t}$, it does so in order to minimize the impact of its information losses on its personalized certainty equivalent. The way information losses affect a household's personalized certainty equivalent depends on its location within a network, so we see that a household's beliefs depend on its location within a network structure.

The following theorem shows how a household optimally selects its personal beliefs by choosing a vector of divergences, $\boldsymbol{\nu}_{h,t}$.

Proposition 1. *If the investment opportunity set is constant, then the date- t personal certainty equivalent of date- $t + dt$ household utility based on the personal belief \mathbb{Q}^{ν^h} is given by*

$$\mu_{h,t}^{\boldsymbol{\nu}}[U_{h,t+dt}] = \mu_{h,t}[U_{h,t+dt}] + U_{h,t} \times \left(\frac{W_{h,t} U_{W_{h,t}}}{U_{h,t}} \boldsymbol{\nu}_{h,t}^{\top} \boldsymbol{\omega}_{h,t} + L_{h,t} \right) dt, \quad (7)$$

where

$$\mu_{h,t}[U_{h,t+dt}] = E_t[U_{h,t+dt}] - \frac{1}{2} \gamma_h U_{h,t} E_t \left[\left(\frac{dU_{h,t}}{U_{h,t}} \right)^2 \right] \quad (8)$$

and $U_{W_{h,t}} = \frac{\partial U_{h,t}}{\partial W_{h,t}}$ is the partial derivative of the utility of household h with respect to its wealth. At date t , a household optimally selects its personal belief \mathbb{Q}^{ν^h} by choosing the vector of personal divergences $\boldsymbol{\nu}_{h,t}$ which minimizes its date- t personal certainty equivalent, $\mu_{h,t}^{\boldsymbol{\nu}}[U_{h,t+dt}]$.

The presence of the term $\frac{W_{h,t} U_{W_{h,t}}}{U_{h,t}} \boldsymbol{\nu}_{h,t}^{\top} \boldsymbol{\omega}_{h,t}$ gives a household the desire to make its divergences in portfolio-return expectations, i.e. $\boldsymbol{\nu}_{h,t}^{\top} \boldsymbol{\omega}_{h,t}$, as negative possible. This desire is clearly a departure from rationality, but it is tempered by the size of its network-weighted information losses, represented by $L_{h,t} = \frac{1}{2\gamma_h} \widehat{L}_{h,t}$. For example, in the special case where $\phi_{hn} = 1$ for all $n \in \{1, \dots, N\}$, the network-weighted information losses from any divergences become infinitely large, so a household chooses not to have any personal divergences,

i.e. $\boldsymbol{\nu}_{h,t}$ is the zero vector. In this case, its personal certainty equivalent reduces to the standard certainty equivalent, i.e. a mean with a penalty for risk that depends on risk aversion γ_h and variance, as given in (8).

In general, network-weighted information losses are not infinitely large, and so a household faces a trade off between more negative personal divergences in portfolio return expectations and larger network-weighted information losses. Because this optimization problem is linear-quadratic it has the following closed-form solution.

Corollary 1. *If the investment opportunity set is constant, a household's personal divergence vector is given by*

$$\boldsymbol{\nu}_{h,t} = -\gamma_h \frac{W_{h,t} U_{W_{h,t}}}{U_{h,t}} (\boldsymbol{\sigma}^{-1} F_h \boldsymbol{\sigma}^{-1})^{-1} \boldsymbol{\omega}_{h,t}, \quad (9)$$

i.e.

$$\nu_{hn,t} = -\gamma_h \frac{W_{h,t} U_{W_{h,t}}}{U_{h,t}} \sigma_n^2 \frac{1 - \phi_{hn}}{\phi_{hn}} \omega_{hn,t}.$$

We can now see how differences in network location across households create heterogeneity in households' expectations. When a household's proximity to a firm is low, it responds by biasing downwards its point estimate for the expected return for that firm. For example, household h will change the expected return for firm n from α to $\alpha + \nu_{hn,t}$, thereby reducing the magnitude of the firm's expected risk premium ($\nu_{hn,t} \leq 0$ if $\omega_{hn,t} > 0$ and $\nu_{hn,t} \geq 0$ if $\omega_{hn,t} < 0$). The size of the reduction depends on each household's proximity to a particular firm—the reduction is smaller for firms to which the household is closer. Thus, differences in proximity across households lead them to use different estimates of expected returns in their decision making.

3 Portfolio-Consumption Choice with Endogenous Beliefs

In this section, we solve the portfolio problem of an individual household whose beliefs are determined endogenously by its network location.

3.1 The Intertemporal Choice Problem with Endogenous Beliefs

If a household's beliefs coincided with the objective physical measure, it would choose its consumption rate, $C_{h,t}$, and portfolio policy, $\omega_{h,t}$, to solve the standard choice problem, which is:

$$\sup_{C_{h,t}} \mathcal{A}\left(C_{h,t}, \sup_{\omega_{h,t}} \mu_{h,t}[U_{h,t+dt}]\right), \quad (10)$$

where $\mu_{h,t}[U_{h,t+dt}]$ is the standard certainty equivalent, given in (8).

In general, with endogenous household beliefs that do not coincide with rational expectations, the time aggregator $\mathcal{A}(\cdot)$ in (4) is unchanged—all we need to do is to replace the maximization of the standard certainty-equivalent in (10), $\sup_{\omega_{h,t}} \mu_{h,t}[U_{h,t+dt}]$, with the combined maximization and minimization of the personal certainty equivalent,⁷ $\sup_{\omega_{h,t}} \inf_{\nu_{h,t}} \mu_{h,t}^{\nu}[U_{h,t+dt}]$ to obtain

$$\sup_{C_{h,t}} \mathcal{A}\left(C_{h,t}, \sup_{\omega_{h,t}} \inf_{\nu_{h,t}} \mu_{h,t}^{\nu}[U_{h,t+dt}]\right). \quad (11)$$

A household, because of the impact of its network location, chooses $\nu_{h,t}$ to *minimize* its personal certainty equivalent; i.e. the household adjusts expected returns more for firms for which its degree of separation is larger, which acts to reduce the personal certainty equivalent. By comparing (10) and (11), we can see that once a household has chosen the vector of personal divergences, $\nu_{h,t}$, to adjust the expected returns of each firm to account for its beliefs, it makes consumption and portfolio choices in the standard way. The solution for the choice of the vector of personal divergences, $\nu_{h,t}$ is given in terms of the portfolio choice $\omega_{h,t}$ in Corollary 1 in the appendix.

⁷Our max-min characterization of the objective function is consistent with the multi-prior approach advocated by Gilboa and Schmeidler (1989) and developed in a static setting by Dow and Werlang (1992), in dynamic discrete-time by Epstein and Wang (1994), and in continuous time by Chen and Epstein (2002). The alternative approach of Hansen and Sargent (2007), which our formulation builds on, assumes that investors allow for the possibility that their model may not be correct and hence consider deviations from the reference model, where the relative likelihood of the two models is measured using entropy; these preferences are called multiplier preferences. Maccheroni et al. (2006) show the relation between multiple-priors and multiplier preferences; they also show that both are nested in a new class called “divergence preferences.” Cerreia-Vioglio, Maccheroni, Marinacci, and Montrucchio (2011) provide a common representation that unifies these preferences.

To solve a household's consumption-portfolio choice problem under subjective beliefs we use Ito's Lemma to derive the continuous-time limit of (11), which leads to the Hamilton-Jacobi-Bellman equation shown in the proposition below.

Proposition 2. *The utility function of a household with endogenous beliefs is given by the following Hamilton-Jacobi-Bellman equation:*

$$0 = \sup_{C_{h,t}} \left(\delta u_\psi \left(\frac{C_{h,t}}{U_{h,t}} \right) + \sup_{\omega_{h,t}} \inf_{\nu_{h,t}} \frac{1}{U_{h,t}} \mu_{h,t}^\nu \left[\frac{dU_{h,t}}{dt} \right] \right), \quad (12)$$

where the function

$$u_\psi(x) = \frac{x^{1-\frac{1}{\psi}} - 1}{1 - \frac{1}{\psi}}, \psi > 0,$$

and

$$\mu_{h,t}^\nu [dU_{h,t}] = \mu_{h,t}^\nu [U_{h,t+dt} - U_{h,t}] = \mu_{h,t}^\nu [U_{h,t+dt}] - U_{h,t},$$

with $\mu_{h,t}^\nu [U_{h,t+dt}]$ given in (7).

The Hamilton-Jacobi-Bellman equation can be decomposed into a portfolio-optimization problem and an intertemporal consumption-choice problem. Given the assumption of a constant risk-free rate, homotheticity of preferences combined with constant returns to scale for production lead to an investment opportunity set that is constant over time. This implies that maximized household utility is a constant multiple of the household's wealth. In this case, the Hamilton-Jacobi-Bellman equation can be decomposed into two parts: a *single-period mean-variance* optimization problem for a household with endogenous beliefs and an intertemporal consumption-choice problem, as shown in the proposition below.

Proposition 3. *The household's optimization problem consists of two parts, a mean-variance optimization*

$$\sup_{\omega_{h,t}} \inf_{\nu_{h,t}} MV_h(\omega_{h,t}, \nu_{h,t}),$$

and an intertemporal consumption choice problem

$$0 = \sup_{C_{h,t}} \left(\delta_h u_\psi \left(\frac{C_{h,t}}{U_{h,t}} \right) - \frac{C_{h,t}}{W_{h,t}} + \sup_{\omega_{h,t}} \inf_{\nu_{h,t}} MV_h(\omega_{h,t}, \nu_{h,t}) \right), \quad (13)$$

where

$$MV_h(\boldsymbol{\omega}_{h,t}, \boldsymbol{\nu}_{h,t}) = i + (\boldsymbol{\alpha} - i\mathbf{1})^\top \boldsymbol{\omega}_{h,t} - \frac{1}{2}\gamma \boldsymbol{\omega}_{h,t}^\top V \boldsymbol{\omega}_{h,t} + \boldsymbol{\nu}_{h,t}^\top \boldsymbol{\omega}_{h,t} + \frac{1}{2\gamma_h} \boldsymbol{\nu}_{h,t}^\top \boldsymbol{\sigma} F_h^{-1} \boldsymbol{\sigma} \boldsymbol{\nu}_{h,t}, \quad (14)$$

$\boldsymbol{\alpha} = (\alpha_1, \dots, \alpha_N)^\top$, and $\mathbf{1}$ denotes the $N \times 1$ unit vector.

In the above proposition, $MV(\boldsymbol{\omega}_{h,t}, \boldsymbol{\nu}_{h,t})$ is the objective function of a single-period mean-variance household with endogenous beliefs: $i + (\boldsymbol{\alpha} - i\mathbf{1})^\top \boldsymbol{\omega}_{h,t}$ is the expected portfolio return under the objective physical probability measure, $-\frac{1}{2}\gamma \boldsymbol{\omega}_{h,t}^\top V \boldsymbol{\omega}_{h,t}$ is the penalty for portfolio variance, $\boldsymbol{\nu}_{h,t}^\top \boldsymbol{\omega}_{h,t}$ is the divergence in the portfolio's expected return arising from household h 's expectations, and $\frac{1}{2\gamma_h} \boldsymbol{\nu}_{h,t}^\top (\boldsymbol{\sigma}^{-1} F_h \boldsymbol{\sigma}^{-1})^{-1} \boldsymbol{\nu}_{h,t}$ is the penalty for adjusting expected returns, i.e. the information loss incurred in deviating from the objective physical probability measure.⁸

In the mean-variance problem under household h 's beliefs, the firm-level expected returns are optimally adjusted downward. The magnitude of the adjustments depend on the degrees of separation of a household from the firms in the economy. Because each household's utility is a constant multiple of wealth and the matrices $\boldsymbol{\sigma}$ and F_h are diagonal, the expression for the vector of personal divergences in equation (9) simplifies to:

$$\boldsymbol{\nu}_{h,t} = -\gamma_h \boldsymbol{\sigma}^2 F_h^{-1} \boldsymbol{\omega}_{h,t}. \quad (15)$$

Note that the expression for $\boldsymbol{\nu}_{h,t}$ above is in terms of $\boldsymbol{\omega}_{h,t}$; in Proposition 4 below, we will derive also an explicit expression for the optimal vector of personal divergences $\boldsymbol{\nu}_{h,t}$.

Substituting the expression for $\boldsymbol{\nu}_{h,t}$ in (15) into (14), we see that the household faces the following mean-variance portfolio problem:

$$\sup_{\boldsymbol{\omega}_{h,t}} MV(\boldsymbol{\omega}_{h,t}) = \left(i + \left(\boldsymbol{\alpha} + \frac{1}{2} \boldsymbol{\nu}_{h,t} - i\mathbf{1} \right)^\top \boldsymbol{\omega}_{h,t} \right) - \frac{1}{2} \gamma_h \boldsymbol{\omega}_{h,t}^\top V \boldsymbol{\omega}_{h,t}.$$

For the extreme case in which the measure of separation for a household with respect to each firms is zero, network-weighted information loss in deviating from the physical measure \mathbb{P}

⁸The vector of personal divergences adjustment is obtained from a minimization problem, so the associated penalty is positive, in contrast with the penalty for return variance.

becomes infinite, and so the vector of divergences is the zero vector. Hence, the adjustment to expected returns is zero and the portfolio weights are exactly the standard mean-variance portfolio weights under rational expectations. For the other extreme in which the measure of separation for a household with respect to each firm is larger than the threshold \bar{d} , the household's personal expected risk premium for each firm becomes zero and it avoids any investment in risky firms, i.e. $\boldsymbol{\omega}_h = \mathbf{0}$; in this case, we obtain non-participation in the stock market for this household.

3.2 Solution to the Choice Problem of an Individual Investor

In this section, we present the solution to the choice problem of an individual household.

Proposition 4. *The optimal vector of personal divergences is*

$$\boldsymbol{\nu}_h = -(I + VF_h\boldsymbol{\sigma}^{-2})^{-1}(\boldsymbol{\alpha} - i\mathbf{1}), \quad (16)$$

and the vector of optimal portfolio weights is

$$\boldsymbol{\omega}_{h,t} = \frac{1}{\gamma_h}V^{-1}(\boldsymbol{\alpha} - i\mathbf{1} + \boldsymbol{\nu}_h) = \frac{1}{\gamma_h}(V + \boldsymbol{\sigma}^2F_h^{-1})^{-1}(\boldsymbol{\alpha} - i\mathbf{1}). \quad (17)$$

For the special case in which the correlation between assets $\rho_{nm} = 0$, the optimal divergence in household h 's expected return for firm n is

$$\nu_{hn} = -(\alpha_n - i)(1 - \phi_{hn}), \quad (18)$$

and the optimal proportion of wealth invested in firm n by household h is

$$\omega_{hn} = \frac{1}{\gamma_h} \frac{\alpha_n - i + \nu_{hn}}{\sigma_n^2} = \frac{1}{\gamma_h} \frac{\alpha_n - i}{\sigma_n^2} \phi_{hn}. \quad (19)$$

From (18), we can see that the size of a household's deviation in a firm's expected return is smaller when the proximity measure, ϕ_{hn} , is larger; if $\phi_{hn} = 1$, then the deviation vanishes altogether and the household holds the portfolio weight that would be optimal under rational expectations. From (19), we see that the standard mean-variance portfolio weight for firm n , $\frac{1}{\gamma_h} \frac{\alpha_n - i}{\sigma_n^2}$, is scaled by the proximity measure for household h with respect to

firm n , ϕ_{hn} . As a household's proximity measure with respect to a particular firm decreases, the proportion of its wealth that it chooses to invest in that firm also decreases.

In summary, when a household is further away from a firm (a reduction in proximity), its beliefs about the firm's return diverge more from the objective physical expectation. Hence, it tilts its portfolio away from the fully diversified portfolio, leading to a reduction in portfolio diversification.

Our portfolio weight specification in equation (19) is similar to that estimated using panel regressions in Grinblatt and Keloharju (2001). Our specification provides additional insight about how to interpret the firm and location fixed effects in those regressions: The firm fixed effects represent distributional properties of stock returns, and the location fixed effects represent risk aversion, which we will measure at the postal code level. Our model thereby provides a microfoundation for the empirical tests and results in Grinblatt and Keloharju (2001).

It is straightforward to show that under the extended specification, which allows for idiosyncratic variation in household beliefs, equation (19) becomes

$$\omega_{hn} = \frac{1}{\gamma_h} \frac{\alpha_n + \xi_{nh} - i}{\sigma_n^2} \phi_{hn}.$$

This specification thus allows for variation of portfolio holdings between households at the same location (with the same ϕ_{hn}), for some households to choose higher weights than $\frac{1}{\gamma_h} \frac{\alpha_n - i}{\sigma_n^2}$ (those with ξ_{hn} sufficiently high), and for a some households to shortsell stock (those with ξ_{hn} sufficiently low).

4 Empirical Results

Our belief-based portfolio-choice model leads to several testable predictions, some of which are unique to our approach. These predictions follow immediately from our earlier results, and therefore, are presented below without formal proofs.

In our model, households will tend to be underdiversified in that they will hold only a subset of the stocks in the market (those at a distance smaller than \bar{d}). The effect will be especially severe for households that are located far away from firms.

Prediction 1. *The portfolio of a household located far away from firms is more underdiversified—i.e., contains fewer stocks—than the portfolio of a household located close to firms.*

A related prediction is that the stocks in a household’s portfolio will be located closer to the household than stocks that are not part of the household’s portfolio.

Prediction 2. *The distances from a household to firms that are included in the household’s portfolio are lower than the distances to firms that are not included in the portfolio.*

The previous predictions relate a household’s portfolio to the locations of firms. Our model also predicts that households which are located close to each other hold similar portfolios.

Prediction 3. *The portfolios of two nearby households contain more common stock constituents than if these constituents were randomly selected among the market’s stocks.*

We next turn to the full portfolio implications of the specification in equations (1) and (19). Specifically, the sensitivity parameter κ determines how distance affects household beliefs. The higher is κ , the more households focus on companies in their network vicinity, whereas when $\kappa = 0$, as long as the distance is less than \bar{d} , beliefs and portfolios are unaffected.

We define $g_h = \ln(\gamma_h)$, $v_{hn} = \ln(\omega_{hn})$ and $s_n = \ln\left(\frac{\alpha_n - i}{\sigma_n^2}\right)$, which from (19) then implies

$$v_{hn} = -g_h + s_n - \kappa d_{hn}. \quad (20)$$

Equation (20) provides a strong characterization of the portfolio holdings of individual agents, as a function of relative risk aversion coefficients, stock return characteristics, and distance. Given data on portfolio holdings of the households in a market, $\{\omega_{hn}\}_{hn}$, together with location parameters, $\{(x_h, y_h)\}_h$ and $\{(x^n, y^n)\}_n$, equation (20) can be estimated and the following prediction can be tested

Prediction 4. *The κ -coefficient in equation (20) is strictly positive, $\kappa > 0$.*

In this estimation, $\{g_h\}_h$ and $\{s_n\}_n$ are treated as (unobservable) household and firm fixed effects, respectively, both with clear economic interpretations. In particular, the estimated firm fixed effects should be related to the actual return distribution of stocks in the market.

Prediction 5. *The firm fixed effects coefficients obtained when estimating equation (20) should be informative about*

$$\ln \left(\frac{\alpha_n - i}{\sigma_n^2} \right), \quad n = 1, \dots, N.$$

Prediction 5 could potentially be used to separate our belief-based explanation for observed portfolio holdings among households that are consistent with home bias from other mechanical explanations for the relation between location and portfolio holdings (along the lines “being closer to a firm implies a larger portfolio weight”).

Finally, the extended version of our model, which allows for random heterogeneity in beliefs across households within the same location, leads to predictions about both the long- and short-selling behavior of households.

Prediction 6.

1. *Among households holding long positions in a firm, those located close to that firm hold larger positions than those located far away.*
2. *Among households holding short positions in a firm, those located close to that firm hold larger short positions than those located far away.*

As is the case for the previous prediction, Prediction 6 could be used to separate our belief-based model from other explanations of home bias. A “naive” model for how location affects portfolio holdings would suggest that nearby households are simply “more positive” about a firm than remote households for unmodelled reasons. In this case, the second part of Prediction 6 would naturally be reversed, i.e., shortselling would mainly be an activity among households far away from a firm.

The prediction would also be different under a standard information-based explanation of home bias. Specifically, if nearby households were better informed about the prospects of a firm, they would either hold more long positions and fewer short positions than remote households—in case the information was positive—or less long positions and more short positions—in case the information was negative. Neither case is consistent with Prediction 6.

Our model generates this unique prediction through the effect uncertainty has on ambiguity-averse investors. These investors become more reluctant to take on extreme positions, both long and short, when there is high uncertainty, in contrast to the naive model and the explanation relying on asymmetric information. We do not test Prediction 6 in this paper, because our dataset does not contain information about short-selling, but mention it as an interesting test for future research.

In the rest of this section we test Predictions 1–5. In Section 4.1, we describe the data. The results of our empirical analysis are reported in Section 4.2 and various robustness checks are described in Section 4.3.

4.1 Data

Portfolio holdings for all accounts on the Helsinki Stock Exchange, as of January 2, 2003, were obtained from Euroclear, which acquired the Finnish Central Securities Depository in 2008.⁹ It contains portfolio holdings and postal-code information, as well as further characteristics (age and gender and sector code classification) of all account holders in the market. There are altogether 3,036 valid postal codes in the data set and the data contains over 60 million trades during the time period 1995-2004, and about 1.2 million accounts, most of which represent the household sector.

We obtain geographical coordinates for each postal code area from the Finnish postal services company, Posti Group Corporation. These postal codes make up a fine-grained representation of Finland, as shown earlier in Figure 2. We represent each postal code geographically by its center of gravity.

⁹The dataset has previously been used, e.g., in Grinblatt and Keloharju (2000, 2001) and Walden (2019).

We obtain information about the postal codes of company headquarter from Thomson One Reuters and exclude companies headquartered outside of Finland. We also exclude companies with shares that were not traded within the previous month (i.e. during December 2002). Finally, we exclude the telecom firm Elisa Oyj, which had until 1999 been a privately held mutual association with broad ownership among its association members, and therefore had a quite different ownership history and structure than the rest of the firms.¹⁰ This leaves us with 125 stocks, which are listed in Table 9 of Appendix B.¹¹

We include accounts that are classified as households (these are accounts associated with sector codes between 500 and 599), that are associated with a valid postal code, and that owned shares in at least one of the 125 stocks on January 2, 2003. This leaves us with 405,868 households associated with altogether $P = 2,923$ postal codes.

The postal code associated with household h is denoted p_h . We assume that each agent resides at the center of gravity of his/her respective postal code and also that each firm is headquartered at the center of gravity of its postal code. Thus, all agents within a postal code are assumed to be at the same distance from each of the firms. We can then rewrite equation (20) at the postal-code level as

$$v_{pn} = -\bar{g}_p + s_n - \kappa d_{pn}, \quad (21)$$

where

$$\bar{g}_p = \ln(\bar{\gamma}_p), \quad \text{and} \quad \frac{1}{\bar{\gamma}_p} = \sum_{\{h:p_h=p\}} \frac{1}{\gamma_h},$$

and $\bar{\gamma}_p$ is the harmonic mean of the relative risk aversion coefficients for agents living in postal code p , and d_{pn} is the distance between the center of gravity of postal code area p ,

¹⁰Elisa Oyj was formed on July 1, 2000, with the merger of the Helsinki Telephone Corporation (in Finnish, "Helsingin Puhelin") and its holding company, HPY Holding Corporation. Helsinki Telephone Corporation had been a privately held telephone cooperative with broad ownership among its 550,000 "association members." Subscribers to its telephone services automatically became association members. When the company was listed on the Helsinki Stock Exchange in 1997, these owners became shareholders, which then carried over to Elisa Oyj after the merger (Source: Annual Reports, 1997-2000). Most of these shareholders only held this one stock, and thus seem to be shareholders for different reasons than the rest of the household investor population. Except for predictions 1 and 3, on underdiversification and similarity of holdings of individual households, the results are very similar when Elisa Oyj is included.

¹¹Some of these stocks represent A- and B-shares in the same company. An A-share in Finland typically come with greater voting rights compared with a B-share. There were significant differences in share prices and returns between A- and B-shares of the same companies and we therefore include both A- and B-shares in our sample for companies with both types of shares.

(x_p, y_p) , and the center of gravity of the postal code area in which firm n is located,

$$d_{pn} = D((x_p, y_p), (x^n, y^n)).$$

Equation (21) thus consists of $P \times N$ postal code/company holdings.

We focus on geographical proximity as a measure of network proximity between agents and companies. As discussed in Walden (2019), this is a reasonable assumption for the time period and market that the data covers. One might view geographical distance as an unimportant hurdle in the present time due to almost universal access for households to information via the Internet. However, only about one third of the Finnish population used the Internet in 2000. Moreover, over half of Finland's population resides in rural areas, making it one of the most rural countries in the European Union. It is therefore plausible that there would be a significant link between geographical and network proximity in the early 2000's.

We normalize the distance function, so that all geographical coordinates lie in the unit square, $[0, 1] \times [0, 1]$. The household-firm that are farthest apart are therefore at a distance somewhere between 1 and $\sqrt{2}$ from each other (in our sample, the maximum distance is 1.175), making the interpretation of the sensitivity coefficient κ in equation (21) straightforward.

The geographical coordinates of postal codes (blue dots) and firms (red circles) are shown in Figure 3. As can be seen in the figure, most firms are headquartered in the far south (around the capital, Helsinki, with associated postal codes between 100 and 9900), whereas about 20% of the firms have headquarters elsewhere. Summary statistics of the data we use are provided in Table 1.

4.2 Results

We first test Prediction 1, that households tend to be more under-diversified the farther away they are located from stocks. We define the center of gravity (CoG) of the stocks,

$$(x^C, y^C) = \frac{1}{\sum_{n=1}^N W^n} \left(\sum_{n=1}^N W^n x^n, \sum_{n=1}^N W^n y^n \right),$$

Figure 3: Center of gravity of postal codes for households and firms

Center of gravity of postal codes for households (blue dots) and firms (red circles) in dataset.

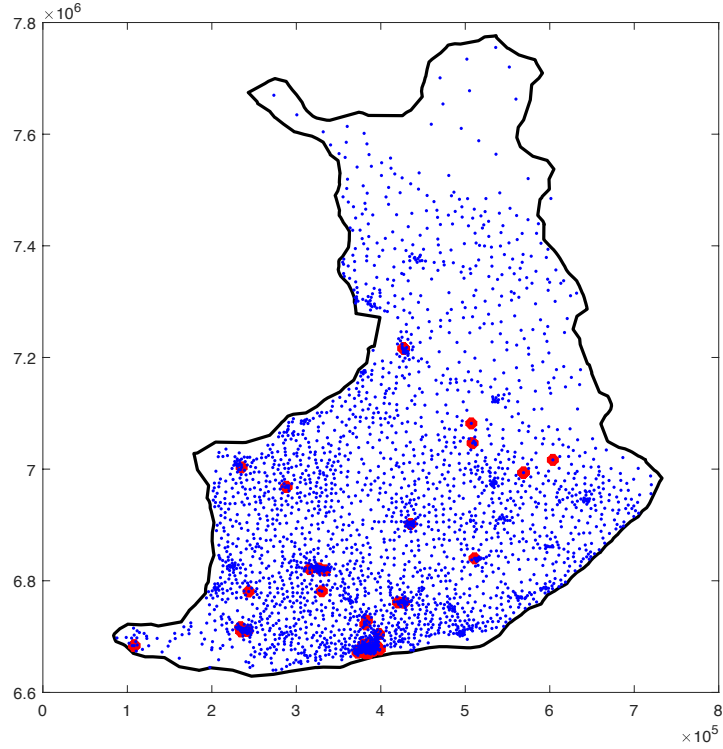


Table 1: Summary statistics

This table gives the summary statistics for the data used in our empirical analysis.

Number of stocks	125
Number of household accounts	405,628
Number of postal codes	2,923
Average number of accounts in postal code	139
Total number of observations	364,750
Number of nonzero observations	132,811
Maximum portfolio holding	FIM 178.7 million
Minimum portfolio holding	FIM 0
Average stock holding by postal code	FIM 30,617
Median number of stocks held by account	2
Mean number of stocks held by account	2.75

where we use both equally-weighted CoG ($W^n = 1$ for all firms) and value-weighted CoG, (a firm's weight is defined as the total value of household portfolio holdings in that firm).

We further define q_p , the average number of stocks in households' portfolios within postal

Table 2: Test of Prediction 1: Underdiversification versus distance from firms

This table shows results from two-sample t-tests of difference in means between the distance from postal codes with zero holdings to firm headquarter and from postal codes with positive holdings. Columns (1-2) are based on equally weighted CoG, whereas columns (3-4) are based on value-weighted CoG. Columns (1) and (3) are univariate, whereas columns (2) and (4) also include average portfolio size in the regressions.

	(1)	(2)	(3)	(4)
Average number of stocks, q_p	-1.011***	-1.024***	-0.306***	-0.313***
Standard error	0.084	0.082	0.072	0.070
Portfolio size, $\ln(W_p)$		0.446***		0.445***
Standard error		0.012		0.012

code area p , and $d_p = D((x_p, y_p), (x^C, y^C))$, the distance between households in postal code area p and firm CoG, and estimate how they are related. The relation, shown in Table 2, is significantly negative, both economically and statistically. A household located at a maximum distance from the firms' CoG is predicted to hold about one fewer stock in its portfolio than a household located right at the CoG—a major effect because the median number of stocks held in a household portfolio is 2. The results are very similar regardless of whether the value-weighted or equally weighted definition of CoG is used.

We also control for portfolio size. Specifically, if wealthier households tend to be more well diversified and are also located closer to firms, similar results would arise, but because of this omitted variable. We therefore include the logarithm of average portfolio-size within a postal code area, $\ln(W_p)$, in the regression. The estimated coefficient decreases by about two thirds when $\ln(W_p)$ is included, both in the value-weighted and equally weighted CoG specification, but still remains highly significant.

We next test Prediction 2, which at the postal code level states that postal codes with zero portfolio holdings in a firm tend to lie geographically further away from that firm than postal codes with positive holdings. For each stock, we perform a two-sample t-test, comparing the average distances of postal codes with zero and with positive holdings from the firm's headquarter. The results are shown in Table 3.

The average distance for postal codes with zero holdings is about 0.332, whereas the average distance for postal codes with positive holdings is about 0.224, corresponding to

Table 3: Test of Prediction 2: Zero versus positive portfolio holdings

This table shows results from two-sample t-tests of difference in means between the distance from postal codes with zero holdings to firm headquarter and from postal codes with positive holdings.

	Zero holding	Positive holding	Difference
Average distance to HQ	0.332	0.224	0.108
Number of firms	6	119	
Average t-stat	-3.047	14.080	
Average t-stat, Total			13.258

a difference of 0.108—about 85 miles. At the individual firm level, the average distance from postal code with zero holdings is higher for 119 of the 125 stocks. For 116 of these firms, the difference is statistically significant at the 0.01% level. For the remaining 6 firms, for which the average distance is higher from postal codes with positive holdings, only one is significant at the 0.01% level. The average t-statistic for the difference of means being positive is 13.3. Thus, the data strongly support the prediction that the further away from a firm’s headquarters, the higher the likelihood of zero portfolio holdings.

We also estimate the cutoff distance, \bar{d} . Specifically, for each of the 125 stocks, we choose the \bar{d} that maximizes the number of correctly classified postal codes with respect to whether the stock holdings in the firm is positive or zero. The results are shown in Table 4. The average estimated \bar{d} is 0.1799, corresponding to a threshold distance of about 143 miles. Beyond this distance to a firm’s headquarter, the familiarity is thus predicted to be so low that an investor completely avoids investing in a stock.

The fraction of holdings that are zero among all postal code/firm observations is slightly less than two thirds, about 64%. When the estimated cutoff thresholds at the firm level are used to predict whether a postal code/firm portfolio holding is zero, the fraction of correct classifications is about 75%, i.e., about three quarters. The model thus captures quite well whether investors in a postal code invest in a stock or not.

Prediction 3, formulated at the postal code level, suggests that that households located in the same postal code should hold portfolios with more overlap (i.e., more common stocks) than if the portfolios were randomly chosen. For example, for two households that each randomly and independently invest in one of N stocks, with probability $\frac{1}{N}$ of choosing each

Table 4: Estimated threshold for zero portfolio holdings

This table shows results the estimated thresholds, \bar{d} , for when portfolio holdings become zero.

	Value
Average threshold, \bar{d}	0.1799
Standard deviation	0.2165
Fraction of zero holdings	0.6359
Fraction correctly classified	0.7534

stock, the probability that the two portfolios overlap would also be $\frac{1}{N}$. In other words, the expected value of the random value \tilde{N}^c which denotes the number common stocks in their portfolios would be $\frac{1}{N}$. More generally, if household h_1 randomly chooses N_{h_1} stocks and household h_2 randomly and independently chooses N_{h_2} stocks, then the expected overlap is

$$E \left[\tilde{N}_{h_1, h_2}^c \right] = \frac{N_{h_1} N_{h_2}}{N}.$$

Prediction 3 can then be tested by studying the actual versus expected overlap of the portfolios of all households located within the same postal code area, and sum up these overlaps over all postal codes, i.e., by calculating the actual total overlap

$$N_{TOT}^c = \sum_p \sum_{\substack{h_1, h_2 \in \mathcal{H}_p \\ h_1 \neq h_2}} N_{h_1, h_2}^c,$$

where $\mathcal{H}_p = \{h : p_h = p\}$ is the set of households in postal code area p , and comparing it with expected total overlap.

The actual total overlap in the data is $N_{TOT}^c = 51,604,247$, whereas the expected total overlap under the independent, $\frac{1}{N}$ probability-per-stock, assumption is 3,478,029. Thus, the likelihood that two randomly chosen households within the same randomly chosen postal code area invest in the same stock is almost 15 times higher than expected.

A limitation of the above test is that it assumes that each stock is chosen with the same probability, $\frac{1}{N}$, whereas in practice some stocks (for example, the telecommunications company Nokia in our dataset) are much more broadly held than others. The higher overlap will therefore partly be a consequence of the $\frac{1}{N}$ assumption. We therefore use a more sophisticated bootstrapping method, based on the actual distribution of portfolio holdings among the households to test the prediction. Briefly, we assume that the empirical distribution of

Table 5: Test of Prediction 3: Overlap of portfolios within a postal code area

This table tests whether portfolio overlaps within a postal code area is higher than expected if households were randomly assigned to the area. Column (1) shows the maximum portfolio size included in test, column (2) the number of households included in the sample, column (3) the expected number of overlaps, column (4) the actual number of overlaps, and column (5) the ratio between actual and expected overlaps.

Max. portf. size	Number of households	Actual overlap	Expected overlap	Ratio
1	188,535	6,324,570	2,649,371	2.387
2	268,268	12,989,232	6,096,314	2.130
5	357,772	28,450,235	16,782,239	1.695
10	393,606	42,315,889	27,935,744	1.515
20	404,277	49,978,463	34,481,911	1.449
125	405,628	51,604,247	35,952,390	1.436

household portfolio holdings represent the underlying data generating process, and compare the expected overlap if households and their portfolios were randomly assigned to postal codes with what is observed in the data. An interesting property of this test is that the maximum portfolio size, measured by number of stocks in the portfolio, can be capped, so that the test is applied to a subpopulation of households whose portfolios contain no more than 1, 2, 5, etc., stocks. Because location, via personal beliefs, drive both underdiversification and the choice of similar portfolios for nearby households, we expect higher-than-expected overlaps to be more pronounced among households that hold few stocks.¹² Further details of the test are provided in Appendix B.

The results of the bootstrapping method are shown in Table 5. The ratio of actual-to-expected overlap (the right-most column in the table) is the highest, 2.387, when only households holding one stock are included in the test, and then gradually decreases as the maximum portfolio size increases. The ratio is 1.436 when the full sample of households is included. These ratios are thus lower than the ratio based on the $\frac{1}{N}$ assumption, but remain highly significant, in support of Prediction 3.

Next, we test Prediction 4, our empirical specification of belief formation in equation (21), which can be viewed as a panel with fixed effects for postal codes (representing the

¹²There is also a purely mechanical effect in that the actual overlap cannot be much higher than the expected overlap when households hold large portfolios. For example, if a household holds *all* stocks in its portfolio, *every* stock in the other household's portfolio is common for the two portfolios, so actual and expected overlap must coincide in that case.

log-risk aversion coefficients, g_p) and companies (representing the log-return characteristics, s_n). We therefore use panel regressions.

The model is not completely identified because the mapping $g_h \mapsto g_h + c$, $s_n \mapsto s_n + c$ yields the same household portfolio weights for an arbitrary constant c . Intuitively, higher risk aversion coefficients are offset by more favorable investment opportunities. We obtain unique identification by normalizing the results so that the average s_n coefficient is 0 ($s_n = 0$ is, for example, obtained with $\alpha_n - i = 0.09$, $\sigma_n = 0.3$).

Finally, we consider two approaches for handling portfolio weights of zero, for which the logarithm is not defined. There are a large number of such observations in the data, even with aggregation at the postal code level. Under the first approach, we include these observations but replace the zero with a small positive threshold, namely one Finnish Mark (corresponding to about USD 0.17). Under the second approach, we exclude such observations and run unbalanced panel regressions. The disadvantage of the former approach is that it introduces an arbitrary lower threshold, whereas the disadvantage of the latter is that it does not use information about zero holdings.

The results are shown in Table 6. We see that the sensitivity coefficient, κ , is highly significant in all regressions (univariate, including risk-aversion fixed effects, including risk-aversion and stock-characteristic fixed effects, and panel regression with robust standard errors double clustered at the firm and postal code level).

The results are also economically significant. The standard deviation of the distance between headquarter and household is 0.312. For the coefficient estimate that includes risk-aversion, stock distribution, and all observations (first row in Panel A, column 4, $\kappa = 3.118$), a one standard deviation decrease in distance to a firm's headquarters predicts an increase in portfolio holdings by a factor $e^{3.1880 \times 0.312} = 2.645$. The R^2 for the univariate regression in Panel A is 0.0654, corresponding to a correlation between network proximity and log-portfolio holdings of about 0.26.

Finally, we test Prediction 5, whether the estimated s_n coefficients (the firm fixed effects) from the belief-based model are informative about stocks return distributions out-of-sample.

Table 6: Test of Prediction 4: Estimated sensitivity coefficient, κ

This table estimates the sensitivity coefficient, κ . Panel A includes observations with $\omega_{pn} = 0$ replaced with $\omega_{pn} = 1$ (one Finnish Mark, corresponding to about USD 0.17). Panel B excludes observations with $\omega_{pn} = 0$. Univariate in column 2, including risk aversion in column 3, including risk aversion and stock distributions in column 4, panel regression with postal code and stock fixed effects and robust standard errors double-clustered at the postal code and firm level in column 5. Statistical significance levels: * = 0.01, ** = 0.001, *** = 0.0001.

	(1)	(2)	(3)	(4)	(5)
<i>Panel A</i>					
Sensitivity coefficient, κ		5.873***	3.861***	3.188***	3.185***
Standard error		0.037	0.058	0.056	0.434
log risk aversion, g					
-average			-4.330	-4.120	
-max			-0.382	-0.325	
-min			-12.887	-12.834	
log distribution, s					
-average				0	
-max				7.258	
-min				-2.658	
R^2		0.065	0.385	0.591	0.591
Adj. R^2		0.065	0.380	0.588	0.588
$N = 368, 298$					
<i>Panel B</i>					
Sensitivity coefficient, κ		2.251***	2.123***	2.669***	2.669***
Standard error		0.035	0.061	0.046	0.403
log risk aversion, g					
-average			-8.572	-7.979	
-max			-4.563	-2.638	
-min			-12.845	-12.883	
log distribution, s					
-average				0	
-max				4.166	
-min				-3.985	
R^2		0.030	0.210	0.626	0.626
Adj. R^2		0.030	0.191	0.618	0.618
$N = 134, 902$					

We calculate daily mean excess returns, $\hat{\alpha}_n - i$, and volatility, $\hat{\sigma}_n$, over a three-year period, from 2003-2005.

The results are shown in Table 7. We first compare the relation between estimated s_n and realized volatility, $\hat{\sigma}_n$ because realized volatility is the part of $\frac{\hat{\alpha}_n - i}{\hat{\sigma}_n^2}$ that is easiest to estimate. This relation should be negative. As shown in Column (1) in Panel A of the table, the relation is strongly negatively significant, with an R^2 of over 50%. We compare this

Table 7: Test of Prediction 5: Predictive power of estimated s_n coefficients

This table tests whether the s_n coefficients estimated from portfolio holdings predict realized volatility (Panel A) and return over variance (Panel B) in subsequent 5-year period. Column (1) uses the belief-based model, column (2) the rational expectations model, and column (3) both models. Realized volatility and returns measured using daily data over 3-year period, 2003-2005. Statistical significance levels: * = 0.01, ** = 0.001, *** = 0.0001.

	(1)	(2)	(3)
<i>Panel A: $\ln(\hat{\sigma}_n)$</i>			
Rational expectations, s_n^{RE}		-0.149***	0.036
Belief-based, s_n^{BB}	-0.185***		-0.219***
R^2	0.506	0.356	0.510
N	125		
<i>Panel B: $\ln\left(\frac{\hat{\alpha}_n - i}{\hat{\sigma}_n^2}\right)$</i>			
Rational expectations, s_n^{RE}		0.075	-0.296*
Belief-based, s_n^{BB}	0.149**		0.426***
R^2	0.068	0.019	0.121
N	108		

with the prediction of the rational expectations model. In the rational expectations model, all households choose the same risky portfolio, $\phi_{hn} \equiv 1$ for all h and n , and by summing (20) over households, it follows that s_n is directly related to a firm's log-size. Column (2) in Panel A of Table 7 shows that log-firm size also is informative about volatility in our data, but with lower explanatory power than the belief-based model. Moreover, when both the belief-based and rational-expectations estimates are included in a bivariate regression, as done in Column (3) of Panel A, the belief-based estimate dominates, remaining significant, whereas the log-firm size coefficient switches sign and becomes insignificant.

In Panel B of the table, we do the same estimation for $\ln\left(\frac{\hat{\alpha}_n - i}{\hat{\sigma}_n^2}\right)$, including the 108 stocks that had positive excess realized returns during the period (for which the logarithm is defined). The explanatory power is much lower both for the belief-based coefficient and the coefficient based on the rational expectation model, due to the well-known challenges of estimating stocks' expected returns from realized returns. However, the belief-based estimate remains significant at the 0.1% level in the univariate regression, and at the 0.01% level in the bivariate regression, while the rational-expectations coefficient switches sign in the bivariate regression. As a robustness check, we also run the same tests using five years of returns. The results (not reported) are similar, the main difference being that the

belief-based coefficient in the bivariate regression in panel B is significant only at the 10% level.

We stress that only data on portfolio holdings and location is used in the estimation of the s_n 's. That these are informative about out-of-sample stock return distributions therefore lends significant support for our belief-based explanation of observed home bias.

4.3 Robustness

A possible concern with the previous results is that they may be driven by different behaviors of households in urban areas (specifically, around Helsinki) relative to rural areas. For example, households in the Helsinki area may have a preference for stocks headquartered in an urban area for some other reason than network proximity, which will then lead to results similar to those reported above. To check if this is indeed the case, we run the regressions *excluding* stocks and postal codes in the Helsinki area (postal codes with fewer than 5 digits). The results are reported in Table 10 in Appendix B. We see that the results are qualitatively similar as before, and specifically, still both statistically and empirically highly significant. We also exclude households in the Helsinki area but not stocks, with similar results (not reported).

A potential alternative explanation for the results is that it may not be geographical distance per se that drives beliefs, but rather employment. That is, if households tend to invest in the firms they work for—which they likely also live close to—similar results may arise. To rule out such an explanation, we exclude observations for which the postal codes of account holder and firm headquarter are close. Specifically, we exclude all observations for which the normalized distance is less than some d_0 . Table 11 in Appendix B, shows that the results remain qualitatively similar when $d_0 = 0.01$ (corresponding to a minimal distance of about 8 miles between postal code of account holder and firm headquarter for an observation to be included), and $d = 0.03$ (corresponding to a minimal distance of about 24 miles). Thus, an employment effect does not seem to be driving the results.

Our approach provides a tightly specified model that links “local bias” to familiarity, through adjustments of investors’ beliefs about a firm as a function of distance—in our

empirical specification captured by geographical distance. Other explanations for local bias that have been put forward in the literature are transaction costs and hedging demand.

We argue that transaction costs are unlikely to play a major role in explaining home bias within a country, especially when all stocks are traded on the same exchange. With respect to hedging demand, an alternative explanation for why agents prefer to invest in nearby stocks is that they provide a hedge against local shocks. For example, if a local firm performs well, prices of services and goods (e.g., housing) may increase because of increased demand from the employees at the firm who are now wealthier. Investing in the local firm provides a hedge against such price shocks. Inasmuch as such hedging demand is related to the age and gender of the population, we can assess its affect, because account level information is available in the data. For example, younger investors—who are less likely to own their home—are likely to be more exposed to real estate price shocks than older investors.

We create variables for the average birth year (YOB) of the investors in a specific stock and postal code, and for their gender (GEN, which is 1 for male and 2 for female). The correlations between the distance to firm headquarter and these variables are both low, $\rho_{d,YOB} = -0.049$, and $\rho_{d,GEN} = -0.056$, suggesting that any hedging demand that varies with age and/or gender is not captured by the distance to firm headquarter. To further explore a potential relation, we rerun the tests from Table 6, Panel B, but including birth year and gender, and also from Table 11 with $d_0 = 0.05$, to rule out hedging demand for local shocks within a 40-mile radius.

As seen in Table 8, the coefficient estimates for κ barely change and are still highly significant. We conclude that it is unlikely that our results are driven by hedging demand against local shocks, at least within a 40-mile radius of the investor, and against shocks that are related to age and/or gender.

A concern may be that headquarter provides a very rough measure of a firm's location. For example, some firms have operations spread out over the whole country and will be familiar to households far away from its headquarter. The challenges of developing an objectively superior alternative measure of firm location are significant though, which is

Table 8: Estimate of sensitivity coefficient with additional controls

This table estimates the sensitivity coefficient, κ , when including investor year of birth (YOB) and gender (GEN). Panel A sets $d_0 = 0$, whereas Panel B sets $d_0 = 0.05$, corresponding to a threshold distance of about 40 miles. No fixed effects in column 2, including risk aversion in column 3, including risk aversion and stock distributions in column 4, panel regression with postal code and stock fixed effects and robust standard errors double-clustered at the postal code and firm level in column 5. Statistical significance levels: * = 0.01, ** = 0.001, *** = 0.0001.

	(1)	(2)	(3)	(4)	(5)
<i>Panel A</i>					
Sensitivity coefficient, κ		2.298***	2.056***	2.658***	2.658***
Standard error		0.034	0.060	0.046	0.398
Year of birth coefficient, YOB		-0.031***	-0.040***	-0.013***	-0.013***
Standard error		0.001	0.001	0.001	0.001
Gender coefficient, GEN		0.597***	0.445***	-0.157***	-0.157***
Standard error		0.019	0.018	0.013	0.033
R^2		0.065	0.249	0.630	0.630
Adj. R^2		0.065	0.233	0.622	0.622
$N = 134,825$					
<i>Panel B</i>					
Sensitivity coefficient, κ		0.809***	0.704***	1.377***	1.377***
Standard error		0.040	0.073	0.056	0.263
Year of birth coefficient, YOB		-0.031***	-0.037***	-0.012***	-0.012***
Standard error		0.001	0.001	0.001	0.001
Gender coefficient, GEN		0.461***	0.345***	-0.169***	-0.169***
Standard error		0.019	0.019	0.014	0.033
R^2		0.039	0.200	0.606	0.606
Adj. R^2		0.039	0.180	0.596	0.596
$N = 115,256$					

why we use the well-established headquarter measure of location. As a robustness check, we ensure that the results do not change when excluding the two “least local” companies during the period: the global telecommunication company Nokia, which made up over half of the stock market value in the early 2000s, and the retail store chain Stockmann, which had stores all over Finland. The results (not reported) are very similar when excluding those two companies.

5 Conclusion

Motivated by empirical evidence that rejects the rational expectations hypothesis and finds instead that beliefs of individuals are influenced by characteristics such as place of residence, culture, and socioeconomic status, we develop a model where a household's beliefs are an *endogenous* outcome of its location in a bipartite network of households and firms. We then evaluate the model empirically using data on portfolio holdings to infer household beliefs. The empirical evidence indicates that geographical distance between the locations of households and firms has a statistically and economically significant effect on the beliefs households have about firm-level stock returns and hence on their portfolio holdings.

A Proofs

In this appendix, we provide all derivations for the results in the main text.

Proof of Proposition 1

The definition of the certainty equivalent in (6) implies that

$$\widehat{\mu}_{h,t}^{\nu}[U_{h,t+dt}] = E_t^{\mathbb{Q}^{\nu_h}} \left[U_{h,t+dt}^{1-\gamma_h} \right]^{\frac{1}{1-\gamma_h}}.$$

Therefore

$$\widehat{\mu}_{h,t}^{\nu}[U_{h,t+dt}] = E_t^{\mathbb{Q}^{\nu_h}} \left[U_{h,t+dt}^{1-\gamma_h} \right]^{\frac{1}{1-\gamma_h}} = E_t^{\mathbb{Q}^{\nu_h}} \left[U_{h,t}^{1-\gamma_h} + d(U_{h,t}^{1-\gamma_h}) \right]^{\frac{1}{1-\gamma_h}}.$$

Applying Ito's Lemma, we obtain

$$\begin{aligned} d(U_{h,t}^{1-\gamma_h}) &= (1-\gamma_h)U_{h,t}^{-\gamma_h}dU_{h,t} - \frac{1}{2}(1-\gamma_h)\gamma_h U_{h,t}^{-\gamma_h-1}(dU_{h,t})^2 \\ &= (1-\gamma_h)U_{h,t}^{1-\gamma_h} \left[\frac{dU_{h,t}}{U_{h,t}} - \frac{1}{2}\gamma_h \left(\frac{dU_{h,t}}{U_{h,t}} \right)^2 \right]. \end{aligned}$$

Therefore

$$\begin{aligned} \widehat{\mu}_{h,t}^{\nu}[U_{h,t+dt}] &= E_t^{\mathbb{Q}^{\nu_h}} \left[U_{h,t+dt}^{1-\gamma_h} \right]^{\frac{1}{1-\gamma_h}} = U_{h,t} \left(E_t \left[1 + (1-\gamma_h) \left[\frac{dU_{h,t}}{U_{h,t}} - \frac{1}{2}\gamma_h \left(\frac{dU_{h,t}}{U_{h,t}} \right)^2 \right] \right] \right)^{\frac{1}{1-\gamma_h}} \\ &= U_{h,t} \left(1 + (1-\gamma_h) \left[E_t^{\mathbb{Q}^{\nu_h}} \left[\frac{dU_{h,t}}{U_{h,t}} \right] - \frac{1}{2}\gamma_h E_t^{\mathbb{Q}^{\nu_h}} \left[\left(\frac{dU_{h,t}}{U_{h,t}} \right)^2 \right] \right] \right)^{\frac{1}{1-\gamma_h}} \\ &= U_{h,t} \left(1 + (1-\gamma_h) \left[E_t^{\mathbb{Q}^{\nu_h}} \left[\frac{dU_{h,t}}{U_{h,t}} \right] - \frac{1}{2}\gamma_h E_t \left[\left(\frac{dU_{h,t}}{U_{h,t}} \right)^2 \right] \right] \right)^{\frac{1}{1-\gamma_h}}. \end{aligned}$$

Hence,

$$\widehat{\mu}_{h,t}^{\nu}[U_{h,t+dt}] = U_{h,t} \left(1 + E_t^{\mathbb{Q}^{\nu_h}} \left[\frac{dU_{h,t}}{U_{h,t}} \right] - \frac{1}{2}\gamma_h E_t \left[\left(\frac{dU_{h,t}}{U_{h,t}} \right)^2 \right] \right) + o(dt).$$

Therefore, in the continuous time limit, we obtain

$$\frac{\widehat{\mu}_{h,t}^{\nu}[dU_{h,t+dt}]}{dt} = \frac{\widehat{\mu}_{h,t}^{\nu}[U_{h,t+dt}] - U_{h,t}}{dt} = U_{h,t} \left(\frac{1}{dt} E_t^{\mathbb{Q}^{\nu_h}} \left[\frac{dU_{h,t}}{U_{h,t}} \right] - \frac{1}{2}\gamma_h \frac{1}{dt} E_t \left[\left(\frac{dU_{h,t}}{U_{h,t}} \right)^2 \right] \right).$$

From Girsanov's Theorem

$$E_t^{\mathbb{Q}^{\nu_h}} \left[\frac{dU_{h,t}}{U_{h,t}} \right] = E_t \left[\frac{dU_{h,t}}{U_{h,t}} \right] + E_t \left[\frac{dU_{h,t}}{U_{h,t}} \frac{dM_{h,t}}{M_{h,t}} \right].$$

Therefore

$$\begin{aligned} \frac{\widehat{\mu}_{h,t}^{\nu} [dU_{h,t}]}{dt} &= \frac{\widehat{\mu}_{h,t}^{\nu} [U_{h,t+dt}] - U_{h,t}}{dt} \\ &= U_{h,t} \left(\frac{1}{dt} E_t \left[\frac{dU_{h,t}}{U_{h,t}} \right] - \frac{1}{2} \gamma_h \frac{1}{dt} E_t \left[\left(\frac{dU_{h,t}}{U_{h,t}} \right)^2 \right] + \frac{1}{dt} E_t \left[\frac{dU_{h,t}}{U_{h,t}} \frac{dM_{h,t}}{M_{h,t}} \right] \right). \end{aligned}$$

It follows from the above expression that the certainty equivalent operator, $\mu_{h,t}[\cdot]$, is given by

$$\begin{aligned} \frac{\mu_{h,t} [dU_{h,t}]}{dt} &= \frac{\mu_{h,t} [U_{h,t+dt}] - U_{h,t}}{dt} \\ &= U_{h,t} \left(\frac{1}{dt} E_t \left[\frac{dU_{h,t}}{U_{h,t}} \right] - \frac{1}{2} \gamma_h \frac{1}{dt} E_t \left[\left(\frac{dU_{h,t}}{U_{h,t}} \right)^2 \right] \right). \end{aligned}$$

Therefore

$$\frac{\widehat{\mu}_{h,t}^{\nu} [dU_{h,t}]}{dt} = \frac{\mu_{h,t} [dU_{h,t}]}{dt} + \frac{1}{dt} U_{h,t} E_t \left[\frac{dU_{h,t}}{U_{h,t}} \frac{dM_{h,t}}{M_{h,t}} \right],$$

i.e.

$$\widehat{\mu}_{h,t}^{\nu} [dU_{h,t}] = \mu_{h,t} [dU_{h,t}] + U_{h,t} E_t \left[\frac{dU_{h,t}}{U_{h,t}} \frac{dM_{h,t}}{M_{h,t}} \right],$$

which implies

$$\widehat{\mu}_{h,t}^{\nu} [U_{h,t+dt}] = \mu_{h,t} [U_{h,t+dt}] + U_{h,t} E_t \left[\frac{dU_{h,t}}{U_{h,t}} \frac{dM_{h,t}}{M_{h,t}} \right].$$

If the investment opportunity set is constant, then $U_{h,t}$ is a function solely of $W_{h,t}$ – there are no other state variables. Hence

$$E_t \left[\frac{dU_{h,t}}{U_{h,t}} \frac{dM_{h,t}}{M_{h,t}} \right] = \frac{W_{h,t} U_{W_{h,t}}}{U_{h,t}} \boldsymbol{\nu}_{h,t}^{\top} \boldsymbol{\omega}_{h,t} dt,$$

and so

$$\widehat{\mu}_{h,t}^{\nu} [U_{h,t+dt}] = \mu_{h,t} [U_{h,t+dt}] + U_{h,t} \frac{W_{h,t} U_{W_{h,t}}}{U_{h,t}} \boldsymbol{\nu}_{h,t}^{\top} \boldsymbol{\omega}_{h,t} dt.$$

From the above expression and (5), (7) follows. □

Proof of Corollary 1

If the investment opportunity set is constant, then the personal divergence vector is obtained from the following optimization problem:

$$\inf_{\nu_{h,t}} \left(\frac{W_{h,t} U_{W_{h,t}}}{U_{h,t}} \nu_{h,t}^\top \omega_{h,t} + \frac{1}{2\gamma_h} \sum_{n=1}^N \frac{\phi_{hn}}{1 - \phi_{hn}} \frac{\nu_{hn,t}^2}{\sigma_n^2} \right).$$

The FOC for the above problem is

$$0 = \frac{W_{h,t} U_{W_{h,t}}}{U_{h,t}} \omega_{hn,t} + \frac{1}{\gamma_h} \frac{\phi_{hn}}{1 - \phi_{hn}} \frac{\nu_{hn,t}}{\sigma_n^2}.$$

(9) follows from the above equation. □

Proof of Proposition 2

Writing out (11) explicitly gives

$$U_{h,t}^{1 - \frac{1}{\psi_h}} = (1 - e^{-\delta_h dt}) C_{h,t}^{1 - \frac{1}{\psi_h}} + e^{-\delta_h dt} (\mu_{h,t}^\nu [U_{h,t+dt}])^{1 - \frac{1}{\psi_h}},$$

where for ease of notation sup and inf have been suppressed. Now

$$\begin{aligned} (\mu_{h,t}^\nu [U_{h,t+dt}])^{1 - \frac{1}{\psi_h}} &= (U_{h,t} + \mu_{h,t}^\nu [dU_{h,t}])^{1 - \frac{1}{\psi_h}} \\ &= U_{h,t}^{1 - \frac{1}{\psi_h}} \left(1 + \mu_{h,t}^\nu \left[\frac{dU_{h,t}}{U_{h,t}} \right] \right)^{1 - \frac{1}{\psi_h}} \\ &= U_{h,t}^{1 - \frac{1}{\psi_h}} \left(1 + \left(1 - \frac{1}{\psi_h} \right) \mu_{h,t}^\nu \left[\frac{dU_{h,t}}{U_{h,t}} \right] \right) + o(dt). \end{aligned}$$

Hence

$$U_{h,t}^{1 - \frac{1}{\psi_h}} = \delta C_{h,t}^{1 - \frac{1}{\psi_h}} dt + U_{h,t}^{1 - \frac{1}{\psi_h}} \left(1 + \left(1 - \frac{1}{\psi_h} \right) \mu_{h,t}^\nu \left[\frac{dU_{h,t}}{U_{h,t}} \right] \right) - \delta_h U_{h,t}^{1 - \frac{1}{\psi_h}} dt + o(dt),$$

from which we obtain (12). □

Proof of Proposition 3

$$\begin{aligned}
\mu_{h,t}^{\nu} [dU_{h,t}] &= E_t [dU_{h,t}] - \frac{\gamma_h}{2} U_{h,t} E_t \left[\left(\frac{dU_{h,t}}{U_{h,t}} \right)^2 \right] + U_{h,t} \inf_{\nu_h} \left(\frac{W_{h,t}}{U_{h,t}} \frac{\partial U_{h,t}}{\partial W_{h,t}} \nu_{h,t}^{\top} \omega_{h,t} + L_{h,t} \right) dt \\
&= W_{h,t} \frac{\partial U_{h,t}}{\partial W_{h,t}} E_t \left[\frac{dW_{h,t}}{W_{h,t}} \right] + \frac{1}{2} W_{h,t}^2 \frac{\partial^2 U_{h,t}}{\partial W_{h,t}^2} E_t \left[\left(\frac{dW_{h,t}}{W_{h,t}} \right)^2 \right] - \frac{\gamma_h}{2} U_{h,t} \frac{W_{h,t}}{U_{h,t}} \frac{\partial U_{h,t}}{\partial W_{h,t}} E_t \left[\left(\frac{dW_{h,t}}{W_{h,t}} \right)^2 \right] \\
&\quad + U_{h,t} \inf_{\nu_h} \left(\frac{W_{h,t}}{U_{h,t}} \frac{\partial U_{h,t}}{\partial W_{h,t}} \nu_{h,t}^{\top} \omega_{h,t} + L_{h,t} \right) dt \\
&= W_{h,t} \frac{\partial U_{h,t}}{\partial W_{h,t}} \left(i_t + (\alpha - i\mathbf{1})^{\top} \omega_{h,t} - \frac{C_{h,t}}{W_{h,t}} \right) dt + \frac{1}{2} W_{h,t}^2 \frac{\partial^2 U_{h,t}}{\partial W_{h,t}^2} \omega_{h,t}^{\top} V \omega_{h,t} \\
&\quad - \frac{\gamma_h}{2} U_{h,t} \frac{W_{h,t}}{U_{h,t}} \frac{\partial U_{h,t}}{\partial W_{h,t}} \omega_{h,t}^{\top} V \omega_{h,t} + U_{h,t} \inf_{\nu_h} \left(\frac{W_{h,t}}{U_{h,t}} \frac{\partial U_{h,t}}{\partial W_{h,t}} \nu_{h,t}^{\top} \omega_{h,t} + L_{h,t} \right) dt
\end{aligned}$$

Substituting (9) into $\frac{W_{h,t}}{U_{h,t}} \frac{\partial U_{h,t}}{\partial W_{h,t}} \nu_{h,t}^{\top} \omega_{h,t} + L_{h,t}$ gives

$$\inf_{\nu_h} \left(\frac{W_{h,t}}{U_{h,t}} \frac{\partial U_{h,t}}{\partial W_{h,t}} \nu_{h,t}^{\top} \omega_{h,t} + L_{h,t} \right) = -\frac{1}{2} \gamma_h \frac{W_{h,t}}{U_{h,t}} \frac{\partial U_{h,t}}{\partial W_{h,t}} \omega_{h,t}^{\top} \sigma F_h^{-1} \sigma \omega_{h,t}$$

Therefore

$$\begin{aligned}
\mu_{h,t}^{\nu} \left[\frac{dU_{h,t}}{U_{h,t}} \right] &= \frac{W_{h,t}}{U_{h,t}} \frac{\partial U_{h,t}}{\partial W_{h,t}} \left(i_t + (\alpha - i\mathbf{1})^{\top} \omega_{h,t} - \frac{C_{h,t}}{W_{h,t}} \right) dt \\
&\quad + \frac{1}{2} \left(\frac{W_{h,t}^2}{U_{h,t}} \frac{\partial^2 U_{h,t}}{\partial W_{h,t}^2} - \gamma_h \frac{W_{h,t}}{U_{h,t}} \frac{\partial U_{h,t}}{\partial W_{h,t}} \right) \omega_{h,t}^{\top} V \omega_{h,t} dt - \frac{1}{2} \gamma_h \frac{W_{h,t}}{U_{h,t}} \frac{\partial U_{h,t}}{\partial W_{h,t}} \omega_{h,t}^{\top} \sigma F_h^{-1} \sigma \omega_{h,t} dt \\
&= \frac{W_{h,t}}{U_{h,t}} \frac{\partial U_{h,t}}{\partial W_{h,t}} \left(i_t + (\alpha - i\mathbf{1})^{\top} \omega_{h,t} - \frac{C_{h,t}}{W_{h,t}} - \frac{1}{2} \gamma_h \omega_{h,t}^{\top} (V + \sigma F_h^{-1} \sigma) \omega_{h,t} \right) dt \\
&\quad + \frac{1}{2} \left(\frac{W_{h,t}^2}{U_{h,t}} \frac{\partial^2 U_{h,t}}{\partial W_{h,t}^2} - \gamma_h \frac{W_{h,t}}{U_{h,t}} \frac{\partial U_{h,t}}{\partial W_{h,t}} \right) \omega_{h,t}^{\top} V \omega_{h,t} dt + \frac{1}{2} \frac{W_{h,t}^2}{U_{h,t}} \frac{\partial^2 U_{h,t}}{\partial W_{h,t}^2} \omega_{h,t}^{\top} V \omega_{h,t} dt \\
&= \frac{W_{h,t}}{U_{h,t}} \frac{\partial U_{h,t}}{\partial W_{h,t}} \left(i_t + (\alpha - i\mathbf{1})^{\top} \omega_{h,t} - \frac{C_{h,t}}{W_{h,t}} - \frac{1}{2} \gamma_h \omega_{h,t}^{\top} (V + \sigma F_h^{-1} \sigma) \omega_{h,t} \right. \\
&\quad \left. - \left(-\frac{W_{h,t}^2}{W_{h,t}} \frac{\partial^2 U_{h,t}}{\partial W_{h,t}^2} \right) \omega_{h,t}^{\top} V \omega_{h,t} \right) dt
\end{aligned}$$

Equation (12) can therefore be rewritten as

$$0 = \sup_{C_{h,t}} \delta u_\psi \left(\frac{C_{ht}}{U_{ht}} \right) + \sup_{\omega_{h,t}} \frac{W_{h,t}}{U_{h,t}} \frac{\partial U_{h,t}}{\partial W_{h,t}} \left(i_t + (\boldsymbol{\alpha} - i\mathbf{1})^\top \boldsymbol{\omega}_{h,t} - \frac{C_{h,t}}{W_{h,t}} - \frac{1}{2} \gamma_h \boldsymbol{\omega}_{h,t}^\top (V + \boldsymbol{\sigma} F_h^{-1} \boldsymbol{\sigma}) \boldsymbol{\omega}_{h,t} \right. \\ \left. - \left(- \frac{W_{h,t}^2 \frac{\partial^2 U_{h,t}}{\partial W_{h,t}^2}}{W_{h,t} \frac{\partial U_{h,t}}{\partial W_{h,t}}} \right) \boldsymbol{\omega}_{h,t}^\top V \boldsymbol{\omega}_{h,t} \right). \quad (\text{A1})$$

We seek a solution to the above Hamilton-Jacobi-Bellman equation of the form

$$U_{h,t} = \kappa_h W_{h,t}, \quad (\text{A2})$$

where κ_h is a constant. Equation (A1) thus reduces to

$$0 = \sup_{C_{h,t}} \delta u_\psi \left(\frac{C_{ht}}{W_{ht}} \right) + \sup_{\omega_{h,t}} \left(i_t + (\boldsymbol{\alpha} - i\mathbf{1})^\top \boldsymbol{\omega}_{h,t} - \frac{C_{h,t}}{U_{h,t}} - \frac{1}{2} \gamma_h \boldsymbol{\omega}_{h,t}^\top (V + \boldsymbol{\sigma} F_h^{-1} \boldsymbol{\sigma}) \boldsymbol{\omega}_{h,t} \right). \quad (\text{A3})$$

We can rewrite the expression $-\frac{1}{2} \gamma_h \boldsymbol{\omega}_{h,t}^\top \boldsymbol{\sigma} F_h^{-1} \boldsymbol{\sigma} \boldsymbol{\omega}_{h,t}$ as $\boldsymbol{\nu}_{h,t}^\top \boldsymbol{\omega}_{h,t} + \frac{1}{2\gamma} \boldsymbol{\nu}_{h,t}^\top \boldsymbol{\sigma} F_h^{-1} \boldsymbol{\sigma} \boldsymbol{\nu}_{h,t}$, and so we obtain (13). \square

Proof of Proposition 4

Substituting (A2) into (9) and exploiting the fact that $\boldsymbol{\sigma}$ and F_h are diagonal matrices and hence commute with each other, we obtain

$$\boldsymbol{\nu}_{h,t} = -\gamma_h \boldsymbol{\sigma}^2 F_h^{-1} \boldsymbol{\omega}_{h,t}. \quad (\text{A4})$$

Again, exploiting the commutativity of diagonal matrices, we can write the portfolio choice problem in (A3) as

$$\sup_{\omega_{h,t}} (\boldsymbol{\alpha} - i\mathbf{1})^\top \boldsymbol{\omega}_{h,t} - \frac{1}{2} \gamma_h \boldsymbol{\omega}_{h,t}^\top (V + \boldsymbol{\sigma}^2 F_h^{-1}) \boldsymbol{\omega}_{h,t}.$$

The above linear-quadratic problem has a unique interior solution given by (17). Substituting (17) into (A4) and simplifying gives (16). We can also rewrite the expression for $\boldsymbol{\omega}_h$ in (17) in terms of the personal divergence measure:

$$\boldsymbol{\omega}_h = \frac{1}{\gamma_h} V^{-1} (\boldsymbol{\alpha} + \boldsymbol{\nu}_h - i\mathbf{1}),$$

where

$$\boldsymbol{\nu}_h = -(I + V F_h \boldsymbol{\sigma}^{-2}) (\boldsymbol{\alpha} - i\mathbf{1}).$$

\square

B Empirical Results

In this appendix, we provide additional information about the empirical tests and results. The stocks in our sample are shown in Table 9. The estimate sensitivity coefficient when excluding the Helsinki area is shown in Table 10. The estimated sensitivity coefficient when including only distant observations are shown in Table 11.

Table 9: Stocks in sample

This table lists the 125 stocks issued by companies that are headquartered in Finland and are included in our sample. For companies with A- and B-shares, both shares are included in our sample.

1	Bank of Aland Plc A	2	Pohjola Group Plc
3	Norvestia Plc	4	Kesko Corporation B
5	Stockmann Plc A	6	Stockmann Plc B
7	Tieto Corporation	8	Amer Sports Corporation
9	Fiskars Corporation	10	Fiskars Corporation K
11	Huhtamki Oyj	12	Instrumentarium
13	Kone Corporation B	14	Metsa Board Oyj A
15	Metsa Board Oyj B	16	Nokia Corporation
17	Tamro Oyj	18	Tamfelt Corp.
19	Tamfelt Corporation Ord. shares	20	Bank of Aland Plc B
21	Uponor Oyj	22	Outokumpu Oyj
23	Citycon Oyj	24	Polar Real Estate Corp.
25	Raisio Plc Vaihto-osake	26	Birka Line Abp B
27	Pohjola Bank A	28	Finnair Oyj
29	Sampo Plc A	30	Stromsdal Corporation
31	Apetit Plc	32	Rautaruukki Corporation
33	Finnlines Plc	34	Silja Oyj Abp
35	Wartsila Corporation A	36	Wartsila Corporation
37	Tiimari Plc	38	Kemira Oyj
39	Ponsse Oyj	40	Viking Line Abp
41	Nokian Tyres Plc	42	Biohit Oyj B
43	Konecranes Plc	44	Stora Enso Oyj A
45	Stora Enso Oyj R	46	UPM-Kymmene Corporation
47	HKScan Oyj A	48	PKC Group Oyj
49	Incap Corporation	50	Atria Plc A
51	Payry PLC	52	Sponda Plc
53	Technopolis Plc	54	Valoe Oyj
55	Alma Media Corporation 1	56	Alma Media Corporation 2
57	Ramirent Plc	58	Fortum Corporation
59	Bittium Corporation	60	Yomi Plc
61	Rapala VMC Corporation	62	Sonera Oyj
63	Eimo Oyj	64	Innofactor Plc
65	Marimekko Corporation	66	SanomaWSOY Corporation A
67	Sanoma Corporation	68	Teleste Corporation
69	Oral Hammaslakarit Plc	70	Perlos Corporation
71	Metso Corporation	72	Talentum Oyj
73	Kesko Corporation A	74	Aldata Solution Oyj
75	Digia Plc	76	Solteq Oyj
77	Ixonos Plc	78	Aspo Plc
79	Aspocomp Group Plc	80	Dovre Group Plc
81	Trainers House Plc	82	Comptel Corporation
83	SSH Communications Security Oyj	84	Basware Corporation
85	Wulff Group Plc	86	Saunalahti Group Oyj
87	Etteplan Oyj	88	QPR Software Plc
89	eQ Oyj	90	Tekla Corporation
91	Sievi Capital plc	92	Sentera Plc
93	Okmetic Oyj	94	CapMan Plc B
95	Vacon Plc	96	eQ Oyj
97	Evox Rifa Group Plc	98	Componenta Corporation
99	Glaston Corporation	100	Tecnotree Corporation
101	Lassila & Tikanoja Plc	102	Suominen Oyj
103	Revenio Group Corporation	104	Biotie Therapies Corp.
105	Ilkka-Yhtyma Oyj 2	106	Neo Industrial Oyj
107	Orion Corporation A	108	Orion Corporation B
109	Raisio Plc K	110	Saga Furs Oyj C
111	YIT Corporation	112	Stonesoft Corporation
113	F-Secure Corporation	114	Chips Corporation B
115	Efore Plc	116	Hackman Oyj Abp
117	Honkarakenne Oyj B	118	Lemminkainen Corporation
119	Evia Oyj	120	Martela Oyj A
121	Olvi Plc A	122	Cramo Oyj
123	Tulikivi Oyj A	124	Elecster Oyj A
125	Vaisala Corporation A		

Table 10: Estimate of sensitivity coefficient excluding Helsinki area

This table estimates the sensitivity coefficient, κ , using only firms and households outside of the Helsinki area (with 5-digit postal codes). Panel A includes observations with $\omega_{pn} = 0$, replacing with $\omega_{pn} = 1$ (one Finnish Mark, corresponding to approximately USD 0.17). Panel B excludes observations with $\omega_{pn} = 0$. Univariate in column 2, including risk aversion in column 3, including risk aversion and stock distributions in column 4, panel regression with postal code and stock fixed effects and robust standard errors double-clustered at the postal code and firm level in column 5. Statistical significance levels: * = 0.01, ** = 0.001, *** = 0.0001.

	(1)	(2)	(3)	(4)	(5)
<i>Panel A</i>					
Sensitivity coefficient, κ		4.228***	3.868***	3.771***	3.771***
Standard error		0.069	0.071	0.073	0.630
log risk aversion, g					
-average			-3.806	-3.771	
-max			-1.097	-1.073	
-min			-12.393	-12.355	
log distribution, s					
-average				0	
-max				5.418	
-min				-1.797	
R^2		0.051	0.308	0.487	0.487
Adj. R^2		0.051	0.283	0.469	0.469
$N = 69,368$					
<i>Panel B</i>					
Sensitivity coefficient, κ		1.824***	1.892***	2.754***	2.754***
Standard error		0.074	0.082	0.071	0.325
log risk aversion, g					
-average			-8.072	-8.084	
-max			-2.681	-2.282	
-min			-12.392	-12.757	
log distribution, s					
-average				0	
-max				1.787	
-min				-3.197	
R^2		0.028	0.249	0.501	0.501
Adj. R^2		0.028	0.159	0.440	0.440
$N = 20,787$					

Table 11: Estimate of sensitivity coefficient including only distant observations

This table estimates the sensitivity coefficient, κ , including only postal code/firm observation at distances larger than d_0 , and excluding $\omega_{pm} = 0$ observations. Panel A sets d_0 to 0.01, corresponding to a minimum distance between postal codes of account holder and firm headquarter of about 8 miles, whereas in Panel B $d_0 = 0.03$, corresponding to a minimal distance of about 24 miles. Univariate in column 2, including risk aversion in column 3, including risk aversion and stock distributions in column 4, panel regression with postal code and stock fixed effects and robust standard errors double-clustered at the postal code and firm level in column 5. Statistical significance levels: * = 0.01, ** = 0.001, *** = 0.0001.

	(1)	(2)	(3)	(4)	(5)
<i>Panel A</i>					
Sensitivity coefficient, κ		1.772***	1.844***	2.397***	2.397***
Standard error		0.036	0.063	0.048	0.392
log risk aversion, g					
-average			-8.480	-7.899	
-max			-4.543	-2.282	
-min			-12.415	-12.609	
log distribution, s					
-average				0	
-max				4.142	
-min				-3.952	
R^2		0.018	0.186	0.614	0.614
Adj. R^2		0.018	0.168	0.605	0.605
$N = 129,802$					
<i>Panel B</i>					
Sensitivity coefficient, κ		0.931***	0.991***	1.662***	1.662***
Standard error		0.039	0.070	0.053	0.326
log risk aversion, g					
-average			-8.187	-7.680	
-max			-4.000	-2.561	
-min			-11.798	-12.117	
log distribution, s					
-average				0	
-max				4.067	
-min				-3.977	
R^2		0.005	0.160	0.599	0.599
Adj. R^2		0.005	0.139	0.589	0.589
$N = 119,874$					

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