Government-induced asset bubbles

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Abstract

We describe a general mechanism by which government overspending can lead to asset bubbles. We show that the government can generate a bubble on its own, without any malevolent intent by financial intermediaries. We consider a two-sector economy with households and the government, where households can purchase an investment good and finance that purchase with government-provided loans. We show that an asset bubble can ensue only when both the government and the households make suboptimal decisions; if at least one sector behaves optimally, inefficiencies never arise. In general, when the government chooses to provide financing to households based on some simple criterion (such as admission to an institution of higher education or compliance with simple mortgage standards), it is only by chance that it will provide the optimal amount of financing. If too much financing is provided, a bubble ensues and households start to default. We also show that government-provided financing creates an endogenous propagation mechanism. If financing exceeds a certain threshold, it is not only the households who receive this excess financing that default on their loans, some households who received financing before it reached the threshold will also default.

Keywords:
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1. Introduction

The narrative of the recent financial crisis often puts blame on imperfections of the financial system and on greed of financial institutions. The U.S. Senate Staff Report of the Permanent Subcommittee on Investigations, for example, states that "the crisis was not a natural disaster, but the result of high risk, complex financial products; undisclosed conflicts of interest; and the failure of regulators, the credit rating agencies, and the market itself to rein in the excesses of Wall Street."[3] This view is echoed by the Financial Crisis Inquiry Commission, which wrote in its final report that "dramatic failures of corporate governance and risk management at many systemically important financial institutions were a key cause of this crisis."[4]

As we show formally in this paper, however, financial intermediaries need not even exist for an asset bubble to emerge. The government can generate it entirely on its own by providing too much financing to households. In addition, we show that the government does not only generate an asset bubble but also acts as a propagating mechanism behind it. This paper provides a formal basis for the view shared by some that government subsidies can lead to financial crises. As Charles Calomiris notes in an April 2012 interview, had the government's aggressive encouragement of high-risk mortgage lending not occurred, that would have been enough to prevent the 2008 financial crisis from happening (Epstein (2012)).[5] This view was also

[3]Source: Wall Street and the Financial Crisis: Anatomy of a Financial Collapse. Majority and Minority Staff Report, Permanent Subcommittee on Investigations, United States Senate, April 13, 2011, p. 1.

[4]Source: The Financial Crisis Inquiry Report, January 2011, p. xviii.

[5]Charles Calomiris also adds in the same interview that it was the combination of government subsidies and lack of prudential regulation that created the crisis. In this paper, we show that

expressed by Peter Wallison in his dissenting statement from the Financial Crisis Inquiry Commission.

We consider partial equilibrium in a two-sector economy with households and the government. There exists an investment good. Households decide whether to acquire the investment good or not. If acquired, the investment good generates an income stream, which depends on the total number of households who acquired the investment good. If households want to acquire the investment good, they have to obtain financing from the government. We assume that the government provides loans to households in the most efficient way possible, in the sense that households who benefit most from government-provided financing receive such financing first. We assume that the income stream generated by the investment good exhibits diminishing marginal returns: the larger the number of households who acquired the investment good, the lower the income stream.

In the above setting, we derive the following results:

1. in order to generate an inefficient outcome, both the government and the households need to make uninformed decisions, if at least one sector has perfect information and behaves optimally, inefficiencies never arise;

2. excess provision of financing by the government can cause an asset bubble;

3. excess provision of financing by the government, in addition to causing the bubble, generates an endogenous propagation mechanism that affects a larger number of households than just the households who received excess financing.

government subsidies alone can in principle lead to financial collapse.

Discussions of the recent financial crisis both in mass media and in academic circles often overlook the simple fact that any financial transaction is a deal between at least two parties. Therefore, a financial crisis cannot arise and propagate if at least one of the parties involved in the underlying financial transactions refuses to participate. Homeowners who obtained subprime loans they could not possibly repay did so willingly and at their own risk, even if they did so because of their misunderstanding of the costs and benefits of home ownership. Some of them might have been defrauded but this does not change the fact that they participated in the bubble. The first result of this paper makes this intuition very clear. We show that as long as rational households are able to perfectly assess the costs and benefits associated with acquiring the investment good, they will acquire it only when it is beneficial to them. Similarly, if the government had perfect information, it would provide financing to households only as long as acquisition of the investment good is beneficial to those households.

When households are not fully aware of the costs (direct and indirect) of acquiring the investment good, it is the amount of government-provided financing that determines whether the equilibrium outcome is efficient or not. If the government provides too much financing, then too many households acquire the investment good. If the amount of financing is large enough, a bubble emerges: some households who obtained loans in order to acquire the investment good will default on those loans with certainty. It happens because the investment good exhibits diminishing marginal returns: as more households acquire it, the income stream that this good generates

(for each household) decreases.[6] At some point, the costs of acquiring the investment good for the marginal household exceed the benefits for this household.[7]

Moreover, as the government provides financing to more and more households and the income stream from the investment good declines, some households who previously acquired the investment good also default on their obligations since their income stream is no longer sufficient to cover their loan payments. Hence, the ensuing defaults affect a larger number of households than just the households who received excess financing.[8] This is what we call an endogenous propagation mechanism created by the government. Excess financing by the government is a double-edged sword that reduces welfare not only for households that receive this excess financing but also for households who received financing before the amount of that financing exceeded the appropriate threshold.

There is no malevolent intent by financial intermediaries in our model, and we

[6]Consider mortgages. When a household acquires a home, it benefits from house price appreciation and also from the flow of services that the house provides. To see clearly that housing purchases exhibit diminishing marginal returns consider two identical households that buy two identical homes at different times, and assume that the hosing market is in expansion. Even though both households will gain from house price appreciation, the household that bought its house first will gain more (since it bought it at a lower price and the flow of housing services is the same for both households). Hence, households that make their purchases later receive lower returns.

[7]For instance, when too many people go to college, the wage premium associated with the college degree falls, and the salaries of relatively less skilled workers increase due to their scarcity. As an example, according to the May 2010 wage estimates by the U.S. Bureau of Labor Statistics, electricians (an occupation that does not require advanced training) made on average $51,810 per year, which is roughly the same or more than what people earned in some occupations that require a bachelor's or even a master's degree (such as tax examiners and collectors, or substance abuse councelors). In addition, electricians do not have the huge debt burden they would have accumulated had they gone to college.

[8]The subprime mortgage crisis illustrates this point very clearly. As households who obtained subprime loans started to default (because ultimately their income could not cover their mortgage payments), house prices began to fall. This, in turn, affected prime homeowners, creating a vicious spiral.

assume that the government allocates financing in the most efficient way possible. All that is needed to generate a bubble are diminishing marginal returns, too much financing provided by the government,[9] and households who are unable to recognize indirect costs of obtaining the investment good (indirect costs arise because not obtaining the investment good also generates an income stream, which for some households can be larger than the income stream after obtaining the investment good). While any additional imperfections will make bubbles more likely, the core mechanism that generates them will remain the same.

The mechanism we describe here is clear and straightforward, and it has important policy implications. Our model suggests that there are two ways to avoid government-induced asset bubbles: self-restraint by the government or self-restraint by households (or both). Government-provided financing for social policies is sometimes prone to generate asset bubbles unless it is accompanied by a strong infrastructure to ensure that the beneficiaries of those social policies clearly understand their costs and benefits. In theory, bubbles can be avoided and social policies implemented as long as the government is able to perfectly allocate financing by correctly estimating the optimal number of households who will benefit from those policies. However, we think that such a scenario is highly unlikely. We conclude that the only feasible way for the government to promote social policies without generating inefficiencies and asset bubbles is to require that people who take advantage of those policies are

[9]While we emphasize the role of the government in creating a bubble, in general any provider of excess financing will generate a bubble in our setting. We simply find it plausible that the government (which spends taxpayer money) is more likely to provide excess financing than private investors (who pledge their own money).

made aware of potential risks associated with them. Hence, consumer education and full and clear disclosure by all market participants may be the most sustainable way to prevent financial collapse stemming from government overspending. Additional regulatory burden on financial intermediaries may be a misguided policy response that creates significant deadweight costs.

The rest of the paper is organized as follows. In section 2 we relate our model to the existing literature. Section 3 describes the model and its implications. In section 4 we provide a numerical example to illustrate our results in a less abstract way. Section 5 discusses our assumptions and the policy implications of our results. Section 6 concludes. All proofs are confined to the appendix.

2. Relation to the existing literature

This paper is broadly related to the general literature on the possibility of rational asset bubbles (Tirole (1985), Santos and Woodford (1997), Farhi and Tirole (2011)). However, both our purpose and the approach we use are significantly different from that literature. Our objective is to show how asset bubbles can arise in a non-rational expectations setting, and what policy response can prevent them. We describe a simple static economy, and the above literature generally concludes that rational asset bubbles are impossible in a static economy (Kreps (1977), Tirole (1982)). Tirole (1982), in particular, provides a set of assumptions that need to be relaxed in order to generate a bubble in a static economy.

We relax the assumption that all agents have rational expectations and investigate the precise mechanism by which it leads to asset bubbles. In particular, irrationality

by some agents is not sufficient to generate a bubble in our economy. By focusing on the interplay between households and the government, we obtain a somewhat stronger result: not only are rational bubbles impossible in our economy, bubbles never arise if at least one sector has full information and behaves rationally, even if the other sector does not act optimally. Another key difference of our setup from the previous literature is the nature of the asset we are considering here. Our paper is concerned with the assets that exhibit diminishing marginal returns with respect to the number of households who acquire those assets. Education and housing are examples of such assets.

Our paper is also broadly related to the literature on banking crises and the literature that emerged in the aftermath of the recent financial crisis (Allen and Gale (2000), Allen, Babus, and Carletti (2009), Gennaioli, Shleifer, and Vishny (2010)). However, the focus of those literatures is on the role of financial intermediaries in generating and propagating asset bubbles. We, on the other hand, show that financial intermediaries may be irrelevant and the government can generate and propagate asset bubbles by acting on its own. We do not suggest that financial intermediaries played no role in the recent financial collapse. However, their role in that collapse may have to be reassessed. The degree to which it was the government that created the bubble and not financial intermediaries is an empirical issue, which needs to be investigated in further research.

3. The model: how excess financing leads to bubbles

In this section we describe what is perhaps the simplest setting that can generate asset bubbles induced by government overspending. We consider a two-sector economy with mass 1 of households who live for 1 period. There is an investment good (such as housing or education), which households can acquire at the beginning of the first period. Each household can acquire only one unit of the investment good. The investment good can be acquired only at the beginning of the first period.[10] If acquired, the investment good generates a payment at the end of the first period, and the value of that payment depends on how many other households also acquired the investment good (specified below). We can think of education as one example of the investment good. Households acquire education to increase their human capital, which will supposedly generate benefits that outweigh the costs of acquiring education. In this case, the income generated by the investment good can be viewed as the salary that skilled workers receive. Housing can be another example. There, the utility from owning a house and the appreciation of housing assets over time represent the income stream that the investment good generates.

Households differ in their ability to acquire the investment good. Let θ denote this ability (θ can be viewed as intellectual ability in the case of education or credit worthiness in the case of housing). We assume that θ is uniformly distributed across households, from 0 to μ. In order to acquire the investment good, households need to pay a cost, determined by the non-negative function $cost(\theta)$, with $cost'(\theta) < 0$

[10]This assumption eliminates the dynamic aspect of the problem we study and helps us maintain the simplest and cleanest setting possible.

(the higher the ability, the lower the cost). We assume that households are born with zero endowment and have to borrow in order to acquire the investment good. If a household with ability level θ_0 decides to acquire the investment good, it has to borrow the full amount of $cost(\theta_0)$. The loan will then have to be fully repaid at the end of the first period.

Households' income is determined by their choice whether to acquire the investment good or not. The investment good generates a payment according to the non-negative continuously differentiable function $s(y)$, with $s'(y) < 0$, where y is the total mass of households who acquired the investment good. For example, when households with abilities from θ_0 to θ_1 acquired the investment good, $y = \frac{\theta_1 - \theta_0}{\mu}$. Households that do not acquire the investment good receive a payment determined by the non-negative continuously differentiable function $u(x)$, with $u'(x) < 0$, where x is the total mass of households who do not acquire the investment good. Since households face a binary choice whether to acquire the investment good or not, $y + x = 1$. For simplicity, we assume that $s(y)$ and $u(x)$ depend only on the total mass of households that acquired the investment good and that didn't acquire it, respectively. It means that the payment received from the investment good is the same for all household who acquired it, regardless of their ability, and all differences between them are reflected in their cost of acquiring the investment good. Functions $s(y)$ and $u(x)$ can be viewed, for example, as wage rates of skilled vis-a-vis unskilled labor or utility from owning a home relative to renting. At the end of their lives, households consume their entire net income in excess of any loan payment they have to make. We assume that consumption cannot be negative and abstract from

households' labor-leisure choice and from their savings decision (there is no bequest motive) since these are not central to the problem we are studying.

We impose the following restrictions on $s(y)$, $u(x)$, and $cost(\theta)$:

$$s(1) - cost(0) < 0, \tag{1}$$

$$s(0) - cost(\mu) > u(1). \tag{2}$$

Restriction (1) states that when all households acquire the investment good, at least one of them defaults. $s(1)$ is the income that the investment good generates when all households choose to acquire it, while $cost(0)$ is the cost of acquiring the investment good for the lowest ability (highest cost) household. This restriction precludes a situation when even the household with the highest cost of acquiring the investment good does not default on its debt after acquiring the investment good. Since $u(x)$ is a non-negative function, restriction (1) also implies that the highest possible payoff from not acquiring the investment good must be greater than the lowest possible payoff from acquiring the investment good. Otherwise, the acquisition of the investment good is so beneficial that it is always optimal to make all households acquire it. Hence, this restriction is necessary to make inefficient acquisitions of the investment good possible.[11] Restriction (2), on the other hand, ensures that some

[11]It is likely that there are some social policies that do not satisfy restriction (1). One example could be the provision of cheap cell phones to farmers in Africa, so that they could communicate with potential buyers and better react to market conditions. In those cases, no matter how large the acquisition of the investment good is, it cannot be suboptimal. We think, however, that many of the markets the government is involved in, such as education and the mortgage market, are likely to be characterized by restriction (1), in the sense that there can exist suboptimal outcomes when too much of the investment good is acquired by households.

households find it beneficial to acquire the investment good. $s(0)$ is the income that the investment good generates when no household chooses to acquire it, $cost(\mu)$ is the cost of acquiring the investment good for the highest ability (lowest cost) household, and $u(1)$ is the income for households who did not acquire the investment good when all households chose not to acquire it.

The second sector of our economy is the government, which can provide financing to households in order for them to acquire the investment good. There are no private financial markets and consumers have to borrow from the government if they wish to acquire the investment good (because they have zero endowment).[12] The government, should it choose to intervene, determines the total amount of financing it is going to provide, which we denote by L. If the government decides to provide financing to some households, it loans them precisely the amount needed to acquire the investment good. For example, if the government provides financing to households with abilities between θ_0 and θ_1, then $L = \int_{\theta_0}^{\theta_1} cost(\theta)d\theta$.

For operational simplicity, we assume that the government provides financing in a sequential manner, starting with the highest ability (lowest cost) households first. In particular, if two households, with abilities θ_1 and θ_2 such that $\theta_1 > \theta_2$, want to

[12]The assumption that there are no private financial markets is for simplicity only. This assumption is motivated by the idea that sometimes financial markets are unable or unwilling to provide funds to households because of asymmetric information. When the market is unwilling to bear the risks associated with acquiring the investment good, it is customary to say that the government provides financing to households (presumably, financing is provided only to solvent households who are simply faced with a financing problem). It is exactly the role of this financing provision in generating asset bubbles that is the central focus of our paper. Relaxing the assumption of no private financial markets will not change the qualitative nature of our results because if the government steps in after some involvement of private financial markets, it will probably have to subsidize relatively less solvent households, making a bubble more likely.

acquire the investment good, the government will provide financing to the household with ability θ_1 first and will finance the other household only if the remaining funds can cover that household's acquisition of the investment good. Formally, the government adopts the following algorithm when it provides financing to households.[13]

Algorithm 1. *The government determines the total amount of financing, L, and provides it to households in a sequential manner, starting from households with the highest ability. It loans funds equal to the cost of acquiring the investment good to households with lower and lower abilities until the total amount L is reached. Hence, $L = \int_{\theta^g}^{\mu} cost(\theta)d\theta$, where θ^g represents the lowest ability level of households who obtain financing from the government.*

In the view of Algorithm 1, in the rest of this paper we will focus only on situations in which households acquire the investment good continuously, starting with μ and until some threshold level of ability θ_0 is reached. We call such allocations continuous and say that all households with abilities between θ_0 and μ acquire the investment good and no other household acquires the investment good.

3.1. Equilibrium when at least one sector has perfect information

We will show in this section that with perfect information bubbles never occur in our model. More importantly, we will show that bubbles never occur as long as either households or the government have perfect knowledge of the costs and benefits associated with acquiring the investment good. As we will show in the next section, a

[13]Notice that any allocation of financing different from Algorithm 1 will make bubbles only more likely.

bubble can emerge only when both the government and households lack information or ability to correctly assess the costs and benefits associated with acquiring the investment good, and when the government provides too much financing.

The problem we have in mind can be described as follows. There exists a set of households, and a decision must be made whether some of them need to acquire the investment good or not. We proceed in a sequential manner, starting with the lowest cost (highest ability) households first. A decision is made whether those households are better off by acquiring the investment good or not. If they acquire the investment good, then households with the second lowest level of cost are considered, and so on. We will show that this sequential process must stop at some point because there exists a maximum mass of households who should acquire the investment good. Any acquisition of the investment good that involves a larger mass of households is inefficient in the sense that some households are better off without the investment good. What it means in real terms is that not every person can live in a mansion and not everybody should obtain a four-year college degree. In particular, there exists a level of ability, θ^*, such that all households with abilities greater than or equal to θ^* are better off by acquiring the investment good while all households with abilities below θ^* are better off by not acquiring the investment good.

Proposition 1. *There exists θ^*, such that $\mu > \theta^* > 0$, and:*

(*i*) *If only households with abilities from θ^* to μ acquire the investment good, then all households are better off than if no household acquired the investment good.*

(*ii*) *If households with abilities from θ' to μ acquire the investment good, where*

$\theta' < \theta^*$, all households are worse off than if only households with abilities from θ^* to μ acquired the investment good.

Proposition 1 states that for all continuous allocations of the investment good there exists a maximum level of investment good acquisition that is Pareto efficient. That level is described by a level of ability, θ^*, so that households with that ability are indifferent between acquiring the investment good and not acquiring it. If more households start acquiring the investment good, all households can be made better off by returning to the situation where only households with abilities higher than or equal to θ^* acquired the investment good. θ^* can determined by a very simple condition that the net discounted benefits to a household from acquiring the investment good, conditional on this households' ability and the total mass of households who acquired the investment good, are equal to zero. For the marginal investor in an asset, its net discounted future cash flows are exactly equal to its price.

The intuition behind θ^* is as follows. Start with a situation when no household acquires the investment good. It is clear in that case that the households with the highest level of ability (and consequently lowest costs) will find it beneficial to acquire the investment good, assuming nobody else acquires it (because of restriction (2)). The income generated by the investment good, which is determined by $s(y)$, will be very high since y is very small. At the same time, since the number of people who did not acquire the investment good goes down, their consumption also rises, even if they decide not to acquire the investment good (since their income stream, determined by $u(x)$, rises as x, the mass of households who do not acquire the investment good, goes down). Households with only slightly lower ability face a similar trade-off, but they

now must take into account the fact that households with the highest level ability acquired the investment good. The payoff to lower ability households will be smaller than that for the highest ability household but may be still high enough to justify acquiring the investment good. And again, as these additional households decide to acquire the investment good, the households who did not do it are becoming better off as well. This process repeats until the marginal household is indifferent between acquiring and not acquiring the investment good, which happens exactly at θ^*. For any household with ability below θ^*, acquiring the investment good makes it strictly worse off. Moreover, it makes some household who previously acquired the investment good worse off as well since their incomes, determined by $s(y)$, fall when y, the mass of households who acquire the investment good, increases.

Think of education. If very few people go to college, the marginal product of skilled labor is extremely high. The marginal product of unskilled labor also rises as more people become skilled because unskilled workers now become relatively more rare. As more an more people become skilled, however, the marginal benefit of going to college diminishes. Hence, it must be the case that at some point further education will bring negative benefits to the people who acquire it. A case in point is the scarcity of manufacturing workers in the United States. Consider the following example. "An aspiring machinist – a popular factory job – can start training at 18 and then do a one- or two-year manufacturing apprenticeship. In five years, he or she could be making more than $50,000. In 10 years, that could double to $100,000."[14] This is more than many college graduates can expect to earn when they turn 28. On

[14]Source: `http://finance.yahoo.com/news/100-000-factory-job-whats-145600750.html`

top of that, this aspiring machinist won't have the huge debt burden he or she would have accumulated while in college.

A similar story applies to the housing market. When there are very few homeowners, housing prices are likely to be very low. Hence, acquiring a house can be a good financial investment. It also brings utility to homeowners from the flow of housing services they obtain. However, when more and more people start to buy houses, housing prices appreciate and have less room to climb further. At some point they reach a value where further price appreciation is impossible. Households who acquire housing after that point are bound to be making a negative net present value investment.

Any acquisition of the investment good by households with abilities below θ^* is inefficient since in this case some households are making welfare-reducing choices. We will show that inefficiencies never arise as long as at least one sector of our economy (households or the government) has perfect information about the costs and benefits associated with the acquisition of the investment good.

Proposition 2. *Assume that the government has no knowledge of $s(y)$, $u(x)$, but knows cost(θ) and the distribution of θ, and provides unlimited financing to households, so that it will extend a loan to buy the investment good to any household that wishes to acquire it. Also assume that households have perfect knowledge of $s(y)$, $u(x)$, cost(θ), and the distribution of θ. Then, in equilibrium, only households with abilities from θ^* to μ acquire the investment good.*

The intuition here is simple. When perfectly rational and fully informed households face the choice of acquiring the investment good, they will do so only when

their future income net of loan repayment is higher than it would be without the investment good (conditioning, of course, on the fact that all households with abilities higher than theirs will also acquire the investment good). Hence, even if the government is willing to provide unlimited financing to households, they will use it up only to the point where they are indifferent between acquiring the investment good and not acquiring the investment good. This is the outcome that will be achieved when rational households possess all relevant information and make choices that maximize their welfare.

We now turn to the government.

Proposition 3. *Assume that households have no knowledge of $s(y)$, $u(x)$, but know $cost(\theta)$ and the distribution of θ, and are willing to acquire the investment good if the government provides them with loans to do so. Also assume that the government has perfect knowledge of $s(y)$, $u(x)$, $cost(\theta)$, and the distribution of θ, and will extend loans to buy the investment good only as long as the households who acquire it improve their welfare. Then, in equilibrium, only households with abilities from θ^g to μ acquire the investment good, where $\theta^g \geq \theta^*$.*

Again, simple intuition applies here. If the government has perfect knowledge of how acquiring the investment good will affect income streams of households, it will provide loans only to those households who will benefit from acquiring the investment good. The exact mass of households who will acquire the investment good will be determined by the total amount of financing the government is willing to provide. However, that amount will never be so high that loans are provided to some households who are better of without the investment good.

Proposition 2 and Proposition 3 show that in order to prevent inefficiencies it is sufficient for either households or the government to correctly estimate the costs and benefits associated with acquiring the investment good. It takes two: in order to generate suboptimal outcomes both the government and the households need to inefficiently participate in the acquisition of the investment good. Sometimes such inefficiencies lead to asset bubbles, as we show in the next section.

3.2. Formation of a bubble

In this section we will show that excess financing by the government can lead to asset bubbles. In fact, given the results of Proposition 2 and Proposition 3, excess financing by the government is a necessary condition to generate an asset bubble. In addition, we show that excess financing creates an endogenous propagation mechanism. When the amount of financial support is provided to households beyond a certain threshold, it is not only these additional households who default, but some households who would not have defaulted had the government-provided financing not crossed the boundary will also default on their debts once that threshold is crossed.

In order to proceed we need to define a bubble in our setting. A common definition of a bubble is when an asset is bought in high volumes at prices that are higher than its intrinsic value. Notice that by Proposition 1, whenever households with abilities below θ^* start acquiring the investment good, those households purchase an asset that makes them worse off. This would be equivalent to making a negative net present value investment. Hence, there exists a bubble whenever households with abilities below θ^* acquire the investment good. However, those households do not necessarily default on their debts. In order to make things somewhat more interesting, we will

focus on situations when households start defaulting on their debts. We assume that consumption cannot be negative and say that a household is in default if that household's income falls below the loan payment that the household has to make. It means that such a household acquired the investment good at a positive price to receive a negative payoff with certainty.

First we show that there exists a default boundary, θ^d, so that as long as only households with abilities above θ^d acquire the investment good, there are no defaults. As soon as households with abilities below θ^d start acquiring the investment good, defaults ensue.

Proposition 4. *There exists θ^d, such that $\theta^* > \theta^d > 0$, and:*

(*i*) *If only households with abilities from θ^d to μ acquire the investment good, no household defaults on its debt.*

(*ii*) *If households with abilities from θ'' to μ acquire the investment good, where $\theta'' < \theta^d$, some households default on their debt.*

The intuition behind Proposition 4 is similar to the intuition of Proposition 1 but uses a different criterion: θ^d is the level of ability at which the income generated by the investment good equals to the amount of loan a household with that ability needs to obtain in order to acquire the investment good.

In the next proposition we will show that a bubble is generated when the government provides too much financing (too much financing means that households with abilities below θ^d are given funds to acquire the investment good).[15]

[15]When households are willing to acquire the investment good as long as the government provides

Proposition 5. *Assume that households have no knowledge of $s(y)$, $u(x)$, but know $cost(\theta)$ and the distribution of θ, and are willing to acquire the investment good if the government provides them with loans to do so. Also assume that the government-provided financing, L, satisfies $L = \int_{\theta^g}^{\mu} cost(\theta)d\theta > \int_{\theta^d}^{\mu} cost(\theta)d\theta$. Then,*

(*i*) *There is an asset bubble.*

(*ii*) *There exists $\theta^{gg} > \theta^d$ so that all households with abilities between θ^g and θ^{gg} default on their debts.*

This is the main result of our paper. Asset bubbles are inevitable when households are unable or unwilling to rationally estimate the costs and benefits associated with the acquisition of the investment good and the government is willing to provide them with too much financing. This highlights both the central role of the government in generating asset bubbles and the complicity of households who are the presumed beneficiaries of government policies. Part (*ii*) of the above Proposition shows that excess financing, in addition to causing the bubble, creates an endogenous propagation mechanism behind it. Once the amount of government-provided financing crosses the default boundary, it is not only the additional households with abilities below that boundary that default. Some households who would have not defaulted had the government-provided financing stayed below the default boundary will also default if that boundary is crossed. This is what we call an endogenous propagation

them with financing, then the amount of this financing uniquely determines the mass of households that acquire the investment good. A close analogy is the decision to go to college in the United States, where students receive federally provided financial aid if they are admitted at an institution of higher learning. In that case, the federal government is willing to provide financing to anyone able to pass entrance tests, regardless of their expected future payoffs.

mechanism for asset bubbles induced by government overspending. It happens because as more and more households acquire the investment good, the income stream that it generates decreases. It is clear that all households with abilities lower than θ^d default if they acquire the investment good. In addition, some households with abilities above θ^d were just about breaking even after acquiring the investment good. Once the income stream generated by the investment good goes down (because some households with abilities below θ^d acquired the investment good), these households will no longer be able to make payments on their loans.[16] Excess financing by the government is a double-edged sword that reduces welfare not only for households that receive this excess financing but also for households who received financing before the amount of that financing exceeded the appropriate threshold.

4. A numerical example: the decision to go to college

In this section, we will present a simple numerical example and demonstrate graphically what happens in our model as we change the level of financing provided by the government. We do this to help the reader clearly understand the intuition behind our results in a visual way. For concreteness, we will focus on the decision of whether to go to college or not.

We describe an economy characterized by a Cobb-Douglas production function. There are two inputs: skilled labor and unskilled labor. Households are born un-

[16]The subprime mortgage crisis illustrates the point of Proposition 5 very clearly. As households who obtained subprime loans started to default (because ultimately their income could not cover their mortgage payments), house prices began to fall. This, in turn, affected prime homeowners, creating a vicious spiral.

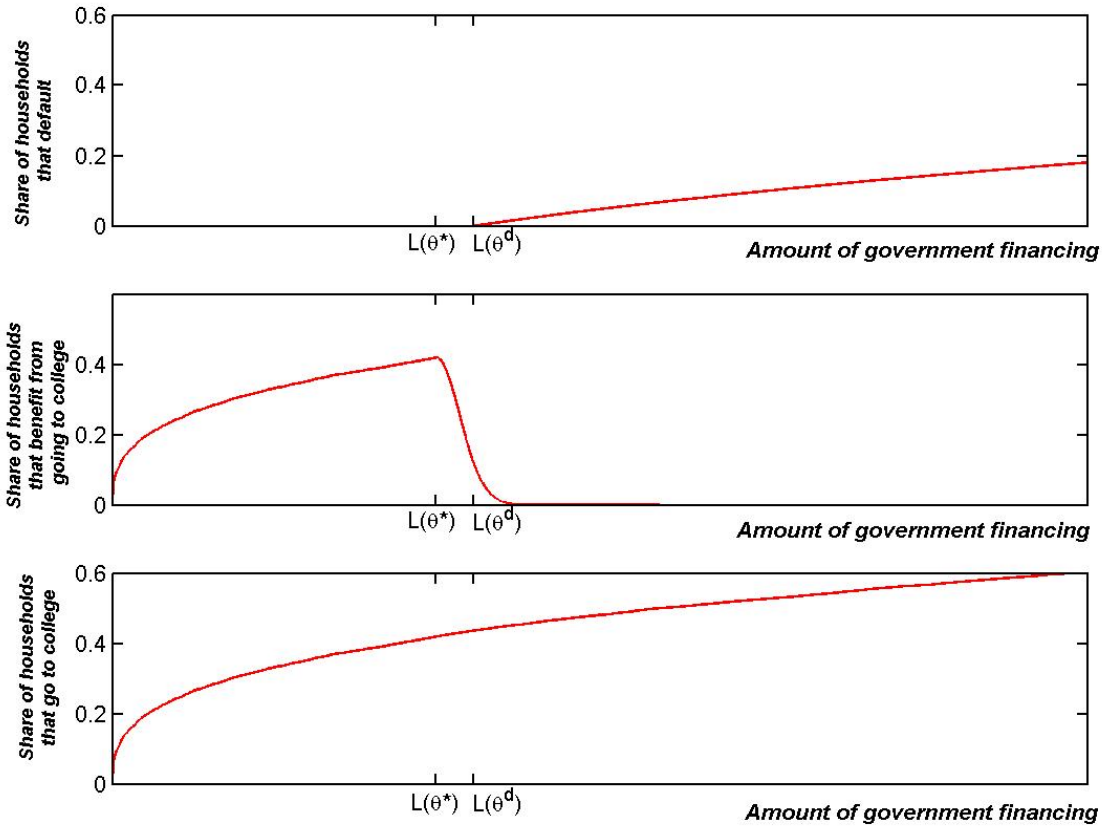
skilled and can decide whether to remain unskilled or to become skilled by acquiring education (going to college). The wage to each input is determined as its marginal product. Each worker is described by his/her ability level at birth, θ; θ is uniformly distributed in the population. We assume that the costs of going to college are quadratic, $cost(\theta) = (\mu - \theta)^2$, where μ is the highest level of ability in the population.

Our simulation is meant for illustrative purposes only, it is not a calibration exercise. We choose the following parameter values: $\alpha = 0.9$ (the share of income paid to skilled labor) and $\mu = 2.2$. We simulate using 10,000 draws from a uniform distribution. We assume, as in Proposition 5, that workers (as well as the government) are fully aware of their abilities and of the distribution of ability in the population. They are also aware of the direct costs of acquiring education. However, we assume that they are unable to assess the benefits and indirect costs of acquiring education and so will acquire education as long as the government provides them with financing. In Figure 1 we depict what happens as the government increases the amount of financing it provides to households.

Figure 1 consists of three panels.[17] The X-axis in all panels is the amount of financing provided by the government. The bottom panel shows the share of population that goes to college, as a function of the amount of financing provided by the government. Unsurprisingly, as the amount of financing grows, so does the number

[17]For clarity, Figure 1 shows what happens to the first 60% of workers only. Depicting it for the entire population will not change any of the conclusions but will make the graph less readable as all effects will be concentrated in the far left part of the graph. That is why we chose to magnify the relevant part of the graph.

Figure 1:



of people who go to college.

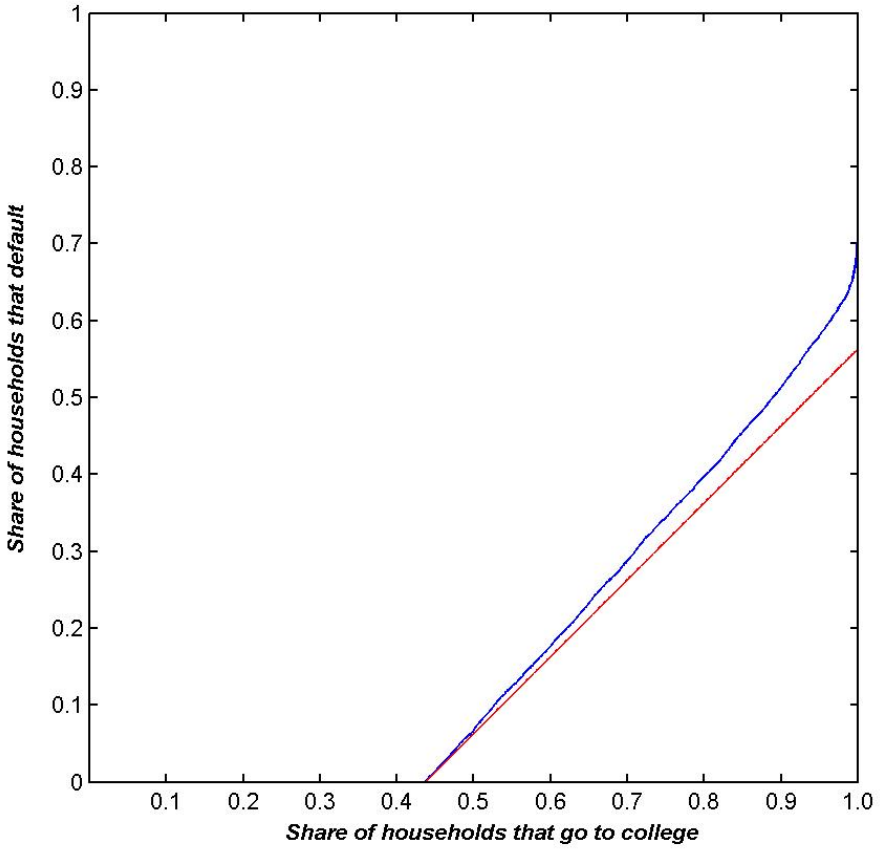
The middle panel shows the share of population that is better off after going to college. To calculate this share we start by comparing, for each individual worker, the income (net of costs) he/she receives after acquiring education to the income he/she would have received if he/she didn't acquire education. We then divide the number of people for whom the income (net of costs) after going to college is greater than

their income had they remained unskilled by the total population (10,000 workers in our case). Initially, this share grows as the number of people who go to college increases. However, after the amount of financing hits $L(\theta^*)$, this share starts to fall. This happens because for all people with abilities below θ^* going to college is dominated by not going to college. Those workers receive less from being skilled, after costs are taken into account, than the wage rate they would have received had they remained unskilled. Still, initially those workers do not default on their loans, as the top panel of Figure 1 demonstrates (being worse off does not necessarily lead to default).

The top panel of Figure 1 depicts the share of households who default on their loans as a function of the total amount of financing provided by the government. It is the share of households whose income after acquiring education is below their costs of acquiring education. Households start to default only when the amount of government-provided financing reaches $L(\theta^d)$, which is higher than $L(\theta^*)$. In simple terms, households who acquire education after the amount of liquidity has reached $L(\theta^d)$ are making a negative value investment: their loan payment will with certainty be greater than the income they will receive after going to college.

In Figure 2 we show the propagation mechanism that government-provided financing produces. It depicts the share of households who default on their loans as a function of the share of households who go to college. Initially, this share is zero: up until a certain point, as long as the amount of government-provided liquidity is below $L(\theta^d)$, the income received by skilled workers is enough to cover their costs of acquiring education. Once households with abilities below θ^d start to acquire educa-

Figure 2:



tion, defaults ensue. What is more important, however, is that the share of people who default grows faster than the share of people who go to college (the relevant line is always above the 45 degree line and is diverging upwards away from it). It means that every additional worker who acquires education causes not only his/her own default but also cases defaults of some people who would not have defaulted had this worker not acquired education. This happens because the marginal product of

skilled labor goes down as the number of skilled workers increases. Hence, the wage rate of skilled labor also goes down and some workers who were previously able to cover their loan payments will no longer be able to do so.

5. Discussion and policy implications

We made several simplifying assumptions to maintain a clean setting and make it crystal clear where our results are coming from. However, our simplifications made it more difficult for us to obtain the results reported here, not easier. We assumed no private financial markets. However, it is natural to assume that private investors (who pledge their own money) are better incentivized to screen borrowers than the government (which spends taxpayer money). In that case, the government will be providing financing to the relatively higher risk individuals, and inefficiencies and asset bubbles will be more likely to occur than in our setting. We also implicitly assumed that all households obtained loans at the same interest rate. However, the higher risk lower ability borrowers are likely to be charged higher interest rates, which will make their costs of acquiring the investment good even higher, and so they will be even more likely to default than in our setting. We also assumed that the government can distribute financing in the most efficient way, starting with the highest ability (lowest cost) households first. This is unlikely to happen in reality but any deviation from this mechanism will make inefficiencies and bubbles only more likely.

Notice that in all of our propositions we assumed that all relevant parties are fully aware of the direct costs of acquiring the investment good (via $cost(\theta)$) and of

the distribution of θ. What is driving all of our results is lack of knowledge about the benefits and indirect costs of acquiring the investment good ($s(y)$ and $u(x)$). Indirect costs arise in our setting due to the fact that not acquiring the investment good also entails a income ($u(x)$), which for some households may be larger than their income after acquiring the investment good.

Our paper makes it very clear that the 2008 subprime mortgage collapse could not have happened without consumers willingly obtaining mortgages they could not possibly repay, most likely due to their misunderstanding of the costs and benefits associated with home ownership. It also could not have happened without the excess financing that stemmed from the U.S. government's desire to increase home ownership in the United States. A similar situation may now be taking place in the U.S. system of higher education. There, the government provides loans to students once they get accepted to an accredited institution of higher learning. Those loans are provided irrespective of students' ability to pay them back after graduation. Unless college applicants perfectly understand their prospects after graduation and are able to assess the costs and benefits associated with obtaining student loans, they are prone to take out loans they cannot possibly repay. Various U.S. media describe multiple stories of students going broke after graduation in recent years.

Our model has important policy implications. The general regulatory response to the recent crisis has been to increase oversight of financial intermediaries and put additional regulatory burden on them. However, our paper suggests that this does not address the core problem behind government-induced asset bubbles. As long as the government provides excess financing, no amount of regulatory oversight will

prevent eventual financial collapse. Malevolent intent by financial intermediary is in no way necessary to generate bubbles, although it can make them more likely.

Our model suggests that there are two ways to avoid government-induced asset bubbles: self-restraint by the government or self-restraint by households (or both). Government-provided financing for social policies is sometimes prone to generate asset bubbles unless it is accompanies by a strong infrastructure to ensure that the beneficiaries of those social policies clearly understand their costs and benefits. In theory, bubbles can be avoided and social policies implemented as long as the government is able to perfectly allocate financing by correctly estimating the optimal number of households who will benefit from those policies. However, we think that such a scenario is highly unlikely. We conclude that the only feasible way for the government to promote social policies without generating inefficiencies and asset bubbles is to require that people who take advantage of those policies are made aware of potential risks associated with them. Hence, consumer education and full and clear disclosure by all market participants may be the most sustainable way to prevent financial collapse stemming from government overspending. Additional regulatory burden on financial intermediaries may be a misguided policy response that creates significant deadweight costs.

6. Conclusion

This paper describes a general mechanism by which government overspending can lead to asset bubbles. In particular, under very general conditions, we derive the following results:

1. in order to generate an inefficient outcome, both the government and the households need to make uninformed decisions, if at least one sector has perfect information and behaves optimally, inefficiencies never arise;

2. excess provision of financing by the government can cause an asset bubble;

3. excess provision of financing by the government, in addition to causing the bubble, generates an endogenous propagation mechanism that affects a larger number of households than just the households who received excess financing.

In general, when the government chooses to provide financing to households based on some simple criterion (such as admission to an institution of higher education or compliance with simple mortgage standards), it is only by chance that it will provide the optimal amount of financing. If too much financing is provided, households start to default and a bubble ensues. We also show that government-provided financing creates an endogenous propagation mechanism. If financing exceeds a certain threshold, it is not only the households who receive this excess financing that default on their loans: some households who received financing before it reached the threshold will also default.

Avoiding welfare loss and an asset bubble is possible if households clearly understand the terms of financing provision and the associated risks and can calculate the costs and benefits of taking advantage of the government's policy. It seems that full and clear disclosure by all market participants is the most sustainable way to avoid financial collapse induced by government overspending.

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Appendix A. Appendix: Proofs of propositions

We adopt the following notation that we will use in all of the proofs. Let $y(\theta_0) = \frac{\mu-\theta_0}{\mu}$ denote that the mass of households who acquired the investment good when all households with abilities from θ_0 to μ acquired the investment good. In particular, $y(0) = 1$ and $y(\mu) = 0$. Similarly, $x(\theta_0) = \frac{\theta_0}{\mu}$ is the mass of households who did not acquire the investment good when all households with abilities from θ_0 to μ acquired the investment good. Since households make a binary choice, $y(\theta_0) + x(\theta_0) = 1$.

Proposition 1. *There exists θ^*, such that $\mu > \theta^* > 0$, and:*

(*i*) *If only households with abilities from θ^* to μ acquire the investment good, then all households are better off than if no household acquired the investment good.*

(*ii*) *If households with abilities from θ' to μ acquire the investment good, where $\theta' < \theta^*$, all households are worse off than if only households with abilities from θ^* to μ acquired the investment good.*

Proof. Part (*i*). Define θ^* such that for a household with ability θ^* the value of its consumption if it acquires the investment good is equal to the value of its consumption if it does not acquire the investment good, conditional on all households with abilities above θ^* acquiring the investment good:

$$s(y(\theta^*)) - cost(\theta^*) = u(1 - y(\theta^*)). \tag{A.1}$$

It is clear that $\mu > \theta^* > 0$. First notice that the left-hand side of (A.1) is monotonically increasing in θ^* while the right-hand side of (A.1) is monotonically decreasing in θ^*. Then notice that if we let $\theta^* = 0$, then the left-hand side of equation (A.1) is smaller than its right-hand side by restriction (1) and the fact that $u(x)$ is a non-negative function. If we let $\theta^* = \mu$, then the left-hand side of equation (A.1) is larger than its right-hand side by restriction (2). Hence, (A.1) must hold with equality for some $\theta^* \in (0, \mu)$.

Now consider any household with ability θ_0, such that $\theta_0 > \theta^*$. The value of this household's consumption, conditional on mass $y(\theta^*)$ households acquiring investment good, is given by $s(y(\theta^*)) - cost(\theta_0)$. The value of this household's consumption, conditional on no household acquiring the investment good, is given by $u(1)$. Since $1 - y(\theta^*) < 1$, it follows that $u(1 - y(\theta^*)) > u(1)$. Moreover, since $\theta_0 > \theta^*$, we have that $cost(\theta_0) < cost(\theta^*)$, and it follows from equation (A.1) that $s(y(\theta^*)) - cost(\theta_0) > u(1 - y(\theta^*)) > u(1)$. Hence, for this household acquiring the investment good, conditional on mass $y(\theta^*)$ households acquiring the investment good, dominates not acquiring the investment good, conditional on no household acquiring the investment good.

Lastly, consider any θ_1, such that $\theta_1 < \theta^*$. Observe that $u(1 - y(\theta^*)) > u(1)$, and hence this household's consumption is higher when households with abilities from θ^* to μ acquire the investment good than when no household acquires the investment good.

Part (*ii*). Pick any household with ability θ_0, such that $\theta_0 \geq \theta^*$. We will show that this household is worse off when all households with abilities between θ' and μ acquire the investment good than when only households with abilities between θ^* and μ acquire the investment good. The value of this household's consumption, if all households with abilities between θ' and μ acquire the investment good, is given by $s(y(\theta')) - cost(\theta_0)$. The value of this household's consumption, if only households with abilities between θ^* and μ acquire the investment good, is given by $s(y(\theta^*)) - cost(\theta_0)$. Since $\theta^* > \theta'$, we have that $s(y(\theta')) < s(y(\theta^*))$, and it follows that $s(y(\theta')) - cost(\theta_0) < s(y(\theta^*)) - cost(\theta_0)$.

Pick any household with ability θ_1, such that $\theta^* > \theta_1 \geq \theta'$. We will show that this household is also worse off when all households with abilities between θ' and μ acquire the investment good than when only households with abilities between θ^* and μ acquire the investment good. The value of this household's consumption, if all households with abilities between θ' and μ acquire the investment good, is given by $s(y(\theta')) - cost(\theta_1)$. The value of this household's consumption, if only households with abilities between θ^* and μ acquire the investment good, is given by $u(1 - y(\theta^*))$. Since $\theta^* > \theta_1 \geq \theta'$, we have that $cost(\theta_1) > cost(\theta^*)$, $s(y(\theta')) < s(y(\theta^*))$, and it follows from equation (A.1) that $s(y(\theta')) - cost(\theta_1) < s(y(\theta^*)) - cost(\theta^*) = u(1 - y(\theta^*))$.

Pick any household with ability θ_2, such that $\theta' > \theta_2$. We will show that this household is also worse off when all households with abilities between θ' and μ acquire the investment good than when only households with abilities between θ^* and μ acquire the investment good. The value of this household's consumption, if all households with abilities between θ' and μ acquire the investment good, is given by $u(1 - y(\theta'))$. The value of this household's consumption, if only households with abilities between θ^* and μ acquire the investment good, is given by $u(1 - y(\theta^*))$. Since $\theta^* > \theta'$, it follows that $u(1 - y(\theta')) < u(1 - y(\theta^*))$. □

Proposition 2. *Assume that the government has no knowledge of $s(y)$, $u(x)$, but knows $cost(\theta)$ and the distribution of θ, and provides unlimited financing to households, so that it will extend a loan to buy the investment good to any household that wishes to acquire it. Also assume that households have perfect knowledge of $s(y)$, $u(x)$, $cost(\theta)$, and the distribution of θ. Then, in equilibrium, only households with abilities from θ^* to μ acquire the investment good.*

Proof. Households have perfect knowledge of $s(y)$, $u(x)$, $cost(\theta)$, and the distribution of θ. Consider any household with ability θ_0, such that $\theta^* \leq \theta_0 < \mu$. Since the government distributes its financing continuously (via Algorithm 1), this household conditions its payoff on the fact that all households with higher abilities also have to acquire the investment good if it acquires the investment good. The value of consumption of a household with ability θ_0 from acquiring investment good, conditional on mass $y(\theta_0)$ of households acquiring the investment good, is equal to $s(y(\theta_0)) - cost(\theta_0)$. Notice that $s(y(\theta_0)) - cost(\theta_0) > s(y(\theta^*)) - cost(\theta^*) = u(1 - y(\theta^*)) \geq u(1 - y(\theta_0))$, where the last term is the value of consumption of a household with ability θ_0 if it does not acquire the investment good, conditional on mass $y(\theta_0)$ of households acquiring the investment good. Hence, this household is better

off acquiring the investment good. It follows that all households with abilities between θ^* and μ acquire the investment good. Analogously, all households with abilities from 0 to θ^* are worse off by acquiring the investment good than by not acquiring the investment good. Consider any θ_1, such that $\theta_1 < \theta^*$. Observe that $s(y(\theta_1)) - cost(\theta_1) < s(y(\theta^*)) - cost(\theta^*) = u(1 - y(\theta^*)) < u(1 - y(\theta_1))$. Thus, in equilibrium only households with abilities from θ^* to μ acquire the investment good. □

Proposition 3. *Assume that households have no knowledge of $s(y)$, $u(x)$, but know $cost(\theta)$ and the distribution of θ, and are willing to acquire the investment good if the government provides them with loans to do so. Also assume that the government has perfect knowledge of $s(y)$, $u(x)$, $cost(\theta)$, and the distribution of θ, and will extend loans to buy the investment good only as long as the households who acquire it improve their welfare. Then, in equilibrium, only households with abilities from θ^g to μ acquire the investment good, where $\theta^g \geq \theta^*$.*

Proof. The government uses Algorithm 1 to provide loans to households in order for them to acquire the investment good. We denote the total amount of financing the government is willing to provide by $L(\theta^g) = \int_{\theta^g}^{\mu} cost(\theta) d\theta$, where θ^g is the lowest ability household that receives government-provided financing. First consider the situation when $\theta^g < \theta^*$. Since the government has perfect knowledge of $s(y)$, $u(x)$, $cost(\theta)$, and the distribution of θ, it can determine the value of θ^* based on Proposition 1. Hence, it will provide the amount of financing that exactly covers the loans for households with abilities from θ^* to μ, $L(\theta^*) = \int_{\theta^*}^{\mu} cost(\theta) d\theta$. If $\theta^g \geq \theta^*$, all households with abilities from θ^g to μ will receive financing since all of them will be strictly better off by acquiring investment good, conditional on mass $y(\theta^g)$ of households acquiring the investment good. In addition, the households who do not receive financing and hence do not acquire the investment good are also better off when households with abilities from θ^g to μ acquire the investment good than if no household acquires the investment good. To see this clearly consider a household with ability θ_0, such that $\theta^g \leq \theta_0 \leq \mu$. The value of this household's consumption, conditional on mass $y(\theta^g)$ of households acquiring the investment good, is equal to $s(y(\theta^g)) - cost(\theta_0)$. Notice that $s(y(\theta^g)) - cost(\theta_0) > s(y(\theta^*)) - cost(\theta^*) = u(1 - y(\theta^*)) \geq u(1 - y(\theta^g))$, where the last term is the value of consumption of a household with ability θ_0 if it does not acquire the investment good, conditional on mass $y(\theta^g)$ of households acquiring the investment good. Hence, the government provides financing to all households between θ^g and μ. □

Proposition 4. *There exists θ^d, such that $\theta^* > \theta^d > 0$, and:*

(*i*) *If only households with abilities from θ^d to μ acquire the investment good, no household defaults on its debt.*

(*ii*) *If households with abilities from θ'' to μ acquire the investment good, where $\theta'' < \theta^d$, some households default on their debt.*

Proof. Part (i). Define θ^d such that the for a household with ability θ^d the value of its consumption, conditional on the mass $y(\theta^d)$ of households acquiring the investment good is equal to 0:

$$s(y(\theta^d)) - cost(\theta^d) = 0. \tag{A.2}$$

It is clear that $\theta^* > \theta^d > 0$. First notice that the left-hand side of (A.2) is monotonically increasing in θ^d. Then notice that if we let $\theta^d = 0$, then the left-hand side of equation (A.2) is smaller than zero (its right-hand side) by restriction (1). If we let $\theta^d = \theta^*$, then the left-hand side of equation (A.2) is greater than zero (its right-hand side) by equation (A.1) and the fact that $u(1-y(\theta^*)) > 0$ since $\theta^* < \mu$. Hence, (A.2) must hold with equality for some $\theta^* \in (0, \theta^*)$.

Pick any household with ability θ_0, such that $\theta_0 > \theta^d$. The value of this household's consumption when households with abilities from θ^d to μ acquire the investment good is equal to $s(y(\theta^d)) - cost(\theta^0)$. Since $\theta_0 > \theta^d$, we have that $cost(\theta_0) < cost(\theta^d)$ and $s(y(\theta^d)) - cost(\theta^0) > 0$. Hence, this household doesn't default.

Now pick any household with ability θ_1, such that $\theta_1 < \theta^d$. The value of this household's consumption when households with abilities from θ^d to μ acquire the investment good is equal to $u(1-y(\theta^d))$. Since $u(x)$ is a nonnegative function, this household doesn't default.

Part (ii). Pick a household with ability θ_0 such that $\theta'' \leq \theta_0 < \theta^d$. The value of this household's consumption when households with abilities from θ'' to μ acquire the investment good is equal to $s(y(\theta'')) - cost(\theta_0)$. Since $\theta_0 < \theta^d$, we have that $cost(\theta_0) > cost(\theta^d)$, and $s(y(\theta^d)) - cost(\theta_0) < s(y(\theta^d)) - cost(\theta^d) = 0$ by equation (A.2). Hence, this household defaults. □

Proposition 5. *Assume that households have no knowledge of $s(y)$, $u(x)$, but know $cost(\theta)$ and the distribution of θ, and are willing to acquire the investment good if the government provides them with loans to do so. Also assume that the government-provided financing, L, satisfies $L = \int_{\theta^g}^{\mu} cost(\theta)d\theta > \int_{\theta^d}^{\mu} cost(\theta)d\theta$. Then,*

- (i) *There is an asset bubble.*
- (ii) *There exists $\theta^{gg} > \theta^d$ so that all households with abilities between θ^g and θ^{gg} default on their debts.*

Proof. Part (i). The government provides financing to households with abilities between θ^g and μ, where θ^d is such that $L(\theta^g) = \int_{\theta^g}^{\mu} cost(\theta)d\theta$. If $L(\theta^g) > \int_{\theta^d}^{\mu} cost(\theta)d\theta$, it follows that $\theta^g < \theta^d$. Thus, by Proposition 4 (part (i) states that $\theta^s < \theta^*$), there is an asset bubble. In addition, according to part (ii) of Proposition 4, some households default.

Part (ii). Pick a household with ability θ^g. The value of this household's consumption when households with abilities from θ^g to μ acquire the investment good is equal to $s(y(\theta^g)) - cost(\theta^g)$. Since $\theta^g < \theta^d$, we have that $cost(\theta^g) > cost(\theta^d)$, $s(y(\theta^g)) < s(y(\theta^d))$, and it follows that $s(y(\theta^d)) - cost(\theta^g) < s(y(\theta^d)) - cost(\theta^d) = 0$ by equation (A.2). Hence, this household defaults.

Define θ^{gg} such that

$$s(y(\theta^g)) - cost(\theta^{gg}) = 0. \tag{A.3}$$

It is clear that $\theta^{gg} > \theta^d$. First notice that the left-hand side of (A.3) is monotonically increasing in θ^{gg}. Also notice that $\theta^g < \theta^d$ by assumption and hence $s(y(\theta^g)) < s(y(\theta^d))$. Thus, it follows from equation (A.2) that if we let $\theta^{gg} = \theta^d$, then the left-hand side of equation (A.3) is smaller than zero (its right-hand side). If $s(y(\theta^g)) - cost(\mu) \leq 0$, then all households with abilities between θ^g and μ default, and hence $\theta^{gg} = \mu$. If $s(y(\theta^g)) - cost(\mu) > 0$, then the left-hand side of equation (A.3) is greater than zero (its right-hand side) if we let $\theta^{gg} = \mu$. Hence, (A.3) must hold with equality for some $\theta^{gg} \in (\theta^d, \mu)$, and all households with abilities between θ^g and θ^{gg} default. □