Rare Disasters and the Term Structure of Interest Rates*

Jerry Tsai

The Wharton School, University of Pennsylvania

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Abstract

This paper offers an explanation for the properties of the nominal term structure of interest rates and time-varying bond risk premia based on a model with rare consumption disaster risk. In the model, expected inflation follows a mean reverting process but is also subject to possible large (positive) shocks when consumption disasters occur. The possibility of jumps in inflation increases nominal yields and the yield spread, while time-variation in the inflation jump probability drives time-varying bond risk premia. Predictability regressions offer independent evidence for the model's ability to generate realistic implications for both the stock and bond markets.

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1 Introduction

Empirical work has documented the failure of the expectations hypothesis. The average nominal term structure of interest rates on government bonds is upward-sloping, and the excess bond returns are predictable by variables such as yield spread. This indicates that bond risk premia are on average positive and vary over time. This paper presents a representative agent asset pricing model in which the aggregate endowment is subject to large negative shocks (disasters). Earlier work has shown that models with time-varying disaster risk can account for the high equity premium, high stock market volatility and aggregate market return predictability observed in the aggregate stock market.[1] In addition to the aggregate market results shown in previous work, my model accurately captures the shape of the nominal yield curve and the time-varying bond risk premia.

This paper provides an explanation for these features of the nominal bonds in a time-varying rare disaster model. In particular, consumption disasters may co-occur with high inflation, implying that nominal bonds are risky because their real values during bad times can be very low. Table 1 provides evidence for the co-occurrence of consumption disaster and high inflation. In this paper, a consumption disaster is defined as a consumption decline of more than 10%, and I consider a period as having high inflation if the average annual inflation rate during the period is greater than 10%. In recorded history, 17 of the 53 consumption disasters in OECD countries, and 30 of the 89 consumption disasters among all countries, were accompanied by inflation rates greater than 10%.[2] Furthermore, in 18 of the 30 inflation disasters, inflation rates exceeded the real consumption declines.[3] Figure 1

[1]For example, Rietz (1988), Longstaff and Piazzesi (2004), and Barro (2006) obtain high equity premium, Gabaix (2012), Gourio (2008), and Wachter (2012) also obtain high volatility and predictability.

[2]One might argue that consumption disasters are accompanied by large deflation. However, only 10 of the OECD disasters, and 17 of all disasters coincide with deflation. Furthermore, none of these disasters had an abnormally large annual deflation rate; for example, the Great Depression had an annual deflation rate of 6.4%.

[3]One of the most extreme examples is the hyperinflation that occurred in Germany after World War I. Between 1922 and 1923, real consumption declines by 12.7%, but the inflation rate in the corresponding

shows that the historical distribution of annual inflation rates has a fat tail. Furthermore, these jumps in inflation rates do not happen all at once, they were gradual processes that lasted a number of years.

Motivated by this evidence, I model the aggregate endowment in the model as subject to two types of disasters. These disasters are modeled as negative jumps in the realized consumption process. When the first type of disaster occurs, aggregate endowment drops, but expected inflation is unaffected. When the second type of disaster occurs, not only does aggregate endowment drop, but expected inflation increases. There were no consumption disasters in the United States in the period following World Was II. In the 1970s, however, the U.S. experienced a period of high inflation. To accommodate this possibility in the model, I allow for a third type of jump, one which affects expected inflation but not aggregate consumption growth.

Because government bonds are nominally denominated, they are subject to inflation jump risks. Investors require compensation for bearing these risks. The shape of the nominal yield curve in the model is mostly determined by the inflation jump risk since bonds with longer maturities are more sensitive to these risks. In particular, the yield spread increases in inflation jump risks, thus the model accurately predicts an upward-sloping nominal yield curve. Furthermore, the time-varying nature of disaster probability implies a time-varying bond risk premium.

This paper makes two main contributions to the existing literature. First, it provides a parsimonious model that jointly explains the stock and bond markets. Second, it can account for the time-series behavior of the bond premium and its relation to the equity premium. While the model is only calibrated to match aggregate consumption growth, inflation, and aggregate stock market moments, it generates realistic implications for the nominal term structure. Similar to the findings of Litterman and Scheinkman (1991), the first three principal components explain almost all the variations in nominal yields in the model, furthermore, each of these three principal component is highly correlated with

period is 3450%.

one of the three state variables in the model. This model also generates bond premium predictability because bond premium are mainly affected by the time-varying risk of the co-occurrence of a consumption disaster and high inflations. In particular, this model is able to reproduce the findings in Campbell and Shiller (1991) and Cochrane and Piazzesi (2005). Nominal bond excess returns are predictable by the yield spread and a linear combination of forward rates.

Besides the shape of the nominal term structure and the time-series behavior of the bond risk premium, this model can also account for the interaction between the stock and nominal bond markets. Duffee (2012) suggests that while term structure variables can predict the bond premium, they are not good predictors for the equity premium. In particular, I show that the price-dividend ratio predicts excess returns on the aggregate market (Campbell and Shiller (1988)) and that it has some predictive power for excess returns on the bond market. Term structure variables predict excess returns on the nominal bond market (Fama and Bliss (1987)) yet they are less effective at predicting excess returns on the aggregate market. In this model, the prices of risk have a two-factor structure, and the model is thus capable of explaining these results.

Several other papers also provide joint explanations for stock and bond market prices. Gabaix (2012) also considers a model with rare disasters. In that model, rather than time-variation in the disaster probability, it is time-variation in the expected size of an inflation jump that drives the bond premium. Furthermore, I allow fewer degrees of freedom in the calibration so that none of the parameters are chosen to match the yield curve. Wachter (2006), Bekaert, Engstrom, and Grenadier (2010) and Buraschi and Jiltsov (2007) consider extensions to the model with external habit formation (Campbell and Cochrane (1999)).Bakshi and Chen (1996) study monetary models in which the money supply directly enters the utility function. The economic mechanisms behind this model differ from those in the papers mentioned here. The shape of the nominal term structure is driven by the time-varying probability of the co-occurrence of a large consumption decline and high inflations. Furthermore, this paper provides evidence of the interaction between stock and bond markets by studying cross-market predictability. It is likely that the term structure of interest rates and bond premia are affected by multiple mechanisms, and this paper provides another possible way to jointly explain the aggregate market and bond market in a single model.

This paper is also related to a stream of literature that focuses on the term structure of interest rates, but does not address equity prices. Piazzesi and Schneider (2006) focus on the negative effects of surprise inflation on future consumption growth. Bansal and Shaliastovich (2012) build on the Bansal and Yaron (2004) long-run risk framework with stochastic volatility. Similar to Piazzesi and Schneider (2006) and Bansal and Shaliastovich (2012), in this model, when the risk of the co-occurrence of a consumption disaster and high inflations is high, expected consumption growth is low and expected inflation is high. However, high inflations and low consumption growth only co-occur when this type of consumption disasters are realized. Bekaert, Hodrick, and Marshall (2001) evaluate the violation of the expectations hypothesis using a Peso problem explanation. Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2012) study the effect of differences in beliefs about expected inflation when investors have habit-formation preferences.

Finally Dai and Singleton (2002) study three-factor term structure models in the essentially affine class (Duffee (2002)) and show that a statistical model of the stochastic discount factor can resolve the expectations hypothesis puzzle. Many other recent papers also consider the role of macroeconomic variables in the term structure by introducing macroeconomic time series into the stochastic discount factor (Ang and Piazzesi (2003), Ang, Dong, and Piazzesi (2007), Bikbov and Chernov (2010), Duffee (2006), and Rudebusch and Wu (2008)).

The remainder of the paper is organized as follows. Section 2 describes and solves the model. Section 3 discusses the quantitative results of the model. Section 4 concludes.

2 Model

2.1 Endowment, inflation, and preferences

The economy is populated with a representative agent. Assume that aggregate real consumption solves the following stochastic differential equation:

$$\frac{dC_t}{C_{t^-}} = \mu\, dt + \sigma_C\, dB_{Ct} + (e^{Z_{ct}} - 1)\, dN_{ct} + (e^{Z_{cq,t}} - 1)\, dN_{cq,t},$$

where B_{Ct} is a standard Brownian motion. Aggregate consumption is subject to two types of large shocks, and the arrival times of these shocks have a Poisson distribution, given by N_{ct} and $N_{cq,t}$. I will discuss the size and intensity of these Poisson jumps after I specify the inflation process.

To model nominal assets, I assume an exogenous process for the price level:

$$\frac{dP_t}{P_{t^-}} = q_t\, dt + \sigma_P\, dB_{Pt}, \tag{1}$$

where B_{Pt} is a standard Brownian motion, that is independent of B_{Ct}.

The expected inflation process, q_t, is time-varying. Specifically, it follows

$$dq_t = \kappa_q \left(\bar{q} - q_t\right)\, dt + \sigma_q\, dB_{qt} - Z_{cq,t}\, dN_{cq,t} - Z_{qt}\, dN_{qt}, \tag{2}$$

where B_{qt} is a standard Brownian motion, that is independent of B_{Ct} and B_{Pt}. The expected inflation process is also subject to two types of large shocks, and the arrival time of these shocks follow Poisson distributions, given by $N_{cq,t}$ and N_{qt}.

The magnitude of an N_c–type jump is determined by Z_c, the magnitude of an N_{cq}–type jump is determined by Z_{cq}, and that of an N_q–type jump is determined by Z_q. I will consider all three types of Poisson shocks to be negative, that is $Z_c < 0$, $Z_{cq} < 0$, and $Z_q < 0$; furthermore, these jump sizes are random and have time-invariant distributions ν_c, ν_{cq}, and ν_q, respectively. In what follows, I use the notation E_{ν_j} to denote expectations taken over the distribution ν_j for $j \in \{c, cq, q\}$. The intensities of these Poisson shocks are time-varying, and each follows a square-root process as in Cox, Ingersoll, and Ross (1985). In what follows, I will assume that inflation spike probability is perfectly correlated with inflation disaster probability.[4] Specifically, for $j \in \{c, cq\}$, the intensity for N_j is denoted by λ_{jt}, and it is given by

$$d\lambda_{jt} = \kappa_{\lambda_j}(\bar{\lambda}_j - \lambda_{jt})\, dt + \sigma_{\lambda_j}\sqrt{\lambda_{jt}}\, dB_{\lambda_j t}.$$

$B_{\lambda_{ct}}$ and $B_{\lambda_{cq,t}}$ are independent Brownian motions, and each is independent of B_{Ct}, B_{Pt}, and B_{qt}. Furthermore, assume that the Poisson shocks are independent of each other, and of the Brownian motions. Define $\lambda_t = [\lambda_{ct},\ \lambda_{cq,t}]^\top$, $\bar{\lambda} = [\bar{\lambda}_c,\ \bar{\lambda}_{cq}]^\top$, $\kappa_\lambda = [\kappa_{\lambda_c},\ \kappa_{\lambda_{cq}}]^\top$, $B_{\lambda t} = [B_{\lambda_c t},\ B_{\lambda_{cq} t}]^\top$, and $B_t = [B_{Ct},\ B_{Pt},\ B_{qt},\ B_{\lambda t}^\top]^\top$.

In what follows, a *disaster* (or *consumption disaster*) is a Poisson shock that affects realized consumption growth. In particular, I will refer to the N_c–type shock as a *non-inflation disaster* and the N_{cq}–type shock as an *inflation disaster*. The N_q–type shock only affects expected inflation and I refer to it as an *inflation spike*. Furthermore, I will refer to λ_c as the *non-inflation disaster probability* and λ_{cq} as the *inflation disaster probability*. Though the latter also governs the intensity of inflation spikes, the majority of its effects comes from inflation disasters rather than inflation spikes.

Following Duffie and Epstein (1992), I define the utility function V_t for the representative agent using the following recursion:

$$V_t = E_t \int_t^\infty f(C_s, V_s)\, ds, \tag{3}$$

[4] Inflation spikes in this model attempt to speak to the period of high inflation in the 1970s and early 1980s. During this period, consumption growth was low, and the outlook for future consumption growth was uncertain. Therefore not modeling inflation spike probability as an independent process is realistic. To simplify the model, I assume that the inflation spike probability equals inflation disaster probability.

where

$$f(C_t, V_t) = \beta(1-\gamma)V_t\left(\log C_t - \frac{1}{1-\gamma}\log\left((1-\gamma)V_t\right)\right). \tag{4}$$

The above utility function is the continuous-time analogue of the recursive utility defined by Epstein and Zin (1989) and Weil (1990), which allows for preferences over the timing of the resolution of uncertainty. Furthermore, equation (4) is a special case when the elasticity of intertemporal substitution (EIS) equals one. In what follows, γ is interpreted as risk aversion and β as the rate of time preference. I assume $\gamma > 0$ and $\beta > 0$ throughout the rest of the paper.

2.2 The value function and risk-free rates

Let $J(W_t, \lambda_t)$ denote the value function, where W_t denotes the real wealth of the representative agent. In equilibrium $J(W_t, \lambda_t) = V_t$.

Theorem 1. *Assume*

$$(\kappa_{\lambda_c} + \beta)^2 > 2\sigma^2_{\lambda_c} E_{\nu_{cq}}\left[e^{(1-\gamma)Z_c} - 1\right] \text{ and } \left(\kappa_{\lambda_{cq}} + \beta\right)^2 > 2\sigma^2_{\lambda_{cq}} E_{\nu_{cq}}\left[e^{(1-\gamma)Z_{cq}} - 1\right]. \tag{5}$$

The value function J takes the following form:

$$J(W_t, \lambda_t) = \frac{W_t^{1-\gamma}}{1-\gamma} I(\lambda_t), \tag{6}$$

where

$$I(\lambda_t) = \exp\left\{a + b_c\lambda_c + b_{cq}\lambda_{cq}\right\}. \tag{7}$$

The coefficients a and b_j for $j \in \{c, cq\}$ take the following form:

$$a = \frac{1-\gamma}{\beta}\left(\mu - \frac{1}{2}\gamma\sigma^2\right) + (1-\gamma)\log\beta + \frac{1}{\beta}b^\top\left(\kappa_\lambda * \bar{\lambda}\right), \tag{8}$$

$$b_j = \frac{\kappa_{\lambda_j} + \beta}{\sigma^2_{\lambda_j}} - \sqrt{\left(\frac{\kappa_{\lambda_j} + \beta}{\sigma^2_{\lambda_j}}\right)^2 - 2\frac{E_{\nu_j}\left[e^{(1-\gamma)Z_j} - 1\right]}{\sigma^2_{\lambda_j}}}, \tag{9}$$

Here and in what follows, we use $*$ to denote element-by-element multiplication of vectors of equal dimension. The signs of b_c and b_{cq} determine how disaster probabilities λ_c and λ_{cq} affect the investor's value function. The following corollary shows that the investor is made worse by an increase in the disaster probabilities.

Corollary 2. *For $j \in \{c, cq\}$, if $Z_j < 0$, then $b_j > 0$.*

The following two corollaries provide expressions for the real and nominal risk-free rates in this economy.

Corollary 3. *Let r_t denote the instantaneous real risk-free rate in this economy, r_t is given by*

$$r_t = \beta + \mu - \gamma\sigma^2 + \underbrace{\lambda_{ct} E_{\nu_c}\left[e^{-\gamma Z_c}(e^{Z_c} - 1)\right]}_{\textit{non-inflation disaster risk}} + \underbrace{\lambda_{cq,t} E_{\nu_{cq}}\left[e^{-\gamma Z_{cq}}(e^{Z_{cq}} - 1)\right]}_{\textit{inflation disaster risk}}. \tag{10}$$

The terms multiplying λ_{ct} and $\lambda_{cq,t}$ in (10) arise from the risk of a disaster. For $Z_j < 0$, the risk-free rate falls in λ_j: Recall that both non-inflation and inflation disasters affect consumption, therefore high disaster risk increases individuals' incentive to save, and thus lowers the risk-free rate.

Corollary 4. *Let $r_t^\$$ denote the instantaneous nominal risk-free rate on the nominal bond in the economy, $r_t^\$$ is given by*

$$r_t^\$ = r_t + q_t - \sigma_P^2. \tag{11}$$

The nominal risk-free rate is affected by expected inflation; when expected inflation is high, investors require additional compensation to hold the nominal risk-free asset.

2.3 Nominal government bonds

This section provides expressions for the prices, yields, and premia for nominal zero-coupon government bonds.

2.3.1 Prices and yields

Nominal bond prices are determined using no-arbitrage conditions and the state-price density. Duffie and Skiadas (1994) show that the real state-price density, π_t, equals

$$\pi_t = \exp\left\{\int_0^t f_V\left(C_s, V_s\right) ds\right\} f_C\left(C_t, V_t\right), \tag{12}$$

and nominal state-price density, $\pi_t^{\$}$, is given by[5]

$$\pi_t^{\$} = \frac{\pi_t}{P_t}. \tag{13}$$

Let $L_t^{\$,(\tau)} = L^{\$}(q_t, \lambda_t, \tau)$ denote the time t nominal price of a nominal government bond that pays off one nominal unit at time$t + \tau$. Then

$$L^{\$}(q_t, \lambda_t, s - t) = E_t\left[\frac{\pi_s^{\$}}{\pi_t^{\$}}\right].$$

The price $L_t^{\$,(\tau)}$ can be solved up to four ordinary differential equations. The following corollary is a special case of Theorem B.4 in Appendix B.

Corollary 5. *The function $L^{\$}$ takes the following form:*

$$L^{\$}(q_t, \lambda_t, \tau) = \exp\left\{a_L^{\$}(\tau) + b_{Lq}^{\$}(\tau) q_t + b_{L\lambda}^{\$}(\tau)^\top \lambda_t\right\}, \tag{14}$$

where $b_{L\lambda}^{\$}(\tau) = \left[b_{L\lambda_c}^{\$}(\tau),\ b_{L\lambda_{cq}}^{\$}(\tau)\right]^\top$. The function $b_{Lq}^{\$}$ takes the form

$$b_{Lq}^{\$}(\tau) = -\frac{1}{\kappa_q}\left(1 - e^{-\kappa_q \tau}\right), \tag{15}$$

[5]Consider a nominal asset that has nominal payoff $X_s^{\$}$ at time $s > t$, the time t nominal price of the asset, $X_t^{\$}$, can be written as $X_t^{\$} = E_t[\frac{\pi_t}{\pi_s}\frac{P_s}{P_t} X_s^{\$}] = E_t[\frac{\pi_s^{\$}}{\pi_t^{\$}} X_s^{\$}]$. Therefore, $\pi_t^{\$} = \frac{\pi_t}{P_t}$.

the function $b^{\$}_{L\lambda_c}$ *solves*

$$\frac{db^{\$}_{L\lambda_c}}{d\tau} = \frac{1}{2}\sigma_{\lambda_c} b^{\$}_{L\lambda_c}(\tau)^2 + \left(b_c\sigma^2_{\lambda_c} - \kappa_{\lambda_c}\right) b^{\$}_{L\lambda_c}(\tau) + E_{\nu_c}\left[e^{-\gamma Z_{ct}}(1 - e^{Z_{ct}})\right], \tag{16}$$

the function $b^{\$}_{L\lambda_{cq}}$ *solves*

$$\frac{db^{\$}_{L\lambda_{cq}}}{d\tau} = \frac{1}{2}\sigma_{\lambda_{cq}} b^{\$}_{L\lambda_{cq}}(\tau)^2 + \left(b_{cq}\sigma_{\lambda_{cq}} - \kappa_{\lambda_{cq}}\right) b^{\$}_{L\lambda_{cq}}(\tau) \\ + E_{\nu_{cq}}\left[e^{-(\gamma + b^{\$}_{Lq}(\tau))Z_{cq,t}} - e^{(1-\gamma)Z_{cq,t}}\right] + E_{\nu_q}\left[e^{-b^{\$}_{Lq}(\tau)Z_{qt}} - 1\right], \tag{17}$$

and the function $a^{\$}_L$ *solves*

$$\frac{da^{\$}_L}{d\tau} = -\beta - \mu + \gamma\sigma^2 + \sigma^2_P + \frac{1}{2}\sigma^2_q b^{\$}_{Lq}(\tau)^2 + b^{\$}_{Lq}(\tau)\kappa_q\bar{q} + b^{\$}_{L\lambda}(\tau)^\top(\kappa_\lambda * \bar{\lambda}), \tag{18}$$

with boundary conditions $a^{\$}_L(0) = b^{\$}_{Lq}(0) = b^{\$}_{L\lambda_c}(0) = b^{\$}_{L\lambda_{cq}}(0) = 0.$

Corollary 5 shows how prices respond to innovations in expected inflation and in changing disaster probabilities. Equation (15) shows that innovations to expected inflation lower prices for nominal bonds of all maturities. Furthermore, the effect will be larger the more persistent it is, that is, the lower is κ_q.

Higher non-inflation disaster probability has a non-negative effect on prices. Consider the ordinary differential equation (16); without the last term $E_{\nu_c}\left[e^{-\gamma Z_{ct}}(1 - e^{Z_{ct}})\right]$, the function $b^{\$}_{L\lambda_c}$ is identically zero. Therefore, this term determines the sign of $b^{\$}_{L\lambda_c}$. This term can be rewritten as: $E_{\nu_c}\left[e^{-\gamma Z_{ct}}(1 - e^{Z_{ct}})\right] = -E_{\nu_c}\left[e^{-\gamma Z_{ct}}(e^{Z_{ct}} - 1)\right]$, which multiplies λ_{ct} in the equation for the nominal risk-free rate (11). Because higher discount rates lower the price, the risk-free rate effect enters with a negative sign. With the boundary condition $b^{\$}_{L\lambda_c}(0) = 0$, this implies that $b^{\$}_{L\lambda_c}(\tau)$ is strictly positive and increasing for all τ. The intuition is straightforward: Non-inflation disaster risks only affect the nominal bonds through the underlying real bonds, and since the real bonds in this economy pay off during consumption disaster periods, they have negative premia.

Unlike non-inflation disasters, the effect of changing inflation disaster probability on bond valuation is more complicated. Recall that this process governs both the probability of an inflation disaster and the probability of an inflation spike. Similarly to the previous argument, the last two terms in ODE (17) determine the sign of $b^{\$}_{L\lambda_{cq}}$. The first expectation arises from inflation disasters, and it can be rewritten as:

$$
\begin{aligned}
&E_{\nu_{cq}}\left[e^{-(\gamma+b^{\$}_{Lq}(\tau))Z_{cq,t}} - e^{(1-\gamma)Z_{cq,t}}\right] = \\
&-\underbrace{E_{\nu_{cq}}\left[e^{-\gamma Z_{cq,t}}(e^{Z_{cq,t}}-1)\right]}_{\text{Risk-free rate effect }(-)} - \underbrace{E_{\nu_{cq}}\left[(e^{-\gamma Z_{cq,t}}-1)(1-e^{-b^{\$}_{Lq}(\tau)Z_{cq,t}})\right]}_{\text{Risk premium effect }(+)} + \underbrace{E_{\nu_{cq}}\left[e^{-b^{\$}_{Lq}(\tau)Z_{cq,t}}-1\right]}_{\text{Nominal price effect }(-)}.
\end{aligned} \tag{19}
$$

The first component is the risk-free rate effect; as previously discussed, this term is multiplied by a negative sign. The second component is part of the bond premium: The nominal bond price drops during periods of inflation disaster, when marginal utility is high; this term captures the premium investors require for bearing these jump risks. This risk premium effect is also multiplied by a negative sign since an increase in the discount rate lowers the bond price. The last term is the nominal price effect, which represents the effect of change in λ_{cq} on expected nominal bond prices through inflation. More specifically, it is the percent change in the price of a nominal bond with maturity τ in the event of an inflation disaster. Because a higher expected bond value raises the price, this term is multiplied by a positive sign.

Given $\gamma > 0$ and $Z_{cq} < 0$, the risk-free rate effect is negative, the risk premium effect is positive and increasing in maturity τ for $\tau > 0$, and the nominal price effect is negative and decreasing in maturity τ for $\tau > 0$. The effect of changing inflation disaster probabilities on bond value depends on the sum of these three effects. Notice that when $\tau = 0$, only the risk-rate effect is non-zero. Together with the boundary condition $b^{\$}_{L\lambda_q}(0) = 0$, this implies that $b^{\$}_{L\lambda_q}(\tau) > 0$ for some small τ: An increase in inflation disaster probability raises prices on bonds with short maturity. As maturity increases, however, risk premium and nominal price effect prevail over the risk-free rate effect, implying that prices on bonds with longer maturity decrease with inflation disaster probability.

The last term in ODE (17) arises from inflation spike risks. Notice that this term represents the nominal price effect, and it enters with a positive sign. Furthermore, it is negative and decreasing in maturity τ for $\tau > 0$; implying that an increase in the chance of an inflation spike lowers nominal bond prices and the effect is stronger for bonds with longer maturity.

Before moving on to discuss bond premia, the following definition and corollary provides expression for the nominal bond yield in the model:

Definition 1. *The yield to maturity for a nominal bond with maturity τ at time t, denoted by $y_t^{\$,(\tau)}$, is defined as:*

$$y_t^{\$,(\tau)} = \frac{1}{\tau} \log\left(\frac{1}{L_t^{\$,(\tau)}}\right). \tag{20}$$

Corollary 5 implies that the yield to maturity in this economy takes a particularly simple form:

Corollary 6. *The nominal yield to maturity for a nominal bond with maturity τ at time t, $y_t^{\$,(\tau)}$, is given by*

$$y_t^{\$,(\tau)} = -\frac{1}{\tau}\left(a_L^{\$}(\tau) + b_{Lq}^{\$}(\tau) q_t + b_{L\lambda}^{\$}(\tau)^{\top} \lambda_t\right), \tag{21}$$

where the coefficients $a_L^{\$}(\tau)$, $b_{Lq}^{\$}(\tau)$, and $b_{L\lambda}^{\$}(\tau)$ are given by (15) - (18).

2.3.2 The bond premium

This section provides an expression for the instantaneous bond premium and discusses its properties. For notation simplicity, I will first define the *jump operator*, which denotes how a process responds to the occurrence of a jump. Let X be a jump-diffusion process. Define the jump operator of X with respect to the jth type of jump as the following:

$$\mathcal{J}_j(X) = X_{t_j} - X_{t_{j-}} \quad j \in \{c, cq, q\},$$

for t_{j-} such that a type-j jump occurs. Then define

$$\bar{\mathcal{J}}_j(X) = E_{\nu_j}\left[X_{t_j} - X_{t_j-}\right] \quad j \in \{c, cq, q\}.$$

The instantaneous nominal expected return on a nominal bond with maturity τ is simply the expected percent change in nominal prices. Let $L_t^{\$,(\tau)} = L^{\$}(q_t, \lambda_t, \tau)$ be the time-t price of a τ-year nominal bond, by Ito's Lemma:

$$\frac{dL_t^{\$,(\tau)}}{L_{t^-}^{\$,(\tau)}} = \mu_{L^{\$,(\tau)},t}\, dt + \sigma_{L^{\$,(\tau)},t}\, dB_t + \frac{1}{L_t^{\$,(\tau)}}\left(\mathcal{J}_c(L_t^{\$,(\tau)})dN_{ct} + \mathcal{J}_{cq}(L_t^{\$,(\tau)})dN_{cq,t} + \mathcal{J}_q(L_t^{\$,(\tau)})dN_{qt}\right).$$

Then the instantaneous expected return can be written as:

$$r_t^{\$,(\tau)} = \mu_{L^{\$,(\tau)},t} + \frac{1}{L_t^{\$,(\tau)}}\left(\lambda_{ct}\bar{\mathcal{J}}_c(L_t^{\$,(\tau)}) + \lambda_{cq,t}\left(\bar{\mathcal{J}}_{cq}(L_t^{\$,(\tau)}) + \bar{\mathcal{J}}_q(L_t^{\$,(\tau)})\right)\right). \tag{22}$$

Corollary 7. *The bond premium relative to the risk-free rate $r^{\$}$ is:*

$$r_t^{\$,(\tau)} - r_t^{\$} = -\lambda_t^{\top}\left(b_{L\lambda}^{\$}(\tau) * b * \sigma_\lambda^2\right) + \lambda_{cq,t}E_{\nu_{cq}}\left[(e^{-\gamma Z_{cq,t}} - 1)(1 - e^{-b_{L\lambda_q}^{\$}(\tau)Z_{cq,t}})\right] \tag{23}$$

The first term in (23) arises from time-varying non-inflation and inflation disaster probabilities (time-varying probability adjustment). Recall that $b_j > 0$ for $j \in \{c, cq\}$, $b_{L\lambda_c}^{\$}(\tau) > 0$ for all τ, $b_{L\lambda_{cq}}^{\$}(\tau) > 0$ for small τ and $b_{L\lambda_{cq}}^{\$}(\tau) < 0$ for larger τ. Therefore, the time-varying non-inflation disaster probability adjustment is negative because the underlying real bond provides a hedge against consumption disasters. On the other hand, the time-varying inflation disaster probability adjustment is negative for bonds with shorter maturities and positive for bonds with longer maturities. The second term arises from the co-movement in nominal bond prices and marginal utility when a disaster occurs. Notice that this term depends on $b^{\$}_{Lq}$: When an inflation disaster occurs, expected inflation rises, which pushes future bond prices down. Given that $b^{\$}_{Lq} < 0$ and the assumption that $\gamma > 0$, $Z_{qt} < 0$, the second term is positive.

In a sample without disasters, but possibly with inflation spikes, the observed return is

$$r^{\$(\tau)}_{\mathrm{nd},t} = \mu_{L^{\$,(\tau)},t} + \frac{1}{L^{\$,(\tau)}_t}\lambda_{cq,t}\bar{\mathcal{J}}_{cq}(L^{\$,(\tau)}_t),$$

where the subscript "nd" is used to denote expected returns in a sample without consumption disasters. The following corollary calculates these expected returns.

Corollary 8. *The observed expected bond excess returns in a sample without disaster is:*

$$r^{\$(\tau)}_{\mathrm{nd},t} - r^{\$}_t = -\lambda_t^\top \left(b^{\$}_{L\lambda}(\tau) * b * \sigma^2_\lambda\right) + \lambda_{cq,t} E_{\nu_{cq}}\left[e^{-\gamma Z_{cq,t}}(1 - e^{-b^{\$}_{Lq}(\tau) Z_{cq,t}})\right]. \tag{24}$$

2.4 The aggregate market

Let D_t denote the dividend on the aggregate market. Assume that total dividends in the economy evolve according to

$$\frac{dD_t}{D_t} = \mu_D\, dt + \phi\sigma\, dB_{Ct} + (e^{\phi Z_{ct}} - 1)\, dN_{ct} + (e^{\phi Z_{qt}} - 1)\, dN_{cq,t}. \tag{25}$$

Under this process, aggregate dividend responds to disasters by a greater amount than aggregate consumption does (Longstaff and Piazzesi (2004)). The single parameter, ϕ, determines how aggregate dividend responds to both normal and disaster shocks. In what follows, ϕ is referred to as leverage as it is analogous to leverage in Abel (1999).

Let $H(D_t, \lambda_t, \tau)$ denote the time t price of a single future dividend payment at time $t + \tau$. Then

$$H(D_t, \lambda_t, s - t) = E_t\left[\frac{\pi_s}{\pi_t} D_s\right],$$

where π is the real state-price density defined by (12). The price H can be solved in closedform up to three ordinary differential equations, and the following corollary is a special case of Theorem B.2 in Appendix B.

Corollary 9. *The function H takes the following form:*

$$H(D_t, \lambda_t, \tau) = D_t \exp\left\{a_\phi(\tau) + \lambda_t^\top b_{\phi\lambda}(\tau)\right\}, \tag{26}$$

where $b_{\phi\lambda} = [b_{\phi\lambda_c} b_{\phi\lambda_{cq}}]^\top$. For $j \in \{c, cq\}$, function $b_{\phi j}$ takes the following form:

$$b_{\phi j}(\tau) = \frac{2E_{v_j}\left[e^{(1-\gamma)Z_{jt}} - e^{(\phi-\gamma)Z_{jt}}\right]\left(1 - e^{-\zeta_{b_j}\tau}\right)}{\left(\zeta_{b_j} + b_j\sigma_j^2 - \kappa_j\right)\left(1 - e^{-\zeta_{b_j}\tau}\right) - 2\zeta_{b_j}}, \tag{27}$$

where

$$\zeta_{b_j} = \sqrt{\left(b_j\sigma_j^2 - \kappa_j\right)^2 + 2\sigma_j^2 E_{\nu_j}\left[e^{(1-\gamma)Z_{jt}} - e^{(\phi-\gamma)Z_{jt}}\right]}. \tag{28}$$

Function $a_\phi(\tau)$ takes the following form:

$$\begin{aligned}
a_\phi(\tau) = \Bigg(&\mu_D - \mu - \beta + \gamma\sigma^2(1-\phi) \\
&- \left(\frac{\kappa_{\lambda_c}\bar{\lambda}_c}{\sigma_{\lambda_c}^2}(\zeta_{b_c} + b_c\sigma_{\lambda_c}^2 - \kappa_{\lambda_c}) + \frac{\kappa_{\lambda_{cq}}\bar{\lambda}_{cq}}{\sigma_{\lambda_{cq}}^2}(\zeta_{b_{cq}} + b_{cq}\sigma_{\lambda_{cq}}^2 - \kappa_{\lambda_{cq}})\right)\Bigg)\tau \\
&- \left(\frac{2\kappa_{\lambda_c}\bar{\lambda}_c}{\sigma_{\lambda_c}^2}\log\left(\frac{(\zeta_{b_c} + b_c\sigma_{\lambda_c}^2 - \kappa_{\lambda_c})(e^{-\zeta_{b_c}\tau} - 1)}{2\zeta_{b_c}}\right)\right. \\
&\left. + \frac{2\kappa_{\lambda_{cq}}\bar{\lambda}_{cq}}{\sigma_{\lambda_{cq}}^2}\log\left(\frac{(\zeta_{b_{cq}} + b_{cq}\sigma_{\lambda_{cq}}^2 - \kappa_{\lambda_{cq}})(e^{-\zeta_{b_{cq}}\tau} - 1)}{2\zeta_{b_{cq}}}\right)\right).
\end{aligned} \tag{29}$$

Let $F(D_t, \lambda_t)$ denote the time t price of the claim to the entire future dividend stream. Then

$$F(D_t, \lambda_t) = \int_0^\infty H(D_t, \lambda_t, \tau)\, d\tau.$$

Equation (27) shows that $b_{\phi j}(\tau) < 0$ for $j \in \{c, cq\}$; therefore the price-dividend ratio,

$$G(\lambda_t) = \int_0^\infty \exp\left\{a_\phi(\tau) + \lambda_{ct} b_{\phi\lambda_c}(\tau) + \lambda_{cq,t} b_{\phi\lambda_{cq}}(\tau)\right\} d\tau, \tag{30}$$

decreases in both non-inflation and inflation disaster probability.

3 Quantitative results

The model is calibrated to match aggregate consumption growth, inflation, and aggregate market moments. To evaluate the quantitative implication of the model, I simulate monthly data for 60,000 years. Furthermore, I simulate 10,000 60-year samples. For each of these small-samples, the initial values of λ_{ct} and $\lambda_{cq,t}$ are drawn from their stationary distributions, and the initial value of q_t is set equal to its mean, $\bar{q}$. In each of the tables that follow, I report the data and population value for each statistic. In addition, I report the 5th-, 50th-, and 95th-percentile values from the small-sample simulations (labelled "All Simulations" in the tables), and the 5th-, 50th-, and 95th-percentile values for the subset of the small-sample simulations that do not contain disasters (labelled "No-Disaster Simulations" in the tables). Samples in this subset do not contain any jumps in consumption, but they may contain jumps in expected inflation.

In the past 60 years, the U.S. did not experience any consumption disasters; however, it experienced a period of high inflation in the late 1970s and early 1980s. The No-Disaster subset from the simulation accommodates the possibility that there was an inflation jump in the country's postwar history; statistics from this subset therefore offer the most interesting comparison for the U.S. postwar data. With this calibration, about 23% of the samples do not experience any type of consumption disaster, and about one-third of these samples contain at least one jump in expected inflation.

3.1 Calibration

3.1.1 Data

The data on bond yields are from the Center for Research in Security Prices (CRSP). Monthly data is available for the period between June 1952 and December 2011. The yield on the three-month government bills is from the Fama risk-free rate, and yields on zero-coupon bonds with maturities between one and five year are from the Fama-Bliss discount bond dataset.

The market return is defined as the gross return on the CRSP value-weighted index. The dividend growth rate is from the dividends on the same index. To obtain real return and dividend growth, I adjust for inflation using changes in the consumer price index, which is also available from CRSP. The price-dividend ratio is constructed as the price divided by the previous 12 months of dividends. The government bill rate is the inflation-adjusted three-month Treasury Bill return. All data are annual. I use data from 1947 to 2010; using only postwar data provides a comparison between U.S. data and the simulated samples without consumption jumps.

3.1.2 Parameter values

Table 2 reports the parameter values. Mean consumption growth and the volatility of consumption growth are both about 2%, which equal their postwar data counterparts. Mean dividend growth is set to 3.48%; it is chosen to match the price dividend ratio instead of the dividend growth in the data: CRSP dividends do not include repurchases; presumably these imply that dividends are likely to be higher sometime in the future, and that the sample mean is not a good indicator of the true mean.

The leverage parameter ϕ governs both the ratio between the volatility of log dividends and the volatility of log consumption, and how dividends response to consumption disasters. In the data, the former ratio suggests leverage to be 4.66; however, I choose a smaller value, $\phi = 3$, so that dividends have a more conservative response to consumption disasters. Rate of time preference β is set to be low to obtain a realistic short-term government bill rate. Relative risk aversion γ is set equal to 3.

Mean expected inflation is set to 2.7%; with this value, the median value of the realized inflation among the simulations with no consumption disaster is 3.65%, the value in the data is 3.74%. The volatility of non-expected inflation σ_p equals 0.8% to match the realized inflation volatility in the data; the median value among the simulations with no consumption disaster is 2.89%, and the value in the data is 3.03%. The volatility of expected inflation σ_q equals 1.3% to match the volatility of short-term bond yield; the volatility of three-month Treasury Bill yield is 3.01% in the data, and the median value among the simulations with no consumption disaster is 2.99%. The mean reversion parameter in the expected inflation process governs the persistence of the inflation process, which is highly persistent and the autocorrelation decays slowly. This parameter it is set to 0.09 to obtain a reasonable first order autocorrelation of the inflation process.

Barro and Ursua (2008) calibrated the average probability of a consumption disaster for OECD countries to be 2.86%, implying that $\bar{\lambda}_c + \bar{\lambda}_{cq} = 2.86\%$.[6] In the data, about one-third of the disasters are accompanied by high inflation (Table 1), therefore I set $\bar{\lambda}_c$ to equal 1.83% and $\bar{\lambda}_{cq}$ to equal 1.03%. The persistence in the price-dividend ratio is mostly determined by the persistence in the disaster probability. I therefore choose a low rate of mean reversion for both inflation and non-inflation disaster probabilities: $\kappa_{\lambda_c} = \kappa_{\lambda_{cq}} = 0.11$. With this choice, the median value of the persistence of the price-dividend ratio among the simulations with no consumption disaster is 0.73; the value in the data is 0.92. The volatilities $\sigma_{\lambda_c} = 0.112$ and $\sigma_{\lambda_{cq}} = 0.103$ lead to a reasonable volatility for the aggregate market.

The disaster distributions Z_c and Z_{cq} are chosen to match the distribution of consumption declines. I consider 10% as the smallest possible disaster magnitude and I assume that Z_c and Z_{cq} follow power law distributions. For non-inflation disasters, I set the power law parameter to equal 10, and for inflation disasters, I set the power law parameter to equal 8. Table 2 plots these power law distributions along with distributions of large consumption declines. In particular, I compare the power law distribution with parameter 8 to the distribution of large consumption declines that are accompanied by high inflation, and the

[6] In this calibration, I calibrate the disaster probability to the OECD subsample but the size of jumps to the full set of samples. This is a more conservative approach as OECD countries have disasters that are rarer but more severe.

power law distribution with parameter 10 to the distribution of large consumption declines that are not accompanied by high inflation. In addition, I will assume that Z_q follows the same distribution as Z_{cq}.

3.2 Yield curves and expected returns as functions of the state variables

3.2.1 Yield curves

It is helpful to understand how the state variables affect the nominal yield curves in order to better understand the simulation results. Equation (21) shows that nominal yields on nominal bonds depend on expected inflation, q; on non-inflation disaster probability, λ_c; and on inflation disaster probability, λ_{cq}. Furthermore, the coefficients on these state variables are functions of maturity τ. Figure 3 shows the term in the expression for the nominal bond yield (21). In particular, it shows the loading on expected inflation, $-b_{Lq}/\tau$; on non-inflation disaster probability, $-b_{L\lambda_c}/\tau$; and on the inflation disaster probability, $-b_{L\lambda_{cq}}/\tau$; all as functions of maturity τ.

The loading on expected inflation is positive and decreases with maturity: High expected inflation lowers bond values and raises bond yields; due to mean-reversion, the effect is larger on bonds with shorter maturities. The loading on inflation spike probability is also positive but increases with maturity: High probability of an expected inflation jump lowers bond values and raises bond yields, and the effect is stronger on bonds with longer maturities.

What is more interesting is the stark distinction between the loading on non-inflation disaster and inflation disaster probabilities. The loading on non-inflation disaster probability is negative and decreases with maturity. While the loading on the inflation disaster probability is also negative for short maturity bonds, it increases with maturity and becomes positive. Disasters in the model affect the nominal yield curves through two channels: They affect realized consumption growth and (possibly) expected inflation. Non-inflation disasters only affects consumption growth, thus high non-inflation disaster risks lower the risk-free rate, which leads to higher bond prices and lower bond yields. Therefore, the coefficient on the non-inflation disaster probability is negative and decreasing with maturity. On the contrary, inflation disaster probability affects the nominal yield curve through both channels, and investors require positive compensation for bearing the risk of jumps on expected inflation. The bottom-right panel of Figure 3 shows that the former effect dominates for bonds with shorter maturity while the latter effect dominates for bonds with longer maturity. Notice that inflation spike risk also affects the shape of the loading on inflation disaster probability: Inflation spike risks further lower bond values and raise bond yields, and the effect is stronger on bonds with longer maturities. Thus the presence of inflation spike risks leads to an even steeper $-b_{L\lambda_{cq}}/\tau$, though the shape is mostly determined by inflation disaster risks.

Figure 4 shows how the yield curve responds to changes in each of the three state variables. In each of the panels, the dashed line represents the yield curve when all state variables are at their means. I then increase the value of one state variable at a time and plot the resulting yield curve. The solid line in the top-left panel shows the yield curve when expected inflation is increased by σ_q: High expected inflation shifts the nominal yield curve up, and the effect is slightly stronger for bonds with short maturities. The solid line in the top-right panel shows the yield curve when non-inflation disaster probability is increased by one standard deviation: High non-inflation disaster probability shifts the nominal yield curve down (the risk-free rate effect), and the effect is slightly stronger for bonds with long maturities. The solid line in the bottom-left panel shows the yield curve when inflation disaster probability is increased by one standard deviation: High inflation disaster probability changes the shape of the nominal yield curve. The yields for short maturity bonds become lower (risk-free rate effect) and the yields for long maturity bonds become higher (risk-premium effect and nominal price effect). The risk of inflation spikes further increases the nominal bond yield (nominal price effect).

From this figure, one can also observe that the primary effect of expected inflation and non-inflation disaster probability is on the level of the yield curve, while non-inflation disaster probability also affects the slope of the yield curve. Furthermore, most of the variations in the term structure variables such as the yield spreads and forward rates come from variations in the probabilities of inflation disasters and inflation spikes.

3.2.2 Risk premia

Figures 5 and 6 plots the bond risk premia as functions of non-inflation disaster probability, λ_c, and inflation disaster probability, λ_q, using Equation (23). Expected inflation q is set equal to 2.8% in all cases. To illustrate the impact of changes in disaster probabilities on bonds with different maturities, I compare the risk premia on one- and five-year bonds.

Figure 5 shows that risk premia decrease with non-inflation disaster probability, and also that bonds with longer maturities are more sensitive to these changes. Equation (23) shows that non-inflation disaster probability implies a negative premium, and that the absolute magnitude of this premium increases with maturity. Figure 6 shows that risk premia increase as a function of the inflation disaster probability and that bonds with longer maturities are more sensitive to these changes. The co-movement of marginal utility and bond prices in inflation disaster periods generates a positive premium for all nominal bonds, and this premium increases with maturity. Time-varying inflation disaster risks generate a small negative premium for short maturity bonds, and this premium increases with maturity and becomes positive when the maturity is longer. Comparing Figures 5 and 6, one can see that bond risk premia are more sensitive to inflation disaster risks than to non-inflation disaster risks, furthermore, one can also see that long-term bonds are more sensitive to these risks than short-term bonds.

Figure 4 – 6 provide evidence of predictable bond premia in the model. Figure 5 and 6 imply that bond excess returns are high when inflation disaster risk is high, or when non-inflation disaster risk is low. Figure 4 shows that yield spread is also high when inflation disaster risk is high, or when non-inflation disaster risk is low. Therefore, one should expect yield spread to have some predictive power on bond excess returns. Furthermore,

since excess returns on long-term bonds are more sensitive to these disaster probabilities than excess returns on short-term bonds are, long-term bond excess returns should be more sensitive to changes in yield spreads than excess returns of short-term bonds are.

3.3 Simulation results

3.3.1 Nominal yields

Figures 7 and 8 show the first two moments of yields for nominal bonds with different maturities. Figure 7 plots the data and model-implied average nominal bond yields, and Figure 8 plot the data and model-implied volatility of nominal bond yields, both as functions of time to maturity. In each figure, I plot the median, the 25th-, and 75th-percentile values drawn from the subset of small-sample simulations that do not contain any consumption disasters. Table 5 reports the data and all model-implied statistics of mean and volatility of yields for nominal bonds with one, two, three, four, and five years to maturity.

The model is capable of explaining the average nominal yield curve. The median values among the simulations with no consumption disasters are close to their data counterparts. Furthermore, the median values increase with time to maturity, implying an upward-sloping average yield curve in the model. The median small-sample value of the mean increases from 5.67% for one-year bonds to 6.03% for five-year bonds; in the data, the average bond yields increase from 5.20% for one-year bonds to 5.82% for five-year bonds. In addition to the first moment, the model also generates realistic implications for the volatility of bond yields. The median values among the simulations having no consumption disasters decreases from 2.79% for one-year bond to 2.61% for five-year bond; in the data, it decreases from 3.02% for one-year bond to 2.78% for five-year bond. Notice that the nominal yields are on average higher and more volatile in the full set of simulations and in population. This is because more jumps in expected inflation (inflation disasters) are realized, and expected inflation are on average higher in these samples.

This model is able to match both the first two moments of the nominal yield curve, while previous literature successfully capture the upward-sloping shape of the nominal yield curve, they do not generate realistic implication for the second moment. In both Piazzesi and Schneider (2006) and Bansal and Shaliastovich (2012), short-term bond yields are also more volatile than long-term bond yields, but the levels are much lower than the data counterparts. The habit formation model in Wachter (2006) implies that short-term yields are more volatile than long-term yields, which is counterfactual. Comparing with the three models, this model also impose a potentially more reasonable requirement on the utility function of the representative agent. In the current calibration, relative risk aversion is set equal to 3. In contrast, Piazzesi and Schneider (2006) set it equal to 43 and Bansal and Shaliastovich (2012) estimate it to be 20.90. The habit formation model in Wachter (2006) assumes a time-varying risk aversion, which is greater than 30 when the state variable is at its long-run mean.

3.3.2 Principal component analysis

Litterman and Scheinkman (1991) find that most of the variations in yield curve can be explained by a three-factor model. Specifically, the first factor affects the level of the yield curve, the second factor affects the slope, and the third factor affects the curvature. To evaluate whether the model also exhibits this feature, I perform a principal component analysis on the data and model-simulated yields. Figure 11 reports the results. For the model, I only report the median values drawn from the subset of small-sample simulations that does not contain any disasters. I plot the loadings on yields with different maturities on each of the first three principal components. Similar to Litterman and Scheinkman (1991), a shock to the first principal component has similar effects across yields of different maturities (*level factor*); a shock to the second principal component raises yields on short-term bonds and reduces yields on long-term bonds (*slope factor*); and a shock to the third principal component raises yields on bonds with median maturity, but lowers yields on short- and long-term bonds (*curvature factor*). In addition, the bottom-right panel also shows that almost all the variations in yield curve are explained by the first three principal components, both in the data and in the model.

Given the three-factor structure of the model, it is natural to ask how these three factors relate to the three state variables in the model. Table 6 reports the correlation between each of the three state variables in the model and each of the three principal components. The level factor is mostly correlated with expected inflation; consistent with Figure 4, an increase in expected inflation or inflation disaster probability raises the yield curve, while an increase in non-inflation disaster probability lowers it. The slope factor is highly negatively correlated with the inflation disaster probability, and slightly positively correlated with expected inflation and non-inflation disaster probability.[7] The curvature factor is mostly correlated with non-inflation disaster risks; a shock to non-inflation disaster risks (also expected inflation and inflation disaster risks) increase the curvature of the yield curve.

Collin-Dufresne and Goldstein (2002) provide evidence of unspanned volatility using data on fixed income derivative. Their findings suggest that interest rate volatility risk cannot be hedged by bonds. Following Collin-Dufresne, Goldstein, and Jones (2009), I simulate the model to obtain 13-year samples at daily frequency. I then regress realized volatility of 6-month yields, constructed using five daily data, on the first three principal components at the beginning of the period. Similar to Collin-Dufresne, Goldstein, and Jones (2009), these regressions yield low R^2-statistics. For example, the median value in the subset of small-samples with out disaster is around 0.03, with the 95th percentile value around 0.19, and Collin-Dufresne, Goldstein, and Jones (2009) find the R^2 to be 0.155 using data from 1990 to 2002. This suggests that the first three principal components do not forecast volatility in the model and in the data.

[7] Note that the loadings on the second principal component decrease with maturity, so a positive shock to this factor reduces the slope.

3.3.3 Time-varying bond risk premia

First I consider the "long-rate" regression in Campbell and Shiller (1991):

$$y_{t+h}^{\$,(n-h)} - y_t^{\$,(n)} = \text{constant} + \beta_n \frac{1}{n-h}\left(y_t^{\$,(n)} - y_t^{\$,(h)}\right) + \text{error}, \tag{31}$$

where n denotes bond maturity and h denotes the holding period. In what follows, I will consider this regression at quarterly frequency, $h = 0.25$, from June 1952 to December 2011.

Table 7 reports the results for regression (31). I consider long-term bonds with maturities of one, two, three, four, and five years, and report the data and model-simulated coefficient of the above regression. Under the expectations hypothesis, excess returns on long-term bonds are unpredictable, which implies that β_n should equal one for all n. As in Campbell and Shiller, the coefficient β_n's are negative and decreasing in maturity n, implying that bond excess returns are predictable by yield spread, and a high yield spread predicts a higher excess return for bonds with longer maturity. The model is capable of capturing this feature. The median value of these coefficients among the simulations that contain no consumption disasters is also negative and decreasing with maturity n, furthermore, the data values are all above the 5th percentile of the values drawn from the model. In what follows, I will discuss how the mechanism drives the model's ability to explain the failure of the expectations hypothesis.

Bond risk premia are not constant in this model; (23) and (24) show that higher inflation disaster risks lead to a higher bond risk premium, and that this premium increases with maturity. Furthermore, Figure 4 shows that variations in inflation disaster risk have a large effect on yield spread, and higher inflation disaster risks lead to higher yield spreads. These imply that bond premia are expected to be high when yield spread is large.[8] However, higher inflation disaster risks also lead to a higher probability of expected inflation jumps,

[8]Non-inflation disaster risks decrease both yield spread and bond premium, which also implies that bond premia will be high when yield spread is high.

and once these jumps are realized, bond prices drop and realized excess returns also fall. In summary, when the variations in yield spread arise from variations in non-inflation and inflation disaster probabilities – and conditional on inflation jumps are not being realized – one should expect a high yield spread to be followed by high bond premia. Furthermore, a high yield spread predicts a larger premium for long-term bonds than it does for short-term bonds. Variations in expected inflation, however, have the opposite effect on the coefficient β_n's. An increase in expected inflation leads to a lower yield spread (Figure 4); furthermore, it leads to a higher bond premium. Therefore, if the variations in yield spread arise from variations in expected inflation, it will have a positive effect on these coefficient β_n's.

In Table 7, one can see that while the median values drawn from the subset of small-sample simulations containing no consumption disasters are negative, the median values among the full set of simulations are positive. This is because there are substantially more inflation jumps among all the small-samples. While the effects of variations in λ_c and λ_{cq} dominate in the subset without disasters, the realizations of inflation jumps and variations in expected inflation dominate among the full set of small-samples.

In addition to the long-rate regressions, I also consider the forward rate regressions performed by Cochrane and Piazzesi (2005) to evaluate the model's success in capturing time-varying bond risk premia. In what follows, I consider the annual forward rate. I denote the n-year forward rate at time t for a loan from time $t+n$ to time $t+n+1$ by f_t^n, defined as:

$$f_t^n = \log L_t^{\$,(n-1)} - \log L_t^{\$,(n)}.$$

As in Cochrane and Piazzesi, these forward rate regressions are done in two steps. First I regress the average annual excess returns on two-, three-, four-, and five-year nominal bonds on all available forward rates:

$$\frac{1}{4}\sum_{n=2}^{5} r_{t+1}^{e,(n)} = \theta^{\top}\mathbf{f_t} + \text{error}, \tag{32}$$

where $r^{e,(n)}_{t+1} = r^{\$,(n)}_{t+1} - r^{\$,(1)}_{t+1}$ is the excess return of a bond with maturity n and $\mathbf{f_t}$ denotes the vector of all forward rates available at time t.

The second step is to form a single factor $\widehat{\text{cp}}_{t+1} = \widehat{\theta}^{\top}\mathbf{f_t}$ and regress the excess returns of bonds with different maturities on this single factor:

$$r^{e,(n)}_{t+1} = \text{constant} + \rho_n \widehat{\text{cp}}_{t+1} + \text{error}. \tag{33}$$

I consider monthly overlapping annual observations from June 1952 to December 2011. In the data, I construct one-, two-, three-, four-, and five-year forward rates. In the model, however, I can only construct three independent forward rates since the model only has three factors. Therefore, I will use all five forward rates in the data, but only one-, three-, and five-year forward rates in the model.

Table 8 reports the results from the second stage regression, (33). In the data the single forward rate factor predicts bond excess returns with an economically significant R^2, furthermore, the coefficient on this factor increases with bond maturity. The model successfully generates these findings: The median values of the R^2 drawn from the subset of the small-sample simulations containing no consumption disasters are slightly smaller than those in the data, but still economically significant. For example, the single forward rate factor predicts excess returns on five-year nominal bonds with $R^2 = 0.18$, and the median values drawn from the subset of samples containing no disasters is 0.17. The median value of the coefficients in these samples increases from 0.40 for one-year bonds to 1.59 for five-year bonds, in the data it increases from 0.44 to 1.47. In the full set of small-samples, the R^2 are lower, but still economically meaningful. The small-sample bias in these regressions, however, is significant: The R^2-statistics are almost zero in population.

One other finding of Cochrane and Piazzesi (see also Stambaugh (1988)) is that in the first stage regression (32), the coefficients exhibit a tent-shaped pattern as a function of maturity. This model is also able to generate these tent-shaped patterns. In about 35% of the subset that contain no consumption disasters, the coefficients from the first stage regression (32) exhibit a tent-shaped pattern. Figure 10 reports the average of these coefficients.

3.3.4 The aggregate market

This model is also successful in matching moments in the aggregate market. Table 9 reports the simulation results. The model is able to explain most of the equity premium, which is 7.25% in the data; the median value from the small-sample containing no consumption disasters is 5.06%, and the data is below the 95th percentile of the values drawn from the model.

To calculate the real three-month Treasury Bill returns, I calculate the realized returns on the nominal three-month Treasury Bill, then adjust them by realized inflation. This model generates reasonable values for the short-term interest rate; this value in the data is 1.25%, and the median value from the small-sample containing no consumption disasters is 2.03%. Furthermore, the data value is above the 5th percentile of the values drawn from the model, indicating that we cannot reject the model at the 10% level.

The model, however, only has limited ability to explain the volatility of the price-dividend ratio. As discussed in Bansal, Kiku, and Yaron (2012) and Beeler and Campbell (2012), this is a limitation shared by models that explain aggregate prices using time-varying moments but parsimonious preferences. Time-varying moments imply cash flow, risk-free rate, and risk premium effects, and one of these generally acts as an offset to the other two, thus limiting the effect time-varying moments have on prices.

3.3.5 Interactions between the aggregate and bond market

In this section, I study the model's implications for the interaction between the aggregate market and the term structure of interest rates. Previous works have shown that variables that predict excess returns in one asset class often fail in another. For example, Duffee (2012) showed that while term structure variables predict bond excess returns, they do not predict stock market excess returns. In this section, I consider two predictor variables, the price-dividend ratio and the linear combination of forward rates that best predicts bond returns. I also consider two excess returns, the aggregate market returns over short-term bonds, and the average long-term bond returns over short-term bonds. The average long-term bond return is defined as the average of the returns on one-, two-, three-, four- and five-year nominal bonds. I calculate the predictive regressions of each excess returns on each predictor variable. Data are annual from 1953 to 2010. Tables 10 – 13 report the results from these predictive regressions.

Tables 10 and 12 show the results of regressing aggregate and bond market excess returns on the price-dividend ratio. It is well known that price-dividend ratio predicts aggregate market excess returns in the data (e.g. Campbell and Shiller (1988), Cochrane (1992), Fama and French (1989) and Keim and Stambaugh (1986)). Equation (30) shows that the price-dividend ratio in the model is governed by both non-inflation and inflation disaster probabilities. In particular, a high disaster probability lowers the price-dividend ratio. Furthermore, investors require a higher-than-average premium when the total disaster risk is high, implying that on average, a high total disaster probability is followed by high returns. Notice that predictability still exists in the full set of simulations, though the R^2-statistics are smaller. This is because realized returns are much lower when disasters actually occur. In population, the predictability is even smaller, reflecting the well-known small-sample bias in predictive regressions.

In the data, the price-dividend ratio also has some predictive power on long-term bond excess returns, though the t-statistics are not significant and R^2 values are low, as shown in Tables 10 and 12. The model generates similar implications. An increase in either non-inflation or inflation disaster probability leads to a low price-dividend ratio; bond excess return, however, is governed mainly by inflation disaster probability. Therefore, investors require a higher-than-average bond premium only when inflation disaster risk is high, implying that on average, high inflation disaster probability is followed by high bond returns. Furthermore, a high non-inflation disaster probability lowers the expected bond excess returns; nonetheless, this effect is substantially smaller. Therefore, if the variation in the price-dividend ratio comes from the inflation disaster probability, then the price-dividend ratio predicts long-term bond excess returns with a negative sign. On the other hand, if the movement in the price-dividend ratio comes from the non-inflation disaster probability, then the price-dividend ratio predicts long-term bond excess returns with a small but positive sign. Notice that predictability still exists in the full set of simulations, but disappears in population.

As shown in previous section, long-term bond excess returns can be predicted using a linear combination of forward rates. Tables 11 and 13 report the results of the long-horizon regression. Unsurprisingly, the model successfully generates the long-term bond excess return predictability found in the data. In the model, both the shape of the term structure and bond excess returns are largely determined by the inflation disaster probability: A high inflation disaster probability leads to a steeper term structure and also a higher bond premium.

On the other hand, the linear combination of forward rates has less predictive power on the aggregate market excess returns (Duffee (2012)). In the full sample from 1953 to 2010, the linear combination of forward rates appears to have no predictive power.[9] In the model, forward rates depend on inflation disaster probability, and high inflation disaster probability is, on average, followed by high returns. Therefore, the linear combination of forward rates still predicts aggregate market excess returns. However, comparing Panel A and Panel B of Tables 11 and 13, one can see that the linear combination of forward rates predicts the long-term bond excess returns with a much higher R^2 value than for the aggregate market excess returns, implying that the forward rate factor has a stronger predictive power on bond excess returns.

Lettau and Wachter (2011) also consider these regressions; the single forward rate factor

[9]The magnitude of the R^2-statistics depends on the subsample. For example, Cochrane and Piazzesi (2005) find that the linear combination of forward rates predicts one-year aggregate market excess returns with an $R^2 = 0.07$ in the sample from 1964 through 2003. In the corresponding period, the R^2 is 0.36 for one-year nominal bond excess returns.

in their model predicts bond excess returns and aggregate market excess returns with similar R^2 values. In the data, even though the R^2 depends on the sample period, the forward rate factor has a stronger predictive power on bond excess returns. Wachter (2006) and Gabaix (2012) also study both the stock and bond markets. The model of Wachter (2006), however, implies that the risk premia on stocks and bonds move together. In Gabaix (2012), the time-varying risks in stock and bond market are unrelated, where in this paper, the underlying risks are the same, but they have different effect on the premia. The model in this paper is able to generate more realistic implications for these predictive regressions because the prices of risks in the model have a two-factor structure, and these factors have differential effects on the stock and bond markets.

4 Conclusion

Why is the average term structure upward-sloping? Why are excess returns on nominal bonds predictable? This paper provides an explanation for these questions using a model with time-varying rare disaster risks. Previous research has shown that a model that includes time-varying disaster risks can generate high equity premium and excess returns volatility. Motivated by historical data, disasters in this model affect not only aggregate consumption, but also expected inflation. A jump in expected inflation pushes down the real value of nominal bonds, and investors require compensation for bearing these inflation disaster risks. Furthermore, this premium increases with bond maturity, which leads to an upward-sloping nominal term structure. Time-varying bond risk premia arise naturally from time-varying disaster probabilities, and prices of risk in this model follow a two-factor structure.

The model is calibrated to match the aggregate consumption, inflation, and equity market moments, and the quantitative results show that this model produces realistic means and volatilities of nominal bond yields. The three state variables in the model are highly correlated with the first three principal components, which explain almost all of the variations in the nominal yield curve both in the model and in the data. This model can also account for the violation of the expectations hypothesis. In particular, I show that the yield spread and a linear combination of forward rates can predict long-term bond excess returns. Furthermore, the model is capable of capturing the joint predictive properties of the aggregate market returns and of the bond returns. Aggregate market variables have higher predictive powers for equity excess returns while the term structure variables have higher predictive powers for bond excess returns.

Appendix

A Model derivation

A.1 Notation

Definition A.1. *Let X be a jump-diffusion process. Define the jump operator of X with respect to the jth type of jump as the following:*

$$\mathcal{J}_j(X) = X_{t_j} - X_{t_{j-}} \quad j \in \{c, cq, q\},$$

for t_{j-} such that a type-j jump occurs. Then define

$$\bar{\mathcal{J}}_j(X) = E_{\nu_j}\left[X_{t_j} - X_{t_j-}\right] \quad j \in \{c, cq, q\},$$

and

$$\bar{\mathcal{J}}(X) = [\bar{\mathcal{J}}_c(X),\ \bar{\mathcal{J}}_{cq}(X),\ \bar{\mathcal{J}}_q(X)]^\top.$$

A.2 The value function

Proof of Theorem 1 Let S denote the value of a claim to aggregate consumption, and conjecture that the price-dividend ratio for the consumption claim is constant:

$$\frac{S_t}{C_t} = l,$$

for some constant l. This relation implies that S_t satisfies

$$dS_t = \mu S_{t^-} dt + \sigma S_{t^-} dB_{ct} + (e^{Z_{ct}} - 1) S_t dN_{ct} + (e^{Z_{cq,t}} - 1) S_t dN_{cq,t}. \tag{A.1}$$

Consider an agent who allocates wealth between S and the risk-free asset. Let α_t be the fraction of wealth in the risky asset S_t, and let c_t be the agent's consumption. The wealth process is then given by

$$dW_t = \left(W_t\alpha_t\left(\mu - r_t + l^{-1}\right) + W_t r_t - c_t\right)dt + W_t\alpha_t\sigma dB_{ct} \\ + \alpha_t W_t\left((e^{Z_{ct}} - 1)S_t dN_{ct} + (e^{Z_{cq,t}} - 1)S_t dN_{cq,t}\right),$$

where r_t denotes the instantaneous risk-free rate. Optimal consumption and portfolio choices must satisfy the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\sup_{\alpha_t, C_t}\Bigg\{J_W\left(W_t\alpha_t\left(\mu - r_t + l^{-1}\right) + W_t r_t - c_t\right) + \kappa_{\lambda_c}\left(\bar{\lambda}_c - \lambda_{ct}\right) + \kappa_{\lambda_{cq}}\left(\bar{\lambda}_{cq} - \lambda_{cq,t}\right) \\ + \frac{1}{2}J_{WW}W_t^2\alpha_t^2\sigma^2 + \frac{1}{2}\left(J_{\lambda_c\lambda_c}\sigma_{\lambda_c}^2\lambda_{ct} + J_{\lambda_{cq}\lambda_{cq}}\sigma_{\lambda_{cq}}^2\lambda_{cq,t}\right) \\ + \lambda_{ct}E_{\nu_c}\left[J\left(W_t\left(1 + \alpha_t\left(e^{Z_{ct}} - 1\right)\right), \lambda_t\right) - J\left(W_t, \lambda_t\right)\right] \\ + \lambda_{cq,t}E_{\nu_{cq}}\left[J\left(W_t\left(1 + \alpha_t\left(e^{Z_{cq,t}} - 1\right)\right), \lambda_t\right) - J\left(W_t, \lambda_t\right)\right] + f\left(c_t, V_t\right)\Bigg\} = 0, \quad \text{(A.2)}$$

where J_n denotes the first derivative of J with respect to variable n, for n equal to λ_i or W, and J_{nm} denotes the second derivative of J with respect to n and m.

In equilibrium, $\alpha_t = 1$ and $c_t = W_t l^{-1}$. Substituting these policy functions into (A.2) implies

$$J_W W_t\mu + J_{\lambda_c}\kappa_{\lambda_c}\left(\bar{\lambda}_c - \lambda_{ct}\right) + J_{\lambda_{cq}}\kappa_{\lambda_{cq}}\left(\bar{\lambda}_{cq} - \lambda_{cq,t}\right) + \frac{1}{2}J_{WW}W_t^2\sigma^2 \\ + \frac{1}{2}\left(J_{\lambda_c\lambda_c}\sigma_{\lambda_c}^2\lambda_{ct} + J_{\lambda_{cq}\lambda_{cq}}\sigma_{\lambda_{cq}}^2\lambda_{cq,t}\right) + \lambda_{ct}E_{\nu_c}\left[J\left(W_t e^{Z_{ct}}, \lambda_t\right) - J\left(W_t, \lambda_t\right)\right] \\ + \lambda_{cq,t}E_{\nu_{cq}}\left[J\left(W_t e^{Z_{cq,t}}, \lambda_t\right) - J\left(W_t, \lambda_t\right)\right] + f\left(c_t, V_t\right) = 0. \quad \text{(A.3)}$$

By the envelope condition $f_C = J_W$, we obtain $\beta = l^{-1}$. Given the consumption-wealth ratio, it follows that

$$f\left(c_t, V_t\right) = f\left(W_t l^{-1}, J(W_t, \lambda_t)\right) = \beta W_t^{1-\gamma}\left(\log\beta - \frac{\log I(\lambda_t)}{1-\gamma}\right). \quad \text{(A.4)}$$

Substituting (A.4) and (6) into (A.3) and dividing both sides by $W_t^{1-\gamma} I(\lambda_t)$, we find

$$
\begin{aligned}
\mu + I^{-1}(1-\gamma)^{-1} \left(I_{\lambda_c} \kappa_{\lambda_c} (\bar{\lambda}_c - \lambda_{ct}) + I_{\lambda_{cq}} \kappa_{\lambda_{cq}} (\bar{\lambda}_{cq} - \lambda_{cq}) \right) - \frac{1}{2}\gamma\sigma^2 \\
+ \frac{1}{2} I^{-1} \left(I_{\lambda_c \lambda_c} \sigma^2_{\lambda_c} \lambda_{ct} + I_{\lambda_{cq}\lambda_{cq}} \sigma^2_{\lambda_{cq}} \lambda_{cq,t} \right) \\
+ (1-\gamma)^{-1} \left(\lambda_c E_{\nu_c} \left[e^{(1-\gamma)Z_c} - 1 \right] + \lambda_{cq} E_{\nu_{cq}} \left[e^{(1-\gamma)Z_{cq}} - 1 \right] \right) \\
+ \beta \left(\log\beta - \frac{\log I(\lambda_t)}{1-\gamma} \right) = 0,
\end{aligned}
$$

where I_{λ_j} denotes the first derivative of I with respect to λ_j and $I_{\lambda_j \lambda_j}$ denotes the second derivative for $j \in \{c, cq\}$.

Collecting terms in λ_{jt} results in the following quadratic equation for b_j:

$$
\frac{1}{2}\sigma^2_{\lambda_j} b_j^2 - (\kappa_{\lambda_j} + \beta) b_j + E_{\nu_j} \left[e^{(1-\gamma)Z_j} - 1 \right],
$$

for $j \in \{c, cq\}$, implying

$$
b_j = \frac{\kappa_{\lambda_j} + \beta}{\sigma^2_{\lambda_j}} \pm \sqrt{ \left(\frac{\kappa_{\lambda_j} + \beta}{\sigma^2_{\lambda_j}} \right)^2 - 2 \frac{E_{\nu_j} \left[e^{(1-\gamma)Z_j} - 1 \right]}{\sigma^2_{\lambda_j}} },
$$

Collecting constant terms results in the following characterization of a in terms of b:

$$
a = \frac{1-\gamma}{\beta} \left(\mu - \frac{1}{2}\gamma\sigma^2 \right) + (1-\gamma)\log\beta + \frac{1}{\beta} b^\top \left(\kappa_\lambda * \bar{\lambda} \right).
$$

Here and in what follows, I use $*$ to denote element-by-element multiplication of vectors of equal dimension. Given the form of $I(\lambda)$, $I_{\lambda_j} = b_j I$ and $I_{\lambda_j \lambda_j} = b_j^2 I$ for $j \in \{c, cq\}$. Because there are no interaction terms, the solution takes the same form as when there is only a single type of jump. As in Wachter (2012, Appendix A.1) we take the negative root of the corresponding equation for b_j to find:

$$b_j = \frac{\kappa_{\lambda_j} + \beta}{\sigma_{\lambda_j}^2} - \sqrt{\left(\frac{\kappa_{\lambda_j} + \beta}{\sigma_{\lambda_j}^2}\right)^2 - 2\frac{E_{\nu_j}\left[e^{(1-\gamma)Z_j} - 1\right]}{\sigma_{\lambda_j}^2}}.$$

$\square$

Proof of Corollary 2 Since $\gamma > 1$, if $Z_j < 0$, then the second term in the square root of (9) is positive. Therefore the square root term is positive but less than $\frac{\kappa_j+\beta}{\sigma_j^2}$, and $b_j > 0$. Similarly, if $Z_j > 0$ then the second term in the square root of (9) is negative. Therefore the square root term is positive and greater than $\frac{\kappa_j+\beta}{\sigma_j^2}$, and $b_j < 0$. $\square$

Proof of Corollary 3 The risk-free rate is obtained by taking the derivative of the HJB (A.2) with respect to α_t, evaluating at $\alpha_t = 1$, and setting it equal to 0. The result immediately follows. $\square$

A.3 The state-price density

Duffie and Skiadas (1994) show that the state-price density π_t equals

$$\pi_t = \exp\left\{\int_0^t f_V\left(C_s, V_s\right) ds\right\} f_C\left(C_t, V_t\right),$$

where f_C and f_V denote derivatives of f with respect to the first and second argument respectively. Note that the exponential term is deterministic. From equation (4), I obtain

$$f_C\left(C_t, V_t\right) = \beta\left(1-\gamma\right)\frac{V}{C}.$$

From the equilibrium condition $V_t = J\left(\beta^{-1}C_t, \lambda_t\right)$, together with the form of the value function (6), I get

$$f_C\left(C_t, V_t\right) = \beta^{\gamma} C_t^{-\gamma} I(\lambda_t). \tag{A.5}$$

Applying Ito's Lemma to (A.5) implies

$$\frac{d\pi_t}{\pi_{t^-}} = \mu_{\pi t} dt + \sigma_{\pi t} dB_t + \left(e^{-\gamma Z_{ct}} - 1\right) dN_{ct} + \left(e^{-\gamma Z_{cq,t}} - 1\right) dN_{cq,t}, \tag{A.6}$$

where

$$\sigma_{\pi t} = \left[-\gamma\sigma,\ 0,\ 0,\ b_c \sigma_c \sqrt{\lambda_{ct}},\ b_{cq} \sigma_{cq} \sqrt{\lambda_{cq,t}}\right]. \tag{A.7}$$

It also follows from no-arbitrage that

$$\begin{aligned} \mu_{\pi t} &= -r_t - \left(\lambda_{ct} E_{\nu_c}\left[e^{-\gamma Z_{ct}} - 1\right] + \lambda_{cq,t} E_{\nu_{cq}}\left[e^{-\gamma Z_{cq,t}} - 1\right]\right) \\ &= -\beta - \mu + \gamma\sigma^2 - \left(\lambda_{ct} E_{\nu_c}\left[e^{(1-\gamma) Z_{ct}} - 1\right] + \lambda_{ct} E_{\nu_{cq}}\left[e^{(1-\gamma) Z_{cq,t}} - 1\right]\right). \end{aligned} \tag{A.8}$$

From (A.6) we can see that in the event of a disaster, marginal utility (as represented by the state-price density) jumps upward. This implies that investors require compensation for bearing disaster risks. The first element of (A.7) implies that the standard diffusion risk in consumption is priced; more importantly, changes in λ_{jt} are also priced as reflected by the last two elements of (A.7).

The nominal state-price density $\pi^\$$ equals

$$\pi_t^\$ = \frac{\pi_t}{P_t}. \tag{A.9}$$

The nominal state-price density follows

$$\frac{d\pi_t^\$}{\pi_{t-}^\$} = \mu_{\pi t}^\$ dt + \sigma_{\pi t}^\$ dB_t + \left(e^{-\gamma Z_{ct}} - 1\right) dN_{ct} + \left(e^{-\gamma Z_{cq,t}} - 1\right) dN_{cq,t}, \tag{A.10}$$

where

$$\sigma_{\pi t}^\$ = \left[-\gamma\sigma,\ -\sigma_P,\ 0,\ b_c \sigma_{\lambda_c} \sqrt{\lambda_{ct}},\ b_{cq} \sigma_{\lambda_{cq}} \sqrt{\lambda_{cq,t}}\right], \tag{A.11}$$

and

$$\mu^{\$}_{\pi t} = -\beta - \mu + \gamma\sigma^2 - q_t + \sigma_P^2 - \left(\lambda_{ct} E_{\nu_c}\left[e^{(1-\gamma)Z_{ct}} - 1\right] + \lambda_{ct} E_{\nu_{cq}}\left[e^{(1-\gamma)Z_{cq,t}} - 1\right]\right). \quad \text{(A.12)}$$

By comparing (A.11) to (A.7), we can see that the second element is no longer zero. This implies that the diffusion risk in inflation is also priced in the nominal state-price density. By comparing (A.12) to (A.8), we can see that the expected inflation and volatility of realized inflation also affect the drift of the nominal state-price density.

Proof of Corollary 4 It follows from no-arbitrage that

$$\mu^{\$}_{\pi t} = -r^{\$}_t - \left(\lambda_{ct} E_{\nu_c}\left[e^{-\gamma Z_{ct}} - 1\right] + \lambda_{cq,t} E_{\nu_{cq}}\left[e^{-\gamma Z_{cq,t}} - 1\right]\right),$$

where $\mu^{\$}_{\pi t}$ is given by (A.12). Therefore the nominal risk-free rate on a nominal bond, $r^{\$}_t$ is

$$r^{\$}_t = \beta + \mu - \gamma\sigma^2 + q_t - \sigma_P^2 + \lambda_{ct} E_{\nu_c}\left[e^{-\gamma Z_{ct}}\left(e^{Z_{ct}} - 1\right)\right] + \lambda_{qt} E_{\nu_{cq}}\left[e^{-\gamma Z_{cq,t}}\left(e^{Z_{cq,t}} - 1\right)\right].$$

□

B Pricing general zero-coupon equity

This section provides the price of a general form of a zero-coupon equity, both in real terms and in nominal terms. The dividend on the aggregate market and the face value on the bond market will be special cases.

B.1 Real assets

First I will consider the price of a real asset. Consider a stream of cash-flow that follows a jump-diffusion process:

$$\frac{dD_t}{D_{t^-}} = \mu_D\, dt + \sigma_D\, dB_t + (e^{\phi_{D,c} Z_{ct}} - 1)\, dN^D_{ct} + (e^{\phi_{D,cq} Z_{cq,t}} - 1)\, dN^D_{cq,t}. \quad \text{(B.1)}$$

This stream of cash-flow is subject to Poisson shocks dN_{jt}^D, $j \in \{c, cq\}$. The arrival time of these Poisson shocks are linked to the arrival time of consumption disasters.

Assumption B.1. *When a consumption disaster happens, this cash-flow stream experiences a jump with probability p_D; that is, for $j \in \{c, cq\}$.*

- *If $dN_{jt} = 0$, then $dN_{jt}^D = 0$.*
- *If $dN_{jt} = 1$, then*

$$dN_{jt}^D = \begin{cases} 1 & \textit{with probability } p_D \\ 0 & \textit{otherwise.} \end{cases}$$

With this assumption, $\phi_{D,j}$ denotes the jump multiplier for a type-j jump, for $j \in \{c, cq\}$.

Lemma B.1. *Let $H(D_t, \lambda_t, \tau)$ denote the time t price of a single future cash-flow at time $s = t + \tau$:*

$$H(D_t, \lambda_t, s - t) = E_t\left[\frac{\pi_s}{\pi_t} D_s\right].$$

By Ito's Lemma, we can write

$$\frac{dH(D_t, \lambda_t, \tau)}{H(D_t, \lambda_t, \tau)} = \mu_{H(\tau),t} dt + \sigma_{H(\tau),t}^\top dB_t + \mathcal{J}_c(\pi_t H(D_t, \lambda_t, \tau)) dN_{ct} + \mathcal{J}_{cq}(\pi_t H(D_t, \lambda_t, \tau)) dN_{cq,t}.$$

for a scalar process $\mu_{H(\tau),t}$ and a vector process $\sigma_{H(\tau),t}$. Then, no-arbitrage implies that:

$$\mu_{\pi,t} + \mu_{H(\tau),t} + \sigma_{\pi,t}\sigma_{H(\tau),t}^\top + \frac{1}{\pi_t H_t(\tau)} \lambda_t^\top \bar{\mathcal{J}}^{real}(\pi_t H(D_t, \lambda_t, \tau)) = 0. \tag{B.2}$$

Proof No-arbitrage implies that $H(D_s, \lambda_s, 0) = D_s$ and that

$$\pi_t H(D_t, \lambda_t, \tau) = E_t\left[\pi_s H(D_s, \lambda_s, 0)\right].$$

To simplify notation, let $H_t = H(D_t, \lambda_t, \tau)$, $\mu_{H,t} = \mu_{H(\tau),t}$, and $\sigma_{H,t} = \sigma_{H(\tau),t}$. It follows from Ito's Lemma that

$$\frac{dH_t}{H_{t^-}} = \mu_{H,t}dt + \sigma_{H,t}dB_t + (e^{\phi_{D,c}Z_{ct}} - 1)dN_{ct} + (e^{\phi_{D,cq}Z_{cq,t}} - 1)dN_{cq,t}.$$

Applying Ito's Lemma to $\pi_t H_t$ implies that the product can be written as

$$\begin{aligned}\pi_t H_t = \pi_0 H_0 + \int_0^t \pi_s H_s \left(\mu_{H,s} + \mu_{\pi,s} + \sigma_{\pi,s}\sigma_{H,s}^\top\right) + \int_0^t \pi_s H_s(\sigma_{H,s} + \sigma_{\pi,s})dB_s \\ \sum_{0<s_{ci}\le t}\left(\pi_{s_{ci}}H_{s_{ci}} - \pi_{s_{ci}^-}H_{s_{ci}^-}\right) + \sum_{0<s_{cq,i}\le t}\left(\pi_{s_{cq,i}}H_{s_{cq,i}} - \pi_{s_{cq,i}^-}H_{s_{cq,i}^-}\right), \quad \text{(B.3)}\end{aligned}$$

where $s_{ji} = \inf\{s : N_{js} = i\}$ (namely, the time that the ith time type-j jump occurs, where $j \in \{c, cq\}$).

We use (B.3) to derive a no-arbitrage condition. The first step is to compute the expectation of the jump terms $\sum_{0<s_{ji}\le t}\left(\pi_{s_{ji}}H_{s_{ji}} - \pi_{s_{ji}^-}H_{s_{ji}^-}\right)$. The pure diffusion processes are not affected by the jump. Adding and subtracting the jump compensation terms from (B.3) yields:

$$\begin{aligned}\pi_t H_t = \pi_0 H_0 + \int_0^t \pi_s H_s \left(\mu_{H,s} + \mu_{\pi,s} + \sigma_{\pi,s}\sigma_{H,s}^\top + \frac{1}{\pi_s H_s}\left(\lambda_c \bar{\mathcal{J}}_c(\pi_s H_s) + \lambda_{cq}\bar{\mathcal{J}}_{cq}(\pi_s H_s)\right)\right) ds \\ + \int_0^t \pi_s H_s(\sigma_{H,s} + \sigma_{\pi,s})dB_s + \sum_{0<s_{ci}\le t}\left(\left(\pi_{s_{ci}}H_{s_{ci}} - \pi_{s_{ci}^-}H_{s_{ci}^-}\right) - \int_0^t \pi_s H_s \lambda_c \bar{\mathcal{J}}_c(\pi_s H_s) ds\right) \\ + \sum_{0<s_{cq,i}\le t}\left(\left(\pi_{s_{cq,i}}H_{s_{cq,i}} - \pi_{s_{cq,i}^-}H_{s_{cq,i}^-}\right) - \int_0^t \pi_s H_s \lambda_{cq}\bar{\mathcal{J}}_{cq}(\pi_s H_s) ds\right) \quad \text{(B.4)}\end{aligned}$$

Under mild regularity conditions analogous to those given in Duffie, Pan, and Singleton (2000), the second and the third terms on the right hand side of (B.4) are martingales. Therefore the first term on the right hand side of (B.4) must also be a martingale, and it follows that the integrand of this term must equal zero:

$$\mu_{\pi,t} + \mu_{H(\tau),t} + \sigma_{\pi,t}\sigma_{H(\tau),t}^\top + \frac{1}{\pi_t H_t(\tau)}\lambda_t^\top \bar{\mathcal{J}}^{\text{real}}(\pi_t H(D_t, \lambda_t, \tau)) = 0.$$

□

Theorem B.2. *The function H takes an exponential form:*

$$H(D_t, \lambda_t, \tau) = D_t \exp\left\{a_\phi(\tau) + \lambda_t^\top b_{\phi\lambda}(\tau)\right\}, \tag{B.5}$$

where $b_{\phi\lambda} = [b_{\phi\lambda_c},\ b_{\phi\lambda_{cq}}]^\top$. Function $b_{\phi\lambda_j}$ for $j \in \{c, cq\}$ solves

$$\begin{aligned}\frac{db_{\phi\lambda_j}}{d\tau} = &\frac{1}{2}\sigma_{\lambda_j}^2 b_{\phi\lambda_j}(\tau)^2 + \left(b_j\sigma_{\lambda_j}^2 - \kappa_{\lambda_j}\right) b_{\phi\lambda_j}(\tau) \\ &+ p_D E_{\nu_j}\left[e^{\left(\phi_{D,j}-\gamma\right)Z_{jt}} - e^{(1-\gamma)Z_{jt}}\right] + (1-p_D)E_{\nu_j}\left[e^{-\gamma Z_{jt}} - e^{(1-\gamma)Z_{jt}}\right],\end{aligned} \tag{B.6}$$

and function a_ϕ solves

$$\frac{da_\phi}{d\tau} = \mu_D - \mu - \beta + \gamma\sigma\left(\sigma - \sigma_D\right) + b_{\phi\lambda}(\tau)^\top\left(\kappa_{\lambda_j} * \bar{\lambda}_j\right). \tag{B.7}$$

The boundary conditions are $a_\phi(0) = b_{\phi\lambda_c}(0) = b_{\phi\lambda_{cq}}(0) = 0$.

Proof See proof of Theorem B.4. □

B.2 Nominal asset

Similar no-arbitrage conditions can be derived for nominally denominated assets. Suppose cash-flow that follows:

$$\frac{dD_t^\$}{D_{t^-}^\$} = \mu_{D^\$}\,dt + \sigma_{D^\$}\,dB_t + (e^{\phi_{D,c}^\$ Z_{ct}} - 1)\,dN_{ct}^D + (e^{\phi_{D,cq}^\$ Z_{cq,t}} - 1)\,dN_{cq,t}^D,$$

where the process N_{jt}^D is given by Assumption B.1 and $\phi_{D,c}^\$$ and $\phi_{D,cq}^\$$ are the jump multipliers for the N_c– and N_{cq}–type jumps, respectively.

Lemma B.3. *Let $H^\$(D_t^\$, q_t, \lambda_t, \tau)$ denote the time t price of a single future dividend payment at time $t+\tau$:*

$$H^\$(D_t^\$, q_t, \lambda_t, s-t) = E_t\left[\frac{\pi_s^\$}{\pi_t^\$}D_s^\$\right].$$

By Ito's Lemma, we can write

$$\frac{dH^{\$}(D_t^{\$}, q_t, \lambda_t, \tau)}{H^{\$}(D_t^{\$}, q_t, \lambda_t, \tau)} = \mu_{H^{\$}(\tau),t}dt + \sigma_{H^{\$}(\tau),t}^{\top}dB_t + \mathcal{J}_c(\pi_t^{\$}H^{\$}(D_t^{\$}, q_t, \lambda_t, \tau))dN_{ct}$$
$$+ \mathcal{J}_{cq}(\pi_t^{\$}H^{\$}(D_t^{\$}, q_t, \lambda_t, \tau))dN_{cq,t} + \mathcal{J}_q(\pi_t^{\$}H^{\$}(D_t^{\$}, q_t, \lambda_t, \tau))dN_{qt}.$$

for a scalar process $\mu_{H^{\$}(\tau),t}$ and a vector process $\sigma_{H^{\$}(\tau),t}$. Then, no-arbitrage implies that:

$$\mu_{\pi^{\$},t} + \mu_{H^{\$}(\tau),t} + \sigma_{\pi^{\$},t}\sigma_{H^{\$}(\tau),t}^{\top} + \frac{1}{\pi_t^{\$}H_t^{\$}(\tau)}\Bigg(\lambda_{ct}\bar{\mathcal{J}}_c(\pi_t^{\$}H^{\$}(D_t^{\$}, q_t, \lambda_t, \tau))$$
$$+ \lambda_{cq,t}\left(\bar{\mathcal{J}}_{cq}(\pi_t^{\$}H^{\$}(D_t^{\$}, q_t, \lambda_t, \tau)) + \bar{\mathcal{J}}_q(\pi_t^{\$}H^{\$}(D_t^{\$}, q_t, \lambda_t, \tau))\right)\Bigg) = 0, \quad \text{(B.8)}$$

Proof See proof of Lemma B.1. □

Theorem B.4. *The function $H^{\$}$ takes an exponential form:*

$$H^{\$}(D_t^{\$}, q_t, \lambda_t, \tau) = D_t^{\$} \exp\left\{a_{\phi^{\$}}(\tau) + b_{\phi^{\$}q}(\tau)q_t + b_{\phi^{\$}\lambda}(\tau)^{\top}\lambda_t\right\}, \quad \text{(B.9)}$$

where $b_{\phi^{\$}\lambda} = \left[b_{\phi^{\$}\lambda_c},\ b_{\phi^{\$}\lambda_{cq}}\right]$. Function $b_{\phi^{\$}q}$ solves

$$\frac{db_{\phi^{\$}q}}{d\tau} = -\kappa_q b_{\phi^{\$}q}(\tau) - 1; \quad \text{(B.10)}$$

function $b_{\phi^{\$}\lambda_c}$ solves

$$\frac{db_{\phi^{\$}\lambda_c}}{d\tau} = \frac{1}{2}\sigma_{\lambda_c}^2 b_{L\lambda_c}(\tau)^2 + \left(b_c\sigma_{\lambda_c}^2 - \kappa_{\lambda_c}\right)b_{\phi^{\$}\lambda_c}(\tau)$$
$$+ p_D E_{\nu_c}\left[e^{(\phi_{D,c}^{\$}-\gamma)Z_{ct}} - e^{(1-\gamma)Z_{ct}}\right] + (1-p_D)E_{\nu_c}\left[e^{-\gamma Z_{ct}} - e^{(1-\gamma)Z_{ct}}\right]; \quad \text{(B.11)}$$

function $b_{\phi^{\$}\lambda_{cq}}$ solves

$$\frac{db_{\phi^{\$}\lambda_{cq}}}{d\tau} = \frac{1}{2}\sigma^2_{\lambda_{cq}} b_{\phi^{\$}\lambda_{cq}}(\tau)^2 + \left(b_{cq}\sigma^2_{\lambda_{cq}} - \kappa_{\lambda_{cq}}\right) b_{\phi^{\$}\lambda_{cq}}(\tau) + E_{\nu_q}\left[e^{-b_{\phi^{\$}q}(\tau))Z_{qt}} - 1\right]$$
$$+ p_D E_{\nu_{cq}}\left[e^{\left(\phi^{\$}_{D,cq} - (\gamma + b_{\phi^{\$}q}(\tau))\right)Z_{cq,t}} - e^{(1-\gamma)Z_{cq,t}}\right]$$
$$+ (1-p_D)E_{\nu_{cq}}\left[e^{-(\gamma + b_{\phi^{\$}q}(\tau))Z_{cq,t}} - e^{(1-\gamma)Z_{cq,t}}\right]; \quad \text{(B.12)}$$

and function a_L solves

$$\frac{da_{\phi^{\$}}}{d\tau} = \mu_D - \beta - \mu + \gamma\sigma(\sigma - \sigma_D) + \sigma_P^2 + \frac{1}{2}\sigma_q^2 b_{\phi^{\$}q}(\tau)^2 + b_{\phi^{\$}q}(\tau)\kappa_q\bar{q} + b_{\phi^{\$}\lambda}(\tau)^\top(\kappa_\lambda * \bar{\lambda}). \quad \text{(B.13)}$$

The boundary conditions are $a_{\phi^{\$}}(0) = b_{\phi^{\$}q}(0) = b_{\phi^{\$}\lambda_c}(0) = b_{\phi^{\$}\lambda_{cq}}(0) = 0$.

Proof It follows from Ito's Lemma that

$$\frac{dH_t^{\$}}{H_{t^-}^{\$}} = \mu_{H^{\$},t}dt + \sigma_{H^{\$},t}dB_t + \frac{1}{H_{t^-}^{\$}}\Big(\mathcal{J}_c(H_t^{\$}) + \mathcal{J}_{cq}(H_t^{\$}) + \mathcal{J}_q(H_t^{\$})\Big),$$

where $\mu_{H^{\$}}$ and $\sigma_{H^{\$}}$ are given by

$$\mu_{H^{\$},t} = \frac{1}{H^{\$}}\left(\frac{\partial H^{\$}}{\partial q}\Big(\bar{q} - q_t\Big) + \frac{\partial H^{\$}}{\partial \lambda_c}\left(\bar{\lambda}_c - \lambda_{ct}\right) + \frac{\partial H^{\$}}{\partial \lambda_{cq}}\left(\bar{\lambda}_c - \lambda_{cq,t}\right) - \frac{\partial H^{\$}}{\partial \tau}\right.$$
$$\left. + \frac{1}{2}\frac{\partial^2 H^{\$}}{\partial q_j^2}\sigma_q^2 + \frac{1}{2}\left(\frac{\partial^2 H^{\$}}{\partial \lambda_c^2}\sigma^2_{\lambda_c} + \frac{\partial^2 H^{\$}}{\partial \lambda_c^2}\sigma^2_{\lambda_c}\right)\right)$$
$$= b_{\phi^{\$}q}(\tau)\kappa_q\left(\bar{q} - q_t\right) + b_{\phi^{\$}\lambda_c}(\tau)\kappa_{\lambda_c}\left(\bar{\lambda}_c - \lambda_{ct}\right) + b_{\phi^{\$}\lambda_{cq}}(\tau)\kappa_{\lambda_{cq}}\left(\bar{\lambda}_{cq} - \lambda_{cq,t}\right)$$
$$+ \frac{1}{2}b_{\phi^{\$}q}(\tau)^2\sigma_q^2 + \frac{1}{2}\left(b_{\phi^{\$}\lambda_c}(\tau)^2\sigma^2_{\lambda_c}\lambda_{ct} + b_{\phi^{\$}\lambda_{cq}}(\tau)^2\sigma^2_{\lambda_{cq}}\lambda_{cq,t}\right)$$
$$- \left(\frac{da_{\phi^{\$}}}{d\tau} + \frac{db_{\phi^{\$}q}}{d\tau}q_t + \sum_j \frac{db_{\phi^{\$}\lambda_j}}{d\tau}\lambda_{jt}\right), \quad \text{(B.14)}$$

and

$$\sigma_{H^\$,t} = \frac{1}{L}\left(\frac{\partial H^\$}{\partial q_t}[0,\ 0,\ \sigma_q\sqrt{q_t},\ 0,\ 0] + \frac{\partial H^\$}{\partial \lambda_c}[0,\ 0,\ 0,\ \sigma_{\lambda_c}\sqrt{\lambda_{ct}},\ 0] + \frac{\partial H^\$}{\partial \lambda_{cq}}[0,\ 0,\ 0,\ 0,\ \sigma_{\lambda_{cq}}\sqrt{\lambda_{cq,t}}]\right)$$
$$= \left[0,\ 0,\ b_{\phi^\$ q}(\tau)\sigma_q\sqrt{q_t},\ b_{\phi^\$\lambda_c}(\tau)\sigma_{\lambda_c}\sqrt{\lambda_{ct}},\ b_{\phi^\$\lambda_{cq}}(\tau)\sigma_{\lambda_{cq}}\sqrt{\lambda_{cq,t}}\right]. \quad \text{(B.15)}$$

Furthermore,

$$\frac{\bar{\mathcal{J}}_c(\pi_t^\$ H_t^\$)}{\pi_t^\$ H_t^\$} = p_D E_{\nu_c}\left[e^{(\phi^\$_{D,c}-\gamma)Z_{ct}} - 1\right] + (1-p_D)E_{\nu_c}\left[e^{-\gamma Z_{ct}} - 1\right], \quad \text{(B.16)}$$

$$\frac{\bar{\mathcal{J}}_{cq}(\pi_t^\$ H_t^\$)}{\pi_t^\$ H_t^\$} = p_D E_{\nu_{cq}}\left[e^{\left(\phi^\$_{D,cq}-(\gamma+b_{\phi^\$ q}(\tau))\right)Z_{cq,t}} - 1\right] + (1-p_D)E_{\nu_{cq}}\left[e^{-(\gamma+b_{\phi^\$ q}(\tau))Z_{cq,t}} - 1\right], \quad \text{(B.17)}$$

and

$$\frac{\bar{\mathcal{J}}_q(\pi_t^\$ H_t^\$)}{\pi_t^\$ H_t^\$} = E_{\nu_q}\left[e^{-b_{\phi^\$ q}(\tau))Z_{qt}} - 1\right]. \quad \text{(B.18)}$$

Recall that $\lambda_q = \lambda_{cq}$. Substituting (B.14) – (B.17) along with (A.11) and (A.12) into the no-arbitrage condition (B.8) implies that functions $a_{\phi^\$}$, $b_{\phi^\$ q}$, $b_{\phi^\$\lambda_c}$, and $b_{\phi^\$\lambda_{cq}}$ solve the following ordinary differential equation:

$$\begin{aligned}
& b_{\phi^\$ q}(\tau)\kappa_q(\bar{q} - q_t) + b_{\phi^\$\lambda_c}(\tau)\kappa_{\lambda_c}\left(\bar{\lambda}_c - \lambda_{ct}\right) + b_{\phi^\$\lambda_{cq}}(\tau)\kappa_{\lambda_{cq}}\left(\bar{\lambda}_{cq} - \lambda_{cq,t}\right) \\
&\quad + \frac{1}{2}b_{\phi^\$ q}(\tau)^2\sigma_q^2 + \frac{1}{2}\left(b_{\phi^\$\lambda_c}(\tau)^2\sigma_{\lambda_c}^2\lambda_{ct} + b_{\phi^\$\lambda_c}(\tau)^2\sigma_{\lambda_c}^2\lambda_{ct}\right) - \beta - \mu + \gamma\sigma^2 - q_t + \sigma_P^2 \\
&\quad + b_{\phi^\$\lambda_c}(\tau)b_j\sigma_{\lambda_c}^2\lambda_{ct} + b_{\phi^\$\lambda_{cq}}(\tau)b_j\sigma_{\lambda_{cq}}^2\lambda_{cq,t} + p_D\lambda_{ct}E_{\nu_c}\left[e^{(\phi^\$_{D,c}-\gamma)Z_{ct}} - e^{(1-\gamma)Z_{ct}}\right] \\
&\quad + (1-p_D)\lambda_{ct}E_{\nu_c}\left[e^{-\gamma Z_{ct}} - e^{(1-\gamma)Z_{ct}}\right] + p_D\lambda_{cq,t}E_{\nu_{cq}}\left[e^{\left(\phi^\$_{D,cq}-(\gamma+b_{\phi^\$ q}(\tau))\right)Z_{cq,t}} - e^{(1-\gamma)Z_{cq,t}}\right] \\
&\quad + (1-p_D)\lambda_{cq,t}E_{\nu_{cq}}\left[e^{-(\gamma+b_{\phi^\$ q}(\tau))Z_{cq,t}} - e^{(1-\gamma)Z_{cq,t}}\right] + \lambda_{cq,t}E_{\nu_q}\left[e^{-b_{\phi^\$ q}(\tau))Z_{qt}} - 1\right] \\
&\quad - \left(\frac{da_{\phi^\$}}{d\tau} + \frac{db_{\phi^\$ q}}{d\tau}q_t + \frac{db_{\phi^\$\lambda_c}}{d\tau}\lambda_{ct} + \frac{db_{\phi^\$\lambda_{cq}}}{d\tau}\lambda_{cq,t}\right) = 0. \quad \text{(B.19)}
\end{aligned}$$

Collecting q_t terms results in the following ordinary differential equation:

$$\frac{db_{\phi^\$ q}}{d\tau} = -\kappa_q b_{\phi^\$ q}(\tau) - 1;$$

collecting terms multiplying λ_c results in the following ordinary differential equation for $b_{\phi^\$ \lambda_c}$

$$\begin{aligned}\frac{db_{\phi^\$ \lambda_c}}{d\tau} = \frac{1}{2}\sigma_{\lambda_c}^2 b_{L\lambda_c}(\tau)^2 + \left(b_c\sigma_{\lambda_c}^2 - \kappa_{\lambda_c}\right) b_{\phi^\$ \lambda_c}(\tau) \\ + p_D E_{\nu_c}\left[e^{(\phi^\$_{D,c}-\gamma)Z_{ct}} - e^{(1-\gamma)Z_{ct}}\right] + (1-p_D)E_{\nu_c}\left[e^{-\gamma Z_{ct}} - e^{(1-\gamma)Z_{ct}}\right];\end{aligned}$$

collecting terms multiplying λ_{cq} results in the following ordinary differential equation for $b_{\phi^\$ \lambda_{cq}}$

$$\begin{aligned}\frac{db_{\phi^\$ \lambda_{cq}}}{d\tau} = \frac{1}{2}\sigma_{\lambda_{cq}}^2 b_{\phi^\$ \lambda_{cq}}(\tau)^2 + \left(b_{cq}\sigma_{\lambda_{cq}}^2 - \kappa_{\lambda_{cq}}\right) b_{\phi^\$ \lambda_{cq}}(\tau) + E_{\nu_q}\left[e^{-b_{\phi^\$ q}(\tau))Z_{qt}} - 1\right] \\ + p_D E_{\nu_{cq}}\left[e^{\left(\phi^\$_{D,cq}-(\gamma+b_{\phi^\$ q}(\tau))\right)Z_{cq,t}} - e^{(1-\gamma)Z_{cq,t}}\right] \\ + (1-p_D)E_{\nu_{cq}}\left[e^{-(\gamma+b_{\phi^\$ q}(\tau))Z_{cq,t}} - e^{(1-\gamma)Z_{cq,t}}\right];\end{aligned}$$

and collecting constant terms results in the following ordinary differential equation for a_L:

$$\frac{da_{\phi^\$}}{d\tau} = \mu_D - \beta - \mu + \gamma\sigma(\sigma - \sigma_D) + \sigma_P^2 + \frac{1}{2}\sigma_q^2 b_{\phi^\$ q}(\tau)^2 + b_{\phi^\$ q}(\tau)\kappa_q\bar{q} + b_{\phi^\$ \lambda}(\tau)^\top(\kappa_\lambda * \bar{\lambda}).$$

The boundary conditions are $a_{\phi^\$}(0) = b_{\phi^\$ q}(0) = b_{\phi^\$ \lambda_c}(0) = b_{\phi^\$ \lambda_{cq}}(0) = 0$. □

C Nominal bond pricing

Proof of Corollary 6

$$y_t^{\$,(\tau)} = \frac{1}{\tau}\log\left(\frac{f_t^\$}{L_t^{\$,(\tau)}}\right),$$

where $L_t^{\$,(\tau)}$ is given by (14), then the results immediately follows. □

Proof of Theorem 7 By the no-arbitrage condition (B.8) and the definition of $\mu_{\pi^\$}$ (A.12), we can rewrite the premium in population (22) as

$$r_t^{\$,(\tau)} - r_t^{\$} = -\sigma_{\pi^\$,t}\sigma_{L,t}^{\top} - \lambda_{ct}\left(\frac{\bar{\mathcal{J}}_c\left(\pi_t^\$ L_t^\$\right)}{\pi_t^\$ L_t^\$} - \frac{\bar{\mathcal{J}}_c(\pi_t^\$)}{\pi_t^\$} - \frac{\bar{\mathcal{J}}_c(L_t^\$)}{L_t^\$}\right)$$
$$- \lambda_{cq,t}\left(\frac{\bar{\mathcal{J}}_{cq}\left(\pi_t^\$ L_t^\$\right)}{\pi_t^\$ L_t^\$} - \frac{\bar{\mathcal{J}}_{cq}(\pi_t^\$)}{\pi_t^\$} - \frac{\bar{\mathcal{J}}_{cq}(L_t^\$)}{L_t^\$}\right) - \lambda_{qt}\left(\frac{\bar{\mathcal{J}}_q\left(\pi_t^\$ L_t^\$\right)}{\pi_t^\$ L_t^\$} - \frac{\bar{\mathcal{J}}_q(\pi_t^\$)}{\pi_t^\$} - \frac{\bar{\mathcal{J}}_q(L_t^\$)}{L_t^\$}\right).$$

From (A.10), we know that for $j \in \{c, cq\}$,

$$\frac{\bar{\mathcal{J}}_j(\pi_t^\$)}{\pi_t^\$} = E_{\nu_j}\left[e^{-\gamma Z_{jt}} - 1\right],$$

and $\frac{\bar{\mathcal{J}}_q(\pi_t^\$)}{\pi_t^\$} = 0$. Furthermore, recall that the N_q type of jump (inflation spike) does not affect $\pi^\$$, therefore; $\frac{\bar{\mathcal{J}}_q\left(\pi_t^\$ L_t^\$\right)}{\pi_t^\$ L_t^\$} = \frac{\bar{\mathcal{J}}_q\left(L_t^\$\right)}{L_t^\$}$. From (B.16) – (B.17) we know that

$$\frac{\bar{\mathcal{J}}_c\left(\pi_t^\$ L_t^\$\right)}{\pi_t^\$ L_t^\$} = p_D E_{\nu_c}\left[e^{(1-\gamma)Z_{ct}} - 1\right] + (1-p_D)E_{\nu_c}\left[e^{-\gamma Z_{ct}} - 1\right],$$
$$\frac{\bar{\mathcal{J}}_{cq}\left(\pi_t^\$ L_t^\$\right)}{\pi_t^\$ L_t^\$} = p_D E_{\nu_{cq}}\left[e^{1-(\gamma + b_{L^\$ q}(\tau))Z_{cq,t}} - 1\right] + (1-p_D)E_{\nu_{cq}}\left[e^{-(\gamma + b_{L^\$ q}(\tau))Z_{cq,t}} - 1\right].$$

Furthermore,

$$\frac{\bar{\mathcal{J}}_c\left(L_t^\$\right)}{L_t^\$} = p_D E_{\nu_c}\left[e^{Z_{ct}} - 1\right],$$
$$\frac{\bar{\mathcal{J}}_{cq}\left(L_t^\$\right)}{L_t^\$} = p_D E_{\nu_{cq}}\left[e^{1-b_{L^\$ q}(\tau)Z_{cq,t}} - 1\right] + (1-p_D)E_{\nu_{cq}}\left[e^{-b_{L^\$ q}(\tau)Z_{cq,t}} - 1\right].$$

Together with (A.11) and (B.15), we obtain:

$$r_t^{\$(\tau)} - r_t^\$ = -\lambda_t^\top\left(b_{L^\$\lambda}(\tau) * b * \sigma_\lambda^2\right) + \lambda_c p_D\, E_{\nu_c}\left[(e^{-\gamma Z_{ct}} - 1)(1 - e^{Z_{ct}})\right]$$
$$+ \lambda_{cq}\Bigg((1-p_D)E_{\nu_{cq}}\left[(e^{-\gamma Z_{cq,t}} - 1)(1 - e^{-b_{L^\$ q}(\tau)Z_{cq,t}})\right]$$
$$+ p_D E_{\nu_{cq}}\left[(e^{-\gamma Z_{cq,t}} - 1)(1 - e^{(1-b_{L^\$ q}(\tau))Z_{cq,t}})\right]\Bigg).$$

□

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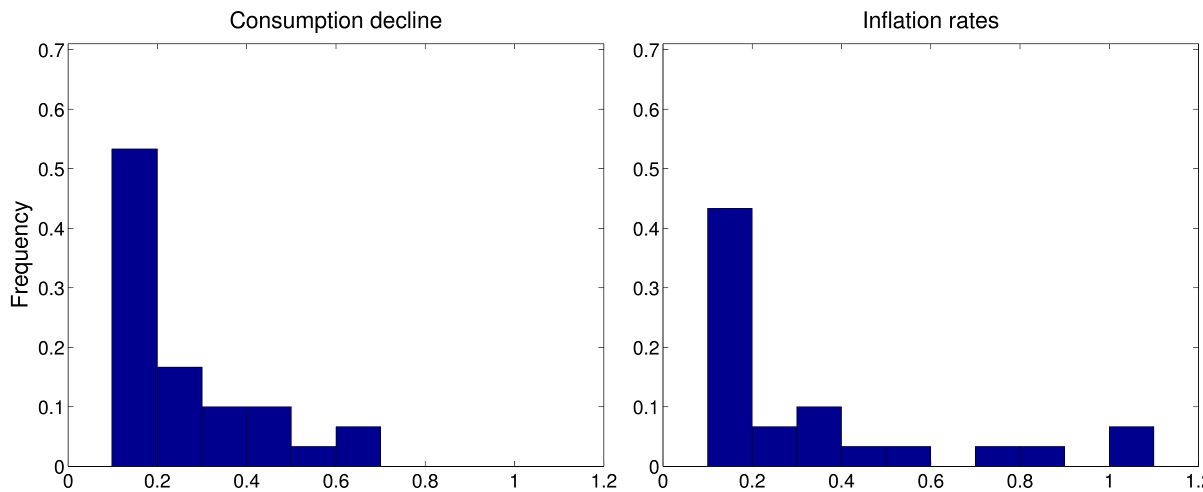
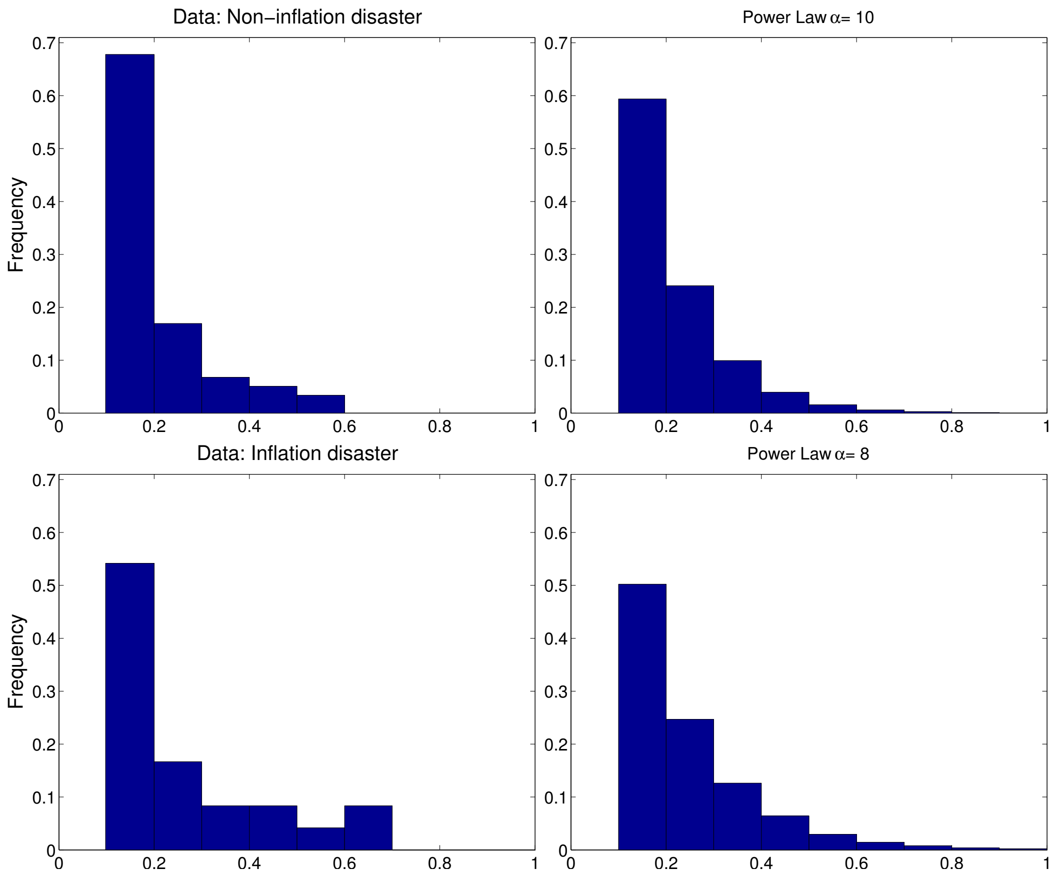
Figure 1: Inflation disasters: Distribution of consumption declines and inflation rates



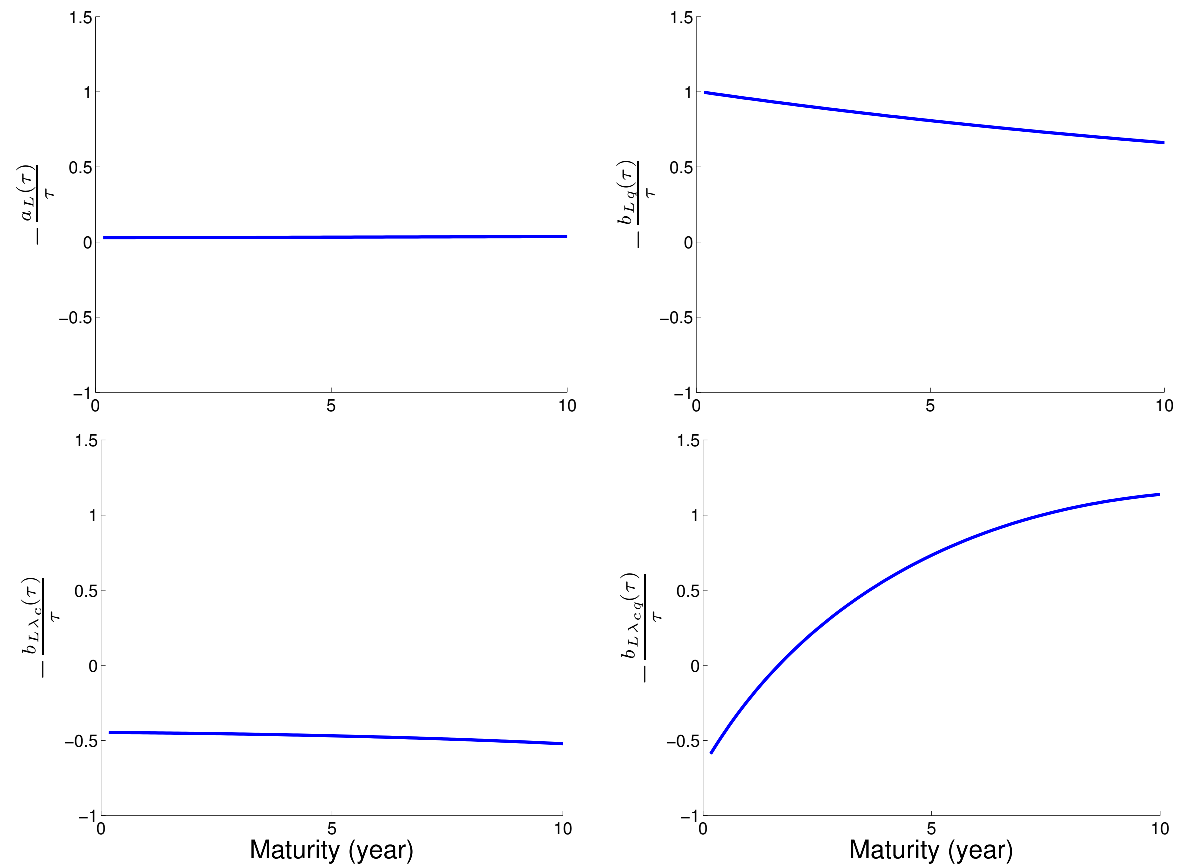
Notes: Histograms show the distribution of large consumption declines (peak-to-trough measure) and high inflation (average annual inflation rate) in periods where large consumption declines and high inflation co-occur. These figures exclude eight events in which average annual inflation rates exceeded 100%. Data from Barro and Ursua (2008).

Figure 2: Data vs. model consumption declines



Notes: This figure plots the distributions of large consumption declines in the data and the power law distribution used in the model. The top-left panel plots the distributions of large consumption declines that do not co-occur with high inflation and the top-right panel plots the power law distribution with parameter 10. The bottom-left panel plots the distributions of large consumption declines that co-occur with high inflation and the bottom-right panel plots the power law distribution with parameter 8. Data from Barro and Ursua (2008).

Figure 3: Solution for the nominal bond yield

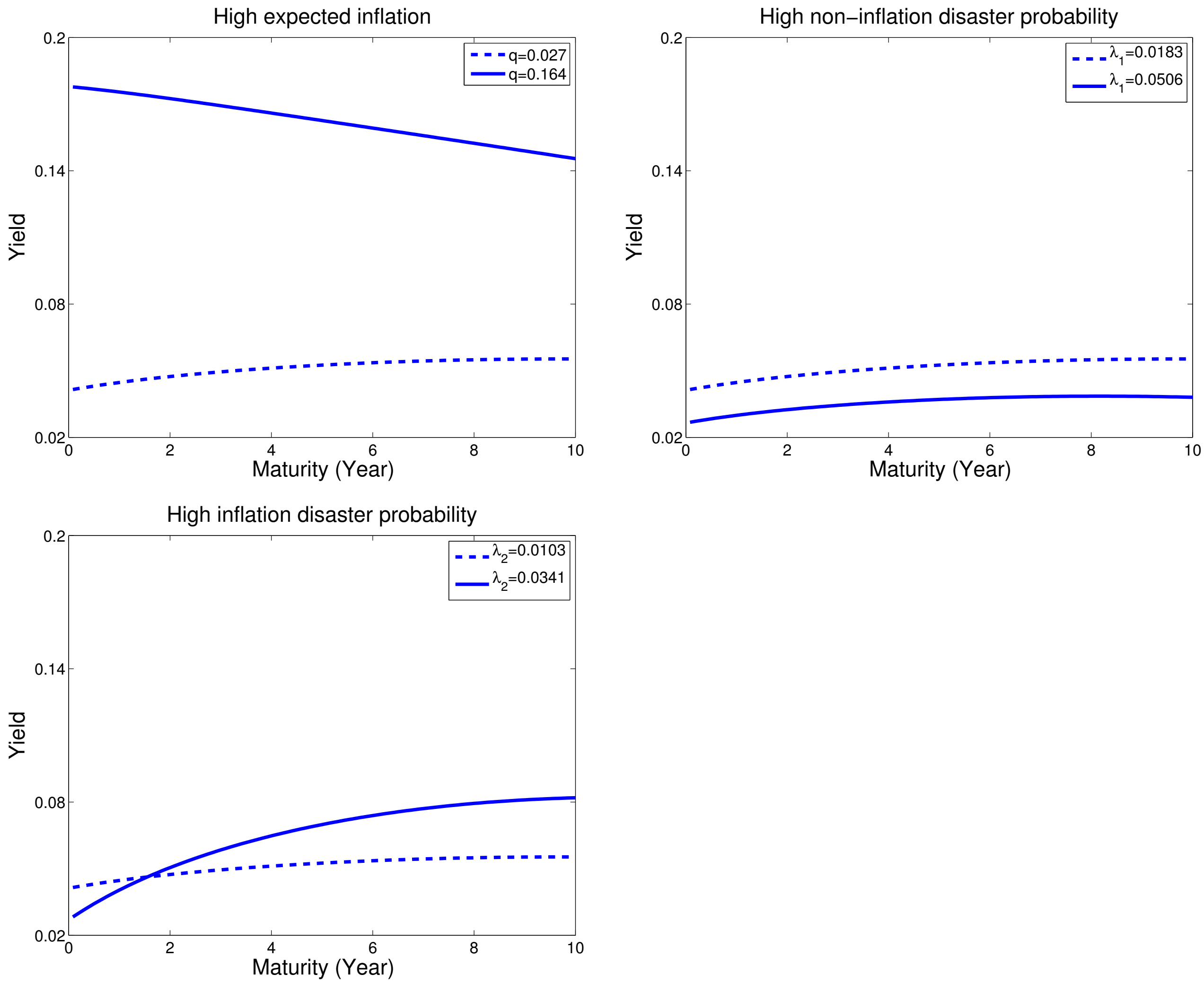


Notes: The nominal yield of a bond with maturity τ is

$$y_t^{\$,(\tau)} = -\frac{1}{\tau}\left(a_L(\tau) + b_{Lq}(\tau)q_t + b_{L\lambda}(\tau)^\top \lambda_t\right).$$

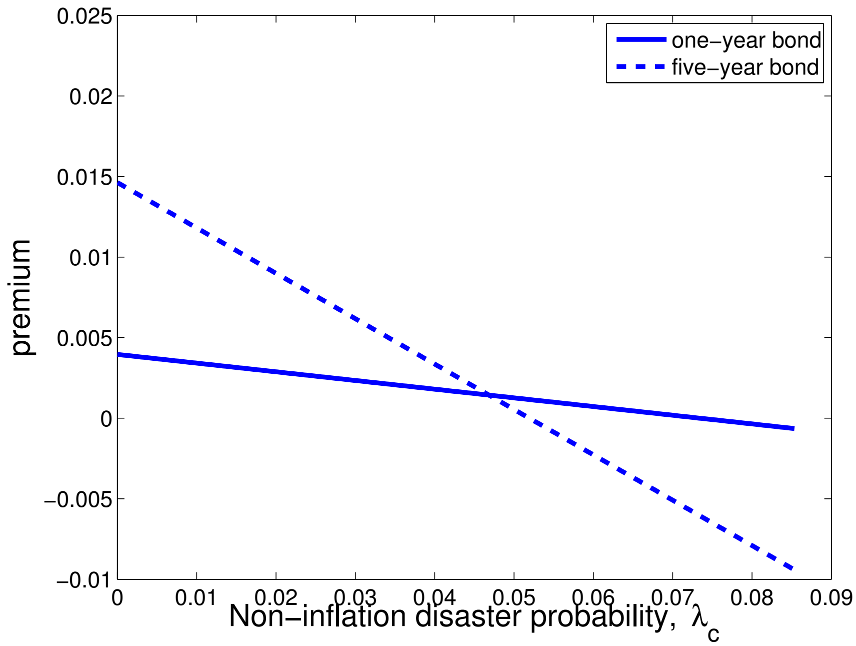
The top-left panel plots the constant term, the top-right panel plots the coefficient multiplying q_t (expected inflation), the bottom-left panel plots the coefficient multiplying λ_c (non-inflation disaster probability), and the bottom right panel plots the coefficient multiplying λ_{cq} (inflation disaster probability). All are plotted as functions of years to maturity (τ).

Figure 4: Yield curve as functions of the state variables



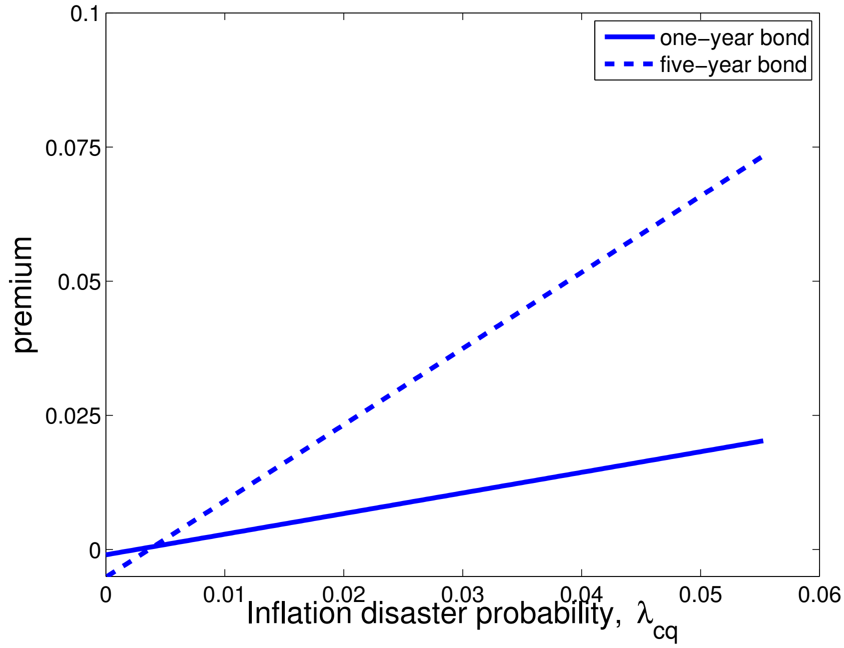
Notes: The figure plots the responses of the nominal yield curve to a shock of standard deviation on each of the three state variables. The dashed line represents the yield curve when all variables are fixed at their means. The solid line in the top-left panel represents high expected inflation; the solid line in the top-right panel represents high non-inflation disaster probability; and the solid line in the bottom-left panel represents high inflation disaster probability.

Figure 5: Risk premiums as a function of non-inflation disaster probability



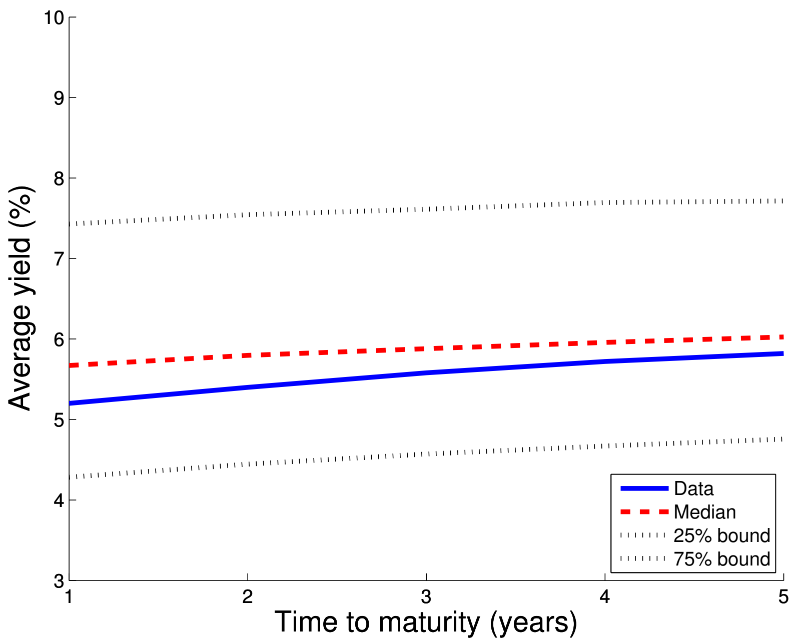
Notes: This figure shows the instantaneous expected nominal return on a one-year nominal zero coupon bond above the nominal risk-free rate (solid line) and the analogous premium for the five-year nominal zero coupon bond (dashed line). Premiums are shown as a function of the non-inflation disaster probability, λ_1, while λ_2 is fixed at its mean of 1.03%. Premiums are in annual terms.

Figure 6: Risk premiums as a function of inflation disaster probability



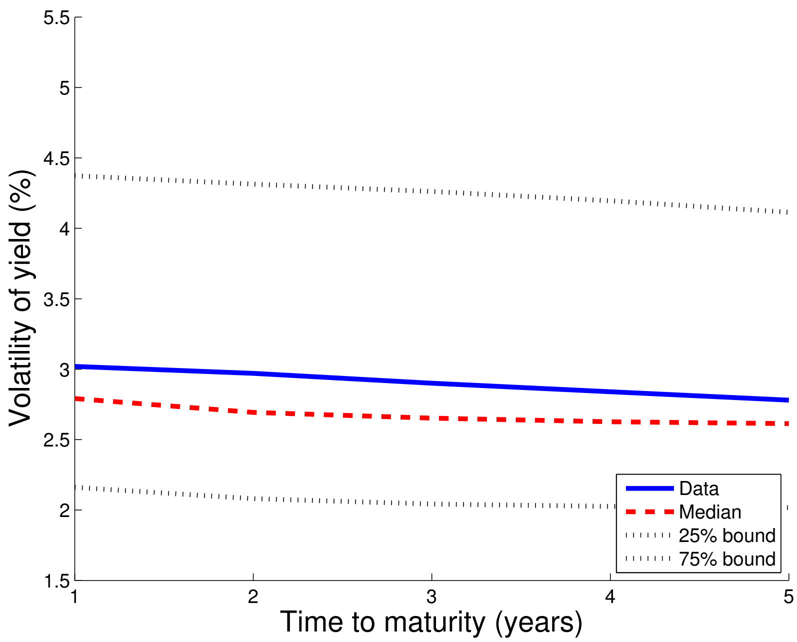
Notes: This figure shows the instantaneous expected nominal return on a one-year nominal zero coupon bond above the nominal risk-free rate (solid line) and the analogous premium for the five-year nominal zero coupon bond (dashed line). Premiums are shown as a function of the disaster probability, λ_2, while λ_1 is fixed at its mean of 1.83%. Premiums are in annual terms.

Figure 7: Average bond yield



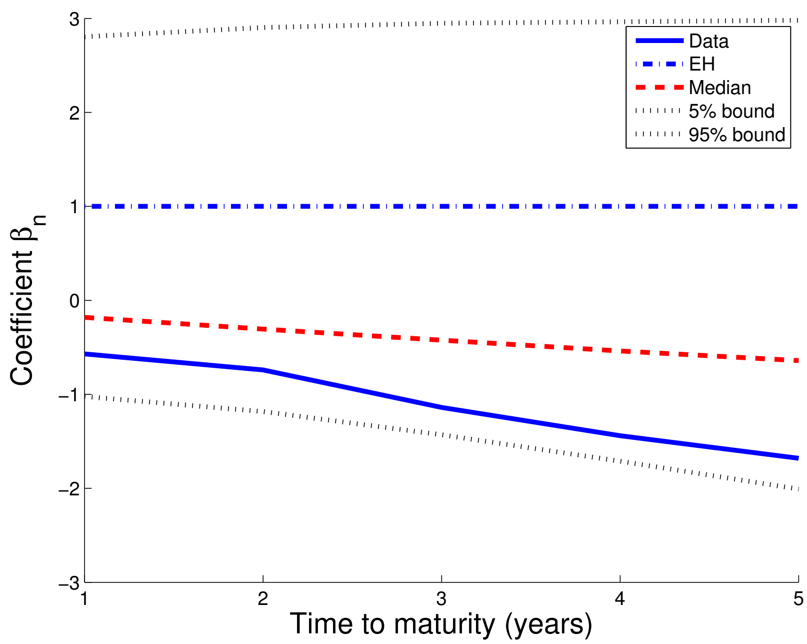
Notes: This figure plots the data and model-implied average nominal bond yield as a function of years to maturity. The solid line plots the average nominal bond yields in the data. The dashed line plots the median average bond yields in the small sample containing no consumption disasters, and the dotted lines plot the 25% and 75% bounds. Data moments are calculated using monthly data from 1952 to 2011. Data are constructed using the Fama-Bliss dataset from CRSP. All yields are in annual terms.

Figure 8: Volatility of bond yield



Notes: This figure plots the data and model-implied volatility of nominal bond yield as a function of years to maturity. The solid line plots the volatility of nominal bond yields in the data. The dashed line plots the median volatility of bond yields in the small-samples containing no consumption disasters, and the dotted lines plot the 25% and 75% bounds. Data moments are calculated using monthly data from 1952 to 2011. Data are constructed using the Fama-Bliss dataset from CRSP. All yields are in annual terms.

Figure 9: Campbell-Shiller long rate regression

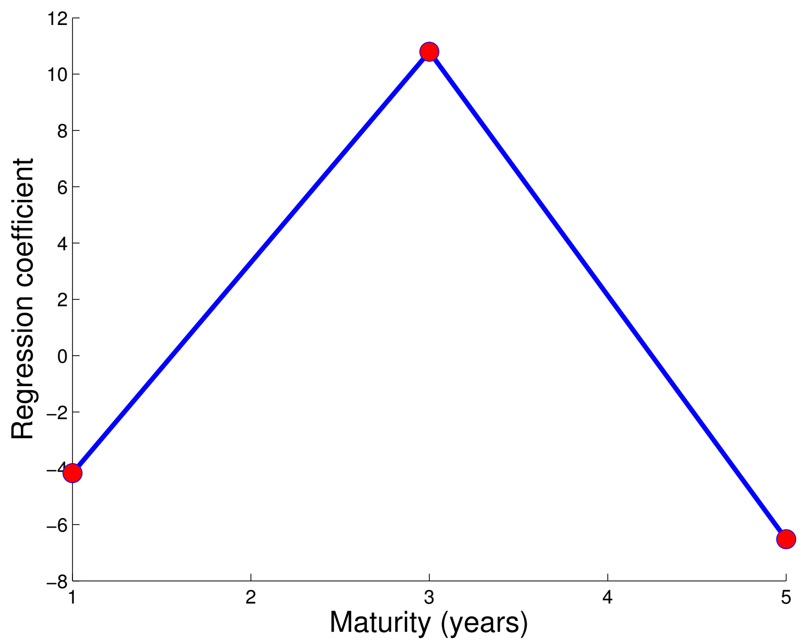


Notes: This figure reports the coefficients of the Campbell-Shiller regression.

$$y_{t+h}^{\$,(n-h)} - y_t^{\$,(n)} = \text{constant} + \beta_n \frac{1}{n-h}\left(y_t^{\$,(n)} - y_t^{\$,(h)}\right) + \text{error},$$

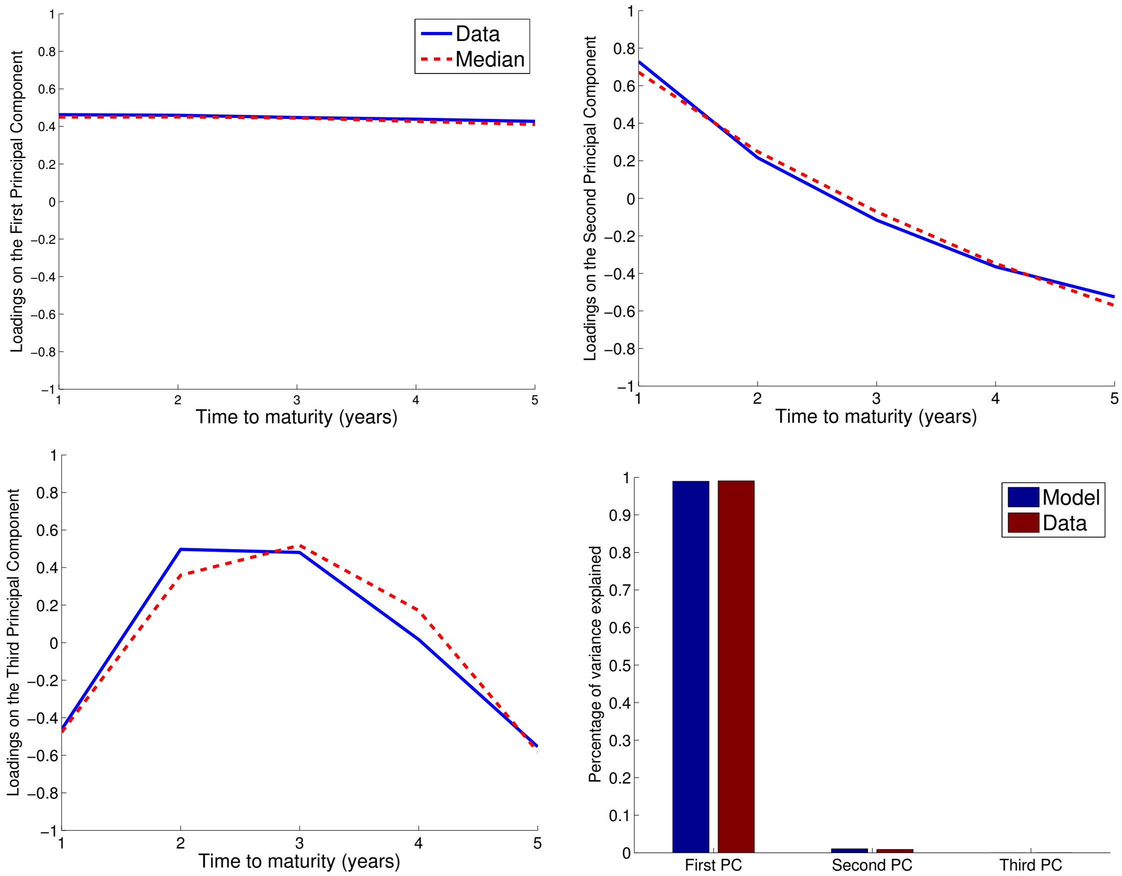
where $h = 0.25$. The solid line plots the coefficients in the data. The dash-dotted line plots the coefficients under the expectation hypothesis. The dashed line plots the median value of the coefficients in the small-samples containing no consumption disasters, and the dotted lines plot the 5% and 95% bounds. Data moments are calculated using monthly data from 1952 to 2011. Data are constructed using Fama-Bliss dataset from the CRSP.

Figure 10: Forward rate regression - First stage estimates



Notes: This figure plots the coefficient from regressing average excess bond returns on forward rates in the model. Average annual returns on two-, three-, four-, and five-year nominal bonds, in excess of the return on the one-year bond, are regressed on the one-, three-, and five-year forward rates. The figure shows the resulting coefficients as a function of the forward-rate maturity. About 35% of the small-sample having no consumption disaster have coefficients that form a tent shape, and this figure plots the average of the coefficients in these samples.

Figure 11: Principal component analysis



Notes: This figure plots the results from the principal component analysis. I report the median values from the subset of small-sample simulations that do not contain any disasters. The top-left panel plots the loadings on the first principal component, the top-right panel plots the loadings on the second principal component, and the bottom-left panel plots the loadings on the third principal component. The bottom-right panel shows the percentage of variance explained by each of the principal components. Data are available at monthly frequency from June 1952 to December 2011.

Table 1: Summary statistics of consumption disasters

Panel A: All countries	
Number of consumption disasters	89
Number of consumption disasters with high inflation	30
Percentage of consumption disasters with high inflation (%)	33.71
Panel B: OECD countries	
Number of consumption disasters	53
Number of consumption disasters with high inflation	17
Percentage of consumption disasters with high inflation (%)	32.08

Table 2: Parameters

Panel A: Basic parameters	
Average growth in consumption (normal times) $\bar{\mu}$ (%)	2.02
Average growth in dividend (normal times) μ_D (%)	3.48
Volatility of consumption growth (normal times) σ (%)	2.00
Leverage ϕ	3.0
Rate of time preference β	0.010
Relative risk aversion γ	3.0
Panel B: Inflation parameters	
Average inflation $\bar{q}$ (%)	2.70
Volatility of expected inflation σ_q (%)	1.30
Volatility of realized inflation σ_p (%)	0.80
Mean reversion in expected inflation κ_q	0.09
Panel C: Non-inflation disaster parameters	
Average probability of non-inflation disaster $\bar{\lambda}_c$ (%)	1.83
Mean reversion in non-inflation disaster probability κ_{λ_c}	0.11
Volatility parameter for non-inflation disaster σ_{λ_c}	0.112
Minimum non-inflation disaster (%)	10
Power law parameter for non-inflation disaster	10
Panel D: Inflation disaster parameters	
Average probability of inflation disaster $\bar{\lambda}_{cq}$ (%)	1.03
Mean reversion in inflation disaster probability $\kappa_{\lambda_{cq}}$	0.11
Volatility parameter for inflation disaster $\sigma_{\lambda_{cq}}$	0.103
Minimum inflation disaster (%)	10
Power law parameter for inflation disaster	8

Table 3: Log consumption and dividend growth moments

Panel A: Consumption growth

		No-Disaster Simulations			All Simulations			
	Data	0.05	0.50	0.95	0.05	0.50	0.95	Population
mean	1.91	1.57	2.00	2.42	−0.19	1.45	2.22	1.29
standard deviation	1.41	1.68	1.99	2.29	1.85	3.86	9.02	5.05
skewness	−0.48	−0.51	−0.01	0.49	−6.06	−3.20	0.22	−6.61
kurtosis	3.49	2.22	2.82	3.97	2.50	16.18	43.29	69.83

Panel B: Dividend growth

		No-Disaster Simulations			All Simulations			
	Data	0.05	0.50	0.95	0.05	0.50	0.95	Population
mean	1.78	2.01	3.29	4.56	−3.28	1.64	3.97	1.16
standard deviation	6.57	5.05	5.97	6.86	5.56	11.57	27.06	15.14
skewness	−0.01	−0.51	−0.01	0.49	−6.06	−3.20	0.22	−6.61
kurtosis	5.26	2.22	2.82	3.97	2.50	16.18	43.29	69.83

Notes: Data moments are calculated using annual data from 1947 to 2010. Population moments are calculated by simulating data from the model at a monthly frequency for 60,000 years and then aggregating monthly growth rates to an annual frequency. I also simulate 10,000 60-year samples and report the 5th-, 50th- and 95th-percentile for each statistic both from the full set of simulations and for the subset of samples for which no consumption disasters occur.

Table 4: Inflation moments

		No-Disaster Simulations			All Simulations			
	Data	0.05	0.50	0.95	0.05	0.50	0.95	Population
Mean	3.74	0.27	3.65	12.59	0.91	6.12	28.51	11.16
Standard deviation	3.03	1.77	2.89	13.55	1.92	5.54	31.34	20.63
AC(1)	0.66	0.61	0.84	0.93	0.65	0.87	0.95	0.95

Notes: Data moments are calculated using annual data from 1947 to 2010. Population moments are calculated by simulating data from the model at a monthly frequency for 60,000 years and then aggregating monthly growth rates to an annual frequency. I also simulate 10,000 60-year samples and report the 5th-, 50th- and 95th-percentile for each statistic both from the full set of simulations and for the subset of samples for which no consumption disasters occur. All numbers are in annual level terms.

Table 5: Nominal Yield Moments

Panel A: Average nominal bond yield

		No-Disaster Simulations			All Simulations			
Maturity	Data	0.05	0.50	0.95	0.05	0.50	0.95	Population
1-year	5.20	2.39	5.67	13.09	2.49	7.39	23.26	10.86
2-year	5.40	2.66	5.80	13.06	2.71	7.51	23.27	10.92
3-year	5.58	2.88	5.88	12.93	2.89	7.60	23.15	10.92
4-year	5.72	3.03	5.96	12.85	3.03	7.65	22.97	10.89
5-year	5.82	3.18	6.03	12.71	3.14	7.67	22.68	10.83

Panel B: Volatility of nominal bond yield

		No-Disaster Simulations			All Simulations			
Maturity	Data	0.05	0.50	0.95	0.05	0.50	0.95	Population
1-year	3.02	1.67	2.79	10.96	1.90	5.09	20.67	12.97
2-year	2.97	1.62	2.69	10.51	1.83	4.96	20.18	12.72
3-year	2.90	1.58	2.65	10.22	1.80	4.88	19.59	12.46
4-year	2.84	1.56	2.63	9.87	1.77	4.80	19.13	12.20
5-year	2.78	1.54	2.61	9.58	1.76	4.73	18.67	11.93

Notes: Panel A reports the average nominal bond yield and Panel B reports the volatility of the nominal bond yield. Data moments are calculated using monthly data from 1952 to 2011. Population moments are calculated by simulating data from the model at a monthly frequency for 60,000 years. I also simulate 10,000 60-year samples and report the 5th-, 50th- and 95th-percentile for each statistic both from the full set of simulations and for the subset of samples for which no consumption disasters occur. All yields are in annual terms.

Table 6: Correlation between principal components and state variables

	PC1	PC2	PC3
expected inflation	0.92	0.09	0.11
non-inflation disaster risks	−0.05	0.07	0.82
inflation disaster risks	0.06	−0.90	0.07

Notes: This table reports the correlation between each principal component and each state variable in the model. I report the median value drawn from the subset of small-sample simulations having no consumption disasters.

Table 7: Campbell-Shiller long rate regression

		No-Disaster Simulations			All Simulations			
Maturity	Data	0.05	0.50	0.95	0.05	0.50	0.95	Population
1-year	−0.57	−1.02	−0.18	2.80	−0.93	0.31	3.65	0.44
2-year	−0.74	−1.18	−0.31	2.90	−1.08	0.30	3.76	0.57
3-year	−1.14	−1.43	−0.42	2.95	−1.31	0.27	3.87	0.67
4-year	−1.44	−1.71	−0.54	2.96	−1.56	0.25	3.93	0.74
5-year	−1.68	−2.01	−0.64	2.98	−1.80	0.23	3.96	0.79

Notes: This table reports the coefficients of the Campbell-Shiller regression.

$$y_{t+h}^{\$,(n-h)} - y_t^{\$,(n)} = \text{constant} + \beta_n \frac{1}{n-h}\left(y_t^{\$,(n)} - y_t^{\$,(h)}\right) + \text{error},$$

where $h = 0.25$ and each row represents a bond with a different maturity (n). Data moments are calculated using quarterly data from June 1952 to December 2011.

Table 8: Cochrane-Piazzesi forward rate regression

Panel A: Coefficient

Maturity	Data	No-Disaster Simulations 0.05	0.50	0.95	All Simulations 0.05	0.50	0.95	Population
2-year	0.44	0.32	0.40	0.48	0.33	0.41	0.49	0.54
3-year	0.83	0.73	0.80	0.87	0.74	0.81	0.88	0.90
4-year	1.26	1.19	1.20	1.21	1.19	1.20	1.21	1.17
5-year	1.47	1.46	1.59	1.73	1.44	1.57	1.71	1.39

Panel B: R^2-statistics

Maturity	Data	No-Disaster Simulations 0.05	0.50	0.95	All Simulations 0.05	0.50	0.95	Population
2-year	0.16	0.02	0.15	0.48	0.01	0.11	0.41	0.01
3-year	0.17	0.03	0.16	0.47	0.02	0.11	0.41	0.01
4-year	0.20	0.03	0.16	0.44	0.02	0.12	0.39	0.01
5-year	0.18	0.03	0.17	0.41	0.02	0.12	0.37	0.01

Notes: This table reports the results from the second stage of the Cochrane-Piazzesi single factor regression. It reports the coefficient on the linear combination of forward rates on nominal bonds and the R^2-statistics from regressing excess bond return on the single forward rate factor. I consider bonds with maturities of two, three, four and five years. Data are monthly from 1952 to 2011.

Table 9: Market moments

	Data	No-Disaster Simulations 0.05	0.50	0.95	All Simulations 0.05	0.50	0.95	Population
$E[R^{(0.25)}]$	1.25	1.03	2.03	2.54	-0.56	1.57	2.40	1.35
$\sigma(R^{(0.25)})$	2.75	0.90	1.26	2.13	0.98	1.64	3.29	2.15
$E[R^m - R^{(0.25)}]$	7.25	3.12	5.06	7.87	2.09	4.78	8.57	5.04
$\sigma(R^m)$	17.79	9.68	13.75	19.91	11.21	17.80	27.44	18.91
Sharpe ratio	0.41	0.25	0.37	0.50	0.11	0.28	0.45	0.27
$\exp(E[p-d])$	32.51	30.71	36.13	39.06	24.48	33.77	38.35	32.66
$\sigma(p-d)$	0.43	0.07	0.15	0.30	0.09	0.21	0.44	0.29
$\text{AR1}(p-d)$	0.92	0.46	0.73	0.90	0.53	0.79	0.92	0.88

Notes: Data moments are calculated using annual data from 1947 to 2010. Population moments are calculated by simulating monthly data from the model for 60,000 years and then aggregating to an annual frequency. We also simulate 10,000 60-year samples and report the 5th-, 50th-, and 95th-percentile for each statistic from the full set of simulations and for the subset of samples for which no disasters occur. $R^{(0.25)}$ denotes the three-month Treasury Bill return where $R^{(0.25)} = R_t^{\$,(0.25)} \frac{P_{t+1}}{P_t}$. R^m denotes the return on the aggregate market, and $p-d$ denotes the log price-dividend ratio.

Table 10: Long-horizon regressions of returns on the price-dividend ratio (One-year holding period)

			Panel A: Aggregate Market						
			No-Disaster Simulations			All Simulations			
	Data	t-stat	0.05	0.50	0.95	0.05	0.50	0.95	Population
Coef.	−0.12	[−1.89]	−0.63	−0.34	−0.17	−0.53	−0.23	0.03	−0.13
R^2	0.07		0.07	0.17	0.29	0.00	0.08	0.23	0.04
			Panel B: Bond Market						
			No-Disaster Simulations			All Simulations			
	Data	t-stat	0.05	0.50	0.95	0.05	0.50	0.95	Population
Coef.	0.02	[1.19]	−0.15	−0.00	0.13	−0.13	0.02	0.18	0.02
R^2	0.02		0.00	0.03	0.23	0.00	0.02	0.19	0.00

Notes: This table reports the results from regressing one-year aggregate market excess returns and average nominal bond excess return on the price-dividend ratios. Data are annual from 1953 to 2010. For the data coefficients, I report t-statistics constructed using Newey-West standard errors. Population moments are calculated by simulating monthly data from the model for 60,000 years and then aggregating to an annual frequency. I also simulate 10,000 60-year samples and report the 5th-, 50th- and 95th-percentile for each statistic from the full set of simulations and for the subset of samples for which no disasters occur.

Table 11: Long-horizon regressions of returns on the linear combination of forward rates (One-year holding period)

			Panel A: Aggregate Market						
			No-Disaster Simulations			All Simulations			
	Data	t-stat	0.05	0.50	0.95	0.05	0.50	0.95	Population
Coef.	0.72	[0.48]	−3.14	0.39	2.84	−3.06	0.54	3.24	−0.28
R^2	0.00		0.00	0.03	0.16	0.00	0.02	0.14	0.00
			Panel B: Bond Market						
			No-Disaster Simulations			All Simulations			
	Data	t-stat	0.05	0.50	0.95	0.05	0.50	0.95	Population
Coef.	1.03	[3.19]	0.86	1.17	1.55	0.81	1.20	1.68	1.59
R^2	0.20		0.02	0.17	0.45	0.01	0.12	0.40	0.01

Notes: This table reports the results from regressing one-year aggregate market excess returns and average nominal bond excess return on the linear combination of forward rates. Data are annual from 1953 to 2010. For the data coefficients, I report t-statistics constructed using Newey-West standard errors. Population moments are calculated by simulating monthly data from the model for 60,000 years and then aggregating to an annual frequency. I also simulate 10,000 60-year samples and report the 5th-, 50th- and 95th-percentile for each statistic from the full set of simulations and for the subset of samples for which no disasters occur.

Table 12: Long-horizon regressions of returns on the price-dividend ratio (Five-year holding period)

Panel A: Aggregate Market									
			No-Disaster Simulations			All Simulations			
	Data	t-stat	0.05	0.50	0.95	0.05	0.50	0.95	Population
Coef.	−0.28	[−2.87]	−1.55	−1.05	−0.57	−1.51	−0.84	0.13	−0.52
R^2	0.13		0.13	0.44	0.69	0.01	0.25	0.61	0.12
Panel B: Bond Market									
			No-Disaster Simulations			All Simulations			
	Data	t-stat	0.05	0.50	0.95	0.05	0.50	0.95	Population
Coef.	0.07	[1.83]	−0.54	0.00	0.43	−0.51	0.08	0.62	0.07
R^2	0.09		0.00	0.07	0.47	0.00	0.06	0.46	0.01

Notes: This table reports the results from regressing five-year aggregate market excess returns and average nominal bond excess return on the price-dividend ratios. Data are annual from 1953 to 2010. For the data coefficients, I report t-statistics constructed using Newey-West standard errors. Population moments are calculated by simulating monthly data from the model for 60,000 years and then aggregating to an annual frequency. I also simulate 10,000 60-year samples and report the 5th-, 50th- and 95th-percentile for each statistic from the full set of simulations and for the subset of samples for which no disasters occur.

Table 13: Long-horizon regressions of returns on the linear combination of forward rates (Five-year holding period)

			Panel A: Aggregate Market						
			No-Disaster Simulations			All Simulations			
	Data	t-stat	0.05	0.50	0.95	0.05	0.50	0.95	Population
Coef.	2.02	[0.68]	−11.03	0.97	8.61	−11.61	1.41	10.89	−1.21
R^2	0.01		0.00	0.08	0.45	0.00	0.07	0.40	0.00
			Panel B: Bond Market						
			No-Disaster Simulations			All Simulations			
	Data	t-stat	0.05	0.50	0.95	0.05	0.50	0.95	Population
Coef.	1.83	[2.53]	1.04	3.37	5.83	0.84	3.66	7.18	6.45
R^2	0.11		0.02	0.26	0.65	0.01	0.24	0.64	0.05

Notes: This table reports the results from regressing five-year aggregate market excess returns and average nominal bond excess return on the linear combination of forward rates. Data are annual from 1953 to 2010. For the data coefficients, I report t-statistics constructed using Newey-West standard errors. Population moments are calculated by simulating monthly data from the model for 60,000 years and then aggregating to an annual frequency. I also simulate 10,000 60-year samples and report the 5th-, 50th- and 95th-percentile for each statistic from the full set of simulations and for the subset of samples for which no disasters occur.