Finance-thy-Neighbor
Trade Credit Origins of Aggregate Fluctuations

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Abstract. Trade credit in the form of a delay of inputs payments is an important source of financing for all types of firms. In this paper, I study the role of credit linkages for the propagation of financial shocks in a production network where firms finance their working capital requirements using bank and trade credit. To this end, I build a quantitative multisector model with endogenous credit linkages between representative firms in each sector. The endogenous adjustment in the volume and cost of trade credit captures two counteracting mechanisms: (1) Firms smooth interest rate shocks by substituting bank and supplier finance. (2) An increase in the interest rate that a firm charges on trade credit tightens the financing terms of its customers thereby amplifying financial shocks. Quantitatively, the model accounts for 30% of the variation in aggregate output in the US-economy. Model simulations show that the existence of the trade credit network doubled the drop in aggregate output during the 2008-2009 crisis relative to an equivalent economy with bank-finance only. Furthermore, the ratio of total outstanding payments owed by customers for already delivered goods and services to bank credit is a good proxy for the systemic importance of a sector in propagating liquidity shocks.

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1. Introduction

The flow of payments from customers to their suppliers plays a crucial role in maintaining the liquidity and turnover of products in a complex network of trade relations between firms. However, the time lag between the purchase of inputs and the receipt of payments for realized sales leads to a cash-flow mismatch for the producer and creates demand for ex-ante liquidity. In day-to-day operations, it is thus common practice for suppliers to offer payment terms in the form of trade credit, that allow customers to delay payments until after the delivery of the product. (see Cuñat and García-Appendini, 2012)

Trade credit as a form of short- and medium-term debt "gives [firms] and [their] suppliers more flexibility to manage [their] businesses effectively through better cash flow management"[1] and represents an alternative source of financing to bank and financial market debt for all types of firms (see e.g. Peterson and Rajan, 1997). However, during the financial crisis, the market for trade credit experienced a severe contraction, consequently forcing firms to use other sources of credit to fund their operations. (see Costello, 2017; Ivashina and Scharfstein, 2010) Anecdotal and empirical evidence[2] thus highlights two countervailing features of trade credit: (1) Firms smooth interest rate shocks by substituting bank and supplier finance. (2) A tightening of supplier financing terms deteriorates the credit conditions for customers and has adverse and exacerbating effects on maintaining production.

In this paper, I investigate the following two questions: Do trade credit linkages amplify or dampen the propagation of financial shocks? To what extent did the trade credit network contribute to the drop in output during the 2008-2009 Financial Crisis? For this purpose, I first build a quantitative multisector model where representative firms in each sector face working capital constraints and which explicitly accounts for both the substitutability of bank- and supplier credit and the input-output relations between sectors. In particular, I contribute to the literature by endogenizing the trade credit intensity between firms in order to explicitly capture both the substitution and

[1] Kris Charles, Kellogg spokeswoman cited in Strom (2015), www.nytimes.com, 10/26/2018

[2] In 2008, there was considerable concern about the insolvency of GM and Chrysler and the resulting domino effect through the supply chain: "I don't think that suppliers will be able to get through the month without continued payments on their receivables" N.De Koker, CEO of the Original Equipment Suppliers Association. (see Vlasic and Wayne, 2008, www.nytimes.com, 10/26/2018); As sales have been declining since 2011, SEARS - an American retail staple - faced a considerable tightening of payment terms offered by their suppliers: "We cut their credit line and shortened payment terms [...] If they pay one day late, we will cut them off." I.Larian, CEO of MGA Entertainment Inc. (see Kapner, 2017, www.wsj.com, 10/26/2018)

amplification mechanism of trade credit outlined before. In order to provide an answer to both questions, I then apply the model to the US-economy at a sector level and quantitatively assess the importance of these two opposing effects. Furthermore, I derive a new credit measure - the net-lending position of a sector - which is defined as the ratio of accounts receivable[3] to the difference between total cost of production and accounts payable. The latter is equal to total bank credit requirements since firms do not have any internal funds in this model. It is shown that this novel measure helps to identify sectors which generate most spillovers through interfirm credit linkages.

This paper makes three contributions to the literature:(1) I present stylised facts on business cycle patterns and the heterogeneity of trade credit usage in the US. (2) I then introduce a model which explicitly emphasises the smoothing and amplifying features of trade credit linkages in the role of interlinked endogenous distortions for the propagation of shocks; and (3) I quantify the effect of trade credit linkages in the US-economy on aggregate output.

Facts. In the first part of the paper, I present stylised facts on business cycle patterns of aggregate trade credit in the US-economy. Using yearly balance sheet data from Compustat of a panel of publicly-traded firms from 2001 to 2016, I first calculate that trade accounts payable of non-financial US-firms account for 11.3% of total corporate debt and 5.3% of US-GDP.[4] It is then shown that, the growth rate of the volume of trade finance is pro-cyclical with and more volatile than the growth rate of current real GDP. Trade credit is also more volatile than the growth rate of total liabilities. In addition, bank and trade credit are substitutes.[5] Finally, I use the novel credit measure described above to show that while there is heterogeneity across firms, the majority of US-firms receive relatively more trade credit than they extend to their customers. The model introduced in the second part of this paper is evaluated based on its ability to reproduce both qualitatively and quantitatively, these observed patterns.

Theory. In order to further understand the role trade credit plays in the propagation of financial shocks, I then build a static quantitative multisector general equilibrium model with trade in intermediate inputs and endogenous credit linkages between per-

[3]Accounts payable(receivable) are the total outstanding payments owed to suppliers (by customers) for already delivered goods and services.

[4]The sample includes all Compustat firm-year observations from 2001 to 2016 of non-financial firms with their head-quarter in the US and positive and non-missing observations of the respective variables of interest. The sales of the firms included in the sample represent approximately 25% of total gross output in the US. For details on the sample, see Appendix D.

[5]These patterns are in line with the findings in the literature. (see e.g. Cuñat, 2007, for an overview)

fectly competitive intermediate good producing firms in each sector. The banking sector is introduced in a reduced form way by means of a sector-specific interest rate on bank credit which contains a risk-premium over the federal funds rate. The risk-premium is subject to financial shocks and increases in the average trade share extended to customers. Since any sales are only realized after production has taken place, firms face working capital constraints and finance production using both bank and supplier credit.

At the beginning of a period, both productivity and financial shocks are realized. Profit-maximizing firms choose the composition of their borrowing portfolio to minimize their cost of production and optimally set the quantity produced and the average trade credit share extended to their customers for given prices and interest rates. The endogenous adjustment of the volume and cost of trade credit or "trade credit channel" then captures the two counteracting mechanisms presented earlier as follows: (1) On the demand side, firms respond to shocks to their bank interest rate risk premium by optimally trading-off credit costs on bank and trade credit and choosing the payment terms associated with the transaction.[6] Hence, firms are able to smooth out any interest rate shocks by adjusting their borrowing portfolio which mitigates the negative effect of an increase in the bank interest rate on output. (2) On the supply side, a firm acts both as a supplier of goods and as a financial intermediary. Consider a firm which experiences an increase in its bank risk premium. Since the risk premium is increasing in the average trade credit share extended to customers, a firm will reduce its optimal trade credit share extended. Consequently, the interest rate charged on trade credit increases, which directly affects the cost of credit and thus production of downstream customers. Similarly, a shift in the borrowing portfolio composition of a firm towards trade credit increases the cost of bank finance of upstream suppliers. This creates an amplification mechanism by which idiosyncratic shocks to the cost of bank credit are propagated both up- and downstream.

In equilibrium, it is shown that the working capital constraint introduces a credit wedge between the firm's marginal revenue and costs thereby distorting a firm's optimal input and output choice away from its optimal scale. The credit wedges in this paper are a weighted average of both interest rates on bank and supplier credit and the weights are the optimally chosen link specific trade credit shares. The financial distortions manifest themselves in equilibrium as (1) an aggregate efficiency wedge decreasing Total Factor

[6]The optimal payment schedule is defined as the cost-minimizing share of input expenditures financed via supplier credit. The effective price a customer pays is a bundle of the actual goods price and the cost of the financial service provided by its supplier.

Productivity (TFP) and as (2) an aggregate labor wedge introducing a wedge between the household's marginal rate of substitution between consumption and labor and the economy's marginal rate of transformation as common to models with distortions. (see Bigio and La'O, 2017; Baqaee and Farhi, 2018b) However, the interdependency of credit wedges affects the propagation of financial shocks and it is shown that to a first order approximation, the net-lending position of a sector determines the relative importance of the insurance and amplification mechanism of the trade credit channel.

Quantitative Application. Whether financial linkages amplify or dampen the effect of credit cost shocks on (aggregate) output is ambiguous and thus remains a quantitative question as the answer clearly depends on the relative strength of the substitution and amplification effects outlined before. To this end, I first calibrate the production structure and the inter-industry credit flows of the model economy to the US at a sector level. I then simulate the model using only the financial shocks to a sector's risk premium based on sector-level bond spreads derived in Gilchrist and Zakrajšek (2012), GZ-spreads hereafter. Thereby, I exclude any additional source of variation affecting the economy such as productivity shocks. Simulations then show that the model reproduces - both qualitatively and quantitatively - business cycle patterns of trade credit as observed in the data. In particular, the model featuring the endogenous adjustment of the volume and cost of trade credit captures approximately 30% of the variation in aggregate output while solely taking into account financial shocks. I then quantify the role of trade credit for the propagation of liquidity shocks during the 2008-2009 Great Recession. The main results are as follows:

The model predicts that in response to an increase in sector-specific bank risk premia during the crisis, bank and supplier credit rates rose by approximately 47bps(20.7%) and 115bps(26.6%) on average. Since bank and trade credit are treated as substitutes, the model implies a drop in the share of inputs purchased on supplier credit by 3.8%, which accounts for 22.5% of the decline observed in the data. The increase in the cost of credit and the resulting adjustment of the credit portfolio of sectors also lead to a decline in both the aggregate efficiency and labor wedge, which accounts for 52.7% and 6.0% of the decline in the respective data counterpart. Ultimately, the increase in the cost of bank finance translated into a 0.9% and a 0.6% decline in GDP and labor, respectively, accounting for approximately 28.8% and 11.0% of observed GDP and labor movements during the crisis. As the model predictions are based on financial shocks only, this highlights the quantitative importance of changes in financial frictions and their effect on aggregate output which corresponds to the findings in Christiano et al. (2015).

In order to evaluate the aggregate effect of the trade credit network, I define the *trade credit multiplier* as the ratio between the percentage drop in the variable of interest generated by an economy with both trade and bank finance and an equivalent economy with bank finance only. The latter represents the benchmark economy discussed in BL(2017). The counterfactual exercise predicts an output credit multiplier of almost 2, implying that the existence of a trade credit network almost doubled the decline in output during the crisis. Overall, the model suggests that the existence of trade credit linkages among firms increases aggregate fluctuations.

The contribution of the *trade credit channel* - the endogenous adjustment of the volume and cost of trade credit - to changes in aggregate output is evaluated by decomposing the general equilibrium response of the variables of interest into their partial equilibrium counterpart derived by keeping both trade credit interest rates and shares at their steady state level. Consequently, the difference between the general and partial equilibrium response can be attributed to the trade credit channel. The model suggests that the trade credit channel reduces aggregate volatility by 1.78%. Lastly, I quantify the main result of the theory section of this paper and show that the trade credit multiplier implied by a financial shock to the top five sectors with the highest *net-lending ratio* is significantly higher as predicted by model than the trade credit multiplier generated by the same financial shock to the five sectors with the lowest net-lending ratio.

Related Literature. This paper relates to three strands of literature: First, this paper is related to an extensive literature investigating the *aggregate effects of micro-level distortions* on aggregates outcomes. This strand can be broadly classified into two substrands: The first sub-strand abstracts or limits the extent of inter-sectoral trade (see i.a. Chari et al., 2007), the second sub-strand explicitly accounts for (some degree of) intermediate goods trade (see i.a. Jones, 2011). More recent contributions by Baqaee and Farhi (2018a,b) develop a more unified framework for the aggregation of micro-level distortions. Since my model builds on Bigio and La'O (2017), BL(2017) hereafter, it is clearly related to the second strand. While BL(2017) treat the distortions as exogenous, my contribution to this literature lies in emphasising the role of interdependent endogenous distortions for the propagation of shocks in the form of credit linkages among firms due to working capital constraints.

Second and foremost, this paper is related to the growing literature which studies *distortions in the context of a production network*. Since the seminal contribution of Long and Plosser (1983), a growing literature investigates the importance of production networks - the structure of intersectoral trade - for understanding how idiosyncratic shocks affect aggregate dynamics in an economy. (see Carvalho, 2014, for an overview) Following the 2008-2009 recession, the interconnection of banking institutions and their role in the propagation of financial shocks have been studied extensively (see i.a. Acemoglu et al., 2015). The financial crisis also spurred empirical contributions on the real effects of credit shocks by focusing on the link between banks and firms (see i.a. Chodorow-Reich, 2014; Iyer et al., 2014; Cingano et al., 2016; Alfaro et al., 2018).

However, the financial aspect of inter-firm trade as a transmission mechanism is a relatively new research agenda. Recent empirical contributions by Raddatz (2010) at a sectoral level and Jacobson and von Schedvin (2015), Costello (2017), Cortes et al. (2018), Dewachter et al. (2018) at a firm level confirm the relevance of trade credit linkages among production units for the propagation of liquidity shocks. Despite their quantitative importance, trade credit linkages have received little attention in the existing theoretical literature on business cycle fluctuations.

The importance of the network of trade credit relationships for the propagation and amplification of financial shocks across the economy was first explicitly highlighted in Altinoglu (2018). However, in contrast to this paper, Altinoglu (2018) assumes the extend of credit links to be a fixed proportion of firms' sales. In order to endogenize the credit link intensity between firms, all firms engage in Nash-bargaining in Zhang (2017), while Luo (2018) introduces the credit link intensity as a choice variable. The model set-up of this paper differs from both studies along multiple dimensions: First, in contrast to Zhang (2017) firms choose the input-specific credit mix in order to minimize cost of production similar to Luo (2018) such that trade credit in this model is introduced by affecting the total cost of production via prices rather than collateral constraints. Second, while the extension of trade credit tightens the borrowing constraint of a firm in Zhang (2017) and Luo (2018), I explicitly impose the timing restriction which separates the financing stage from the production stage such that at the time a firm needs to finance its input expenditures, no sales have been realized. Third, in order to generate both up- and downstream propagation patterns Luo (2018) explicitly models the financial sector following Gertler and Karadi (2011) such that bank credit costs are only indirectly affected by the credit portfolio choice of firms. Contrary to Luo (2018), I impose that the cost of short term bank credit lines is increasing in the amount of trade credit extended to customers, based on the positive empirical relationship[7] between sectoral GZ-credit spreads and the share of sales made on credit. In particular, this assumption

[7] For details see Appendix D.

introduces a direct upstream cost-effect independent of any additional frictions in the banking sector, through which trade credit affects bank interest rates directly. Fourth, unlike Luo (2018) and Zhang (2017), I explicitly model the cost of trade credit extended to a firm's customers while firms take both input prices and the cost of supplier credit as given.

I contribute to the literature on endogenous trade credit linkages in a production network by providing a tractable model that allows to study (a) the role of trade credit as both an insurance and a contagion device of financial shocks and (b) the relationship between bank and supplier finance over the business cycle by explicitly modelling the price of trade credit.

This paper is also related to the important contribution by Kiyotaki and Moore (1997), who highlight the relevance of trade credit for the propagation of liquidity shocks due to trade credit defaults. In this spirit, the trade credit multiplier investigated in this paper is also related to the concept of the *financial accelerator* (Bernanke et al., 1996). However, in the case of the financial accelerator, distortions in financial markets are at the origin of transmitting financial shocks to the real economy by affecting the borrowing constraints of firms. Although related, in my model I abstract from both investment decision and associated collateral constraints as well as default risk in order to focus on studying the direct interplay between bank- and trade credit finance, the relevance of which for the macroeconomy is still understudied.

To the best of my knowledge, this paper is the first to explicitly emphasise and quantify non-linearities in the effect of interlinked distortions on aggregate outcomes while explicitly taking into account the direct interaction between bank and supplier credit via the price channel in a simplified framework.

Outline. The remainder of the paper is organized as follows. Section 2 discusses empirical regularities of trade credit over the business cycle and of the heterogeneity of the net-lending position across sectors. Section 3 introduces the model. In Section 3.1 I characterize the equilibrium of this economy and Section 3.2 derives the main results of my model on the business cycle implications of trade credit linkages in an economy. Section 4 presents a quantitative assessment of the role of trade credit in the US-economy during the Great Recession and Section 5 concludes.

2. Empirical Observations

The 2008-2009 Financial Crisis was characterized by a global collapse of credit markets that quickly transmitted to the corporate sector and led to a contraction of real (US) GDP in advanced economies by 3.4(2.5)%[8]. An important role in the transmission of the liquidity shock from the banking to the real sector was played by trade credit relations among firms. (see e.g. Jacobson and von Schedvin, 2015; Costello, 2017) In order to incorporate credit linkages into a multisectoral general equilibrium model, I first summarize stylized facts on the relevance and cyclical properties of trade credit in the US-economy at an aggregate level, that will be informative for the set-up of the model in Section 3. For this purpose, I obtain yearly balance sheet data from Compustat of a panel of publicly-traded firms[9] from 2001 to 2016, whose nominal sales represent approximately 25% of total gross output in the US. Although trade credit is more intensively used by small and medium-sized enterprises (SMEs) with a lower degree of access to both bank finance and financial markets (see i.a. Peterson and Rajan, 1997), supplier credit still represents a non-negligible source of financing for large publicly-traded firms. In particular, total accounts payable (receivable) account for approximately 11.3(8.9)% of total liabilities (assets) and make up approximately 5.4(7.1)% of US-GDP[10]. Even though these magnitudes represent a lower bound for the usage of trade credit by US-firms, they highlight the quantitative importance of supplier credit for the aggregate US-economy.

Trade Credit Over the Business Cycle. In the following, I first illustrate cyclical features of trade credit and its relation to other external financing sources such as bank and financial market debt in the US-economy at an aggregate level for the time-period 2001-2016. To this end, Panel (a) of Figure 1 plots the log changes of real GDP (Y), accounts payable (AP) and accounts receivable (AR) in terms of 2007 dollars using the implied GDP-deflator provided by the BEA. Panel (b) presents the log changes of real accounts payable and both, total (LT) and current (LC) liabilities. In addition, I also report the standard-deviation and the pairwise correlation of the respective series in Table 1. Panel (a) and (b) of Figure 1 highlight three business cycle features of trade

[8]Source: World Economic Outlook and BEA

[9]For more details on the sample, see Appendix D.

[10]Since both accounts payable and receivable are likely to contain trade credit volumes from foreign transactions, I also calculate the share of the respective balance sheet item in US-GDP adjusted for exports and imports, respectively. Then, accounts payable (receivable) make up approximately 3.1(4.1)% of US-GDP including imports (exports). Notably, total R&D expenditures of the same sample of US-firms account for 1% of US-GDP. Overall, the BIS (2010) estimates that at a global scale two thirds of world trade are supported by inter-firm credit.

credit in the US:

(F1) The growth rate of the volume of trade finance is pro-cyclical with the growth rate of current real GDP. In other words, the growth rate of accounts payable and receivable increases during expansions and decreases during recessions.

(F2) Trade credit is more volatile than the growth rate of total value added.

(F3) Trade credit is more volatile than firms' total liabilities and exhibits a volatility of similar magnitude of current liabilities[11].

The same cyclical patterns of trade credit have been found in Cun et al. (2018) for a sample of Chinese industrial enterprises, which suggests similarities in the usage of trade credit of firms in advanced and emerging markets. (see i.a. Love et al., 2007; Love and Zaidi, 2010)

Given that the focus of this paper is the role of trade credit for the transmission of liquidity shocks during the Great Recession, I now discuss the relationship between the usage of supplier and bank credit during this period of financial turmoil. For this purpose, I first calculate the log change of the share of accounts payable in current liabilities (θ^T) as a proxy for the evolution of the composition of short-term borrowing. I then obtain two aggregate measures of frictions in the financial market. The first measure is the aggregate credit spread index derived in Gilchrist and Zakrajšek (2012). The "GZ-spread" (GZ) is defined as the average difference in the yields on corporate bonds and yields on Treasury securities of comparable maturities and represents an important indicator of the degree of tensions in the financial system. The second measure reports the tightening in lending standards (LS) by banking institutions based on the Senior Loan Officer Opinion Survey on Bank Lending Practices conducted by the Federal Reserve. The series corresponds to the net percentage of domestic respondents tightening their standards for commercial and industrial (C&I) loans.

Panel (c) of Figure 1 plots both measures of financial frictions (left axis) as well as the log-change in the share of accounts payable in current liabilities (right axis). Together with the evolution of accounts payable, Figure 1c implies the following:

(F4) As credit spreads rose and lending standards tightened considerably at the onset of the financial crisis in 2008, liquidity in the supplier credit market contracted immediately and firms drew down their bank credit lines. The composition of

[11] The latter result on the comparison of the volatility of current liabilities and accounts payable, however, depends on the composition of the sample of Compustat firms. In other words, if less strict sample selection criteria with respect to the number of observation are applied, current liabilities are also more volatile than supplier credit.

short-term borrowing shifted towards bank credit as firms substituted supplier with bank credit.

This observation is consistent with the empirical evidence on the evolution of bank lending during 2008 documented in Ivashina and Scharfstein (2010) and on supplier credit presented in Costello (2017). Using data on syndicated loans from Reuter's Dealscan, Ivashina and Scharfstein (2010) show that while syndicated lending fell, C&I loans as reported on the balance sheets of US-banks rose due to an increase in drawdowns of existing credit lines at the onset of the financial crisis. At the same time, receivables contracted significantly along the intensive margin as documented in Costello (2017) using detailed transaction data at a firm level in the US. Thus, the compositional shift of short-term borrowing towards bank credit in 2008 was due to the joint occurrence of the reduction in the provision of supplier credit and drawdowns of unused credit-lines. However, the increase in C&I loans by approximately 17% in 2008 was followed by a sharp drop of 6.5% in 2009[12], as the tightening of lending standards in 2008 translated into a considerable decline in the availability of new credit-lines. Simultaneously, accounts payable and receivable increased such that the compositional shift reversed and firms substituted bank with trade credit as evident from Figures 1a and 1c.

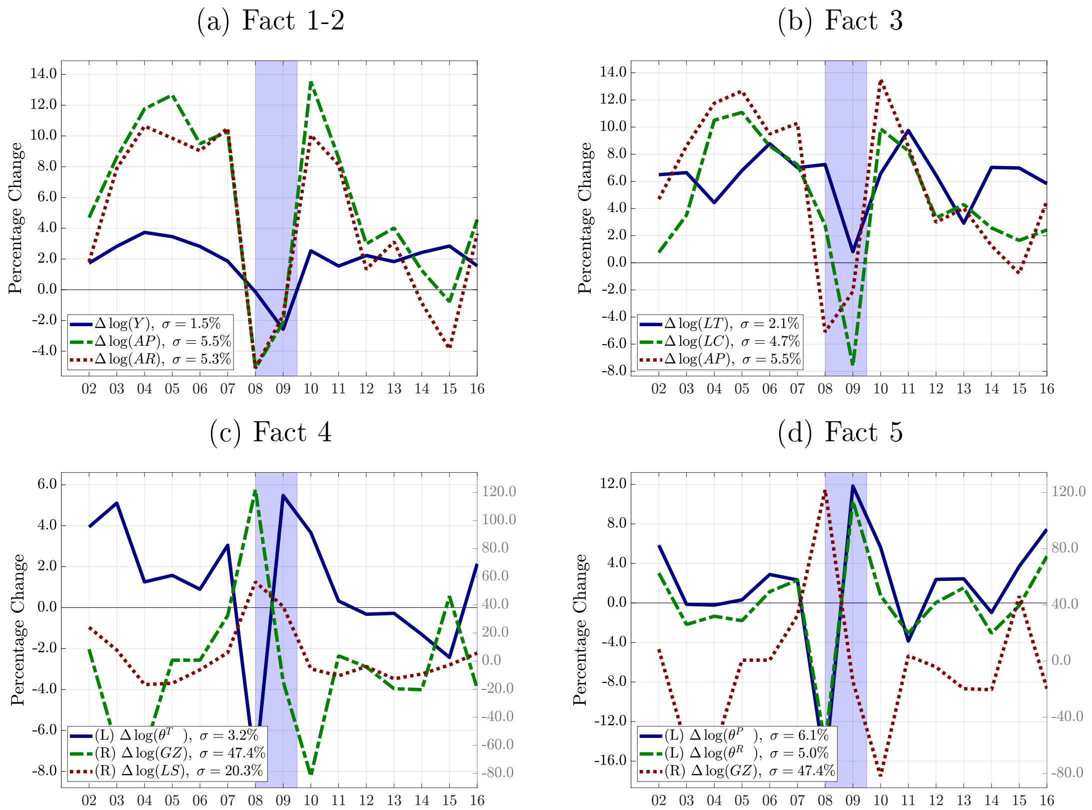
A reasonable explanation for the differences in the speed of adjustment between credit markets in response to a deterioration of financial conditions is the contractual enforceability or rather the lack thereof in the case of supplier credit. While existing credit-lines are prior commitments by banks to lend to corporations any amount up to a preset limit at prespecified rates (see Ivashina and Scharfstein, 2010), trade credit is not subject to formal contracts (see Cuñat, 2007). The empirical observation on the substitutability[13] of supplier and financial market debt is consistent with the findings of a large body of literature on the relationship between trade and bank credit over the business cycle starting with Meltzer (1960). It is argued that during a contractionary period, firms with access to liquidity will increase the amount of trade credit extended to customers, thereby providing funds to credit rationed firms. (see i.a. Meltzer, 1960;

[12]Source: Board of Governors of the Federal Reserve System (US), Commercial and Industrial Loans, All Commercial Banks [BUSLOANS], retrieved from FRED, Federal Reserve Bank of St. Louis; October 6, 2018

[13]It should be noted, that a few papers find evidence of a complementarity between bank and trade credit (see Giannetti et al., 2011) consistent with a theoretical argument of the signalling function of trade credit on the solvency of borrowers (see Biais and Gollier, 1997). In other words, the extension of trade credit conveys a positive signal on the creditworthiness of a customer, which induces banks to lend. The co-existence of the substitutability and complementarity of bank and trade credit and its cyclical pattern is investigated further in a recent contribution by Huang et al. (2011).

Schwartz, 1974; Kohler et al., 2000; Nilsen, 2002) As a result, trade credit serves as a liquidity insurance across firms (see Cuñat, 2007; Wilner, 2000). In particular, Amberg et al. (2016) show that firms manage liquidity shortfalls by increasing trade credit obtained from suppliers and rationing credit extended to customers. This observed pattern, will be exploited in the set up of the model presented in Section 3.

Figure 1: Business Cycle Properties of Trade Credit in the US



Note: The panels in this figure plot the evolution of the log change in percent of aggregate US-GDP (Y), Accounts Payable (AP), Accounts Receivable (AR), Total (LT) and Current (LC) Liabilities, the share of AP in Current Liabilities (θ^T), the aggregate credit spread index - GZ-spread (GZ) - derived in Gilchrist and Zakrajšek (2012), the net percentage of domestic banking institutions reporting a tightening their standards for C&I loans (LS), the share of AP in Total Costs of Goods Sold (θ^P) and the share of AR in Total Sales (θ^R). The figures also report the standard deviation of the respective series in percent. The sample includes all Compustat firm-year observations from 2001 to 2016 of firms with their head-quarter in the US and positive and non-missing observations of the respective variables of interest. Financial firms (NAICS 52 and 53) are excluded, yielding a panel of 21,504 firm-year observations for 1,344 unique firms. For details on the sample, see Appendix D.

Heterogeneity in Trade Credit-Policies. Panel (d) of Figure 1 plots the log change of the share of accounts payable in total costs of production and the share of accounts receivable in revenues at an aggregate level on the right axis as well as the change in logs of credit spreads on the left axis. As evident from Figure 1 and Table 1:

(F5) The share of accounts payable and receivable in total costs of production and sales are strongly positively correlated. In addition, the shares are strongly negatively correlated with aggregate credit spreads in the economy.

While the evolution of log change of total payables and receivables is informative for the business cycle properties of trade credit, they contain little information on the heterogeneity of the usage of trade credit across firms. In order to provide a summary measure of the trade credit usage of a firm from both its perspective as a lender and a borrower, I first define the net-lending position of a firm (θ^τ).

Definition 1. *The net-lending position of a firm is defined as the ratio of total trade credit extended to customers (accounts receivable) and the difference between total cost of production and accounts payable. A higher ratio implies that a firm provides relatively more trade credit to its customers than it takes up from its suppliers.*

Table 1: Time Series Correlations

(a) Fact 1-2		Y_t	Y_t	AP_t
	Y_t	1.000		
	AP_t	0.652	0.652	
	AR_t	0.544	0.544	0.974

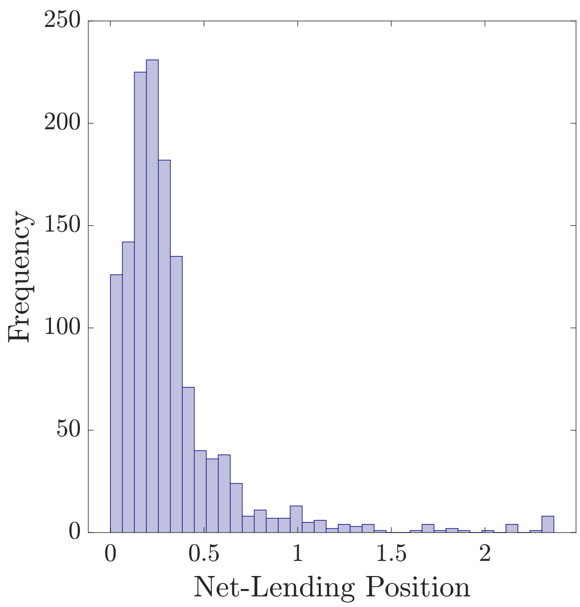
(b) Fact 3		Y_t	LT_t	LC_t
	LT_t	0.486		
	LC_t	0.771	0.545	
	AP_t	0.652	0.280	0.808

(c) Fact 4		Y_t	θ_t^T	GZ_t
	θ_t^T	-0.011		
	GZ_t	-0.349	-0.712	
	LS_t	-0.784	-0.175	0.566

(d) Fact 5		Y_{t+1}	θ_t^P	θ_t^R
	θ_t^P	0.549		
	θ_t^R	0.495	0.970	
	GZ_t	-0.780	-0.560	-0.484

Note: Each subtable presents the pairwise correlations between the log-changes of the time series plotted in the corresponding panel of Figure 1.

Figure 2: Fact 6



Note: The figure plots the distribution of the average net-lending position defined as the ratio of accounts receivable and the difference between total cost of production and accounts payable over 2004-2007 of the sample of Compustat firms described in Appendix D.

Figure 2 plots the distribution of the average net-lending position in 2004-2007 of the sample of firms and highlights one key pattern of trade credit usage by US-firms:

(F6) There is heterogeneity in the trade credit usage of US-firms: The distribution of the net-lending position is heavily skewed to the left. In other words, the majority of US-firms receive relatively more trade credit than they extend to their customers.

By taking a closer look at the industry-affiliation of firms, it becomes apparent that firms which are more upstream in the production chain (e.g. primary-industries, manufacturing) tend to have a higher net-lending position than more downstream firms (e.g. retail, services). This observation highlights that the structure of intersectoral trade plays a crucial role in determining which aspect of trade finance dominates and confirms the findings in Kalemli-Özcan et al. (2014), that upstream firms have higher accounts receivable compared to final product firms. The relationship between the net-lending position of firms (sectors) and the production network will be investigated in more detail in the context of the calibration of the model in Section 4.1.

This section has discussed the cyclical properties of supplier credit as well as the heterogeneity in the lending and borrowing behavior across firms. I now build a model, which focuses on the contraction of the liquidity in the trade credit market at the onset of the financial crisis in order to investigate the role of credit linkages in the propagation of the financial shock. To this end, I abstract from any dynamics by imposing the timing restriction that financial markets contracted at the same time as aggregate output such that the share of accounts payable (receivable) in total production costs (sales) is now positively correlated with current rather than next period's output. Although output declined with a time lag in response to the deterioration of credit conditions as shown in Figure 1a, this simplification may be justifiable as (1) the sharp increase in credit spreads occurred in the second half of 2008 and (2) a firm's production plans and therefore intermediate demand might by pre-determined. This allows me to focus on the effect of a decline in supplier credit on aggregate output during the crisis.

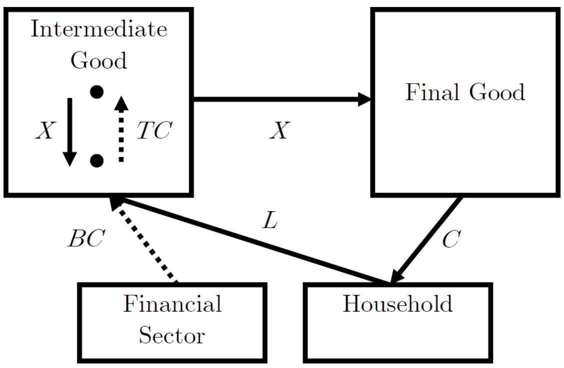
In particular, I build a model in which firms face working capital constraints and finance their input expenditures using both bank- and supplier credit. At this point it should be noticed that in order to keep the model tractable, I only consider a firm's trade credit decision along the intensive rather than the extensive margin. In other words, I do not explicitly model a firm's decision to enter the trade credit market in the first place. A growing theoretical literature investigates both the characteristics and motives of firms to engage in financial intermediation. Contributions by Emery (1984), Smith (1987), Biais and Gollier (1997), Burkart and Ellingsen (2004), Cuñat (2007) among others rationalize the existence of trade credit with the presence of transaction costs, imperfect market competition, information asymmetries or moral hazard problems. A detailed overview of the different strands of theoretical literature is provided in Cuñat and García-Appendini (2012) and is beyond the scope of this paper. I now describe the set-up of the model.

3. A Multisector Model with Financial Frictions

In this section, I introduce a static quantitative multisector general equilibrium model in the tradition of Long and Plosser (1983) with trade in intermediate inputs and endogenous credit linkages between sectors. The model nests the economy introduced in BL(2017) if no credit linkages are considered. The main novelty of this paper is the introduction of endogenous credit linkages among sectors by, in contrast to previous work by Zhang (2017) and Luo (2018), explicitly modelling the price of trade credit as well as introducing a direct link between the cost of bank finance and the amount of trade credit extended to customers. The model set-up is as follows.

Production Structure. The economy consists of M intermediate sectors indexed by $k = 1, ..., M$ producing M differentiated goods, a final good sector indexed by 0 producing a composite final good and a representative household. A continuum of perfectly competitive firms within each sector produce an identical good using the same technology such that there exists a representative firm per sector. Therefore, I use the words firm and sector interchangeably. The production structure of the economy is depicted in Figure 3.

Figure 3: Flow Chart of the Model Economy



Note: The figure depicts the flow of intermediate goods (X), the final consumption good (C), labor (L) as solid lines, and the flow of supplier (TC) and bank credit (BC) as dashed lines.

An *intermediate goods* firm k produces output q_k using capital k_k, labour ℓ_k^Q and a composite of intermediate inputs X_k with the Cobb-Douglas technology

$$q_k = \left(A_k (k_k^{\alpha_k} \ell_k^{Q,1-\alpha_k})^{\eta_k} X_k^{1-\eta_k}\right)^{\chi_k} \tag{1}$$

where A_k is the sector-specific productivity. The intermediate technology exhibits decreasing returns to scale $\chi_k \in (0,1)$ in capital and in its sector-specific intermediate composite input, X_k, defined as

$$X_k = \prod_{s=1}^{M} x_{ks}^{\omega_{ks}^X}. \tag{2}$$

Due to the Cobb-Douglas technology, the production parameter $\omega_{ks}^X \in [0,1]$ denotes the share of good s in the total intermediate input use of sector k and it is assumed that $\sum_{s=1}^{M} \omega_{ks}^X = 1 \, \forall k \in \{1, ..., M\}$. The output of firm k is used both as an intermediate input in production and to produce a composite final good F consumed by the household such that $F = C$. The final good firm assembles the consumption good using the constant returns to scale (CRS) technology

$$F = A_0 \prod_{m=1}^{M} q_{0m}^{\omega_m^F} \tag{3}$$

with productivity A_0, where $\sum_{m=1}^{M} \omega_m^F = 1$. Similarly, the production parameters $\omega_m^F \in [0,1]$ denote the expenditure share on good m by the final good firm. Productivity in the intermediate and final sector is given by

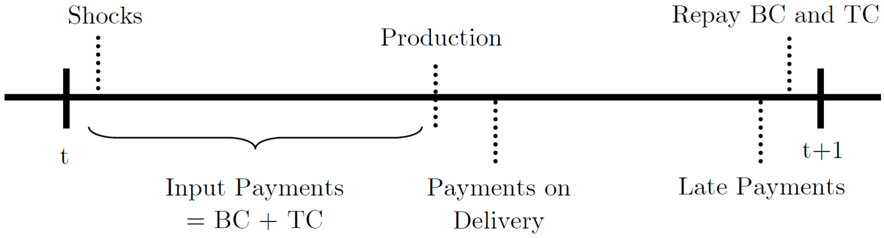
$$A_k = \exp(z_k^Q) \left(\chi_k \eta_k^{\eta_k} (1-\eta_k)^{(1-\eta_k)} \alpha_k^{\alpha_k} (1-\alpha_k)^{(1-\alpha_k)} \prod_{m=1}^{M} (\omega_{ks}^X)^{\omega_{ks}^X (1-\eta_k)} \right)^{-1} \tag{4}$$

$$A_0 = \exp(z_0^Q) \left(\prod_{m=1}^{M} (\omega_m^F)^{\omega_m^F} \right)^{-1} \tag{5}$$

where z_k^Q and z_0^Q are productivity shocks. For the purpose of the model, I assume that $z_0^Q = 0$. The timing of events is as follows

At the beginning of a period both productivity and banking shocks are realized. Within period I consider two stages: the pre-production stage and the post-production stage. Due to the working capital constraint, firms make the production and borrowing portfolio decisions prior to producing their output. Once firms produced, they sell their

Figure 4: Overview and Timing of the Intermediate Godds Firm's Problem



output to both intermediate and final good producers and retrieve the share of sales paid on delivery. At the end of the period, firms repay their debt obligations and receive the remaining share of their revenues. This motivates the first assumption of the model

Assumption 1. *The production and delivery of products along the supply chain within period is sequential such that any sales are only realized after production has taken place.*

Financing Production. The representative intermediate good producing firm in sector k faces a cash flow mismatch between input payments at the beginning of the period and the realization of revenues. Wlog, I assume that firms have no internal funds available such that firm k needs to finance its working capital using

(1) an intraperiod bank loan, BC_k at an interest rate r_k^B, and

(2) trade credit from its suppliers (net accounts payable)[14] at an interest rate r_s^T

$$AP_k \; = \; \sum_{s=1}^{\mathcal{S}} AP_{ks} \; = \; \sum_{s=1}^{\mathcal{S}} \theta_{ks} p_s x_{ks} \tag{7}$$

where $\theta_{ks} \in [0,1]$ represents the share of payments to supplier s that firm k postpones paying until after its sales are realized. Thus, the financial constraint of firm k can be written as

$$w\left(\ell_k^Q + \ell_k^T\right) + \sum_{s=1}^{\mathcal{S}} p_s x_{ks} \; \leq \; BC_k + AP_k \tag{8}$$

which is binding in equilibrium. Furthermore, I abstract from a proper microfoundation of (1) the incentives of firms to lend in kind to their customers and (2) of why firms obtain

[14]Similarly, AR_k is the total amount of trade credit extended to firm k's customers (net accounts receivable)

$$AR_k \; = \; \sum_{c=1}^{\mathcal{C}} AR_{ck} \; = \; \sum_{c=1}^{\mathcal{C}} \theta_{ck} p_k x_{ck} \tag{6}$$

both bank and trade credit in the first place, even if the interest costs of bank finance are cheaper than supplier credit as discussed in Cuñat and García-Appendini (2012). In line with the empirical observations presented in Section 2, I therefore assume that

Assumption 2. *Bank and supplier credit are substitutes. Firms simultaneously demand bank and trade credit to finance their working capital requirements. At the same time, while taking demand for trade credit as given, firms choose the price for trade credit by optimally trading off the associated costs and benefits discussed below.*

In order to ensure the tractability of a firm's optimization problem while capturing two main features of trade credit: (1) A firm's short-term credit portfolio is composed of both bank and supplier credit. (Fact 3) and (2) Accounts Payable and total (current) liabilities are positively correlated. (Fact 2), I impose the following two assumptions:

Assumption 3. *Firms face additional management costs of bank and supplier credit lines in the form of a non-productive labour input.*

Assumption 4. *The cost of bank credit is an increasing function in the average trade share extended to customers.*

I now discuss both assumptions in greater detail below.

Management Costs of Credit Lines. In order to manage its credit lines, a firm needs to hire non-productive labour (accountants, sales people and managers), ℓ_k^T, which introduces an additional cost component into the firm's problem. The adjustment of a firm's credit portfolio is subject to a combination of convex and nonconvex frictions. Formally, the total costs of credit adjustment is given by

$$C_k^T(\{\theta_{ks}\}_s) \;=\; w\ell_k^T \;=\; \kappa_k^B + \sum_{s=1}^{\mathcal{S}} \kappa_{0,ks}^T \theta_{ks} + \frac{\kappa_{1,ks}^T}{2} \left(\frac{\theta_{ks} - \overline{\theta}_k^S}{\overline{\theta}_k^S} \right)^2 \tag{9}$$

where I adapt the findings of a strand of literature related to the functional form of the adjustment costs of capital (see Cooper and Haltiwanger, 2006). The term $\overline{\theta}_k^S$ denotes the average share of intermediate input payments obtained on trade credit. The first term implies that there are fixed costs, κ_k^B, involved in managing credit lines such that even if sector k does not obtain any supplier credit, it still faces fixed management costs. Similar to Luo (2018), the quadratic adjustment cost part captures the fact that it is costly to change the credit composition. In addition, I further assume that while κ_k^B and $\{\kappa_{1,ks}^T\}_s$ $\forall k$ are always positive, the linear cost parameter, $\kappa_{0,ks}^T$, may take on both positive and negative values. This should highlight, that in adjusting the credit relationship with one's supplier s, firm k may undergo an organizational restructuring of its supplier relationship thereby increasing (e.g. switching suppliers within sector) or decreasing (e.g. intensifying the business relationship) the management costs. Notably, I assume the variable adjustment cost parameters to be specific to a firm-supplier pair.

Costs of Bank Credit Lines. In order to ensure the analytical tractability of the model while capturing key features of the relationship between bank and supplier credit, I introduce the banking sector in a highly reduced form by imposing the following functional form on the interest rate charged on bank-credit

$$r_k^B \;=\; \underline{r}_0^B + r_k^Z \;=\; \underline{r}_0^B + \exp(z_k^B)(\overline{\theta}_k^D + \theta_k^C)^{\mu}\underline{r}_0^B \tag{10}$$

where $\frac{\partial r_k^B}{\partial \underline{r}_0^B}, \frac{\partial r_k^B}{\partial \theta_k^C} > 0$ and $\frac{\partial^2 r_k^B}{(\partial \theta_k^C)^2} > 0$. In other words, I assume that each sector is charged a risk premium, r_k^Z, over the federal funds rate, $\underline{r}_0^B$, which is a convex function in the aggregate default probability, $\overline{\theta}_k^D$ and in the average trade credit share extended to firm k's customers, θ_k^C. While the positive relationship between the interest rate charged on bank loans and the probability of default is a common modelling assumption (see i.a. Khan et al., 2016) and is supported by empirical evidence (see i.a. Angbazo, 1997), I abstract from including a microfoundation thereof in order to keep the model tractable. Clearly, this set up introduces a direct upstream credit link between the cost of bank credit of firm k and the trade credit extended to its customers: the higher the share of delayed payments by firm k's customers, the higher the interest rate on bank credit that firm k is charged. I am now able to formulate the intermediate goods firm's profit maximization problem.

Intermediate Production. Firms are price takers in both goods and credit markets. I assume that in the short run capital is exogenously given and at its steady state value. The objective of the representative firm in sector k is to choose production inputs and the credit portfolio ($\mathcal{V}(t)$) to maximise profits/dividends, where $\mathcal{V}(t)$ denotes the set of choice variables[15]. The intermediate goods firm's problem profit maximization

[15] Let $\mathcal{V}(t)$ be the set of static - $\mathcal{V}(t) = \{\ell_{k,t}, \{x_{ks,t}\}_s, \mathcal{V}(m,t), \mathcal{V}(c,t)\}$ choice variables, where $\mathcal{V}(m,t) = \{\{x_{ck,t}\}_c(t), q_{0k,t}, \theta_{k,t}^C\}$ is the set of choice variables associated with the market structure of perfect competition and $\mathcal{V}(c,t) = \{\{\theta_{ks,t}\}_s\}$ is the set of choice variables related to trade credit. As the model is static, I drop the time subscript in the remainder of the text.

problem can be formulated as (see Appendix A.1 for details)

$$\mathrm{V}(z_k^Q, z_k^B) \;=\; \max_{\mathcal{V}} \; (1 + r_k^T \theta_k^C) p_k q_k - (1 + r_k^B) BC_k - \sum_{s=1}^{M} (1 + r_s^T) AP_{ks} \tag{11}$$

subject to the production function (1), total supplier (7) and bank-credit (8) and

- the production constraint

$$q_{0k} + \sum_{c=1}^{\mathcal{C}} x_{ck} \leq \; q_k \tag{12a}$$

- the feasibility constraint with respect to the trade credit shares

$$0 \leq \theta_{ks} \leq 1 \;\forall k, s \tag{12b}$$

and non-negativity constraints $\ell_k^Q, \ell_k^T, x_{ks} \;\geq\; 0 \;\forall k, s$.

Final Demand. The representative final good producer is required to pay its input expenditures at the time of the delivery of the product. Since I assume that the final goods producer does not face any working capital constraints, the profit maximization problem is simply given by

$$\max_{\{q_{0m}\}_m} \; PF - \sum_{m=1}^{M} p_m q_{0m} \tag{13}$$

subject to the production function (3) and a non-negativity constraint $q_{0m} \;\geq\; 0 \;\forall m$. The *household* maximizes utility

$$U(C, L) \;=\; \frac{C^{1-\epsilon_C}}{1-\epsilon_C} - \frac{L^{1+\epsilon_L}}{1+\epsilon_L}. \tag{14}$$

subject to the budget constraint

$$PC \;\leq\; wL + \sum_{m=1}^{M} \pi_m + \sum_{m=1}^{M} r_m^B BC_m \tag{15}$$

The parameters $\epsilon_C \;>\; 0$ denotes the income elasticity for labor supply and $\epsilon_L \;>\; 0$ denotes the inverse Frisch elasticity of labor supply. The budget constraint of the household indicates that total income of the household - total wage bill, profits and interest income from extending bank-credit to firms - is spent on the aggregate consumption good. Ultimately, I assume that banks are owned by foreign households such that any interest rate income while initially rebated to households, is treated as an import in the calculation of aggregate GDP.

Market Clearing. As depicted in Figure 3 the intermediate good of sector k is used both in the production of intermediate goods as well as in the production of the final good such that the market clearing for sector k is given by

$$q_k \ = \ q_{0k} + \sum_{c=1}^{M} x_{ck} \tag{16}$$

The labor market clears if $L = \sum_{k=1}^{M} \ell_k^Q + \ell_k^T$ holds. Similarly, the interest rate on trade credit charged by sector k is set such that

$$\theta_k^C p_k q_k \ = \ \sum_{c=1}^{M} \theta_{ck} p_k x_{ck} \tag{17}$$

holds. In other words, total accounts receivable equal total accounts payable of sector k. A competitive equilibrium in this economy is then defined as follows

Definition 2. *A competitive equilibrium in this economy is a set of aggregate ($\{P\}$) and sector level ($\{p_m, r_m^T\}_m$) prices, aggregate ($\{C, F, L\}$) and sector level ($\{\ell_m, q_m, q_{0m}, \{x_{ms}\}_{s=1}^M\}_{m=1}^M$) quantities, sector level trade credit shares ($\{\{\theta_{ms}\}_{s=1}^M\}_{m=1}^M$) such that*

(i) The representative household maximizes utility.

(ii) Intermediate and Final-producers maximize profits.

(iii) Goods and Factor markets clear.

(iv) Financial markets clear.

3.1. Equilibrium Characterization

Having introduced the model-set up in the previous section, I now discuss the effect of working capital constraints and credit links on the optimal intermediate input choice and credit composition before characterizing the equilibrium of the economy.

Firm Optimality and Credit Composition. In particular, I first describe the effect of distortions on the optimal input demand by the representative intermediate good producing firm while taking both credit costs and the composition of the borrowing portfolio as given.

Lemma 1 (Optimal Input Choice)**.** *Given a vector of prices, interest rates, credit links and the real wage, firm k's optimal demand for intermediate input (x_{ks}) and labor (ℓ_k^Q) is given by*

$$p_s \;=\; \omega^X_{ks}(1-\eta_k)\chi_k \frac{\phi^R_k}{\phi^X_{ks}} \frac{p_k q_k}{x_{ks}} \quad (18) \qquad w \;=\; (1-\alpha)\eta_k\chi_k \frac{\phi^R_k}{\phi^L_k} \frac{p_k q_k}{\ell^Q_k} \quad (19)$$

where the credit wedges are given by

$$\phi^L_k \;=\; 1 + r^B_k \quad (20) \qquad \phi^X_{ks} \;=\; 1 + (1-\theta_{ks})r^B_k + \theta_{ks} r^T_s \quad (21)$$

and the revenue wedge is $\phi^R_k = 1 + r^T_k \theta^C_k$.

Due to the Cobb-Douglas production technology, expenditures on any production input are proportional to sector k's revenues. However, as evident from Equation (18) and (19) in Lemma 1, the requirement to finance total input expenditures prior to the realization of any sales introduces a credit wedge between the firm's marginal cost and marginal revenue of the respective input, thereby distorting the first order conditions. While labor expenditures are exclusively financed via bank credit, firm k finances its expenditures on intermediate input obtained from supplier s using both bank and supplier credit such that the respective credit wedge is a weighted average of both credit costs and the weights are equal to the trade credit share. In addition, Lemma 1 also highlights that an increase in trade credit extended to customers ceteris paribus increases sector k's revenues due to an increase in the effective price charged and thus also increases sector k's demand for production inputs.

Corollary 1 (Marginal Costs of Production)**.** *Given credit links, the marginal cost of production,* p^V_k*, can be decomposed into a combined credit wedge,* ϕ^V_k*, and a composite of the wage rate and the intermediate input prices and is given by*

$$p^V_k \;=\; \phi^V_k mc^V_k \;=\; \left(\phi^L_k\right)^{\upsilon_k} \left(\prod_{s=1}^{\mathcal{S}} \left(\phi^X_{ks}\right)^{\omega^X_{ks}}\right)^{(1-\upsilon_k)} \left(w\right)^{\upsilon_k} \left(\prod_{s=1}^{\mathcal{S}} \left(p_s\right)^{\omega^X_{ks}}\right)^{(1-\upsilon_k)} \quad (22)$$

The combined sectoral credit wedge $\phi^V_k > 1$ *is a Cobb-Douglas composite of the individual credit costs. The marginal cost of production is increasing in the cost of bank,* r^B_k*, and supplier credit,* r^T_k*, and increasing (decreasing) in the trade credit share taken from supplier* s*,* θ_{ks} *if* $r^B_k < (>) r^T_k$*.*

Credit costs associated with the working capital constraint thus aggregate to a marginal cost wedge ϕ^V_k as shown in Corollary 1 and thus increase both, the total cost of production and the optimal goods price charged as shown in Lemma 2. However, if a firm also extends trade credit to its customers and thereby increases the marginal revenue generated by an additional unit sold ,ϕ^R_k, the optimal price charged on the actual good decreases.

Lemma 2 (Optimal Price)**.** *The optimal goods price equals a mark-up over marginal costs*

$$p_k = \frac{MC_k^V}{MP_k^V} = \frac{\phi_k^V}{\phi_k^R} \frac{mc_k^V}{(1-\alpha\eta_k)\,\chi_k q_k V_k^{-1}} \tag{23}$$

The optimality conditions were derived while taking both interest rates as well as trade credit shares as given. Therefore, I now describe the profit maximizing supplier credit share and the optimal interest rate charged on trade credit while taking both prices and demand for trade credit as given.

Lemma 3 (Optimal Demand for TC)**.** *Firm k chooses $\{\theta_{ks}\}_s$ to maximise profits. The FOC associated with $\{\theta_{ks}\}_s$ imply that the optimal demand for trade credit is given by*

$$\frac{\theta_{ks}}{\overline{\theta}_k^S} = 1 + \frac{\overline{\theta}_k^S}{\kappa_{1,ks}^T} \left(-\kappa_{0,ks}^T + \frac{p_s x_{ks} \Delta_{ks}}{(1+r_k^B)w} \right) \tag{24}$$

In other words, firm k chooses $\{\theta_{ks}\}_s$ such that the combined change in the cost of production and managing credit lines associated with changing the share of trade credit obtained from k's supplier is zero at the optimum. However, it should be noted that the sign of the interest-rate differential governs the trade-off that a firm faces when choosing the composition of its credit portfolio. In particular, if the interest differential $\Delta_{ks} = r_k^B - r_s^T$ is positive such that the interest rate on trade credit offered by supplier s is cheaper than the interest rate on bank credit, then an increase in the trade credit share obtained from supplier s reduces the marginal cost of production but increases the credit management costs if $\kappa_{0,ks}^T, \kappa_{1,ks}^T > 0$. However, if the interest differential $\Delta_{ks} = r_k^B - r_s^T$ is negative and both management cost parameters are positive, then an increase in the trade credit share obtained from supplier s increases total cost of production such that firm k chooses the trade credit share to minimize total costs.

The first order condition (24) determining the optimal trade credit share obtained from supplier s exhibits the following properties. It holds that, ceteris paribus, an increase in the interest rate on bank credit increases both the marginal cost of production and the gross-non-productive labor costs of sector k, increasing the optimal share of purchases obtained on credit from sector s. On the other hand, an increase in the interest rate on trade credit decreases the optimal trade credit share. If the interest differential for firm k is negative such that obtaining trade credit from firm s is more expensive than bank credit ($\Delta_{ks} < 0$) then an increase in either, the price of good s

or intermediate goods obtained from firm s, decreases the optimal trade credit share obtained from supplier s.

The question remains: what is the optimal interest rate charged for extending trade credit to firm k's customers? As discussed in Section 3, firms operate under perfect competition and therefore take the demand for trade credit as given. However, the firm faces the following trade off: An increase in the demand for trade credit by its customers increases its revenues due to an increase in the interest income from lending to its customers on the one hand. On the other hand it also increases total marginal costs of production due to the increase in the interest rate on bank credit by Assumption 4. Thus, the interest rate on trade credit, r_k^T, is set to equalize the marginal revenue to the marginal costs of extending trade credit to customers.

Lemma 4 (Optimal Interest Rate on Trade Credit)**.** *The optimal interest rate on trade credit extended by sector k is given by*

$$r_k^T \;=\; \mu\left(\overline{\theta}_0^D + \theta_k^C\right)^{-1}\left(r_k^B - \underline{r}_0^B\right)\left(\frac{\phi_k^{\partial V} p_k^V V_k}{p_k q_k} + \frac{w\ell_k^T}{p_k q_k}\right) \tag{25}$$

Ceteris paribus, an increase in the share of credit management costs in total sales net of interest income from trade credit as well as an increase in the total change of marginal costs of production in response to an increase in the trade credit share extended to customers increases the interest rate charged on trade credit. Similarly, an increase in the responsiveness of the bank interest rate to changes in extended supplier credit will also increase the interest rate charged on trade credit.

As firm k sets a *common* contract rather than a link-specific contract, this implies that a tightening of bank credit of firm k's customer c and the resulting increase in lending to c also increases the trade credit rate charged to everyone. In other words the existence of common suppliers may lead to interest rate shocks spilling over from one customer of supplier s to another via an increase in the interest rate of trade credit. A common contract is assumed for simplicity - this should capture that it is costly to maintain link-specific contracts. Even if, an increase in the borrowing of one customer might affect the ability of firm k to lend to others (e.g. shifting monitoring sources etc.) such that spill-overs can be justified.

Partial Equilibrium. I now characterize the partial equilibrium of the economy. I first make the following two assumptions: (1) The nominal wage rate is taken as the numeraire. and (2) Capital is at its steady state level and investment is equal to zero in equilibrium. It should be noted at this point that the model only admits an analytical solution of its partial equilibrium when taking both interest rates on trade credit as well as trade credit linkages as given. Consequently, there exists almost a one to one mapping of the partial equilibrium in this section to the general equilibrium analysis presented in BL(2017). To summarize, as shown in BL(2017), the distortions manifest themselves as an aggregate efficiency and labour wedge:

Lemma 5 (Aggregate Efficiency and Labor Wedge)**.** *An economy consisting of individual sectors operating with Cobb-Douglas production technologies and engaging in intersectoral trade aggregates to a Cobb-Douglas aggregate production function characterized by decreasing returns to scale*

$$Y = Z(\boldsymbol{z})\Phi(\boldsymbol{\phi})L^{(1-\lambda)} \tag{26}$$

where $Z(\boldsymbol{z})$ denotes aggregate productivity and $\Phi(\boldsymbol{\phi})$ represents the aggregate efficiency wedge which is a non-linear combination of all sectoral distortions. The aggregate labor wedge, Φ^L, is given by

$$-\frac{L^{\epsilon_L}}{C^{-\epsilon_C}} = \Phi^L(\boldsymbol{\phi})(1-\lambda)\frac{Y}{L} \tag{27}$$

and is defined as a wedge between the household's marginal rate of substitution between consumption and labor and the aggregate marginal product of labor.

As evident from Equation (26) and (27), the presence of distortions - in this paper working capital constraints - leads to misallocation and efficiency loss. However, Equations (26) and (27) mask that the aggregate wedges are in fact not only a function of the interest rates on bank and trade credit but also a function of the credit network, $\boldsymbol{\Theta}$, which in equilibrium is the outcome of firms minimizing their total cost of production. Since firms are both lenders and borrowers of trade credit at the same time, clearly this implies that distortions in this economy are interlinked and will crucially affect the propagation of liquidity shocks in this economy. I will now discuss the role of endogenous credit links for the propagation of liquidity shocks the next section.

3.2. Business Cycle Interpretation

It has been shown that - as common to models with distortions - working capital constraints introduce an aggregate efficiency and labor wedge thereby generating an efficiency loss as resources are diverted from being used for production. However, the nature of trade credit generates an inter-dependency between sector-specific distortions. In particular, as firms adjust both their lending rates and their borrowing portfolio, credit cost of production and credit linkages are subject to changes along the intensive margin, thereby distorting the transmission of shocks.

In order to highlight the role of endogenous credit links on the propagation of liquidity shocks, I log-linearize the model around its steady state. This allows for a decomposition of the log-change of all variables of interest into changes attributed to (1) productivity shocks, (2) general equilibrium adjustments in the aggregate labor supply and (3) distortions introduced as credit wedges. The effect of the log-change of each component is determined by the entries of the corresponding elasticity matrices $\boldsymbol{E}$, which are non-linear functions of the steady state of the economy. The credit wedges summarize the composite effect of changes in credit costs and the composition of credit portfolios on sectoral sales, prices and output. Therefore, I first present the decomposition of log-changes of sectoral credit wedges into effects attributed to changes in interest rates on both bank and trade credit and changes in trade credit shares. I then discuss the effect of changes in interest rates and credit shares on one another each at a time. This allows me to define the trade credit multiplier summarizing the total effect of shocks to the bank risk-premium on the cost of credit and the composition of the borrowing portfolio. At the end of this section, I discuss the structural output response following a financial shock. In particular, it is shown that to a first order approximation the structural elasticities are functions of equilibrium expenditures, accounts payable and accounts receivables, which determines the strength of the trade credit channel on output.

For illustrative purposes, I now abstract from productivity shocks and consider the partial equilibrium case only, assuming that both productivity and aggregate labor remain at their steady state levels. In addition, I further simplify the analysis by treating the share of quantities sold to intermediate and final good producers in total production as constant. Lemma 6 describes the effect of interest rates and trade credit shares on the input-specific cost of production and revenues:

Lemma 6 (Revenue and Credit Wedges)**.** *The log-linearisation of the revenue wedge for each sector k implies that*

$$\widehat{\phi}_k^R \;=\; [\boldsymbol{E}^T_{\phi(R)}]_{kk}\widehat{r}_k^T + \sum_{c=1}^{M}[\boldsymbol{E}^{\theta}_{\phi(R)}]_{ck}\widehat{\theta}_{ck} \tag{28}$$

The labor and intermediate credit wedge deviations for each sector k are given by

$$\widehat{\phi}_k^L = [\boldsymbol{E}_{\phi(L)}^B]_{kk}\widehat{r}_k^B \quad and \quad \widehat{\phi}_{ks}^X = [\boldsymbol{E}_{\Phi}^B]_{ks}\widehat{r}_k^B + [\boldsymbol{E}_{\Phi}^T]_{ks}\widehat{r}_s^T - sgn(\Delta_{ks})|[\boldsymbol{E}_{\Phi}^{\theta}]_{ks}|\widehat{\theta}_{ks} \tag{29}$$

All entries of the elasticity matrices $\boldsymbol{E}$ are positive. While an increase in the revenue wedge of sector k increases sector k's revenues, an increase in the input-specific credit wedges increases sector k's cost of hiring labor and obtaining inputs from supplier s.

An increase in the trade credit interest rate charged by sector k as well as an increase in the average trade credit share extended to customers increase the revenue wedge of sector k. Similarly, an increase in the cost of credit increases both the labor and the intermediate credit wedge of sector k related to hiring workers and purchasing sector s's output. However, the sign of the effect on changes in the credit link on intermediate credit costs depends on the sign of the interest rate differential $\overline{\Delta}_{ks} = \overline{r}_k^B - \overline{r}_s^T$. If in equilibrium sector k's bank interest rate is cheaper than supplier credit from sector s, then an increase in the trade credit share obtained from supplier s increases sector k's credit costs associated with obtaining inputs from supplier s.

The intermediate sales credit wedge, $\phi_{\kappa,k}^S$, summarizes the effect of changes in credit costs and the composition of credit on sector k's sales.

Lemma 7 (Sales Wedge)**.** *The sales wedge $\boldsymbol{\phi}_{\kappa}^S$ is defined in Equation (30). Assuming that all firms sell to the final good producer such that the effect of credit costs on final sales dominates, the wedge response can be written as*

$$\widehat{\phi}_{\kappa,k}^S = -\sum_{m=1}^{M}\underbrace{[\boldsymbol{E}_{\phi(S)}^B]_{km}}_{>0}\widehat{r}_m^B + \sum_{m=1}^{M}\underbrace{[\boldsymbol{E}_{\phi(S)}^T]_{km}}_{>0}\widehat{r}_m^T + \sum_{m=1}^{M}\sum_{s=1}^{M}\underbrace{[\boldsymbol{E}_{\phi(S)}^{\theta}]_{k,ms}}_{>0}\widehat{\theta}_{ms} \tag{30}$$

An increase in the combined sales wedge of sector k reduces sector k's revenues.

Let sector k be a supplier of sector m. The elasticity of the sales wedge of sector k with respect to changes in the *bank interest rate* of sector m is the result of two channels: An increase in the bank rate of sector m (1) increases the interest rate income of households and therefore final demand, and (2) increases sector m's costs of production such that the demand for k's output declines. If the effect on final revenues dominates the cost effect, then the composite effect of an increase in the bank rate of sector m decreases the sales wedge and hence increases sector k's sales. An increase in sector m's *trade credit rate* increases the intermediate input costs of production of m's customers, thereby reducing both intermediate and final sales. If $m = k$, then the increase in sector

k's trade credit rate also increases sector k's revenues from extending trade credit to customers, crowding out the increase in the sales wedges implied by the cost effect. An increase in the *trade credit share* of sector m to sector $s \neq k$ reduces the profit income of households, thereby increasing the sales wedge of sector k. If $s = k$, an increase in the trade credit share of sector m to sector k increases (decreases) the intermediate input costs of production of m if the interest differential Δ_{mk} is negative (positive), consequently increasing (decreasing) k's sales wedge. At the same time, an increase in the trade credit share of sector m to sector k increases sector k's revenues from extending trade credit to customers. If the effect on final demand outweighs the effect on intermediate costs and revenues, then sector k's sales wedge increases. Now impose that Assumption 5 holds.

Assumption 5. *Define* $[\boldsymbol{W}_P^P]^{-1} = \boldsymbol{I} - diag\left(\boldsymbol{\chi} \circ (\boldsymbol{\iota} - \boldsymbol{\eta})\right) \boldsymbol{\Omega}^X$ *and assume that* $\boldsymbol{W}_P^P diag(\boldsymbol{\iota} - \overline{\boldsymbol{\chi}})\boldsymbol{\iota} < \boldsymbol{\iota}$ *holds.*

The price credit wedge, $\phi^P_{\kappa,k}$, is a composite of the effect of credit costs and trade credit shares on marginal cost of production and sales due to decreasing returns to scale.

Lemma 8 (Price Wedge)**.** *The price wedge* $\widehat{\boldsymbol{\phi}}^P_\kappa$ *is defined in Equation (31) and is a combination of the direct effect of credit costs and links on prices and the sales wedge (30) due to the presence of decreasing returns to scale.*

$$\phi^P_{\kappa,k} = \sum_{m=1}^{M} \underbrace{[\boldsymbol{E}^B_{\phi(P)}]_{km}}_{>0} \widehat{r}^B_m + \sum_{m=1}^{M} \underbrace{[\boldsymbol{E}^T_{\phi(P)}]_{km}}_{>0} \widehat{r}^T_m - \sum_{m=1}^{M}\sum_{n=1}^{M} \underbrace{[\boldsymbol{E}^\theta_{\phi(P)}]_{k,mn}}_{\lessgtr 0} \widehat{\theta}_{mn} \tag{31}$$

An increase in the combined price wedge of sector k increases sector k's price.

An increase in the *bank interest rate* of sector m increases marginal cost of production of m and affects intermediate and final revenues by increasing and decreasing the sales wedge of sector m, respectively. Due to Assumption 5, the cost dominates the revenue effect such that price wedge of k increases. Similarly, an increase in the *interest rate on trade credit* of sector m increases marginal costs and thus the price charged on output produced by m's customers. In addition, the revenues of m are affected as follows: An increase in m's trade credit rate (1) increases m's revenues from extending trade credit to its customers and (2) decreases the demand for m's output and hence the profit income of households. If the cost effect on prices dominates, an increase in the interest rate on trade credit of sector m also increases the price wedge and price of k. The total effect of an increase in the *credit share* of sector m to sector n on the price wedge of sector k is ambiguous and depends on the relative size of the effect on (1) the production costs of sector m and on (2) the sales of sector n discussed below. First, the shift of m's supplier-specific borrowing portfolio towards trade finance increases (decreases) the marginal cost of production and thus the price wedge of sector m if the interest differential Δ_{mn} is negative (positive). At the same time, the profit income of households decreases, thereby crowding out (reinforcing) the increase (decrease) in sector m's price wedge due to DRS. Second, sector n's revenues increase from (1) extending credit to sector m and from (2) an increase in the demand for n's output if trade credit offered by sector n is cheaper than the bank interest rate faced by sector m. Consequently, sector n's price wedge increases due to DRS.

The total effect of credit costs and linkages on sectoral output is given by the output wedge ϕ_k^Q which is a combination of the sales and prices wedges. Consequently, and increase in either wedge decreases sectoral output via the an increase in the cost of production which reduces input demand, sales and ultimately household's income.

Lemma 9 (Output Wedge)**.** *The sectoral output wedge $\widehat{\phi}_k^Q$ is a combination of the sales (30), the revenue (28) and the price wedges (31)*

$$\widehat{\phi}_k^Q = \sum_{m=1}^{M} \underbrace{[\boldsymbol{E}_{\phi(Q)}^B]_{km}}_{>0} \widehat{r}_m^B + \sum_{m=1}^{M} \underbrace{[\boldsymbol{E}_{\phi(Q)}^T]_{km}}_{>0} \widehat{r}_m^T - \sum_{m=1}^{M} \sum_{s=1}^{M} \underbrace{[\boldsymbol{E}_{\phi(Q)}^\theta]_{k,ms}}_{\lesseqgtr 0} \widehat{\theta}_{ms} \tag{32}$$

An increase in the output wedge of sector k reduces sector k's output.

Following from the discussion above, an increase in the *bank interest rate* of sector m, increases both the price and sales wedge of sector m, thereby also increasing sector k's output wedge. In particular since Assumption 5 holds, the cost effect on prices of an increase in the interest rate on bank credit dominates the income effect on final demand and the output wedge of sector k increases. An increase in the *interest rate on trade credit* of sector m increases the price wedge of all other sectors and the sales wedge of m. As the cost effect dominates, the output wedge of sector k rises. The effect of an increase in the credit link intensity between sector m and its supplier s on the output wedge of sector k is ambiguous and crucially depends on the interest rate differential and the relative effect of the change on prices and revenues as discussed above.

Lemma 10 (Trade Credit Share)**.** *The log-linearized trade credit share between sector k and s is*

$$\begin{aligned}\widehat{\theta}_{ks} &= +[\underset{>0}{\boldsymbol{E}_\theta^B}]_{ks,k}\widehat{r}_k^B - [\underset{>0}{\boldsymbol{E}_\theta^T}]_{ks,s}\widehat{r}_s^T \\ &\quad + sgn(\Delta_{ks})\left\{+\sum_{m\neq k}^{M}[\underset{>0}{\boldsymbol{E}_\theta^B}]_{ks,m}\widehat{r}_m^B + \sum_{m\neq s}^{M}[\underset{>0}{\boldsymbol{E}_\theta^T}]_{ks,m}\widehat{r}_m^T - \sum_{m=1}^{M}\sum_{n=1}^{M}[\underset{>0}{\boldsymbol{E}_\theta^\theta}]_{ks,mn}\widehat{\theta}_{mn}\right\} \end{aligned} \quad (33)$$

The effect of changes in the cost of bank and supplier credit and of changes in trade credit shares on the share of trade credit obtained from sector s by sector k crucially depends on the interest-rate differentials between bank and trade credit and is thus ambiguous. If supplier s charges a higher interest rate on trade credit than the interest rate on bank credit faced by sector k, then any change in credit costs or links that result in a rise of the cost of production of k typically decreases the share of delayed input payments to sector s. In other words, if bank-finance is cheaper for sector k, then an increase in the overall cost of production - directly or indirectly - implies that sector k will rely more on bank rather than supplier finance. The opposite holds if the interest rate differential is positive.

Now, let the interest differential be negative and consider the effect of an increase in the bank interest rate of sector k and the interest rate on trade credit charged by supplier s on the share of delayed input payment to sector s by sector k. First, an increase in sector k's bank interest rate - ceteris paribus - directly reduces the interest rate differential which implies that sector k increases trade credit obtained from supplier s. Similarly, an increase in sector s's interest rate charged on trade credit reduces the trade credit share. Second, an increase in either interest rate also indirectly increases marginal cost of production of sector k and therefore shifts the input-specific credit portfolio towards bank credit. However, if the direct effect dominates the potentially counteracting indirect effects then an increase in the interest rate on bank (trade) credit faced by sector k increases (decreases) the share of delayed input payments to sector s.

Lemma 11 (Interest Rates)**.** *The log-linearized interest rate on bank and trade credit of sector k is*

$$\widehat{r}_k^B = [\underset{>0}{\boldsymbol{E}_B^{Z(B)}}]_k\widehat{z}_k^B + \sum_{m=1}^{M}[\underset{>0}{\boldsymbol{E}_B^\theta}]_{mk}\widehat{\theta}_{mk} \quad (34)$$

$$\widehat{r}_k^T = -\sum_{m=1}^{M}[\underset{>0}{\boldsymbol{E}_T^B}]_{km}\widehat{r}_m^B + \sum_{m=1}^{M}[\underset{>0}{\boldsymbol{E}_T^T}]_{km}\widehat{r}_m^T + \sum_{m=1}^{M}\sum_{n=1}^{M}[\underset{\leqslant 0}{\boldsymbol{E}_T^\theta}]_{k,mn}\widehat{\theta}_{mn} + [\boldsymbol{E}_T^{Z(B)}]_{kk}\widehat{z}_k^B \quad (35)$$

By definition, sector k's interest rate on bank credit is increasing in shocks to the sector-specific risk premium and is also increasing in the trade credit shares extended to sector k's customers. The response of the interest rate on trade credit charged by sector k depends on the effect of credit cost and shares on the change in the bank interest rate of sector k and the relative change in marginal and credit management costs of sector k. Clearly, an exogenous increase in the risk premium charged on bank credit increases marginal costs of bank borrowing from extending trade credit to sector k's customers such that the optimal interest rate on trade credit increases. The effect of changes in the interest rate on bank and trade credit is ambiguous. However, the effect on revenues (costs) dominates, then an increase in the bank (trade) credit rate of sector m will decrease (increase) the interest rate charged on trade credit by sector k. Similarly, the effect of direct and indirect changes in trade credit shares on the interest rate charged on trade credit by sector k is ambiguous and depends on the overall effect on cost of production, prices and revenues as outlined above.

Assume that Assumption 5 holds such that the cost effect of a change in credit costs and shares on prices dominates any revenue effects on prices and output. To summarize, a rise in the *bank interest rate* increases both the households' income and the cost of production. The cost effect dominates the income effect such that prices increase and output declines in response to an increase in the cost of bank borrowing. While the immediate effect of a rise in k's bank interest rate implies a shift of k's input-specific credit portfolio towards trade credit, the indirect effect of an increase in the cost of sector $m \neq k$'s bank finance is ambiguous and depends on the interest rate differential. An increase in the *interest rate on trade credit* also decreases sales and increases prices such that output declines. Similar to the case of bank-credit, the immediate effect of a rise in s's trade credit interest rate implies a shift of k's input-specific credit portfolio towards bank credit. The indirect effect is again ambiguous and depends on the interest rate differential. An increase in the interest rate on trade credit charged by sector m, increases sector k's interest rate if the relative decline in sales outweighs the decline in the change of marginal costs. The effect of an increase in the *trade credit share* of sector m to sector k decreases k's sales if the decline in final demand is sufficiently strong. The total effect of an increase in the trade credit share θ_{ks} on prices, output, credit costs and credit linkages is ambiguous and depends on the interest rate differential and the relative size of the cost and revenue effect discussed above.

Based on the discussion of the effect of changes in the cost of credit and the input-specific borrowing portfolio, I now define the Trade Credit Channel as follows

Definition 3 (Trade Credit Channel). *Consider an economy with working capital constraints and two sources of external funds: bank and supplier credit. The ability of firms to delay input payments to their suppliers introduces two opposing channels via which trade credit affects economic outcomes:*

(a) (Smoothing) The existence of two external financing sources allows firms to smooth any interest rate shocks via an adjustment of their borrowing portfolio by optimally trading-off credit costs as evident from Lemma 3.

(b) (Amplification) A shift in the borrowing portfolio composition of a firm's customers towards trade credit directly increases its cost of bank finance by Assumption 4. In addition, an increase in the cost of bank finance of a firm directly translates into an increase in interest rate charged on trade credit as shown in Lemma 4 and 11, thereby tightening the financing terms for its customers.

The Trade Credit Channel in an economy implies that idiosyncratic shocks to the cost of bank credit propagate both up- and downstream: On the one hand, the ability of firms to substitute bank and supplier credit implies that the output effect of an increase in bank financing costs is dampened. On the other hand, firms counteract an increase their external financing costs by increasing the cost of trade credit thereby reducing the demand and thus supply of trade credit in the economy. This implies that firms directly transmit changes in external financing costs to their costumers thereby amplifying idiosyncratic shocks to financial shocks. Which effect dominates is ultimately an quantitative question as will be investigated further in Section 4. As evident from the previous discussion, credit costs and the composition of the input-specific credit portfolio are inter-dependent such that there exists a credit multiplier that captures the total effect - direct and indirect - of changes in credit conditions on the variables of interest in addition to the standard feedback effects via the production structure.

Definition 4 (Trade Credit Multiplier). *In partial equilibrium assume that* $[\widehat{\boldsymbol{w}}^C]_{ck} = \widehat{L} = 0 \ \forall c, k$ *and abstract from any changes in productivity such that the response of the vector of credit-costs and shares is given by*

$$\boldsymbol{\tau} \;=\; \boldsymbol{E}_{\tau}^{\tau}\boldsymbol{\tau} + \boldsymbol{E}_{\tau}^{Z(B)}\boldsymbol{\epsilon}_B \tag{36}$$

such that

$$\begin{bmatrix} \widehat{\boldsymbol{r}}^B \\ \widehat{\boldsymbol{r}}^T \\ \widehat{\boldsymbol{\theta}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & +\overset{c}{\boldsymbol{E}}{}^{\theta}_{B} \\ -\boldsymbol{E}^B_T & +\boldsymbol{E}^T_T & +\mathring{\boldsymbol{E}}^{\theta}_T \\ +\mathring{\boldsymbol{E}}^B_{\theta} & +\mathring{\boldsymbol{E}}^T_{\theta} & -\mathring{\boldsymbol{E}}^{\theta}_{\theta} \end{bmatrix} \begin{bmatrix} \widehat{\boldsymbol{r}}^B \\ \widehat{\boldsymbol{r}}^T \\ \widehat{\boldsymbol{\theta}} \end{bmatrix} + \begin{bmatrix} +\boldsymbol{E}^{Z(B)}_B \\ +\boldsymbol{E}^{Z(B)}_T \\ \mathbf{0} \end{bmatrix} \boldsymbol{\epsilon}_B$$

The elasticity matrices $\boldsymbol{E}^{\tau}_{\tau}$ *and* $\boldsymbol{E}^{Z(B)}_{\tau}$ *summarize the equilibrium-network effects via prices and sales on the respective variables of interest. The credit multiplier and its first-order approximation are then given by*

$$\boldsymbol{\Psi}_{\tau} = (\boldsymbol{I} - \boldsymbol{E}^{\tau}_{\tau})^{-1} = \begin{bmatrix} \boldsymbol{\Psi}^{BB}_{\tau} & \boldsymbol{\Psi}^{BT}_{\tau} & \boldsymbol{\Psi}^{B\theta}_{\tau} \\ \boldsymbol{\Psi}^{TB}_{\tau} & \boldsymbol{\Psi}^{TT}_{\tau} & \boldsymbol{\Psi}^{T\theta}_{\tau} \\ \boldsymbol{\Psi}^{\theta B}_{\tau} & \boldsymbol{\Psi}^{\theta T}_{\tau} & \boldsymbol{\Psi}^{\theta\theta}_{\tau} \end{bmatrix} \approx \widetilde{\boldsymbol{\Psi}}_{\tau} = \boldsymbol{I} + \boldsymbol{E}^{\tau}_{\tau} \tag{37}$$

Using the first order approximation of the trade credit multiplier and the results of Lemma (6) to (11) implies that the partial equilibrium structural responses of credit costs, links and sectoral output can be expressed as follows:

Lemma 12. *Consider the partial equilibrium and abstract from any productivity shocks such that* $\widehat{L} = \epsilon^Q_m = 0 \ \forall m$ *. Then, to a first order approximation, the structural response of credit costs and credit links as well as the structural response of output are given by*

$$[\widehat{\boldsymbol{v}}]_i = \sum_{m=1}^{M} [\widetilde{\boldsymbol{\Psi}}^{v}_{B,\tau}]_{im} \epsilon^B_m \tag{38}$$

for $i \in \{k, ks\}$ *and* $\widehat{\boldsymbol{v}} \in \{\widehat{\boldsymbol{r}}^B, \widehat{\boldsymbol{r}}^T, \widehat{\boldsymbol{\theta}}, \widehat{\boldsymbol{q}}\}$ *and* $v \in \{B, T, \theta, Q\}$*. The respective elasticity matrices* $\widetilde{\boldsymbol{\Psi}}^{v}_{B,v}$ *are a linear combination of the equilibrium up-front financing costs* $(\overline{w\ell}_m, \overline{AP}^-_{mj})$ *and accounts receivable* $(\overline{AR}_{jm})$*:*

$$[\widetilde{\boldsymbol{\Psi}}^{\cdot}_{v}]_{im} = [\widetilde{\boldsymbol{\kappa}}_{B,v}]_{im} + [\boldsymbol{E}^{B(L)}_{B,v}]_{km} \overline{w\ell}_m + \sum_{j=1}^{M} [\boldsymbol{E}^{B(X)}_{B,v}]_{i,mj} \overline{AP}^-_{mj} + \sum_{j=1}^{M} [\boldsymbol{E}^{T(X)}_{B,v}]_{i,jm} \overline{AR}_{jm} \tag{39}$$

where $\overline{AP}^-_{mj} = (1 - \overline{\theta}_{mj}) \overline{p}_j \overline{x}_{mj}$ *and* $\overline{AR}_{jm} = \overline{\theta}_{jm} \overline{p}_m \overline{x}_{jm}$*. The elasticity matrices are a combination of the elasticity matrices presented in Lemma (6) to (11).*

As evident from Lemma 12, the relationship between the total up-front financing needs and trade credit extended - the net-lending position of a sector - affects the elasticities of the variables of interest with respect to financial shocks. I therefore define the net-lending position of a sector as the ratio of total trade credit extended to customers (accounts receivable) and the difference between total cost of production and accounts payable (see Definition 1). This immediately raises the following question: What is the effect of a sector's net-lending position on changes in sectoral output in response to a financial shock? In partial equilibrium, the sectoral output response is ultimately a function of the structural output wedge. Combining the results of Lemma (6) to (12), I conclude this section with the following proposition:

Proposition 1. *Let sector m be a net-borrower (net-lender) in the economy. Then, the adjustment of trade credit costs and credit shares will smooth (amplify) the negative effect of an increase in the bank-rate affecting sector m on (partial equilibrium) output of sector k.*

In other words, if a sector is affected by a financial shock whose (weighted) upfront total financing costs are larger than the (weighted) trade credit extended to customers - the sector classifies as a net-borrower (net-lender) according to Definition 1 - then trade credit has a smoothing (amplifying) effect. This section highlights that the existence of trade credit in an economy distorts the transmission of financial shocks and has both smoothing and amplifying effects. In particular, as Proposition 1 highlights, the trade credit usage of a sector also plays an important role for the propagation of idiosyncratic shocks to the risk-premium on bank credit. Ultimately, the question of which aspect of the trade credit channel is more operative in an economy and how it affects aggregate fluctuations needs to be answered quantitatively.

4. A Quantitative Assessment of Trade Credit and Aggregate Fluctuations

In this section, I apply the framework introduced in this paper to the US-Economy to quantitatively evaluate the effects of trade credit linkages on business cycle comovement and aggregate fluctuations during the 2008-2009 financial crisis. In particular, I provide answers to the following two questions: (1) Did the interfirm credit network amplify or smooth financial shocks during the Great Recession in comparison to an economy without trade credit?, and (2) To what extend does the trade credit channel - the endogenous adjustment of trade credit volumes and prices - contribute to aggregate fluctuations?

To this end, I first calibrate the equilibrium of the model at a sectoral level in Section 4.1, using the model's first order conditions. The US-trade credit network is mapped based on the input-output tables provided by the Bureau of Economic Analysis and by using balance sheet data on accounts receivable and payable of a panel of US-firms from Compustat. Sectoral credit spreads derived in Gilchrist and Zakrajšek (2012) are used to calibrate and impute the sector-specific shocks to the risk-premium of the bank interest rate in Equation (10). The imputed financial shocks are then used to simulate the model economy in Section 4.2, while abstracting from any shocks to sectoral productivity. A comparison of the model-implied responses of the credit portfolio of sectors and aggregate output with the fluctuation patterns presented in Section 2, provides a first assessment of the predictive ability of the model in Section 4.2.1. I then quantify the role of trade credit for business cycle comovement and aggregate fluctuations during the crisis in Section 4.2.2. For this purpose, I first simulate an equivalent economy without trade credit linkages and then conduct a partial equilibrium analysis in order to provide an answer to both questions posed at the beginning of the section.

4.1. Calibration Strategy

Production and Financial Network Data. The static nature of the model and its analytical tractability allow me to conduct a period-by-period calibration of the equilibrium of the US-economy at a sectoral level[16] using the model's first order conditions. The input-output tables from the Bureau of Economic Analysis (BEA) are used to map the production structure of the US-economy at the 3-digit NAICS industry level at an annual frequency, covering the time period 1997-2016. In total 45 sectors (excluding FIRE) are included in the analysis. While data on the production structure of the US-economy is readily available, data on trade credit flows between production units at a firm or sectoral level are not[17]. In order to overcome these data-limitations on the inter-firm credit network, I construct a proxy of inter-industry credit flows using the approach suggested in Altinoglu (2018). The balance sheet data of a panel of US-firms[18] from Compustat are used to calculate the share of accounts payable in total input expenditures (θ_k^P) and

[16]Due to the paucity data at a firm level, I conduct the quantitative analysis at a sectoral level, which does not affect the qualitative implications of the model.

[17]An exception for the US is Costello (2017), who employs proprietary transaction data from Credit2B; proprietary Firm-to-Firm transaction data are also used in recent empirical contributions by Jacobson and von Schedvin (2015) for Sweden, Dewachter et al. (2018) for Belgium, Giannetti et al. (2018) for Italy or Cortes et al. (2018) for Brazil.

[18]The sample description and selection criteria are discussed in Appendix D.

the share of accounts receivable in total revenues (θ_s^R). The inter-industry trade credit share from supplier s to customer k is constructed as a (sales) weighted average of the total trade credit shares

$$\theta_{ks} = \frac{R_s}{R_k + R_s}\theta_k^P + \frac{R_k}{R_k + R_s}\theta_s^R \tag{40}$$

and is non-zero if both sectors also engage in trade in intermediate inputs.

The second complication in mapping the model to the data is the consistent assignment of interest rate costs on bank and trade credit for the derivation of the production function parameters. The nominal intermediate input expenditures recorded in the IO-tables are net of any interest payments on bank credit associated with the transactions. Note that any interest payments on trade credit are part of the effective price paid, and are therefore already accounted for in the nominal intermediate expenditures shown in the IO-tables. Bank interest rate expenditures, however, are recorded as part of the gross operating surplus in the IO-tables net of interest-income. (see Horrowitz and Planting, 2009). I thus decompose the gross-operating surplus into capital expenditures, dividend payments and bank interest rate expenditures using the shares of the respective counterparts in gross operating profits calculated from the income statements of the panel of US-firms from Compustat. Only then, using an iterative procedure, I can consistently calibrate the parameters of the production function (1) - the labor, intermediate input shares and returns to scale parameter - using the first order conditions of the model presented in Lemma 1. Details on the calibration procedure and adjustments can be found in Appendix D.

Data on total hours worked and sectoral prices are provided by the Bureau of Labor Statistics (BLS). Total hours worked are then used to infer an aggregate wage rate from total labor expenditures recorded in the IO-tables. In order to ensure consistency with the model, where the wage rate is chosen as the numeraire, all prices are normalized by the common wage rate. The sector specific prices are treated as the input prices net of any interest cost on trade credit and are used to construct the respective quantities. The price of the final good is constructed using the results of profit maximization problem of the final good producer. Since capital owned by firms is included into the model as a constant and set to its steady state level, the real interest rate on capital implied by a time preference rate of $\beta = 0.96$ is an annualized risk-free rate of 4 percent. The household's preference parameters, ϵ_L and ϵ_C are set such that $\epsilon_C : \epsilon_L = 0.4$, which implies that ϵ_L and ϵ_C vary around the values 0.2 and 0.5, a standard calibration used in macroeconomics. While the calibration of the production parameters, prices and quantities is rather straight forward, in the remainder of this section I now discuss (a) the calibration of the parameters of the credit management cost function in Equation (9) and (b) the imputation of the shocks to the sector specific risk-premium on bank credit as in Equation (10).

Credit Costs and Financial Shock Identification. First, the expenditures non-productive labor or credit management costs, $C_k^T = w\ell_k^T$, are calibrated to be a share of total sectoral labor expenditures recorded in the IO-tables. The sector-specific share is set equal to the share of combined intermediate expenditures on management (NAICS = 55) and administrative services (NAICS = 561) in total sectoral intermediate input costs. The remaining parameters of the credit management cost function (9) - $\{\kappa_k^B, \kappa_{0,ks}^T, \kappa_{1,ks}^T\}$ - are calibrated as follows. First, I rearrange Equation (24) by taking the wage rate as a numeraire and replace $\kappa_{0,ks}^T = \overline{\kappa}_0^T \forall k, s$ and $\kappa_{1,ks}^T = \overline{\kappa}_1^T \forall k, s$. Additional manipulation and rearranging yields Equation (41)

$$\theta_k^P \;=\; \left[\overline{\theta}_k^S - (\overline{\theta}_k^S)^2 \frac{\overline{\kappa}_0^T}{\overline{\kappa}_1^T}\right] + \left[\frac{(\overline{\theta}_k^S)^2}{\overline{\kappa}_1^T}\right] p_k^E \;=\; \beta_{0k} + \beta_{1k} p_k^E \tag{41}$$

where

$$\theta_k^P \;=\; \frac{\sum_{s=1}^M \theta_{ks} p_s x_{ks}}{\sum_{s=1}^M p_s x_{ks}} \;\text{ and }\; p_k^E \;=\; \underbrace{\left[\frac{\sum_{s=1}^M (p_s x_{ks})^2 \Delta_{ks}}{(\sum_{s=1}^M p_s x_{ks})^2}\right]}_{(1)} \underbrace{\left[\frac{\sum_{s=1}^M p_s x_{ks}}{p_k^V V_k}\right]}_{(2)} \frac{p_k^V}{(1 + r_k^B)} \tag{42}$$

Note that θ_k^P is simply the share of aggregate accounts payable in total intermediate cost of production excluding interest rate payments. The variable p_k^E can be interpreted as the discounted $(1 + r_k^B)$ marginal cost of producing one unit (p_k^V) multiplied by the (1) difference in the sector-specific credit expenditure herfindal index and (2) the share of intermediate expenditures excluding credit costs in total productive input expenditures. The data-counterparts are derived using the steady-state values of the corresponding variables derived as described above. The (link-specific) parameters are then calibrated using the estimated coefficients $\widehat{\beta}_0$ and $\widehat{\beta}_1$ by running a simple OLS regression of Equation (41) as described in Appendix D. The remaining parameters of the credit management cost function can be calculated using Equation (24) and (9). The interest rate on trade credit is inferred directly from the first order condition. The sector specific interest rate on bank credit is assumed to take on the following functional form as discussed in

Section 3:

$$r_{kt}^{B} \;=\; r_{0}^{B} + \exp(z_{kt}^{B})\left(\overline{\theta}_{0}^{D} + \theta_{kt}^{C}\right)^{\mu} r_{0}^{B} \;=\; r_{0}^{B} + r_{kt}^{GZ} \tag{43}$$

In other words, each sector is charged a risk premium over the federal funds rate. As a baseline measure for the risk-premium, I employ the sectoral credit spreads derived in Gilchrist and Zakrajšek (2012) and provided to me by the authors, adjusted to match the calibrated bank-interest rate expenditures imputed from the IO-tables. The "GZ-spread" is the defined as the sectoral average of differences in the yields on corporate bonds and yields on Treasury securities of comparable cash flows and maturities. As outlined in Section 3 Assumption 4, the risk premium is assumed to be a convex function of the average leverage in the economy and the average trade credit share extended to customers. The components of the risk-premium are calibrated as follows: First, the risk-free interest rate on bank credit, r_0^B, is set by calculating the time average (1997-2016) of the federal funds rate. And second, the average leverage in the economy, $\overline{\theta}_0^D$, as well as the sectoral average trade credit share, θ_{kt}^C, can be directly calculated from the data. The exponent, μ, is estimated using a simple OLS-regression based on Equation (43) and is set to 1.2. The implied shocks to the risk premium on bank credit, z_{kt}^B, can then be constructed directly from Equation (43). As a result, the calibrated equilibrium interest rates on trade credit exceeds the interest rate on bank credit for the majority of sectors, thereby mapping the empirical observation on the relative cost of supplier and bank finance discussed in Cuñat and García-Appendini (2012).

At this point, it should be highlighted that three parameters are central in determining the magnitude of fluctuations in the economy with both bank and trade credit. The degree of convexity of the risk premium in the joint default probability measure, μ, as well as the average leverage in the economy, $\overline{\theta}_0^D$, determine (1) the relative size of the equilibrium interest rate on trade credit and (2) its volatility in response to bank credit shocks. In other words, an increase in the convexity of the mark-up function and an increase in the relative importance of the average trade credit share extended to customers for the risk premium, increases the level and volatility of the trade credit interest rate, thereby reinforcing the trade credit channel. Similarly, a decrease in the adjustment cost parameter $\kappa_{1,ks}^T$, increases the extend to which firms are able to adjust the composition of their borrowing portfolio between bank and trade credit, thereby increasing the substitution effect and therefore smoothing aspect of the trade credit channel. Table D.2 provides summary statistics on the calibrated parameters.

4.1.1. Properties of the US Production and Credit Network

I now summarize selected characteristics of the imputed (endogenous) trade credit network of the US-Economy and contrast these with those of the complementary production-network. Note that while I assume the production structure $\mathbf{\Omega}$ to be constant, the credit network, $\mathbf{\Theta}_t$, is endogenous along the intensive margin and thus varies over time. I first define a sector's average input- and credit-demand-share as the row sum of $\mathbf{\Omega}(\mathbf{\Theta})$ divided by the number of suppliers and a sector's average sales- and credit-supply-share as the column sum of $\mathbf{\Omega}(\mathbf{\Theta})$ normalized by the number of customers. By following standard graph theoretical terminology (see Carvalho, 2010), the first measure is labelled the (weighted) in-degree, d_k^I, and the second measure refers to the (weighted) out-degree of sector k, d_k^O.[19]

Figures 5a and 5b plot the distribution of the average demand- and supply-shares normalized by the median share of both networks in 2007[20], highlighting a well known feature of the US-production network (see Carvalho, 2010): While the US-sectors exhibit a considerable degree of homogeneity along the extensive margin of sectoral demand, US-sectors are heterogeneous in their role as input suppliers. These characteristics translate to the approximation of the US-credit network due to the complementarity of both networks. Panel (c) of Figure 5 plots the distribution of the average net-lending position of sectors in 2007 and highlights that the distribution of sectoral trade credit policies exhibits the same properties as that of the sample of Compustat firms shown in Figure 2. However, while there are a few firms in the sample that extend more credit to their customers than their up-front financing needs, I do not find sectors with a net-lending position strictly greater than one. Nevertheless, I still refer to sectors with a high ratio of accounts receivable to bank credit as net-lenders.

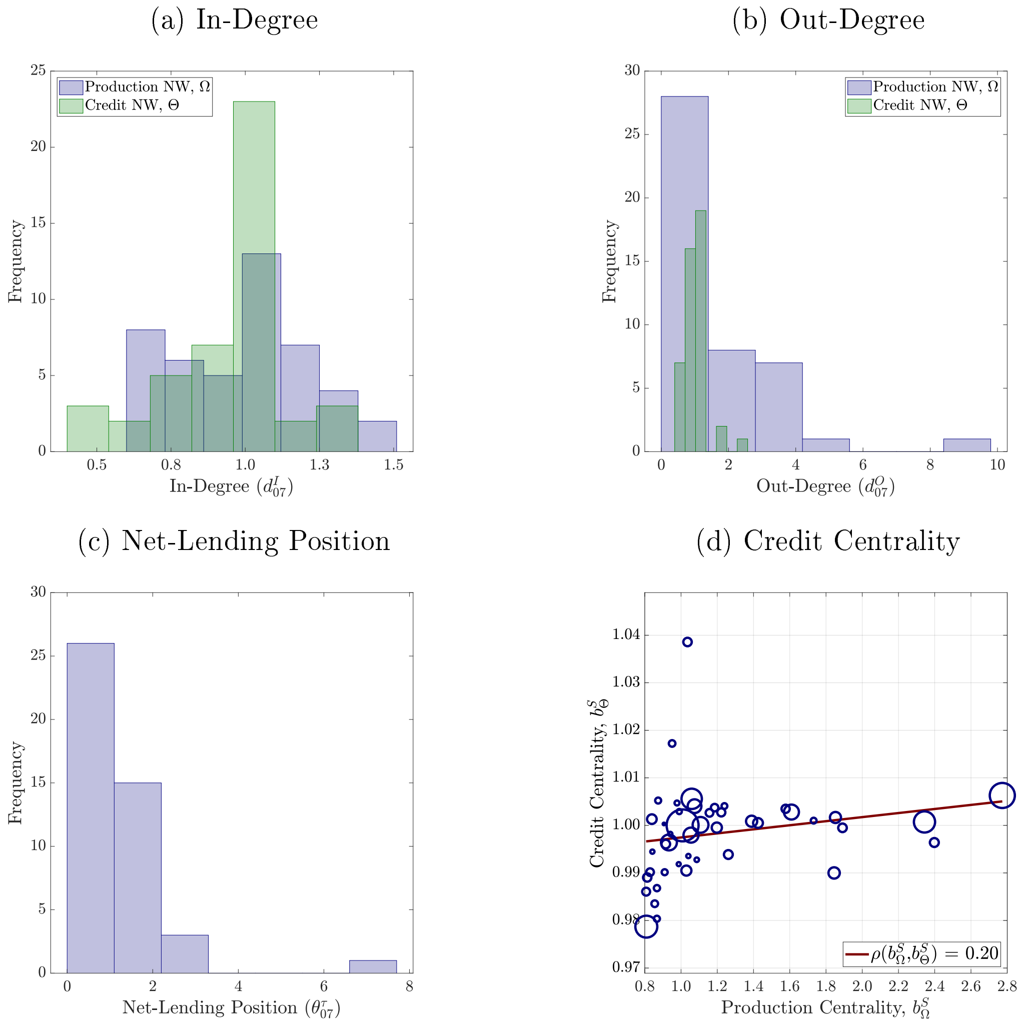
The complementarity of the network structures may raise the concern that mainly the production rather than the credit linkages among sectors affect the propagation of liquidity shocks. To this end, I consider the network concept of the (weighted) Bonacich centrality, $b^{D(S)_k}$. It describes the systemic importance of a sector based on the total

[19] Note that due to the complementarity of the production and the credit network - e.g. a credit link between sector k and s exists if and only if sectors also engage in intermediate input trade - the cardinality of a sector's set of customers and suppliers for each network exhibit (almost) perfect correlation. Therefore, rather than analysing the extensive margin of each network structure, I focus on a combined measure capturing both the extensive and intensive margins of sectoral trade in goods and credit.

[20] It should be noted that while the qualitative features of the US production and credit network are constant over time, I chose the year prior to the crisis to highlight these patterns.

weighted number of walks between two sectors and is similar to the concept of the Leontief-Inverse common to any input-output model. Panel (d) of Figure 5 plots the relation between the production centrality and the credit centrality of sectors in 2007 and Table 2 reports the correlation of network properties.

Figure 5: Properties of the US-Production and Credit Network (2007)



Note: The network statistics have been calculated using the calibrated data on the production parameters and trade credit shares of 2007 as derived in Section 4.1 and have been normalized by the median. Panel (a) and (b) plot the distribution of the average demand- (In-Degree) and supply-shares (Out-Degree) of the production and the credit network. Panel (c) depicts the distribution of the average net-lending position as defined in 1. Panel (d) plots the relation between the production centrality and the credit centrality of sectors. A more detailed definition of the standard graph theoretical statistics can be found in Appendix D.

Table 2: Correlation of Network Properties (2007)

2007	$d^I_{\mathbf{\Omega}}$	$d^I_{\mathbf{\Theta}}$	$d^O_{\mathbf{\Omega}}$	$d^O_{\mathbf{\Theta}}$	$b^D_{\mathbf{\Omega}}$	$b^D_{\mathbf{\Theta}}$	$b^S_{\mathbf{\Omega}}$	$b^S_{\mathbf{\Theta}}$
$d^I_{\mathbf{\Theta}}$	-0.28							
$d^O_{\mathbf{\Omega}}$	-0.36	-0.04						
$d^O_{\mathbf{\Theta}}$	-0.28	0.39	0.09					
$b^D_{\mathbf{\Omega}}$	0.18	0.06	-0.21	0.12				
$b^D_{\mathbf{\Theta}}$	-0.38	0.95	0.01	0.40	-0.02			
$b^S_{\mathbf{\Omega}}$	-0.35	-0.01	0.94	0.03	-0.19	0.00		
$b^S_{\mathbf{\Theta}}$	-0.26	0.31	0.18	0.68	0.21	0.35	0.20	
θ^τ	-0.39	0.34	0.29	0.30	0.00	0.20	0.49	0.35

Note: This table reports the correlation between the following network properties of the calibrated US-production $\mathbf{\Omega}^X$ and credit network $\mathbf{\Theta}$ in 2007 as derived in Section 4.1: weighted demand- (In-Degree,d^I_k) and supply-shares (Out-Degree,d^I_k), demand-(b^D_k) and supply-centrality(b^S_k), net-lending position θ^τ_k as defined in Definition 1. A more detailed description of the standard graph theoretical statistics can be found in Appendix D.

Figure 5 and Table 2 highlight three properties of the production structure and the calibrated trade credit network in the US

(1) There is heterogeneity among sectors when supplying goods and credit to their customers. In particular, the distribution of sales- and credit-supply shares is heavily skewed to the left such that only a few sectors act as major suppliers of goods and credit in the US-economy.

(2) The systemic importance of a sector based on the production structure is weakly positively correlated with it's credit centrality. In other words, a sector which plays a central role in the trade of goods is also more likely to play a central role in the provision of supplier credit.

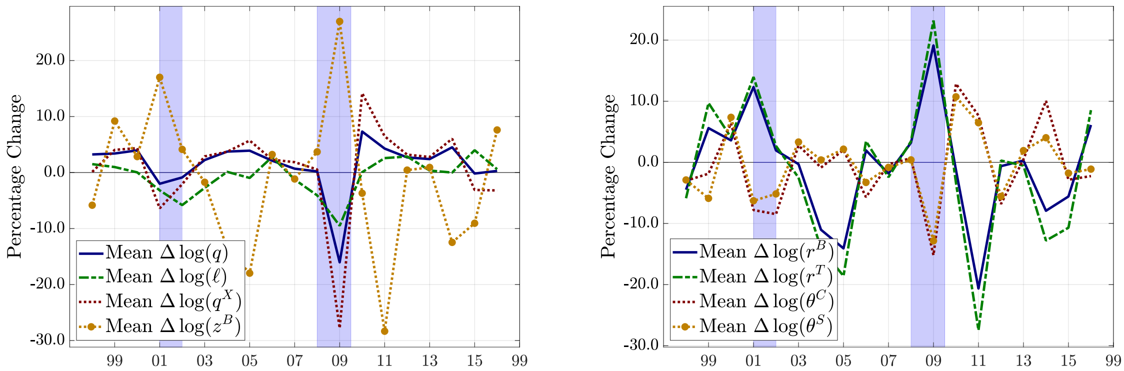
(3) The net-lending position of a sector is positively correlated with the overall systemic importance of a sector as a supplier of credit.

4.1.2. Business Cycle Statistics

Following the period-by-period calibration of the equilibrium of the model discussed at the beginning of this section, I now document business cycle statistics for selected real and financial variables. Panel (a) of Figure 6 plots the sectoral mean of the implied banking shock and of the log-change of selected production inputs across time. Panel (b) of Figure 6 plots the average log changes in the interest rates on bank and trade credit as well as in the trade credit shares. The sample period covers two recessions: the dotcom-crash in 2001 and the 2008-2009 financial crisis.

Figure 6: Data - Mean Changes

(a) Real Variables and Financial Shocks (b) Financial Variables



Note: This figure plots the log change in percent of the calibrated time series of selected real and financial variables derived in Section 4.1 from 1998-2016. Panel (a) plots the sectoral mean of the implied financial shock (z_k^B), the log-change of output (q_k), labor (ℓ_k) and the intermediate input composite (q_k^X). Panel (b) plots the average log change in the interest rate on bank (r_k^B) and trade credit (r_k^T) as well as in the trade credit shares (θ_k^C, $\theta_k^S = \sum_{s=1}^{M} \theta_{ks}/M$).

As documented in Section 2, real US-GDP dropped by 2.5% during the crisis. Figure 6a documents that the implied shock to bank-interest rate risk premia rose by 26.7% lead to an increase of bank and trade credit interest rates by 19.1% and 23.2%, respectively. Average sectoral output declined by approximately 16.0% caused by a drop of total labor and the composite intermediate input of 9.5% and 27.8%, respectively. At the same time, the average trade credit share extended to customers decline by 15.1% and the average share of intermediate expenditures obtained on trade credit dropped by 13.2%. For each sector, I now calculate the standard deviation of log changes in the variables of interest. I also derive the within sector correlation between (a) log changes in output and (b) log changes in the cost of bank credit and the remaining real and financial variables. The cross-sectional mean of the business cycle statistics are reported in Table 3. In addition, I split the sample of sectors based on their net-lending position: a sector is counted as a net-lender if its net-lending position is above the median of the distribution of net-lending shares.

Table 3: Data - Time-Series Correlation

(a) Real Variables

	Total (97-16)				
	VAR	All	NB	NL	p-Value
MEAN	q	0.014	0.017	0.011	0.324
	ℓ	-0.006	0.000	-0.013	0.020
	q^X	0.008	0.016	-0.000	0.068
	z^B	-0.009	-0.010	-0.008	0.767
STDV	q	0.069	0.065	0.072	0.628
	ℓ	0.065	0.049	0.080	0.188
	q^X	0.140	0.131	0.149	0.586
	z^B	0.149	0.137	0.161	0.192
CORR	(q,ℓ)	0.557	0.600	0.512	0.444
	(q,q^X)	0.809	0.781	0.838	0.325
	(q,z^B)	-0.417	-0.382	-0.453	0.315
	#OBS	45	23	22	

(b) Financial Variables

	Total (97-16)				
	VAR	All	NB	NL	p-Value
MEAN	r^B	-0.007	-0.010	-0.005	0.202
	r^T	-0.015	-0.014	-0.016	0.752
	θ^C	-0.006	-0.010	-0.002	0.267
	θ^S	-0.005	-0.008	-0.001	0.134
STDV	r^B	0.115	0.114	0.117	0.822
	r^T	0.160	0.148	0.173	0.301
	θ^C	0.153	0.145	0.161	0.517
	θ^S	0.112	0.098	0.127	0.150
CORR	(r^B,r^T)	0.921	0.957	0.885	0.033
	(r^B,θ^C)	-0.246	-0.286	-0.204	0.311
	(r^B,θ^S)	-0.318	-0.396	-0.237	0.042
	#OBS	45	23	22	

Note: This table reports the time-mean of the cross-sectional mean, the standard deviation and the correlation with output and the bank interest rate of the log-change of the following calibrated variables: output (q_k), labor (ℓ_k), the intermediate input composite (q_k^X), the interest rate on bank (r_k^B) and trade credit (r_k^T), the trade credit shares (θ_k^C, $\theta_k^S = \sum_{s=1}^{M} \theta_{ks}/M$). The first column reports the business cycle statistics for the entire sample. The second and third column report the same statistics for a subgroup of sectors based on the net-lending position Definition 1. The p-values for the differences in means between the two groups are reported in the last column.

Output, Labor and Intermediate Inputs. Over the entire sample period, 1997-2016, average sectoral output exhibits a volatility of around 7%. Total sectoral hours worked and the intermediate composite show a standard deviation of approximately 6.5% and 14.0%, respectively. In other words, on average, labor demand is less volatile whereas the demand for the composite intermediate inputs is more volatile than output. Furthermore, log changes in output are positively correlated with both changes in production inputs. The business-cycle statistics of the calibrated model on sectoral output, labor and the intermediate composite for the entire sample are similar to those reported in BL(2017). I now take a closer look at the mean and the standard deviation of log changes in both subsamples of sectors. The p-values for the differences in means between net-borrowing and net-lending sectors suggest that while there is no significant difference in the volatility or output-correlation, the two groups of sectors seem to differ in the level of employed labor and intermediate inputs at a 10% significance level.

Cost of Credit and Trade Credit. The imputed sectoral interest rates on bank and trade credit exhibit a standard deviation of 11.5% and 16.0%, respectively and are thus both more volatile than sectoral output. Table 3 indicates that the implied interest rate on trade credit is more volatile than the interest rate on bank credit, which relates to the stylised facts presented in Section 2 on the relative volatility of accounts payable and liabilities. In addition, both interest rates comove strongly. The average trade credit share extended to customers, θ^C, and obtained from suppliers, θ^S, are more volatile than sectoral output but less volatile than either costs of credit. In addition, they are negatively correlated with the cost of bank-finance such that an increase in the cost of bank credit decreases either shares. This is due to the fact that while the interest rates on bank and supplier credit comove strongly, the latter exhibits a higher standard deviation. Consequently, firms shift their borrowing portfolio towards bank finance in response to an increase in credit market frictions as discussed in Section 2. Interestingly, the correlation between the cost of bank credit and the interest charged on trade credit as well as the trade credit obtained from suppliers seems to be significantly higher for the group of sectors classified as net-borrowers at a 5% significance level. In other words, as net-borrowers face a higher cost of bank finance, they are more likely to increase their lending rates and shift more towards bank-finance. This suggests that the substitution effect of trade credit discussed in Section 2 may be more pronounced for net-borrowers.

4.2. Quantitative Application

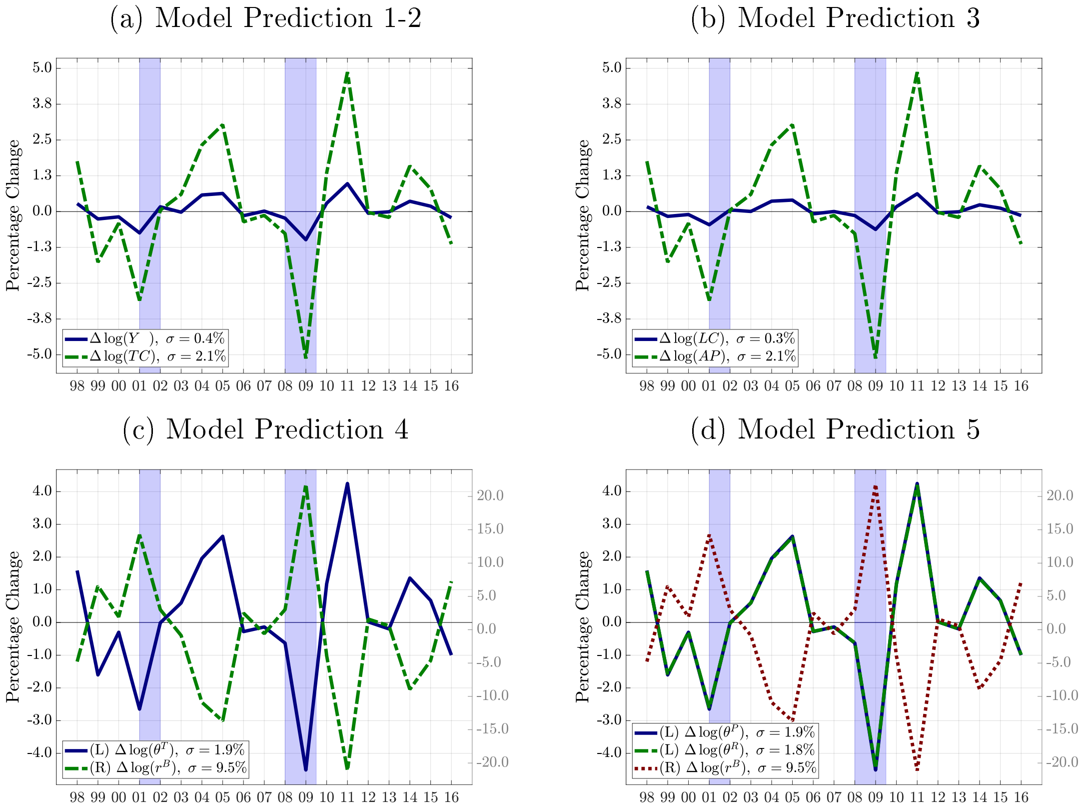
This section evaluates the role of interlinked distortions implied by trade credit linkages among firms for business cycle comovement and aggregate fluctuations through the lens of the model. The model-implied time series are obtained by feeding in the financial shock series imputed from the GZ-credit spreads and solving numerically for the equilibrium of the static economy. Any additional variation originating from changes in (1) production and financial parameters, (2) capital and (3) productivity and foreign trade shocks is excluded by keeping the respective variables at their four-year-average (2004-2007) prior to the crisis. The data-counterparts of the variables of interest are obtained via direct period-by-period calibration of the equilibrium of the static economy presented in Section 3 as described in Section 4.1.

I first examine the model performance based on its ability to re-produce fluctuation patterns of the variables of interest observed in the data for the sample period (1997-2016) discussed in Section 2. With a particular focus on the Financial Crisis of 2008-2009, I then quantitatively assess the role of trade credit linkages for the propagation of financial shocks by introducing the concept of the Trade Credit Multiplier.

4.2.1. Business Cycle Properties Through the Lens of the Model

In order to provide a first assessment of the ability of the model to generate business cycle patterns of both real and financial variables shown in Section 2, I first reproduce Figure 1 using the model implied series only.

Figure 7: Model Implied Business Cycle Properties of Trade Credit



Note: The panels in this figure replicate the graphs presented in Figure 1 and plot the evolution of the log change in percent of the simulated time series of aggregate US-GDP (Y), Accounts Payable (AP), Accounts Receivable (AR), Current (LC) Liabilities, the share of AP in Current Liabilities (θ^T), the aggregate GZ-spread (GZ), the share of AP in Total Costs of Goods Sold (θ^P) and the share of AR in Total Sales (θ^R). The model simulations are based on financial shocks only. The figures also report the standard deviation of the respective series in percent.

As evident from Figure 7, the model indeed reproduces *qualitatively* the business cycle features of trade credit and the changes in the short-term borrowing portfolio observed in the data when accounting for the timing restrictions discussed Section 2. Thus, the model simulated series imply that in response to financial shocks to the bank risk premium only: (M1) The growth rate of the volume of trade finance is pro-cyclical with the growth rate of current real GDP; Trade credit is more volatile than the growth rate of (M2) total value added and (M3) firms' liabilities. In particular, the model also replicates a key feature of the recent financial crisis: (M4) As credit spreads rose during the financial crisis, the supplier credit market contracted immediately and firms drew down their bank credit lines substituting supplier with bank credit. Furthermore, the model also predicts that (M5) the share of accounts payable and receivable in total costs of production and sales are strongly positively correlated and are strongly negatively correlated with aggregate credit spreads in the economy.

Quantitatively, the average simulated IO-adjusted risk-premium features 19.9% of the volatility of the aggregate GZ-spread. The model simulations based on the adjusted financial shock series demonstrate that the model is able to account for 29.8% of the variation in output, 13.3(6.0)% of the variation in total (current) liabilities and 39.2% of the variation in supplier credit. Taking a closer look at the credit composition, the model also reproduces 57.7% of the fluctuations in the credit composition of short-term borrowing and approximately 30.6(36.7)% of the variation in the share of trade credit in total costs and sales, respectively. I thus conclude that the model is a reasonable tool for the analysis of trade credit linkages and the effect on business cycle comovements and aggregate fluctuations.

Table 4 presents the time series correlation for selected aggregate real and sector-level financial variables with their counterpart in the data across time, using a 10-year-rolling window. The correlation between the model-implied growth rate of aggregate GDP (labor) and the actual rate observed in the data is approximately 63(61)% on average across time. The model fits particularly well during later years in the sample.

Table 4: Model Fit - Time-Series Correlation

Year	2008	2009	2010	2011	2012	2013	2014	2015	2016	Mean	Std	Min	Max
Crisis	0.22	0.40	0.50	0.40	0.30	0.30	0.30	0.30	0.30	0.34	0.08	2008	2010
$\Delta\log(Y)$	0.43	0.40	0.38	0.75	0.76	0.73	0.73	0.77	0.74	0.63	0.17	2010	2015
$\Delta\log(C)$	0.29	0.26	0.26	0.69	0.75	0.73	0.73	0.78	0.76	0.58	0.24	2009	2015
$\Delta\log(L)$	0.34	0.34	0.34	0.70	0.75	0.79	0.75	0.73	0.74	0.61	0.20	2010	2013
$\Delta\log(\Phi^Z)$	0.61	0.61	0.75	0.76	0.66	0.57	0.58	0.62	0.54	0.63	0.08	2016	2011
$\Delta\log(\Phi^L)$	0.42	0.39	0.46	0.58	0.43	0.45	0.48	0.47	0.41	0.45	0.05	2009	2011
$\Delta\log(AR)$	0.70	0.70	0.76	0.89	0.81	0.80	0.79	0.78	0.81	0.78	0.06	2009	2011
$\Delta\log(AP)$	0.61	0.59	0.69	0.82	0.81	0.78	0.80	0.78	0.80	0.74	0.09	2009	2011
$\Delta\log(r^T)$	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	2010	2013
$\Delta\log(\theta)$	0.64	0.64	0.64	0.77	0.71	0.71	0.69	0.69	0.74	0.69	0.05	2009	2011
$\Delta\log(\theta^C)$	0.53	0.53	0.53	0.76	0.67	0.70	0.66	0.65	0.69	0.63	0.09	2009	2011
$\Delta\log(\phi^X)$	0.98	0.98	0.98	0.99	0.99	0.99	1.00	1.00	1.00	0.99	0.01	2010	2016

Note: This table presents the time series correlation for selected aggregate real and sector-level financial variables with their counterpart in the data across time, a using a 10-year-rolling window. The indicated years represent the end-dates of a 10-year-rolling window used to calculate the time-series correlation. The variable *crisis* shows the frequency the US-economy spent in crisis based on NBER-recession dates during the 10-year-period.

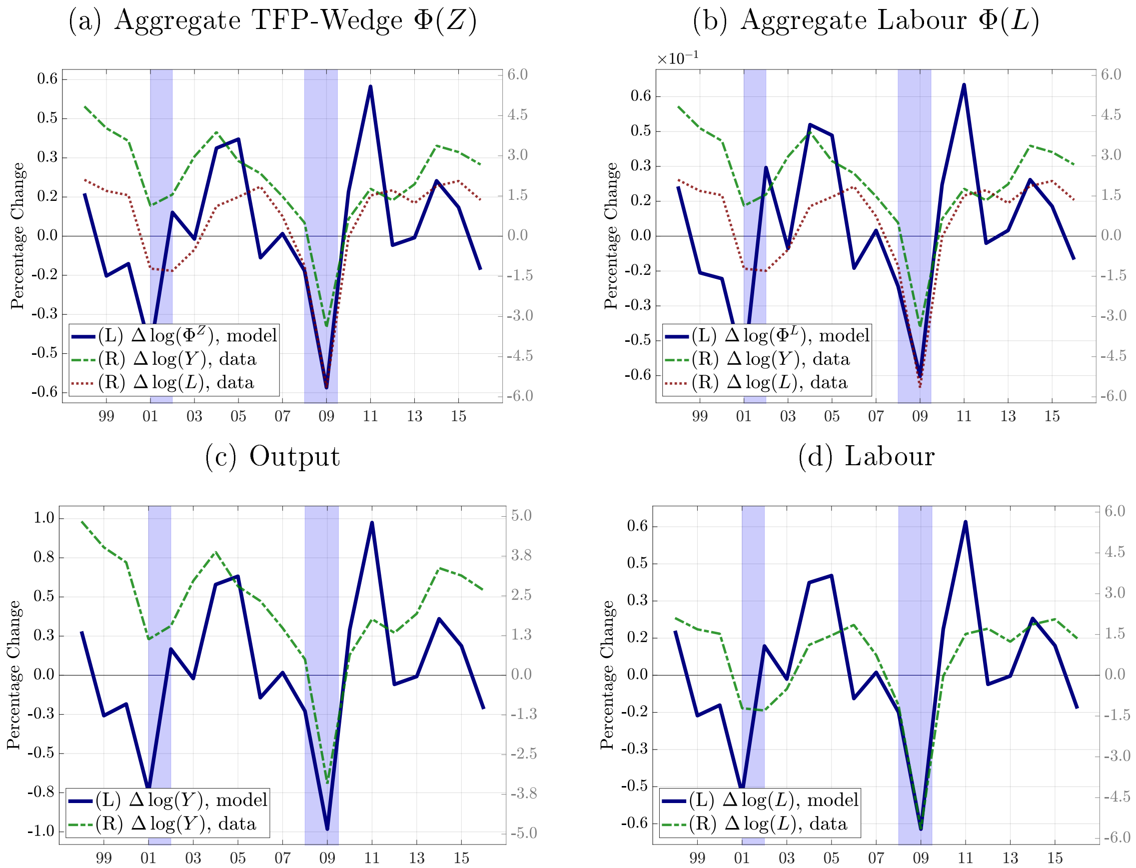
4.2.2. The Role of the Credit Network during the Great Recession

In the previous paragraphs I have shown that the model is able to reproduce business cycle patterns of trade credit similar to those observed in the data. I now evaluate the effects of trade credit linkages on aggregate distortions and business cycle fluctuations in the US-economy during the 2008-2009 financial crisis in order to provide answers to the questions posed at the beginning of the section: (1) Did the interfirm credit network amplify or smooth financial shocks during the Great Recession in comparison to an economy without trade credit?, and (2) To what extend does the trade credit channel contribute to aggregate fluctuations?

Financial Frictions and The Business Cycle. The existence of a working capital constraint for firms generates a demand for ex-ante liquidity which is met by obtaining credit from both banks and suppliers. The costs of drawing credit lines from either divert funds from productive inputs which manifests itself as an aggregate efficiency and labor wedge as shown in Section 3. Panel (a) and (b) of Figure 8 plot the predicted percentage changes in the aggregate TFP and the aggregate labor wedge following in response to a shock to the cost of bank credit as well as the log-changes in observed aggregate output and labor measured against the right axis. As can be seen in the figures, the changes in either wedge co-move strongly with aggregate output and labor in the data. Panel (c) and (d) of Figure 8 present the model-predicted percentage changes of aggregate output and labor on the right axis against those observed in the data. The model predicts that changes in the financial frictions alone account for approximately 28.8% of the actual drop in output and 11.0% of the drop in labor during the 2008-2009 Great Recession.

As emphasised in BL(2017) and further discussed in the theoretical section, the cost of credit affect aggregate output through two channels: changes in the aggregate TFP and the labor wedge. A decomposition of the log changes in aggregate output and labor into contributions of either channel suggests that most of the changes are attributed to changes in the efficiency rather than the labor wedge. This result contrasts the findings in BL(2017) for two reasons. First, differences in the calibration strategy of aggregate prices and financial shocks may affect the relative importance either channel. Second, and more importantly, wedges are inter-dependent.

Figure 8: Model Fit



Note: Panel (a) and (b) in this figure plot the model implied log changes in the aggregate TFP and the aggregate labor wedge in response to shocks to the cost of bank credit only. The log changes of observed aggregate output and labor are measured against the right axis. Panel (c) and (d) plot both the log changes of aggregate output and labor as implied by the model simulations in response to shocks to the cost of bank credit on the right axis against those observed in the data. All log changes are reported in percent.

Having established that financial frictions are able to account for a non-negligible fraction of movements in aggregate variables, I now focus on the quantification of the role played by inter-linked distortions in the form of trade credit linkages among firms during the financial crisis. In particular, I conduct two main exercises: First, I compare the response of the variables of interest to the same financial shocks in an economy with both bank and trade finance to those generated by an otherwise equivalent economy with bank finance only. This allows me to quantitatively evaluate the effect of the existence of a credit network among firms on economic outcomes per se. Second, I decompose the general equilibrium response of the variables into their partial equilibrium counterpart derived by keeping both trade credit interest rates and shares at their steady state level. The difference between the general and partial equilibrium response can be attributed to the trade credit channel, highlighting the effect of the endogenous adjustment of the volume and cost of trade credit on the aggregate economy. In either scenario, the question remains whether credit relations among firms dampen or amplify financial shocks as discussed in Section 3.

TC-Multiplier. Since the model nests the economy presented in BL(2017) if no trade credit linkages are considered, the comparison of the predictions produced by a model with trade credit to the otherwise equivalent model without any credit linkages provides a clear way to disentangle the effects of the credit network from those of the inter-sectoral trade network alone. For this purpose, similar to BL(2017), I first define an equivalent economy, $\mathcal{E}(0)$, and the Trade Credit Multiplier as follows

Definition 5 (Equilvalent Economies). *Let $\mathcal{E}(0)$ be an equivalent economy to an economy with both production and credit linkages, $\mathcal{E}(\theta)$, with production linkages only. Then $\mathcal{E}(0)$ features the same observed input prices net of any credit costs and the same observed nominal sales, input expenditures and value added as in $\mathcal{E}(\theta)$.*

Definition 6 (Trade Credit Multiplier). *Let $\mathcal{E}(\theta)$ be an economy in which firms finance their working capital requirements with both trade and bank finance and let $\mathcal{E}(0)$ be the corresponding equivalent economy. Consider the same sector-specific shocks across both economies. The "trade credit multiplier" is the ratio between the percentage drop in aggregate output generated by an economy with both trade and bank finance and an equivalent economy with bank finance only.*

Then, I simulate both economies, an economy with bank and supplier credit, $\mathcal{E}(\theta)$, and an equivalent economy where firms finance their working capital requirements with bank credit only, $\mathcal{E}(0)$, using the same financial shocks to the sector-specific bank risk premium. The first row of Table 5 reports the percentage change in aggregate output, labor and both the efficiency and labor wedge in 2009 for the economy introduced in Section 3 and its equivalent counterpart. The resulting trade credit multiplier $\mathcal{M}$ ranges from 1.70 for aggregate labor to 2.25 for the aggregate efficiency implying that the credit network itself generates a considerable amplification of distortions. In an economy with bank finance only, a shock to the cost of external funds overall increases the cost production, thereby increasing prices and ultimately decreasing sectoral and aggregate output. As discussed in detail in Section 3.2, in an economy where firms finance their production using both bank and supplier credit, an increase in sectoral bank risk premia also translates into an increase in the cost of trade credit as shown in Lemma 11 such that total credit cost of production increase by more relative to an economy with bank credit only. The adjustment of a firm's trade credit shares towards bank finance during the crisis following Lemma 10 is not enough to undo the exacerbating effecting of credit linkages on the increase in aggregate volatility *relative* to an economy without credit relations among firms. This observation therefore translates into a trade credit multiplier greater than one as recorded in Table 5. Interestingly, the credit linkages reduce the effect of financial shocks on the aggregate labor wedge. This suggests that in the current calibration of the model, the credit network does not only increase the volatility of both the marginal product of labor and the real wage rate, it also increases their comovement, thereby reducing the volatility of the aggregate labor wedge.

Table 5: Counterfactual - Multipliers

CF	EN	$\Delta\%(Y)_{09}$	$\Delta\%(L)_{09}$	$\Delta\%(\Phi^Z)_{09}$	$\Delta\%(\Phi^L)_{09}$
TC0	$\mathcal{E}(\theta)$	-0.982	-0.622	-0.581	-0.060
	$\mathcal{E}(0)$	-0.494	-0.364	-0.258	-0.113
	$\mathcal{M}$	1.989	1.707	2.248	0.532
TCA	$\mathcal{E}(\theta)$	-0.982	-0.622	-0.581	-0.060
	$\mathcal{E}(0)$	-0.989	-0.621	-0.587	-0.053
	$\mathcal{M}$	0.994	1.001	0.989	1.133
NL	$\mathcal{E}(\theta)$	-0.046	-0.025	-0.029	0.003
	$\mathcal{E}(0)$	-0.008	-0.006	-0.004	-0.002
	$\mathcal{M}$	5.431	3.900	6.947	-1.486
NB	$\mathcal{E}(\theta)$	-0.203	-0.137	-0.114	-0.027
	$\mathcal{E}(0)$	-0.140	-0.104	-0.073	-0.033
	$\mathcal{M}$	1.444	1.320	1.561	0.812

Note: This table documents the model simulated log change of aggregate output (Y), labor (L), the aggregate efficiency (Φ^Z) and labor wedge (Φ^L) to shocks to sector-specific bank risk premia in an economy with bank and supplier credit, $\mathcal{E}(\theta)$, in an equivalent economy with bank credit only, $\mathcal{E}(0)$ and in an equivalent economy with symmetric and constant trade credit shares, $\mathcal{E}(\bar{\theta})$. The credit multipliers ($\mathcal{M}$) are calculated as the ratio of responses of the variable in $\mathcal{E}(\theta)$ to their counterparts in $\mathcal{E}(0)$ and $\mathcal{E}(\bar{\theta})$, respectively. The equivalent economies of the four counterfactual exercises considered are an economy with bank finance only (TC0); with constant and symmetric trade credit shares (TCA); (NL/NB) in which only net-lenders (net-borrowers) experience an increase in their bank interest rates using Definition 1. All log-changes are reported in percent.

TC-Mechanism. Having established that the credit network itself considerably amplifies the propagation of financial shocks, the question remains whether in an economy with both bank and trade credit the smoothing or the amplification mechanism of the trade credit channel dominates. In order to quantify the actual trade credit channel, I first calculate the partial equilibrium response of the variables of interest by keeping both the interest rate on trade credit as well as the trade credit shares at their steady state level. The difference between the general and partial equilibrium response can thus be attributed to the trade credit channel. As before, an increase in the cost of bank credit of sector k leads to the following adjustments of trade credit costs and volumes: First, the interest rate on trade credit charged by sector k increases which, in addition to any shocks to the bank risk premium of k's customers, will further increase the cost of production of all customers of sector k. (see Lemma 11 and 6) As such, for given credit shares the increase in the cost of supplier credit amplifies idiosyncratic shocks to the interest rate on bank credit. Second, since the model generates a more volatile response of the cost of trade than the cost of bank credit, firms shifted their borrowing portfolio towards bank finance (see Lemma 10 and Figure 7c) thereby dampening the effect of the increase in the cost supplier finance. The model predicts that the trade credit channel introduced in Section 3.2 reduces aggregate volatility by 1.78% which implies that the ability of firms to substitute trade with bank credit and vice versa dominates the network effect.

Network-TC and Shock Heterogeneity. In the last part of this quantitative exercise, I examine the role of heterogeneity of trade credit linkages on the trade credit multiplier defined in 6 by conducting two additional counterfactual exercises. I first evaluate the importance of the asymmetry of credit linkages for the propagation of liquidity shocks by comparing the model's response against the response of an equivalent economy with constant interest rates on trade credit and constant and symmetric trade credit shares. The second row of Table 5 highlights that the implied trade credit multiplier is close to one which suggests that the asymmetry in the credit link intensity only plays a minor role in the propagation of liquidity shocks. However, as highlighted in Section 2, there is heterogeneity in the net-lending position of sectors defined as the ratio of accounts receivable to bank credit. In order to evaluate the relevance of asymmetries in the trade credit usage of sectors for the propagation of liquidity shocks, I conduct the following exercise: I first identify the top five net-borrowers and the top five net-lenders[21] based on the net-lending position (see Definition 1) of sectors calculated from Compustat directly. Notably, as discussed in Section 2, the set of net-lenders is characterised by being more upstream in the supply chain of the US-Economy while the top net-borrowers are closer to the end consumer. I then feed in a symmetric shock series calculated as the average shock to sectoral risk-premia that affects only one group of sectors at a time. The results of this exercise are reported in the last two rows of Table 5 and highlight that the aggregate credit multiplier is higher if sectors that extend

[21] The sector IDs of the top net-lenders are { 3361MV, 486, 514, 331, 211 } and the top net-borrowers { 55, 445, 452, 62, GOV }.

relatively more trade credit than their upfront financing needs face an increase in their bank risk premium. In other words, sectors which extend a lot of supplier credit in the US-economy play a central role in the propagation of liquidity shocks.

In addition, I also quantify the trade credit channel of the same financial shock to the top five (a) net-lenders and (b) net-borrowers on sectoral output of the remaining sectors. To this end, I conduct a partial equilibrium analysis of both scenarios in the same way as discussed above. However, due to the differences in the proximity of either group of sectors to the end consumer, I normalize the general and partial equilibrium responses of sectoral output by the log change of aggregate labor in both exercises in order to control for equilibrium demand effects. Thus, the response of output attributed to changes in trade credit interest rates and volumes is derived as the difference between the normalized general and partial equilibrium responses. In the case of financial shocks to the group of net-lenders, the response of output ascribed to the trade credit channel exhibits a negative sign while the opposite holds for shocks to the group of net-borrowers. A difference in means test confirms that the sectoral trade credit channel implied by shocks to net-lenders significantly differs from that generated by shocks to net-borrowers. This observation corroborates the predictions of Proposition 1: The adjustment of trade credit costs and credit shares in response to shocks to sectors classified as net-borrowers (net-lenders) will crowd-out (amplify) the negative effect of an increase in the bank-rate affecting output of sector k.

4.3. Comment on Hulten's Theorem with Inter-Dependent Distortions

In the previous section, I have so far shown that (a) inter-linked distortions are quantitatively important for the propagation of liquidity shocks and (b) sectors which extend relatively more trade credit in comparison to their upfront financing needs play a central role in the propagation of liquidity shocks. The latter observation relates to a strand of literature which investigates the impact of microeconomic shocks on aggregate outcomes. In his seminal contribution, Hulten (1978) states that the impact on aggregate TFP of a sector-specific TFP shock is proportional to the sector's sales share or Domar weight. While this equivalence result is true up to a first-order approximation in an efficient economy without any distortions (see i.a. Gabaix, 2011; Acemoglu et al., 2012), Baqaee and Farhi (2018a,b) in recent important contributions highlight that both non-linearities and distortions generally lead to a failure of Hulten's theorem.

I conclude my quantitative analysis by evaluating the relationship between a sector's sales share or Domar weight and its (a) distortion or credit influence and (b) net-lending position. A sector's sales share or TFP influence (see e.g. Acemoglu et al., 2012) is given by

$$\boldsymbol{\tau}(S) \;=\; \left[\boldsymbol{I} - \left(\boldsymbol{\Omega}^X\right)' \operatorname{diag}(\boldsymbol{\chi} \circ (\boldsymbol{\iota} - \boldsymbol{\eta})) - \boldsymbol{\Omega}^F \left(\boldsymbol{\iota}(\boldsymbol{\iota} - \overline{\boldsymbol{\chi}})'\right)\right]^{-1} \boldsymbol{\Omega}^F \tag{44}$$

where $\overline{\boldsymbol{\chi}} = (\boldsymbol{\iota} - \boldsymbol{\alpha} \circ \boldsymbol{\eta})\boldsymbol{\chi}$ and measures the aggregate effect of a sectoral shock. I then use the definition of a sector's distortion influence provided in BL(2017) to define a sector's credit influence, $\boldsymbol{\tau}(.)$, as the change in the aggregate efficiency or labor wedge in response to an increase in the bank risk-premium of sector k. The credit influence of sector k on the aggregate efficiency and labor wedge, respectively is defined as

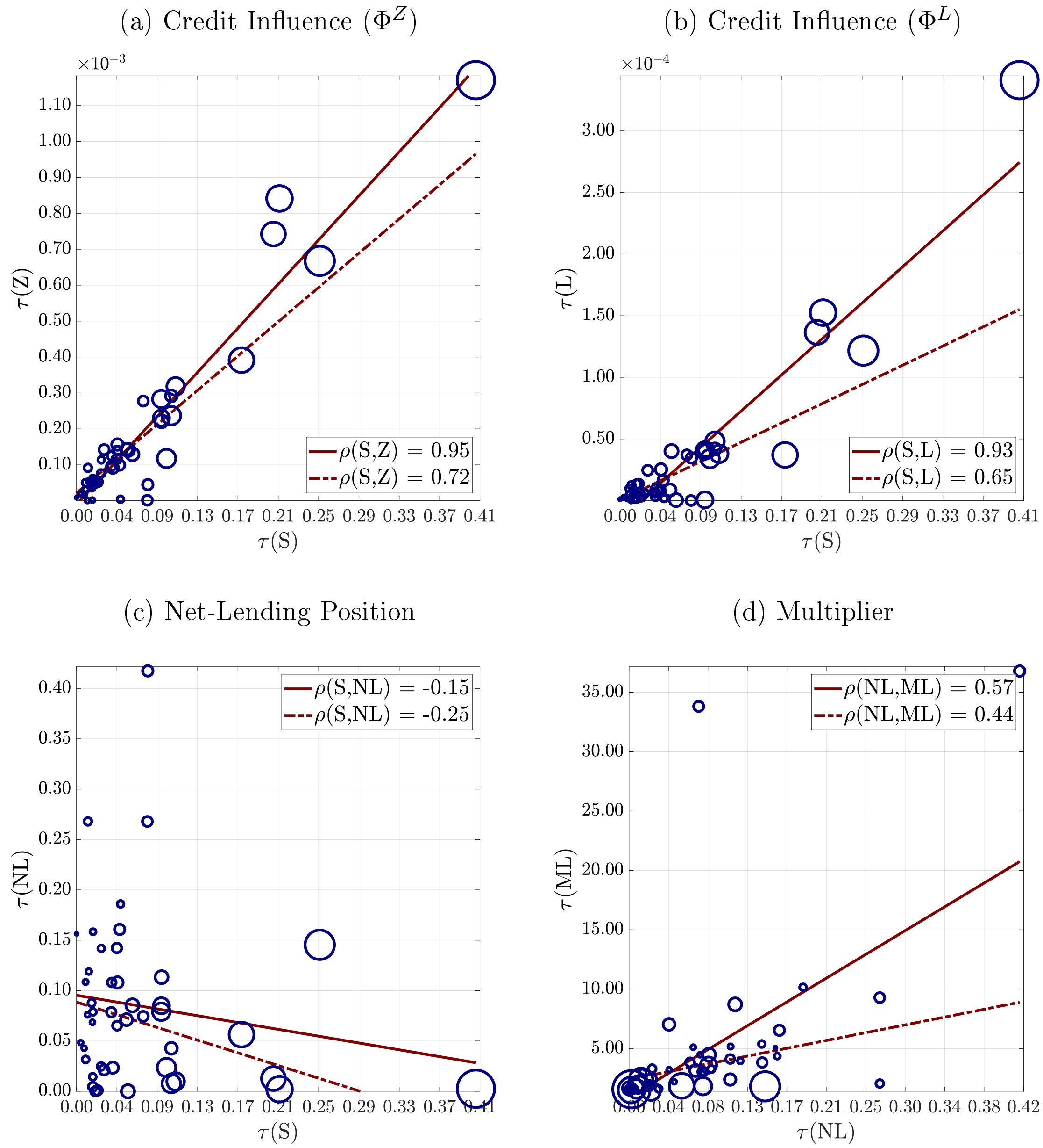
$$\boldsymbol{\tau}(Z) \;=\; \frac{d\log(\Phi^Z)}{d\log(z_k^B)} \quad \text{and} \quad \boldsymbol{\tau}(L) \;=\; \frac{d\log(\Phi^L)}{d\log(z_k^B)} \tag{45}$$

As a baseline for the credit influence vectors, I choose 2007 as the reference year. In particular, I simulate a 10% increase in the bank risk-premium for each sector k and calculate the response of both aggregate wedges. I then plot the correlation between the respective credit influence and the sales share of each sector as shown in Panel (a) and (b) of Figure 9.

As evident from each panel, a sector's TFP- and credit influence appears to be highly positively correlated suggesting that in contrast to the findings in Baqaee and Farhi (2018a) the sales share indeed might be a good predictor for the effect of idiosyncratic financial shocks on aggregate outcomes. However, this result is misleading as it is driven by the sectors with the highest sales shares only (e.g. government). Removing the government sector as an outlier reveals that the Domar weights are only weakly correlated with the credit influence of a sector.

In addition, I plot the relationship between the net-lending position of a sector and the implied trade credit multiplier defined above. Figure 9c shows that the net-lending position of a sector is a good predictor for its systemic importance in the credit network and thus for the propagation of liquidity shocks. Panel (c) of Figure 9 depicts the relationship between the TFP influence of a sector and the proposed measure of its systemic importance in the credit network - the net-lending position. Interestingly, the net-lending position of a sector is weakly negatively correlated with its sales share which implies that the latter is not a good predictor for the relevance of a sector as measured by its trade credit multiplier.

Figure 9: Correlations between Influence Vectors



Note: This figure plots the relationship between the sales share of each sector, $\tau(S)$, and (a) the credit influence related to the aggregate efficiency, $\tau(\Phi^Z)$, and (b) the labor wedge, $\tau(\Phi^L)$ defined in 45 and (c) the net-lending position $\tau(NL) = \theta_k^\tau$ defined in Definition 1. Panel (d) plots the relationship between the net-lending position of sectors and the implied credit multiplier $\tau(ML)$. The influence vectors are calculated based on the response of both aggregate wedges in response to a 10% increase in the bank risk-premium for each sector k in 2007. The size of each observation indicates the share of sectoral value added in total value added. The correlation $\rho(.)$ between the respective series are also reported in each subfigure for the entire sample (solid line) and a sample where the two largest observations are excluded (dotted line).

5. Conclusion

In this paper I study the role of endogenous trade credit linkages in an intersectoral production network for the propagation of liquidity shocks. For this purpose, I build a static quantitative general equilibrium model in which firms finance their working capital requirements using both bank and supplier credit. Profit-maximizing firms (a) choose the composition of their borrowing portfolio to minimize their cost of production and (b) optimally set both the price of the good and the interest rate on trade credit. The model captures two features of trade credit in an economy: trade credit serves both as an insurance and as an amplification device. On the demand side, the existence of two external financing sources allows firms to smooth any interest rate shocks via an adjustment of their borrowing portfolio by optimally trading-off credit costs. On the supply side, an increase in the external financing conditions of a firm directly translates into an increase in cost of trade credit, thereby tightening the financing terms for its customers. I show that the net-lending position of a firm determines its systemic importance in the transmission of liquidity shocks.

In a quantitative application of my model to the US-economy during the crisis, I proxy the increase in risk premia using sector level credit spreads calculated in Gilchrist and Zakrajšek (2012). Model simulations show that the credit network contributed significantly to the drop in aggregate output during the crisis relative to an equivalent economy with bank-finance only. In particular the model predicts that the decline in output doubles when taking into account trade credit linkages among firms. However, I also show that in an economy with both bank and trade finance, the ability of firms to adjust their borrowing portfolio overall decreases aggregate volatility by 0.5%. In a last application, I confirm the predictions derived in the theoretical section of the paper on the relevance of the net-lending position of a firm for aggregate fluctuations: A financial shock to a sector extending more supplier credit relative to its upfront working capital requirements implies that the amplification mechanism of trade credit is more pronounced than the smoothing effect of credit portfolio adjustments. The opposite holds if shocks to sectors receiving a lot supplier credit are considered. This last result also suggest that the net-lending position of firms in an economy may be informative for the transmission of monetary policy which is left for future research.

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A. Model-Derivations

Both intermediate and final goods are produced by a representative, price-taking firm in the respective sector. While final good producing firms only face their profit maximization problem, the intermediate good producing firm faces two maximisation problems each period: (1) Profit Maximization Problem, and (2) Credit Decision Problem. Section A.1 derives and discusses the profit maximization problem of the intermediate good firm will be discussed. The credit decision problem of a representative intermediate goods producing firm is discussed separately in Appendix A.2.

A.1. The Firms' Optimization Problem

Derivation of Optimization Problem. Firm k's customer c generates the following *revenues*

$$R_{ck} = (1-\theta_{ck})\, p_k x_{ck} + (1+r_k^T)\theta_{ck} p_k x_{ck} = \left(1+r_k^T\theta_{ck}\right) p_k x_{ck} \tag{A.1}$$

such that total revenues are

$$R_k = \sum_{c=1}^{\mathcal{C}} \left(1+r_k^T\theta_{ck}\right) p_k x_{ck} + p_k q_{0k} = \left(1+r_k^T\theta_k^C\right) p_k q_k = \phi_k^R p_k q_k \tag{A.2}$$

where ϕ_k^R is the revenue wedge and θ_k^C denotes the weighted average of trade credit extended to firm k's customers

$$\theta_k^C = \sum_{c=1}^{\mathcal{C}} \theta_{ck}\frac{x_{ck}}{q_k} + \theta_{0k}\frac{q_{0k}}{q_k} = \sum_{c=1}^{\mathcal{C}} \theta_{ck} w_{ck}^X + \theta_{0k} w_{0k}^X \tag{A.3}$$

with $\sum_{c=1}^{\mathcal{C}} w_{ck}^X + w_{0k}^X = 1$ and $\theta_{0k} = 0$. The *binding* working capital constraint implies that *total costs of production* including interest payments are

$$(1+r_k^B)BC_k + \sum_{s=1}^{\mathcal{S}}(1+r_s^T)\theta_{ks}p_s x_{ks} = \phi_k^L w\left(\ell_k^Q + \ell_k^T\right) + \sum_{s=1}^{\mathcal{S}} \phi_{ks}^X p_s x_{ks} \tag{A.4}$$

where the respective credit wedges are

$$\phi_k^L \;=\; 1 + r_k^B \qquad \text{(A.5)} \qquad \phi_{ks}^X \;=\; 1 + (1-\theta_{ks})r_k^B + r_s^T\theta_{ks} \quad \text{(A.6)}$$

Note that the intermediate goods credit wedge equals a weighted average of the interest rates on bank and trade credit. Define the interest rate differential between bank and supplier credit as $\Delta_{ks} = r_k^B - r_s^T$. Substituting for the binding working capital constraint implies that dividends or profits can be written as

$$d_k \;=\; \phi_k^R p_k q_k - \phi_k^L w\left(\ell_k^Q + \ell_k^T\right) - \sum_{s=1}^{\mathcal{S}} \phi_{ks}^X p_s x_{ks} - P i_k \tag{A.7}$$

The intermediate goods firm's problem *profit maximization problem* can be formulated recursively as

$$\mathrm{V}(z,k) \;=\; \max_{\mathcal{V},k'}\{d_k + \mathbb{E}_t m' \mathrm{V}(z',k')\}$$

subject to the production function (1), dividends are given by Equation (A.7) and the

- Production Constraint

$$q_{0k} + \sum_{c=1}^{\mathcal{C}} x_{ck} \le\; q_k \tag{A.8a}$$

- Feasibility constraint with respect to TC

$$0 \le \theta_{ks} \le 1 \tag{A.8b}$$

- Law of motion for capital

$$k_k' = i_k + (1-\delta)k_k \tag{A.8c}$$

where $\mathcal{V} = \{\ell_k, \{x_{ks}\}_s, \mathcal{V}(m), \mathcal{V}(c)\}$ is the set of static choice variables. $\mathcal{V}(m) = \{\{x_{ck}\}_c, q_{0k}, \theta_k^C\}$ is the set of choice variables associated with the market structure of perfect competition and $\mathcal{V}(c) = \{\{\theta_{ks}\}_s\}$ is the set of choice variables related to trade credit. In this optimization problem, I have included capital in as a choice variable in order to derive its optimal level for the calibration of the model. Note, the actual model is static, where capital is treated as a constant and equal to its steady state level, such that the profit maximization problem reduces to

$$\mathrm{V}(z) \;=\; \max_{\mathcal{V}} d_k \qquad \text{subject to (1),(A.8a),(A.8b).}$$

Profit Maximization Problem. The firm profit maximization problem is solved as a dual problem in two steps: (1) Given interest rates and trade credit shares, firms choose their production inputs and trade credit shares to minimize total costs of production, (2) Firm's solve for the optimal level of output and the optimal trade credit share extended to customers.

Proof of Lemma 1. (1) *Cost-Minimzation.* For given credit links, the optimal input demand is derived in two steps: given total input expenditures, the firm minimizes expenditures on (a) composites inputs and (b) individual inputs. The production function (1) is first rewritten as

$$q_k = \left(A_k k_k^{\alpha_k \eta_k} V_k^{1-\alpha_k \eta_k}\right)^{\chi_k}$$

where $A_k = \exp(z_k^Q)\kappa_k^Q$ and the composite of (productive) labour and intermediate inputs, V_k, as well as the composite of intermediate inputs, X_k are defined as

$$V_k = \kappa_k^V \left(\ell_k^Q\right)^{\upsilon_k} \left(X_k\right)^{1-\upsilon_k} \quad \text{and} \quad X_k = \kappa_k^X \prod_{s=1}^{\mathcal{S}} x_{ks}^{\omega_{ks}^X}$$

where $\upsilon_k = \frac{(1-\alpha_k)\eta_k}{1-\alpha_k\eta_k}$ and $1-\upsilon_k = \frac{(1-\eta_k)}{1-\alpha_k\eta_k}$ such that both V_k and X_k exhibit CRS and the normalization constants are defined as

$$\begin{aligned}
\left[\kappa_k^V\right]^{-1} &= ((1-\alpha_k)\eta_k)^{\upsilon_k}(1-\eta_k)^{(1-\upsilon_k)}(1-\alpha_k\eta_k)^{-1} \\
\left[\kappa_k^X\right]^{-1} &= \prod_{s=1}^{\mathcal{S}} \left(\omega_{ks}^X\right)^{\omega_{ks}^X} \\
\left[\kappa_k^Q\right]^{-1} &= \chi_k (\alpha_k\eta_k)^{\alpha_k\eta_k}(1-\alpha_k\eta_k)^{(1-\alpha_k\eta_k)}
\end{aligned}$$

(a) Let $p_k^V V_k$ be the cost-minimizing total expenditures on inputs, where p_k^V is a composite of input costs. The corresponding Lagrangian equals

$$\min_{\ell_k^Q, X_k} \mathcal{L} = \phi_k^L w \ell_k^Q + p_k^X X_k + \lambda\left[V_k - \kappa_k^V \left(\ell_k^Q\right)^{\upsilon_k}\left(X_k\right)^{1-\upsilon_k}\right]$$

where p_k^X is the price of the intermediate composite. The FOCs with respect to this problem imply that the minimum expenditures $p_k^V V_k$ to produce one unit of $V_k = 1$ are

$$p_k^V = \left(\phi_k^L w\right)^{\upsilon_k}\left(p_k^X\right)^{(1-\upsilon_k)} \tag{A.9}$$

The optimal demand for labour ℓ_k^Q and the composite intermediate input X_k is given by

$$\ell_k \;=\; \frac{(1-\alpha_k)\eta_k}{1-\alpha_k\eta_k}\left(\frac{\phi_k^L w}{p_k^V}\right)^{-1} V_k \;\; \text{(A.10)} \quad \text{and} \quad X_k \;=\; \frac{(1-\eta_k)}{1-\alpha_k\eta_k}\left(\frac{p_k^X}{p_k^V}\right)^{-1} V_k \quad \text{(A.11)}$$

(b) Similarly, let $p_k^X X_k$ be the cost-minimizing total expenditures on intermediate goods such that the Lagrangian of the corresponding cost-minimization problem equals

$$\min_{\{x_{ks}\}_s} \; \mathcal{L} \;=\; \sum_{s=1}^{\mathcal{S}} \phi_{ks}^X p_s x_{ks} + \lambda_X \left(X_k - \kappa_k^X \prod_{s=1}^{\mathcal{S}} (x_{ks})^{\omega_{ks}^X} \right)$$

The FOCs imply that the aggregate price index of the composite intermediate good X_k is

$$p_k^X \;=\; \phi_k^X \prod_{s=1}^{\mathcal{S}} (p_s)^{\omega_{ks}^X} \qquad \text{(A.12)} \quad \text{where} \qquad \phi_k^X \;=\; \prod_{s=1}^{\mathcal{S}} (\phi_{ks}^X)^{\omega_{ks}^X} \qquad \text{(A.13)}$$

The optimal demand for x_{ks} is given by

$$x_{ks} \;=\; \omega_{ks}^X \left(\frac{\phi_{ks}^X p_s}{p_k^X}\right)^{-1} X_k \;=\; \omega_{ks}^X \frac{(1-\eta_k)}{(1-\alpha_k\eta_k)} \left(\frac{\phi_{ks}^X p_s}{p_k^V}\right)^{-1} V_k \qquad \text{(A.14)}$$

Using Equation (A.10) and (A.14), total costs of *productive* inputs including interest payments are

$$C_k^Q \;=\; \phi_k^L w \ell_k^Q + \sum_{s=1}^{\mathcal{S}} \phi_{ks}^X p_s x_{ks} \;=\; p_k^V V_k \qquad \text{(A.15)}$$

For given credit costs and shares $\{\theta_{ks}\}_{sj}$, the *marginal cost of production (including interest-cost)* equals the price of the labour and intermediate composite and can be written as

$$p_k^V \;=\; \phi_k^V mc_k^V \;=\; \left(\phi_k^L \right)^{\upsilon_k} \left(\prod_{s=1}^{\mathcal{S}} (\phi_{ks}^X)^{\omega_{ks}^X} \right)^{(1-\upsilon_k)} \left(w \right)^{\upsilon_k} \left(\prod_{s=1}^{\mathcal{S}} (p_{ks})^{\omega_{ks}^X} \right)^{(1-\upsilon_k)} \qquad \text{(A.16)}$$

where ϕ_k^V denotes the composite credit wedge which is a function of the credit links between sectors. □

Proof of Corollary 1. Corollary 1 follows directly from Lemma 1. □

Proof of Lemma 2. Dividends of firm k are defined as

$$d_k = \phi_k^R p_k q_k - p_k^V V_k - (1 + r_k^B) w \ell_k^T \tag{A.17}$$

The FOC with respect to V_k is given by

$$\frac{\partial \mathrm{V}_k}{\partial V_k} : \quad p_k^V V_k = (1 - \alpha\eta_k)\chi_k \phi_k^R p_k q_k \tag{A.18}$$

Equation (A.18) implies that total input expenditures (including interest rate costs) are a fraction of total revenues. The optimal goods price equals

$$p_k = \frac{MC_k^V}{MP_k^V} = \frac{\phi_k^V}{\phi_k^R} \frac{mc_k^V}{(1 - \alpha\eta_k)\,\chi_k q_k V_k^{-1}} \tag{A.19}$$

and the implied effective mark-up over marginal costs of production, $\phi_k^V (\phi_k^R)^{-1}$, is a combination of credit and revenue wedges. □

Properties of the Cost-Function. For given input prices, credit costs and links, the marginal cost of production are

- ... *increasing in the cost of bank credit:* $\upsilon_k \in \{\mathbb{r}_k^B, A_k^B, \theta_k^C\}$ *since* $0 < \frac{\partial r_k^B}{\partial \upsilon_k}$

$$\frac{\partial p_k^V}{\partial \upsilon_k} = \left[\frac{\upsilon_k}{\phi_k^L} \frac{\partial \phi_k^L}{\partial r_k^B} + (1 - \upsilon_k) \sum_{s=1}^{\mathcal{S}} \frac{\omega_{ks}^X}{\phi_{ks}^X} \frac{\partial \phi_{ks}^X}{\partial r_k^B} \right] p_k^V \frac{\partial r_k^B}{\partial \upsilon} = \phi_k^{\partial V} p_k^V \frac{\partial r_k^B}{\partial \upsilon_k} \tag{A.20a}$$

- ... *decreasing (increasing) in the share of trade credit taken from supplier s,* θ_{ks}

$$\frac{\partial p_k^V}{\partial \theta_{ks}} = \omega_{ks}^X (1 - \upsilon_k) p_k^V \left(\phi_{ks}^X\right)^{-1} (-\Delta_{ks}) = -\frac{p_s x_{ks}}{V_k} \Delta_{ks} \begin{cases} < 0 & \text{if } \Delta_{ks} > 0 \\ > 0 & \text{otw} \end{cases} \tag{A.20b}$$

where the last line uses Equation (A.14).

The *scale of the changes in marginal costs* following a change in the cost of bank credit (given credit links and costs), $\phi_k^{\partial V}$, and is given by

$$1 \; > \; \phi_k^{\partial V} \;\; = \;\; \frac{\upsilon_k}{\phi_k^L} + (1-\upsilon_k)\sum_{s=1}^{\mathcal{S}} \omega_{ks}^X \frac{(1-\theta_{ks})}{\phi_{ks}^X} \tag{A.21}$$

Straight forward calculations show that the scale of the changes in marginal costs is decreasing in $\upsilon_k \in \{r_k^B, A_k^B, \theta_k^C\}$ and also decreasing in the share θ_{ks} and the interest rate on trade credit r_s^T. In addition, the scale of the change in marginal costs is smaller than one which implies that a change in the cost of bank credit do not change marginal cost of production one-for one. The *curvature* of the cost-function based on the 2nd order derivative with respect to credit shares obtained, θ_{ks}, and the average credit share extended, θ_k^C, is as follows. Given credit links, the marginal cost of production is

- *... concave in the trade credit share to supplier s.*

$$0 \; > \; \frac{\partial^2 p_k^V}{(\partial \theta_{ks})^2} \;\; = \;\; -\omega_{ks}^X (1-\upsilon_k)\left(1-\omega_{ks}^X(1-\upsilon_k)\right) p_k^V \left(\frac{\Delta_{ks}}{\phi_{ks}^X}\right)^2$$

- *... convex*[22] *in the trade credit extended to customers, θ_k^C, if $\mu > \underline{\mu}$ holds.*

$$0 \; < \; \frac{\partial^2 p_k^V}{(\partial \theta_k^C)^2} \;\; = \;\; \left\{\left[\frac{\partial \phi_k^{\partial V}}{\partial \upsilon_k}\left(\frac{\partial r_k^B}{\partial \upsilon_k}\right)^{-1} + \left(\phi_k^{\partial V}\right)^2\right]\left(\frac{\partial r_k^B}{\partial \upsilon_k}\right)^2 + \phi_k^{\partial V}\frac{\partial^2 r_k^B}{\partial \upsilon_k^2}\right\} p_k^V$$

Note that, if $\frac{\partial^2 r_k^B}{\partial \upsilon_k^2} = 0$, then the marginal cost of production p_k^V is a concave function in θ_k^C which is a contradiction to the assumption of firms maximizing profits. The concavity is implied by

$$0 \; > \; \frac{\partial \phi_k^{\partial V}}{\partial \upsilon_k}\left(\frac{\partial r_k^B}{\partial \upsilon_k}\right)^{-1} + \left(\phi_k^{\partial V}\right)^2 \;\; = \;\; -(1-\upsilon_k)\upsilon_k\left\{\frac{1}{\phi_k^L} - \sum_{s=1}^{\mathcal{S}}\omega_{ks}^X\frac{1-\theta_{sk}}{\phi_{kx}^X}\right\}^2 - \mathcal{O}_k \tag{A.22}$$

where due to Jensen's inequality

$$0 \; < \; \mathcal{O}_k \;\; = \;\; (1-\upsilon_k)\left\{\left(\sum_{s=1}^{\mathcal{S}}\omega_{ks}^X\left(\frac{1-\theta_{sk}}{\phi_{kx}^X}\right)^2\right) - \left(\sum_{s=1}^{\mathcal{S}}\omega_{ks}^X\frac{1-\theta_{sk}}{\phi_{kx}^X}\right)^2\right\} \tag{A.23}$$

Therefore, in order for the marginal cost of sector k, p_k^V, to be a convex function in θ_k^C,

[22] For the profit maximization problem to be consistent, the cost function needs to be convex with respect to θ_k^C.

it needs to hold that

$$-\left[\frac{\partial \phi_k^{\partial V}}{\partial \upsilon_k}\left(\frac{\partial r_k^B}{\partial \upsilon_k}\right)^{-1} + (\phi_k^{\partial V})^2\right]\frac{1}{\phi_k^{\partial V}} \;<\; \frac{\partial^2 r_k^B}{\partial \upsilon_k^2}\left(\frac{\partial r_k^B}{\partial \upsilon_k}\right)^{-2}$$

Using Equation (A.22)

$$\kappa_k^{\gamma} \;=\; \frac{(1-\upsilon_k)\upsilon_k\left\{\frac{1}{\phi_k^L} - \sum_{s=1}^{\mathcal{S}} \omega_{ks}^X \frac{1-\theta_{sk}}{\phi_{kx}^X}\right\}^2 + \mathcal{O}_k}{\frac{\upsilon_k}{\phi_k^L} + (1-\upsilon_k)\sum_{s=1}^{\mathcal{S}} \omega_{ks}^X \frac{1-\theta_{sk}}{\phi_{kx}^X}} \;<\; \frac{\partial^2 r_k^B}{\partial \upsilon_k^2}\left(\frac{\partial r_k^B}{\partial \upsilon_k}\right)^{-2}$$

and due to r_k^B being an increasing function in θ_k^C

$$\frac{\partial^2 r_k^B}{\partial \upsilon_k^2}\left(\frac{\partial r_k^B}{\partial \upsilon_k}\right)^{-2} \;=\; \frac{\mu(\mu-1)A_k^B(\overline{\theta}_0^D + \theta_k^C)^{\mu-2}\underline{r}_0^B}{\left(\mu A_k^B(\overline{\theta}_0^D + \theta_k^C)^{\mu-1}(\underline{r}_0^B)\right)^2} \;=\; \frac{(\mu-1)}{\mu A_k^B(\overline{\theta}_0^D + \theta_k^C)^{\mu}\underline{r}_k^B} \;=\; \frac{(\mu-1)}{\mu(r_k^B - \underline{r}_k^B)}$$

The lower bound for μ is thus given by

$$\underline{\mu} \;=\; (1-\kappa_k^{\mu}(r_k^B - \underline{r}_k^B))^{-1} < \mu$$

for $\kappa_k^{\mu}(r_k^B - \underline{r}_k^B) < 1$. In other words, the risk-premium on the interest rate of bank credit has to be a convex enough function in the trade credit share extended to k's customers in order for firm k to be maximizing profits with respect to θ_k^C.

A.2. Optimal Credit Decisions

Proof of Lemma 10. Firm k chooses $\{\theta_{ks}\}_s$ to maximise profits. The FOC associated with $\{\theta_{ks}\}_s$ is given by

$$\frac{\partial \mathrm{V}_k}{\partial \theta_{ks}} : -\left[\frac{\partial p_k^V}{\partial \theta_{ks}} V_k + (1+r_k^B)\frac{\partial w\ell_k^T}{\partial \theta_{ks}}\right] + \lambda_\theta^0 - \lambda_\theta^1 = 0 \tag{A.24}$$

where λ_θ^0 and λ_θ^1 are the Lagrange multipliers associated with the feasibility constraints. In other words, firm k chooses $\{\theta_{ks}\}_s$ in order to minimize total costs of production such that the combined change in the cost of production and managing credit lines associated with changing the share of trade credit obtained from k's supplier is zero at the optimum. Using Equation (A.20b), the derivative of Equation (9) with respect to θ_{ks} and assuming that the optimal $\theta_{ks} \in (0,1)\forall k, s$, such that $\lambda_\theta^0 = \lambda_\theta^1 = 0$, the FOC implies the following *optimal trade credit share*

$$\theta_{ks} \;=\; \left(1 - \frac{\overline{\theta}_k^S}{\kappa_{1,ks}^T}\kappa_{0,ks}^T + \frac{\overline{\theta}_k^S}{\kappa_{1,ks}^T}\frac{p_s x_{ks}\Delta_{ks}}{(1+r_k^B)wV_k}V_k\right)\overline{\theta}_k^S \tag{A.25a}$$

Using Equation (A.14), Equation (A.25a) can be alternatively written as

$$\theta_{ks} \;=\; \left(1 - \frac{\overline{\theta}_k^S}{\kappa_{1,ks}^T}\kappa_{0,ks}^T + \frac{\overline{\theta}_k^S}{\kappa_{1,ks}^T}\frac{\omega_{ks}^X(1-\eta_k)\chi_k R_k\Delta_{ks}}{\phi_{ks}^X\phi_k^L w}\right)\overline{\theta}_k^S \tag{A.25b}$$

The 2nd order derivative of V_k with respect to θ_{ks} is given by

$$\frac{\partial^2 \mathrm{V}_k}{(\partial\theta_{ks})^2} \;=\; -\left[\frac{\partial^2 p_k^V}{\partial(\theta_{ks})^2}V_k + (1+r_k^B)\frac{\partial^2 w\ell_k^T}{\partial(\theta_{ks})^2}\right] \tag{A.26}$$

and - using Equation (A.25a) - is negative at the optimum if

$$\underbrace{\frac{\Delta_{ks}\overline{\theta}_k^S}{\phi_{ks}^X}}_{\in(-1,1)}\left(\frac{\theta_{ks}-\overline{\theta}_k^S}{\overline{\theta}_k^S} + \frac{\overline{\theta}_k^S}{\kappa_{1,ks}^T}\kappa_{0,ks}^T\right) < \underbrace{\left(1-\omega_{ks}^X(1-\upsilon_k)\right)^{-1}}_{>1}$$

$\square$

Properties of the Optimal Trade Credit Share. In equilibrium the following bounds need to hold

$$g(\theta) \;=\; \frac{\theta_{ks}-\overline{\theta}_k^S}{\overline{\theta}_k^S} \;=\; \frac{\overline{\theta}_k^S}{\kappa_{1,ks}^T}\left(-\kappa_{0,ks}^T + \frac{p_s x_{ks}\Delta_{ks}}{(1+r_k^B)w}\right) \;\in\; \left[-1, \frac{1-\overline{\theta}_k^S}{\overline{\theta}_k^S}\right]$$

and

$$g(\theta) \;\begin{cases} < 0 & \dfrac{p_s x_{ks}}{(1+r_k^B)w} < (>)\dfrac{\kappa_{0,ks}^T}{\Delta_{ks}} \quad \text{for } \Delta_{ks} > (<)0 \\ > 0 & \dfrac{p_s x_{ks}}{(1+r_k^B)w} > (<)\dfrac{\kappa_{0,ks}^T}{\Delta_{ks}} \quad \text{for } \Delta_{ks} > (<)0 \end{cases}$$

Note that if firm k can adjust its credit portfolio frictionless such that $\kappa_{1,ks}^T = 0\forall ks$, then the FOC reduces to

$$p_s x_{ks}\Delta_{ks} + (1+r_k^B)w\kappa_{0,ks}^T \;=\; \lambda_\theta^1 - \lambda_\theta^0 \tag{A.27}$$

which implies that the optimal demand for TC is simply given by

$$\theta^*_{ks} = \begin{cases} 1 & \text{if } p_s x_{ks}\Delta_{ks} + (1+r^B_k)w\kappa^T_{0,ks} > 0 \text{ and } \lambda^1_\theta > 0, \lambda^0_\theta = 0 \\ 0 & \text{if } p_s x_{ks}\Delta_{ks} + (1+r^B_k)w\kappa^T_{0,ks} < 0 \text{ and } \lambda^1_\theta = 0, \lambda^0_\theta > 0 \end{cases} \tag{A.28}$$

Thus, the introduction of non-linear management costs of credit lines and respective parameter choices ensures that the demand for TC will have an interior solution. The optimal trade credit obtained from firm k's supplier is ...

- *... increasing (decreasing) in the price of the intermediate good* p_s *if* $\Delta_{ks} > (<)0$.
- *... increasing (decreasing) in intermediate goods obtained from firm* s, x_{ks}, *if* $\Delta_{ks} > (<)0$.
- *... decreasing in the interest rate charged on trade credit* r^T_s.
- *... increasing in the interest rate on bank credit* r^B_k.

In other words if obtaining trade credit from firm s is more expensive than bank credit for firm k ($\Delta_{ks} < 0$) then an increase in the price of good s decreases the share of purchases obtained on credit from sector s. Similarly, if $\Delta_{ks} < 0$, then an increase in the share of intermediates goods obtained from firm s increases the share of purchases obtained on credit from sector s.

Proof of Lemma 4. Firms take the demand for trade credit, θ^C_k, as given due to perfect competition. Therefore, firms choose θ^C_k to maximize profits which implies that the FOC is given by

$$\frac{\partial \mathrm{V}_k}{\partial \theta^C_k} : \frac{\partial \phi^R_k}{\partial \theta^C_k} p_k q_k = \frac{\partial p^V_k}{\partial \theta^C_k} V_k + \frac{\partial r^B_k}{\partial \theta^C_k} w \ell^T_k \tag{A.29}$$

The interest rate on trade credit charged is the solution to Equation (A.29). In other words, firm k sets r^T_k in order to equalize the marginal revenue to marginal costs of extending trade credit to customers. The change in the cost of production associated with extending trade credit equals the total effect of trade credit demand on external borrowing costs of firm k. Using Equation (A.20a), the FOC can be written as

$$r^T_k p_k q_k = \frac{\partial r^B_k}{\partial \theta^C_k} \left(\phi^{\partial V}_k p^V_k V_k + w\ell^T_k\right) \tag{A.30}$$

Dividing by $p_k q_k$ and defining the share of the net change in MC in net revenues (A.31a) and the share of net credit management costs in net revenues (A.31b) as

$$s_k^V \;=\; \frac{\phi_k^{\partial V} p_k^V V_k}{p_k q_k} \qquad \text{(A.31a)} \qquad\qquad s_k^T \;=\; \frac{w\ell_k^T}{p_k q_k} \qquad \text{(A.31b)}$$

implies that the optimal interest rate on trade credit extended by sector k is

$$r_k^T \;=\; \frac{\mu(r_k^B - \mathbb{r}_k^B)}{(\bar{\theta}_k^D + \theta_k^C)} \left(s_k^V + s_k^T\right) \tag{A.32}$$

Alternatively, define the share of the net change in MC in gross revenues (A.33a) and the net credit management costs in gross revenues (A.33b)

$$\widetilde{s}_k^V \;=\; \frac{\phi_k^{\partial V} p_k^V V_k}{R_k} \qquad \text{(A.33a)} \qquad\qquad \widetilde{s}_k^T \;=\; \frac{w\ell_k^T}{R_k} \qquad \text{(A.33b)}$$

implies that Equation (A.32) can be written as

$$r_k^T \;=\; \frac{\mu(r_k^B - \mathbb{r}_0^B)}{(\bar{\theta}_0^D + \theta_k^C)} \phi_k^R \left(\widetilde{s}_k^V + \widetilde{s}_k^T\right) \tag{A.34}$$

Rearranging and solving for r^T yields

$$r_k^T \;=\; \frac{\mathbb{r}_k^\theta \left(\widetilde{s}_k^V + \widetilde{s}_k^T\right)}{1 - \mathbb{r}_k^\theta \theta_k^C \left(\widetilde{s}_k^V + \widetilde{s}_k^T\right)} \;=\; \frac{\mu(r_k^B - \mathbb{r}_0^B)\left(\widetilde{s}_k^V + \widetilde{s}_k^T\right)}{\bar{\theta}_0^D - (1 - \mu(r_k^B - \mathbb{r}_0^B)\left(\widetilde{s}_k^V + \widetilde{s}_k^T\right))\theta_k^C} \tag{A.35}$$

The second order derivative of the value function with respect to θ_k^C

$$\frac{\partial^2 \mathrm{V}_k}{(\partial\theta_k^C)^2} \;=\; -\left[\frac{\partial^2 p_k^V}{(\partial\theta_k^C)^2} V_k + \frac{\partial^2 r_k^B}{(\partial\theta_k^C)^2} w\ell_k^T\right] \;<\; 0 \tag{A.36}$$

since $0 \;<\; \frac{\partial^2 p_k^V}{\partial(\theta_k^C)^2}, \frac{\partial^2 r_k^B}{\partial(\theta_k^C)^2}$. □

Properties of r_k^T. The properties of the equilibrium condition for interest rate on trade credit discussed in the main text follow from straight forward calculations.

B. General Equilibrium

In order to derive the partial equilibrium expressions of revenues, sales, output, aggregate GDP, labor the corresponding aggregate efficiency and labor wedge, I follow the same steps as in BL(2017). The Proof of Lemma 5 follows from straight forward calculations and is available upon request.

C. Log-Linearization

In this section I log-linearize the model around its steady state. First, define $x = \overline{x}\exp\{\widehat{x}\} \approx \overline{x}(1+\widehat{x})$ and $\widehat{x} = \text{dlog}(x) = \log x/\overline{x}$. In the following, I first derive the log-linearized equilibrium responses of revenues, prices and sectoral output in terms of (1) productivity shocks, (2) general equilibrium adjustments in the aggregate labor supply and (3) distortions introduced as credit wedges in Section C.1. The credit wedges summarize the composite effect of changes in credit costs and the composition of credit portfolios on sectoral sales, prices and output. Section C.2 then derives the decomposition of credit wedges into effects attributed to changes in interest rates on bank and trade credit and changes in trade credit shares. The effect of the log-change of each component is determined by the entries of the corresponding elasticity matrices $\boldsymbol{E}$, which are non-linear functions of the steady state of the economy. Section C.4 provides a proof of Proposition 1. To this end, I adopt the following notation:

For any [M×M]-matrix $\boldsymbol{W}$ let the respective [M×M^s]-matrix denote

$$\overset{c}{\boldsymbol{W}} = (\boldsymbol{\iota} \otimes \boldsymbol{W}') \circ (\boldsymbol{I} \otimes \boldsymbol{\iota}') \qquad \text{... the sum over customers}$$

$$\overset{s}{\boldsymbol{W}} = (\boldsymbol{\iota} \otimes \boldsymbol{W}) \circ (\boldsymbol{\iota}' \otimes \boldsymbol{I}) \qquad \text{... the sum over suppliers}$$

where $\circ$ denotes the Hadamard product. In addition, I define

- Net Accounts Payable of m paid to s
$$\overline{AP}_{ms} = \overline{\theta}_{ms}\overline{p}_s\overline{x}_{ms}$$
- Net Cash on Delivery of m paid to s
$$\overline{AP}^-_{ms} = (1-\overline{\theta}_{ms})\overline{p}_s\overline{x}_{ms}$$
- Net Accounts Receivable of m from c
$$\overline{AR}_{cm} = \overline{\theta}_{cm}\overline{p}_m\overline{x}_{cm}$$
- Net Cash on Delivery of c paid to m
$$\overline{AR}^-_{cm} = (1-\overline{\theta}_{cm})\overline{p}_m\overline{x}_{cm}$$

The wage rate is taken as the numeraire and capital is constant such that $\widehat{k}_k = 0$.

I also make the following simplifying assumptions: (1) I abstract from productivity shocks and consider the partial equilibrium case only, assuming that both productivity and aggregate labor remain at their steady state levels. (2) I treat the share of quantities sold to intermediate and final good producers in total production as constant.

C.1. Log-Linearized Equilibrium

(a) The log-linearisation of *intermediate and final revenues* yields

$$\widehat{R}_0 = \left\{ E_F^L + \sum_{m=1}^{M}\sum_{n=1}^{M}[\boldsymbol{W}_F^R]_{nn}[\boldsymbol{W}_R^R]_{nm}[\boldsymbol{E}_R^L\boldsymbol{\iota}]_m \right\} \widehat{L} - \widehat{\phi}_\kappa^F \tag{C.1}$$

$$\widehat{R}_k = -\sum_{m=1}^{M}[\boldsymbol{W}_R^R]_{km}[\widehat{\boldsymbol{\phi}}_\kappa^S]_m + \sum_{m=1}^{M}[\boldsymbol{W}_R^R]_{km}[\boldsymbol{E}_R^L\boldsymbol{\iota}]_m\widehat{L} \tag{C.2}$$

The intermediate and final sales wedges are

$$\widehat{\phi}_\kappa^F = \sum_{m=1}^{M}\sum_{n=1}^{M}[\boldsymbol{W}_F^R]_{nn}[\boldsymbol{W}_R^R]_{nm}[\widehat{\boldsymbol{\phi}}_\kappa^S]_m + \widehat{\phi}_F^S, \text{and} \qquad \widehat{\phi}_{\kappa,k}^S = \widehat{\phi}_{X,k}^S + [\boldsymbol{W}_R^F\boldsymbol{\iota}]_k\widehat{\phi}_F^S \tag{C.3}$$

where

$$\widehat{\phi}_F^S = \sum_{m=1}^{M}\sum_{s=1}^{M}[\boldsymbol{W}_F^T]_{ms}\widehat{r}_s^T + [\boldsymbol{W}_F^\theta]_{ms}\widehat{\theta}_{ms} - [\boldsymbol{W}_F^\Phi]_{ms}\widehat{\phi}_{ms}^X - [\boldsymbol{W}_F^{\phi(L)}]_m\widehat{\phi}_m^L + [\boldsymbol{W}_F^C]_m\widehat{C}_m^T \tag{C.4}$$

$$\widehat{\phi}_{X,k}^S = \sum_{c=1}^{M}[\boldsymbol{W}_R^X]_{ck}\widehat{\phi}_{ck}^X - \widehat{\phi}_k^R \tag{C.5}$$

Define $\left[\boldsymbol{W}_R^R\right]^{-1} = \boldsymbol{I} - (\boldsymbol{W}_R^X)' - \boldsymbol{W}_R^F\boldsymbol{J}\boldsymbol{W}_F^R$. The entries of the elasticity matrices are all positive and given by

$$\begin{aligned}
\overline{R}_0 E_F^L &= \overline{L} \text{ and } [\boldsymbol{E}_R^L]_{kk} = [\boldsymbol{W}_R^F]_{kk}E_F^L & \overline{R}_0[\boldsymbol{W}_F^T]_{ms} &= \overline{r}_s^T\overline{\theta}_{ms}\overline{p}_s\overline{x}_{ms}\\
\overline{R}_0[\boldsymbol{W}_F^R]_m &= \overline{R}_m - \textstyle\sum_{s=1}^{M}(1+\overline{r}_s^T\overline{\theta}_{ms})\overline{p}_s\overline{x}_{ms} - \overline{w\ell}_m^Q & \overline{R}_0[\boldsymbol{W}_F^\Phi]_{ms} &= (1+\overline{r}_s^T\overline{\theta}_{ms})\overline{p}_s\overline{x}_{ms}\\
\overline{R}_0[\boldsymbol{W}_F^{\phi(L)}]_m &= \overline{w\ell}_m^Q & \overline{R}_k[\boldsymbol{W}_R^X]_{ck} &= \overline{\phi}_k^R\overline{p}_k\overline{q}_{ck}\\
\overline{R}_0[\boldsymbol{W}_F^C]_m &= \overline{w\ell}_m^T & \overline{R}_k[\boldsymbol{W}_R^F]_{kk} &= \overline{\phi}_k^R\overline{p}_k\overline{q}_{0k}
\end{aligned}$$

An increase in the sales wedges decrease intermediate and final revenues.

(b) The linearized response of *prices* is

$$\widehat{p}_k = \widehat{\phi}^P_{\kappa,k} + [\boldsymbol{E}^L_P]_{kk}\widehat{L} - \sum_{m=1}^{M}[\boldsymbol{E}^{Z(Q)}_P]_{km}\widehat{z}^Q_m \tag{C.6}$$

with the following price credit wedge

$$\widehat{\phi}^P_{\kappa,k} = \sum_{m=1}^{M}[\boldsymbol{W}^P_P]_{km}\widehat{\phi}^P_{P,m} - [\boldsymbol{W}^R_P]_{km}\widehat{\phi}^S_{\kappa,m} \tag{C.7}$$

where $\widehat{\phi}^P_{P,k} = \overline{\chi}_k\widehat{\phi}^V_k - \widehat{\phi}^R_k$ and $\overline{\chi}_k = (1-\alpha\eta_k)\chi_k$. The elasticity matrices are

$$[\boldsymbol{W}^P_P]^{-1} = \boldsymbol{I} - \mathrm{diag}\left(\boldsymbol{\chi}\circ(\boldsymbol{\iota}-\boldsymbol{\eta})\right)\boldsymbol{\Omega}^X \qquad \boldsymbol{E}^L_P = \boldsymbol{W}^R_P\boldsymbol{W}^F_R E^L_F\boldsymbol{\iota}$$

$$\boldsymbol{W}^R_P = \boldsymbol{W}^P_P\mathrm{diag}(\boldsymbol{\iota}-\overline{\boldsymbol{\chi}})\boldsymbol{W}^R_R \qquad \boldsymbol{E}^{Z(Q)}_P = \boldsymbol{W}^P_P\mathrm{diag}(\boldsymbol{\chi})$$

An increase in the price wedge increases prices.

(c) Using Equation (C.2) and (C.6) the response of *sectoral output* is

$$\widehat{q}_k = \sum_{m=1}^{M}[\boldsymbol{E}^{Z(Q)}_Q]_{km}\widehat{z}_m - \widehat{\phi}^Q_k + [\boldsymbol{e}^L_Q]_k\widehat{L} \tag{C.8}$$

where the output wedge $\widehat{\phi}^Q_k$ is a combination of the sales (C.3), the price (C.7) and the revenue wedge (C.10) and can be defined as

$$\widehat{\phi}^Q_k = \sum_{m=1}^{M}[\boldsymbol{W}^P_P]_{km}[\widehat{\boldsymbol{\phi}}^P_P]_m + \widehat{\phi}^R_k + \sum_{m=1}^{M}\left([\boldsymbol{W}^R_R]_{km} - [\boldsymbol{W}^R_P]_{km}\right)[\widehat{\boldsymbol{\phi}}^S_\kappa]_m \tag{C.9}$$

where

$$[\boldsymbol{E}^{Z(Q)}_Q]_{km} = [\boldsymbol{E}^{Z(Q)}_P]_{km} \quad \text{and} \quad [\boldsymbol{e}^L_Q]_k = \sum_{m=1}^{M}[\boldsymbol{W}^R_R]_{km}[\boldsymbol{E}^L_R\boldsymbol{\iota}]_m - [\boldsymbol{E}^L_P]_{kk}$$

An increase in either wedge decreases sectoral output via an increase in the cost of production which reduces input demand, sales and ultimately household's income. The output wedge thus summarizes the combined effect of credit costs and links on output. An increase in the output wedge decreases output.

C.2. Credit Wedges

Proof of Lemma 6. The log-linearization of the *revenue wedge* for sector k yields

$$\widehat{\phi}_k^R \;=\; [\boldsymbol{E}^T_{\phi(R)}]_{kk}\widehat{r}^T_k + \sum_{c=1}^{M}[\boldsymbol{E}^\theta_{\phi(R)}]_{ck}\widehat{\theta}_{ck} \tag{C.10}$$

The *labor and intermediate credit wedge* deviations for each sector k is given by

$$\widehat{\phi}_k^L \;=\; [\boldsymbol{E}^B_{\phi(L)}]_{kk}\widehat{r}^B_k \quad \text{and} \quad \widehat{\phi}^X_{ks} \;=\; [\boldsymbol{E}^B_\Phi]_{ks}\widehat{r}^B_k + [\boldsymbol{E}^T_\Phi]_{ks}\widehat{r}^T_s - [\boldsymbol{E}^\theta_\Phi]_{ks}\widehat{\theta}_{ks} \tag{C.11}$$

where the entries of the elasticity matrices are defined as follows

$$\begin{aligned}
\overline{R}_k[\boldsymbol{E}^T_{\phi(R)}]_{kk} &= \overline{r}^T_k \textstyle\sum_{c=1}^M \overline{\theta}_{ck}\overline{p}_k\overline{x}_{ck} & \overline{\phi}^X_{ks}\left[\boldsymbol{E}^B_\Phi\right]_{ks} &= (1-\overline{\theta}_{ks})\overline{r}^B_k \\
\overline{R}_k[\boldsymbol{E}^\theta_{\phi(R)}]_{ck} &= \overline{r}^T_k\overline{\theta}_{ck}\overline{p}_k\overline{x}_{ck} & \overline{\phi}^X_{ks}\left[\boldsymbol{E}^T_\Phi\right]_{ks} &= \overline{\theta}_{ks}\overline{r}^T_s \\
\overline{\phi}^L_k\left[\boldsymbol{E}^B_{\phi(L)}\right]_k &= \overline{r}^B_k & \overline{\phi}^X_{ks}\left[\boldsymbol{E}^\theta_\Phi\right]_{ks} &= \left(\overline{r}^B_k - \overline{r}^T_s\right)\overline{\theta}_{ks}
\end{aligned}$$

All elasticities are positive. The sign of the elasticity of the change in the trade credit share from sector k to sector s depends on the sign of the interest rate differential $\overline{\Delta}_{ks} = \overline{r}^B_k - \overline{r}^T_k$. □

Proof of Lemma 7. The *FG-sales wedge*, $\boldsymbol{\phi}^S_F$, is defined in Equation (C.3) and is such that an increase in the final sales credit wedge decreases total revenues of the final good producer. Using Equations (C.11), the wedge response can be written as

$$\widehat{\phi}^S_F \;=\; -\sum_{m=1}^M[\boldsymbol{e}^B_{\phi(S,F)}]_m\widehat{r}^B_m + \sum_{m=1}^M[\boldsymbol{e}^T_{\phi(S,F)}]_m\widehat{r}^T_m + \sum_{m=1}^M\sum_{s=1}^M[\boldsymbol{E}^\theta_{\phi(S,F)}]_{ms}\widehat{\theta}_{ms} \tag{C.12}$$

The elasticities are given by

$$\begin{aligned}
[\boldsymbol{e}^B_{\phi(S,F)}]_m &= \frac{\overline{r}^B_m}{\overline{R}_0}\left\{\frac{1}{\overline{\phi}^L_m}\overline{w\ell}^Q_m + \sum_{s=1}^M\frac{(1+\overline{\theta}_{ms}\overline{r}^T_s)}{\overline{\phi}^X_{ms}}(1-\overline{\theta}_{ms})\overline{p}_s\overline{x}_{ms}\right\} \\
[\boldsymbol{e}^T_{\phi(S,F)}]_s &= \frac{\overline{r}^T_s}{\overline{R}_0}\left\{\sum_{m=1}^M\frac{(1-\overline{\theta}_{ms})\overline{r}^B_m}{\overline{\phi}^X_{ms}}\overline{\theta}_{ms}\overline{p}_s\overline{x}_{ms}\right\} \\
[\boldsymbol{E}^\theta_{\phi(S,F)}]_{ms} &= \frac{(1+\overline{r}^T_s)\overline{r}^B_m}{\overline{\phi}^X_{ms}}\frac{\overline{\theta}_{ms}\overline{p}_s\overline{x}_{ms}}{\overline{R}_0} + \frac{\overline{C}^T_m}{\overline{R}_0}[\boldsymbol{E}^\theta_{C(T)}]_{ms}
\end{aligned}$$

The *IG-sales wedge* $\boldsymbol{\phi}_X^S$ is defined in Equation (C.5) and is such that an increase in the intermediate sales credit wedge decreases revenues of the intermediate good producer. Using Equations (C.10) and (C.11), the wedge response can be written as

$$\widehat{\phi}_{X,k}^S = \sum_{c=1}^{M}[\boldsymbol{E}_{\phi(S,X)}^B]_{ck}\widehat{r}_c^B - [\boldsymbol{E}_{\phi(S,X)}^T]_{kk}\widehat{r}_k^T - \sum_{c=1}^{M}[\boldsymbol{E}_{\phi(S,X)}^\theta]_{ck}\widehat{\theta}_{ck} \tag{C.13}$$

where

$$[\boldsymbol{E}_{\phi(S,X)}^B]_{ck} = \overline{r}_c^B \frac{\overline{\phi}_k^R}{\overline{\phi}_{ck}^X}\frac{(1-\overline{\theta}_{ck})\overline{p}_k\overline{x}_{ck}}{\overline{R}_k}, \quad [\boldsymbol{E}_{\phi(S,X)}^T]_{kk} = \overline{r}_k^T \sum_{c=1}^{M}\left(1-\frac{\overline{\phi}_k^R}{\overline{\phi}_{ck}^X}\right)\frac{\overline{\theta}_{ck}\overline{p}_k\overline{x}_{ck}}{\overline{R}_k}$$

$$[\boldsymbol{E}_{\phi(S,X)}^\theta]_{ck} = \left\{\overline{r}_c^B\frac{\overline{\phi}_k^R}{\overline{\phi}_{ck}^X} + \overline{r}_k^T\left(1-\frac{\overline{\phi}_k^R}{\overline{\phi}_{ck}^X}\right)\right\}\frac{\overline{\theta}_{ck}\overline{p}_k\overline{x}_{ck}}{\overline{R}_k}$$

Note that $\overline{\theta}_k^C < 1$ and

$$1 > \frac{\overline{\phi}_k^R}{\overline{\phi}_{ck}^X} = \frac{1+\overline{r}_k^T\overline{\theta}_k^C}{1+(1-\overline{\theta}_{ck})\overline{r}_c^B + \overline{\theta}_{ck}\overline{r}_k^T} \quad \text{holds if} \quad \frac{\overline{r}_c^B}{\overline{r}_k^T} > \frac{\overline{\theta}_k^C - \overline{\theta}_{ck}}{1-\overline{\theta}_{ck}} \tag{C.14}$$

The *combined-sales wedge* $\boldsymbol{\phi}_\kappa^S$ is defined in Equation (C.3) such that an increase in the combined sales wedge of sector k reduces sector k's revenues. Assuming that the effect of credit costs on the final sales wedge dominates if sector k sells to the final good producing sector, the wedge response can be written as

$$\widehat{\phi}_{\kappa,k}^S = -\sum_{m=1}^{M}[\boldsymbol{E}_{\phi(S)}^B]_{km}\widehat{r}_m^B + \sum_{m=1}^{M}[\boldsymbol{E}_{\phi(S)}^T]_{km}\widehat{r}_m^T + \sum_{m=1}^{M}\sum_{s=1}^{M}[\boldsymbol{E}_{\phi(S)}^\theta]_{k,ms}\widehat{\theta}_{ms} \tag{C.15}$$

The entries of the respective elasticity matrices are a combination of the elasticity matrices of the final and intermediate sales wedge. The response of sector k's combined sales wedge depends on the demand structure in the economy: $\omega_k^F \in [0,1]$. In the following I only consider the case $\omega_k^F > 0$.

- *Bank Credit Rate.* The elasticity of the combined sales wedge of sector k wrt changes in the bank interest rate of sector m is

$$[\boldsymbol{E}_{\phi(S)}^B]_{km} = \overline{r}_m^B[\boldsymbol{W}_{\phi(S)}^{B(L)}]_{km}\overline{w\ell}_m + \overline{r}_m^B\sum_{s=1}^{M}[\boldsymbol{W}_{\phi(S)}^{B(X)}]_{k,ms}(1-\overline{\theta}_{ms})\overline{p}_s\overline{x}_{ms}$$

where

$$[\boldsymbol{W}^{B(L)}_{\phi(S)}]_{km} \;=\; \omega^F_k \frac{\overline{\phi}^R_k}{\overline{R}_k} \frac{1}{\overline{\phi}^L_m}, \quad \text{and} \quad [\boldsymbol{W}^{B(X)}_{\phi(S)}]_{k,ms} \;=\; \frac{\overline{\phi}^R_k}{\overline{R}_k} \left\{\omega^F_k(1+\overline{\theta}_{ms}\overline{r}^T_s) - \mathcal{I}_{s=k}\right\} \frac{1}{\overline{\phi}^X_{ms}}$$

- *Trade Credit Rate.* The elasticity of the combined sales wedge of sector k wrt changes in the trade credit interest rate of sector m is

$$[\boldsymbol{E}^T_{\phi(S)}]_{km} \;=\; \overline{r}^T_m \sum_{n=1}^{M} [\boldsymbol{W}^{T(X)}_{\phi(S)}]_{k,nm} \overline{AR}_{nm}$$

where

$$[\boldsymbol{W}^{T(X)}_{\phi(S)}]_{k,nm} \;=\; \omega^F_k \frac{\overline{\phi}^R_k}{\overline{R}_k} \frac{(1-\overline{\theta}_{nm})\overline{r}^B_n}{\overline{\phi}^X_{nm}} - \left(1 - \frac{\overline{\phi}^R_m}{\overline{\phi}^X_{nm}}\right) \frac{1}{\overline{R}_m} \mathcal{I}_{m=k}$$

- *Trade Credit Share.* The elasticity of the combined sales wedge of sector k wrt changes in the trade credit share of sector m from sector s is

$$[\boldsymbol{E}^\theta_{\phi(S)}]_{k,ms} \;=\; +[\boldsymbol{W}^{\theta(X)}_{\phi(S)}]_{k,ms} \overline{\theta}_{ms} \overline{p}_s \overline{x}_{ms} + [\boldsymbol{W}^{\theta(\mathcal{K})}_{\phi(S)}]_{k,ms} [\boldsymbol{E}^\theta_{C(T)}]_{ms}$$

where

$$\overline{R}_k [\boldsymbol{W}^{\theta(X)}_{\phi(S)}]_{k,ms} \;=\; \omega^F_k \frac{\overline{\phi}^R_k}{\overline{\phi}^X_{ms}} (1+\overline{r}^T_s)\overline{r}^B_m - \left\{ \overline{r}^B_m \frac{\overline{\phi}^R_s}{\overline{\phi}^X_{ms}} + \overline{r}^T_s \left(1 - \frac{\overline{\phi}^R_s}{\overline{\phi}^X_{ms}}\right) \right\} \mathcal{I}_{s=k}$$

$$\overline{R}_k [\boldsymbol{W}^{\theta(\mathcal{K})}_{\phi(S)}]_{k,ms} \;=\; \omega^F_k \overline{\phi}^R_k \overline{C}^T_m$$

Note that for $s = k$, $\overline{R}_k [\boldsymbol{W}^{\theta(X)}_{\phi(S)}]_{k,ms} > 0$ if

$$1 + \frac{\overline{r}^T_k}{\overline{r}^B_m} \frac{\overline{\phi}^X_{mk} - \overline{\phi}^R_k}{\overline{\phi}^R_k} \;=\; 1 + \frac{\overline{r}^T_k}{\overline{r}^B_m} \frac{(1-\overline{\theta}_{mk})\overline{r}^B_m + (\overline{\theta}_{mk} - \overline{\theta}^C_k)\overline{r}^T_k}{1+\overline{\theta}^C_k \overline{r}^T_k} \;<\; \omega^F_k (1+\overline{r}^T_k)$$

□

Proof of Lemma 8. Using Equations (C.11) and (C.10) the price wedge response is

$$\widehat{\phi}^P_{P,k} \;=\; [\boldsymbol{E}^B_{\phi(P,P)}]_k \widehat{r}^B_k + \sum_{s=1}^{M} [\boldsymbol{E}^T_{\phi(P,P)}]_{ks} \widehat{r}^T_s - \sum_{s=1}^{M} [\boldsymbol{E}^{\theta(S)}_{\phi(P,P)}]_{ks} \widehat{\theta}_{ks} - \sum_{c=1}^{M} [\boldsymbol{E}^{\theta(C)}_{\phi(P,P)}]_{ck} \widehat{\theta}_{ck} \quad \text{(C.16)}$$

The typical entries of the corresponding elasticity matrices are defined below

$$[\boldsymbol{E}^{B}_{\phi(P,P)}]_{kk} \;=\; \frac{\overline{r}^{B}_{k}}{\overline{R}_{k}}\left\{\overline{w\ell}_{k}+\sum_{n=1}^{M}(1-\overline{\theta}_{kn})\overline{p}_{n}\overline{x}_{kn}\right\}$$

$$[\boldsymbol{E}^{T}_{\phi(P,P)}]_{ks} \;=\; \overline{r}^{T}_{s}\frac{\overline{\theta}_{ks}\overline{p}_{s}\overline{x}_{ks}}{\overline{R}_{k}} \quad \text{and} \quad [\boldsymbol{E}^{T}_{\phi(P,P)}]_{kk} \;=\; -\frac{\overline{r}^{T}_{k}}{\overline{R}_{k}}\sum_{n\neq k}^{M}\overline{\theta}_{nk}\overline{p}_{k}\overline{x}_{nk}$$

$$[\boldsymbol{E}^{\theta(S)}_{\phi(P,P)}]_{ks} \;=\; (\overline{r}^{B}_{k}-\overline{r}^{T}_{s})\frac{\overline{\theta}_{ks}\overline{p}_{s}\overline{x}_{ks}}{\overline{R}_{k}} \quad \text{and} \quad [\boldsymbol{E}^{\theta(C)}_{\phi(P,P)}]_{ck} \;=\; \overline{r}^{T}_{k}\frac{\overline{\theta}_{ck}\overline{p}_{k}\overline{x}_{ck}}{\overline{R}_{k}}$$

The *combined price wedge* $\widehat{\boldsymbol{\phi}}^{P}_{\kappa}$ is defined in Equation (C.7) and is a combination of the price wedge (C.7) and the combined sales wedge (C.3) due to the presence of decreasing returns to scale such that prices are increasing in revenues. An increase in the combined price wedge of sector k increases sector k's price. The wedge response in terms of the cost of credit can be written as

$$\phi^{P}_{\kappa,k} \;=\; \sum_{m=1}^{M}[\boldsymbol{E}^{B}_{\phi(P)}]_{km}\widehat{r}^{B}_{m}+\sum_{m=1}^{M}[\boldsymbol{E}^{T}_{\phi(P)}]_{km}\widehat{r}^{T}_{m}-\sum_{m=1}^{M}\sum_{n=1}^{M}[\boldsymbol{E}^{\theta}_{\phi(P)}]_{k,mn}\widehat{\theta}_{mn} \tag{C.17}$$

The typical entries of the elasticity matrices are derived below.

- *Bank Interest Rate.* The elasticity is given by

$$[\boldsymbol{E}^{B}_{\phi(P)}]_{km} \;=\; \overline{r}^{B}_{m}[\boldsymbol{W}^{B(L)}_{\phi(P)}]_{km}\overline{w\ell}^{Q}_{m}+\overline{r}^{B}_{m}\sum_{n=1}^{M}[\boldsymbol{W}^{B(X)}_{\phi(P)}]_{k,mn}(1-\overline{\theta}_{mn})\overline{p}_{n}\overline{x}_{mn}$$

where

$$[\boldsymbol{W}^{B(L)}_{\phi(P)}]_{km} \;=\; [\boldsymbol{W}^{P}_{P}]_{km}\frac{1}{\overline{R}_{m}}+\left(\sum_{j=1}^{M}[\boldsymbol{W}^{R}_{P}]_{kj}\omega^{F}_{j}\frac{\overline{\phi}^{R}_{j}}{\overline{R}_{j}}\right)\frac{1}{\overline{\phi}^{L}_{m}}$$

$$[\boldsymbol{W}^{B(X)}_{\phi(P)}]_{k,mn} \;=\; [\boldsymbol{W}^{P}_{P}]_{km}\frac{1}{\overline{R}_{m}}+\left(\sum_{j=1}^{M}[\boldsymbol{W}^{R}_{P}]_{kj}\omega^{F}_{j}\frac{\overline{\phi}^{R}_{j}}{\overline{R}_{j}}\right)\frac{(1+\overline{\theta}_{mn}\overline{r}^{T}_{n})}{\overline{\phi}^{X}_{mn}}-[\boldsymbol{W}^{R}_{P}]_{kn}\frac{\overline{\phi}^{R}_{n}}{\overline{\phi}^{X}_{mn}}\frac{1}{\overline{R}_{n}}$$

- *Trade Credit Interest Rate.* The elasticity is given by

$$[\boldsymbol{E}^{T}_{\phi(P)}]_{km} \;=\; \overline{r}^{T}_{m}\sum_{n=1}^{M}[\boldsymbol{W}^{T(X)}_{\phi(P)}]_{k,nm}\overline{\theta}_{nm}\overline{p}_{m}\overline{x}_{nm}$$

where

$$[\boldsymbol{W}^{T(X)}_{\phi(P)}]_{k,nm} \;=\; \left\{[\boldsymbol{W}^{P}_{P}]_{kn}\frac{1}{\overline{R}_{n}}+[\boldsymbol{W}^{R}_{P}]_{km}\left(1-\frac{\overline{\phi}^{R}_{m}}{\overline{\phi}^{X}_{nm}}\right)\frac{1}{\overline{R}_{m}}\right\} -\left\{[\boldsymbol{W}^{P}_{P}]_{km}\frac{1}{\overline{R}_{m}}+\left(\sum_{j=1}^{M}[\boldsymbol{W}^{R}_{P}]_{kj}\omega^{F}_{j}\frac{\overline{\phi}^{R}_{j}}{\overline{R}_{j}}\right)\frac{(1-\overline{\theta}_{nm})\overline{r}^{B}_{n}}{\overline{\phi}^{X}_{nm}}\right\}$$

- *Trade Credit Share.* The elasticity is given by

$$[\boldsymbol{E}^{\theta}_{\phi(P)}]_{k,mn} \;=\; [\boldsymbol{W}^{\theta(X)}_{\phi(P)}]_{k,mn} AP_{mn} + [\boldsymbol{W}^{\theta(\mathcal{K})}_{\phi(P)}]_{k,mn} [\boldsymbol{E}^{\theta}_{C(T)}]_{mn} \tag{C.18}$$

where

$$\begin{aligned}
[\boldsymbol{W}^{\theta(X)}_{\phi(P)}]_{k,mn} \;=\;& +[\boldsymbol{W}^{P}_{P}]_{kn}\frac{\overline{r}^{T}_{n}}{\overline{R}_{n}} + [\boldsymbol{W}^{P}_{P}]_{km}\frac{(\overline{r}^{B}_{m}-\overline{r}^{T}_{n})}{\overline{R}_{m}} \\
& - [\boldsymbol{W}^{R}_{P}]_{kn}\frac{1}{\overline{R}_{n}}\left[\overline{r}^{B}_{m}\frac{\overline{\phi}^{R}_{n}}{\overline{\phi}^{X}_{mn}} + \overline{r}^{T}_{n}\left(1-\frac{\overline{\phi}^{R}_{n}}{\overline{\phi}^{X}_{mn}}\right)\right] + \left(\sum_{j=1}^{M}[\boldsymbol{W}^{R}_{P}]_{kj}\omega^{F}_{j}\frac{\overline{\phi}^{R}_{j}}{\overline{R}_{j}}\right)\frac{(1+\overline{r}^{T}_{n})\overline{r}^{B}_{m}}{\overline{\phi}^{X}_{mn}} \\
[\boldsymbol{W}^{\theta(\mathcal{K})}_{\phi(P)}]_{k,mn} \;=\;& \left(\sum_{j=1}^{M}[\boldsymbol{W}^{R}_{P}]_{kj}\omega^{F}_{j}\frac{\overline{\phi}^{R}_{j}}{\overline{R}_{j}}\right)\overline{C}^{T}_{m}
\end{aligned}$$

□

Proof of Lemma 9. The sectoral *output wedge* $\widehat{\boldsymbol{\phi}}^{Q}$ is a combination of the sales (30), the revenue (28) and the price wedges (31) and can be written as

$$\widehat{\phi}^{Q}_{k} \;=\; \sum_{m=1}^{M}[\boldsymbol{E}^{B}_{\phi(Q)}]_{km}\widehat{r}^{B}_{m} + \sum_{m=1}^{M}[\boldsymbol{E}^{T}_{\phi(Q)}]_{km}\widehat{r}^{T}_{m} - \sum_{m=1}^{M}\sum_{s=1}^{M}[\boldsymbol{E}^{\theta}_{\phi(Q)}]_{k,ms}\widehat{\theta}_{ms} \tag{C.19}$$

Define

$$[\boldsymbol{\Delta}_{RP}]_{km} \;=\; [\boldsymbol{W}^{R}_{R}]_{km} - [\boldsymbol{W}^{R}_{P}]_{km} \;=\; [\boldsymbol{W}^{R}_{R}]_{km} - \sum_{n=1}^{M}(1-\overline{\chi}_{n})[\boldsymbol{W}^{P}_{P}]_{kn}[\boldsymbol{W}^{R}_{R}]_{nm}$$

and assume that $[\boldsymbol{\Delta}_{RP}]_{km} > 0$ which implies that the effect of the combined sales wedge on revenues dominates the effect on prices.

- *Bank Interest Rate.* The elasticity is given by

$$[\boldsymbol{E}^{B}_{\phi(Q)}]_{km} \;=\; +\overline{r}^{B}_{m}[\boldsymbol{W}^{B(L)}_{Q}]_{km}\overline{w\ell}_{m} + \overline{r}^{B}_{m}\sum_{s=1}^{M}[\boldsymbol{W}^{B(X)}_{Q}]_{k,ms}\overline{AP}^{-}_{ms}$$

where

$$\begin{aligned}
[\boldsymbol{W}^{B(L)}_{Q}]_{km} \;=\;& [\boldsymbol{W}^{P}_{P}]_{km}\frac{1}{\overline{R}_{m}} - \left(\sum_{n=1}^{M}[\boldsymbol{\Delta}_{RP}]_{kn}\omega^{F}_{n}\frac{\overline{\phi}^{R}_{n}}{\overline{R}_{n}}\right)\frac{1}{\overline{\phi}^{L}_{m}} \\
[\boldsymbol{W}^{B(X)}_{Q}]_{k,ms} \;=\;& [\boldsymbol{W}^{P}_{P}]_{km}\frac{1}{\overline{R}_{m}} + [\boldsymbol{\Delta}_{RP}]_{kn}\frac{\overline{\phi}^{R}_{n}}{\overline{\phi}^{X}_{mn}}\frac{1}{\overline{R}_{n}} - \left(\sum_{j=1}^{M}[\boldsymbol{\Delta}_{RP}]_{kj}\omega^{F}_{j}\frac{\overline{\phi}^{R}_{j}}{\overline{R}_{j}}\right)\frac{(1+\overline{\theta}_{mn}\overline{r}^{T}_{n})}{\overline{\phi}^{X}_{mn}}
\end{aligned}$$

- *Trade Credit Interest Rate.* The elasticity is given by

$$[\boldsymbol{E}^T_{\phi(Q)}]_{km} = \overline{r}^T_m \sum_{n=1}^{M} [\boldsymbol{W}^T_Q]_{k,nm} \overline{AR}_{nm}$$

such that $0 < [\boldsymbol{E}^T_{\phi(Q)}]_{km}$ where

$$\begin{aligned}
[\boldsymbol{W}^T_Q]_{k,nm} &= +\frac{\mathcal{I}_{m=k}}{\overline{R}_m} + [\boldsymbol{W}^P_P]_{kn}\frac{1}{\overline{R}_n} + \left(\sum_{j=1}^{M}[\boldsymbol{\Delta}_{RP}]_{kj}\omega^F_j \frac{\overline{\phi}^R_j}{\overline{R}_j}\right)\frac{(1-\overline{\theta}_{nm})\overline{r}^B_n}{\overline{\phi}^X_{nm}} \\
&\quad - \left\{[\boldsymbol{W}^P_P]_{km}\frac{1}{\overline{R}_m} + [\boldsymbol{\Delta}_{RP}]_{km}\left(1 - \frac{\overline{\phi}^R_m}{\overline{\phi}^X_{nm}}\right)\frac{1}{\overline{R}_m}\right\}
\end{aligned}$$

- *Trade Credit Shares.* The elasticity is given by

$$[\boldsymbol{E}^\theta_{\phi(Q)}]_{k,ms} = +[\boldsymbol{W}^{\theta(X)}_Q]_{k,ms}\overline{AP}_{ms} - [\boldsymbol{W}^{\theta(\mathcal{K})}_Q]_{k,ms}[\boldsymbol{E}^\theta_{C(T)}]_{ms}$$

where

$$\begin{aligned}
[\boldsymbol{W}^{\theta(X)}_Q]_{k,ms} &= [\boldsymbol{W}^P_P]_{km}\frac{(\overline{r}^B_m - \overline{r}^T_s)}{\overline{R}_m} + \left([\boldsymbol{W}^P_P]_{ks} - \mathcal{I}_{s=k}\right)\frac{\overline{r}^T_s}{\overline{R}_s} - \sum_{n=1}^{M}[\boldsymbol{\Delta}_{RP}]_{kn}[\boldsymbol{W}^{\theta(X)}_{\phi(S)}]_{n,ms} \\
[\boldsymbol{W}^{\theta(\mathcal{K})}_Q]_{k,ms} &= \sum_{n=1}^{M}[\boldsymbol{\Delta}_{RP}]_{kn}[\boldsymbol{W}^{\theta(\mathcal{K})}_{\phi(S)}]_{n,ms}
\end{aligned}$$

□

C.3. Credit Costs, Links and Interest Rates

Proof of Lemma 10. The *trade credit share* responses are derived as follows. The log-linearization of Equation (24) and tedious algebra yields

$$\widehat{\theta}_{ks} = +\sum_{m=1}^{M}[\boldsymbol{E}^B_\theta]_{ks,m}\widehat{r}^B_m + \sum_{m=1}^{M}[\boldsymbol{E}^T_\theta]_{ks,m}\widehat{r}^T_m - \sum_{m=1}^{M}\sum_{n=1}^{M}[\boldsymbol{E}^\theta_\theta]_{ks,mn}\widehat{\theta}_{mn}$$

The respective elasticities are defined in the following, where

$$[\boldsymbol{W}_\theta]_{ks} = \frac{\overline{\theta}^S_k}{\overline{\theta}_{ks}}\frac{\overline{\theta}^S_k}{\kappa^T_{1,k}}\omega^X_{ks}(1-\upsilon)\frac{\overline{\Delta}_{ks}}{\overline{\phi}^L_k}\frac{\overline{p}^V_k\overline{V}_k}{\overline{\phi}^X_{ks}}$$

- *Bank Interest Rate.* The elasticity is given by

$$[\boldsymbol{E}^B_\theta]_{ks,m} = \overline{r}^B_m[\boldsymbol{\kappa}^B_\theta]_{ks,m}\mathcal{I}_{m=k} + \overline{r}^B_m[\boldsymbol{W}^{B(L)}_\theta]_{ks,m}\overline{w\ell}_m + \overline{r}^B_m\sum_{n=1}^{M}[\boldsymbol{W}^{B(X)}_\theta]_{ks,mn}(1-\overline{\theta}_{mn})\overline{p}_n\overline{x}_{mn}$$

where

$$
\begin{aligned}
[\boldsymbol{W}_\theta]^{-1}_{ks}[\boldsymbol{W}^{B(L)}_\theta]_{ks,m} &= (1-\upsilon_k)\sum_{i=1}^{M}\omega^X_{ki}[\boldsymbol{W}^{B(L)}_{\phi(P)}]_{im} \\
[\boldsymbol{W}_\theta]^{-1}_{ks}[\boldsymbol{W}^{B(X)}_\theta]_{ks,mn} &= (1-\upsilon_k)\sum_{i=1}^{M}\omega^X_{ki}[\boldsymbol{W}^{B(X)}_{\phi(P)}]_{i,mn} \\
[\boldsymbol{W}_\theta]^{-1}_{ks}[\boldsymbol{\kappa}^B_\theta]_{ks,m} &= \left(\frac{\text{sgn}(\Delta_{ks})}{|\overline{\Delta}_{ks}|} - \frac{(1-\overline{\theta}_{ks})}{\overline{\phi}^X_{ks}}\right)\frac{1}{(1-\upsilon_k)} + \sum_{n=1}^{M}\omega^X_{kn}\frac{(1-\overline{\theta}_{kn})}{\overline{\phi}^X_{kn}} - \frac{1}{\overline{\phi}^L_k}
\end{aligned}
$$

- *Trade Credit Interest Rate.* The elasticity is given by

$$
[\boldsymbol{E}^T_\theta]_{ks,m} = -\overline{r}^T_m[\boldsymbol{\kappa}^T_\theta]_{ks,m}\mathcal{I}_{m=s} + \overline{r}^T_m\sum_{n=1}^{M}[\boldsymbol{W}^T_\theta]_{ks,nm}\overline{AR}_{nm}
$$

where

$$
\begin{aligned}
[\boldsymbol{W}_\theta]^{-1}_{ks}[\boldsymbol{W}^T_\theta]_{ks,nm} &= \frac{\mathcal{I}_{n=k}}{\overline{\chi}_k\overline{R}_k} + (1-\upsilon_k)\sum_{i=1}^{M}\omega^X_{ki}[\boldsymbol{W}^{T(X)}_{\phi(P)}]_{i,nm} \\
[\boldsymbol{W}_\theta]^{-1}_{ks}[\boldsymbol{\kappa}^T_\theta]_{ks,m} &= \frac{\text{sgn}(\Delta_{ks})}{|\overline{\Delta}_{ks}|} + \frac{\overline{\theta}_{ks}}{\overline{\phi}^X_{ks}}
\end{aligned}
$$

- *Trade Credit Shares.* The elasticity is given by

$$
[\boldsymbol{E}^\theta_\theta]_{ks,mn} = -[\boldsymbol{\kappa}^\theta_\theta]_{ks}\mathcal{I}_{m=k}\mathcal{I}_{n=s} + [\boldsymbol{W}^{\theta(X)}_\theta]_{ks,mn}\overline{AP}_{mn} + [\boldsymbol{W}^{\theta(\mathcal{K})}_\theta]_{ks,mn}[\boldsymbol{E}^\theta_{C(T)}]_{mn}
$$

where

$$
\begin{aligned}
[\boldsymbol{W}_\theta]^{-1}_{ks}[\boldsymbol{W}^{\theta(X)}_\theta]_{ks,mn} &= \frac{(\overline{r}^B_m - \overline{r}^T_n)}{\overline{\chi}_m\overline{R}_m}\mathcal{I}_{m=k} + (1-\upsilon_k)\sum_{i=1}^{M}\omega^X_{ki}[\boldsymbol{W}^{\theta(X)}_{\phi(P)}]_{i,mn} \\
[\boldsymbol{W}_\theta]^{-1}_{ks}[\boldsymbol{W}^{\theta(\mathcal{K})}_\theta]_{ks,mn} &= (1-\upsilon_k)\sum_{i=1}^{M}\omega^X_{ki}[\boldsymbol{W}^{\theta(\mathcal{K})}_{\phi(P)}]_{i,mn} \\
[\boldsymbol{W}_\theta]^{-1}_{ks}[\boldsymbol{\kappa}^\theta_\theta]_{ks} &= \frac{\left(\overline{r}^B_k - \overline{r}^T_s\right)\overline{\theta}_{ks}}{\overline{\phi}^X_{ks}}
\end{aligned}
$$

□

Proof of Lemma 11. The log-linearization of the *bank interest rate* yields

$$
\widehat{r}^B_k = [\boldsymbol{E}^Z_B]_k\widehat{z}^B_k + \sum_c[\boldsymbol{E}^\theta_B]_{ck}\widehat{\theta}_{ck} \tag{C.20}
$$

where the elasticity matrices are

$$[\boldsymbol{E}_B^Z]_k \;=\; \frac{\Delta_k^B}{\overline{r}_k^B} \quad \text{and} \quad [\boldsymbol{E}_B^\theta]_{ck} \;=\; \frac{\mu\Delta_k^B}{\overline{r}_k^B}\frac{\overline{\theta}_k^C}{\overline{\theta}_0^D+\overline{\theta}_k^C}\frac{\overline{\theta}_{ck}\overline{p}_k\overline{x}_{ck}}{\overline{p}_k\overline{q}_k}$$

The log-linearization of the *trade credit interest rate* yields

$$\widehat{r}_k^T \;=\; -\sum_{m=1}^M[\boldsymbol{E}_T^B]_{km}\widehat{r}_m^B+\sum_{m=1}^M[\boldsymbol{E}_T^T]_{km}\widehat{r}_m^T+\sum_{m=1}^M\sum_{n=1}^M[\boldsymbol{E}_T^\theta]_{k,mn}\widehat{\theta}_{mn}+[\boldsymbol{E}_T^{Z(B)}]_{kk}\widehat{z}_k^B$$

Let $[\boldsymbol{A}_1]_k$, $[\boldsymbol{A}_2]_k$ and $[\boldsymbol{A}_3]_k$ be positive constants, then

- *Bank Interest Rate.* The elasticity is given by

$$[\boldsymbol{E}_T^B]_{km} \;=\; \overline{r}_m^B[\boldsymbol{W}_T^{B(L)}]_{km}\overline{w\ell}_m+\overline{r}_m^B\sum_{n=1}^M[\boldsymbol{W}_T^{B(X)}]_{k,mn}\overline{AP}_{mn}^- \tag{C.21}$$

where

$$[\boldsymbol{W}_T^{B(L)}]_{km} \;=\; +\frac{\overline{\mathbb{r}}_m^\theta}{\overline{r}_m^T}\frac{\overline{\phi}_m^R}{\overline{R}_m}\mathcal{I}_{m=k}+\frac{\overline{\mathbb{r}}_k^\theta\overline{s}_k^T}{\overline{r}_k^T}\sum_{n=1}^M[\boldsymbol{W}_R^R]_{kn}[\boldsymbol{W}_{\phi(S)}^{B(L)}]_{nm}$$
$$[\boldsymbol{W}_T^{B(X)}]_{k,mn} \;=\; +\frac{\overline{\mathbb{r}}_m^\theta}{\overline{r}_m^T}\frac{\overline{\phi}_m^R}{\overline{R}_m}\mathcal{I}_{m=k}+[\boldsymbol{A}_3]_k\sum_{n=1}^M[\boldsymbol{W}_R^R]_{kn}[\boldsymbol{W}_{\phi(S)}^{B(X)}]_{n,ms}$$

- *Trade Credit Interest Rate.* The elasticity is given by

$$[\boldsymbol{E}_T^T]_{km} \;=\; \overline{r}_m^T\sum_{n=1}^M[\boldsymbol{W}_T^{T(X)}]_{k,nm}\overline{AR}_{nm} \tag{C.22}$$

where

$$[\boldsymbol{W}_T^{T(X)}]_{k,nm} \;=\; \frac{([\boldsymbol{A}_2]_k+[\boldsymbol{A}_3]_k)}{\overline{R}_m}\mathcal{I}_{m=k}+[\boldsymbol{A}_3]_k\sum_{i=1}^M[\boldsymbol{W}_R^R]_{ki}[\boldsymbol{W}_{\phi(s)}^{T(X)}]_{i,nm}-\frac{\overline{\mathbb{r}}_n^\theta}{\overline{r}_n^T}\frac{\overline{\phi}_n^R}{\overline{\phi}_{nm}^X}\frac{(1-\overline{\theta}_{nm})}{\overline{R}_n}\mathcal{I}_{n=k}$$

- *Trade Credit Shares.* The elasticity for $m \neq k$ is given by

$$[\boldsymbol{E}_T^{\theta 1}]_{k,mn} \;=\; [\boldsymbol{W}_T^{\theta(X)}]_{k,mn}\overline{AP}_{mn}+[\boldsymbol{W}_T^{\theta(\mathcal{K})}]_{k,mn}[\boldsymbol{E}_{C(T)}^\theta]_{mn}$$

where

$$
\begin{aligned}
[\boldsymbol{W}_T^{\theta(X)}]_{k,mn} &= \frac{\overline{r}_k^\theta}{\overline{r}_k^T} \frac{\overline{\phi}_k^R \overline{C}_k^T}{\overline{R}_k} \sum_{j=1}^M [\boldsymbol{W}_R^R]_{kj} [\boldsymbol{W}_{\phi(S)}^{\theta(X)}]_{j,mn} \\
[\boldsymbol{W}_T^{\theta(\mathcal{K})}]_{k,mn} &= \frac{\overline{r}_k^\theta}{\overline{r}_k^T} \frac{\overline{\phi}_k^R \overline{C}_k^T}{\overline{R}_k} \sum_{j=1}^M [\boldsymbol{W}_R^R]_{kj} [\boldsymbol{W}_{\phi(S)}^{\theta(\mathcal{K})}]_{j,mn}
\end{aligned}
$$

□

C.4. Discussion

Proof of Lemma 12. The *partial equilibrium* ($\widehat{L} = \widehat{w}_{cm}^C = 0$) output response for sector k to credit shock ($\epsilon_k^Q = 0$) is given by $\widehat{q}_k = -\widehat{\phi}_k^Q$. Using Equation (C.19) and applying the first order approximation of the trade credit multiplier defined in Definition 6, the output wedge response to a first order approximation is given by

$$
\widehat{\phi}_k^Q = \sum_{m=1}^M \left\{ [\widetilde{\boldsymbol{\Psi}}_Q^B]_{km} + [\widetilde{\boldsymbol{\Psi}}_Q^T]_{km} - [\widetilde{\boldsymbol{\Psi}}_Q^\theta]_{km} \right\} \epsilon_m^B = \sum_{m=1}^M [\widetilde{\boldsymbol{\Psi}}_{B,\tau}^Q]_{km} \epsilon_m^B
$$

Straight forward but tedious algebra yields the first order approximation of the structural coefficients related to the bank and trade credit interest rate and trade credit shares:

$$
\begin{aligned}
[\widetilde{\boldsymbol{\Psi}}_Q^B]_{km} &= +\overline{r}_m^B [\boldsymbol{S}_{B,Q}^{B(L)}]_{km} \overline{w\ell}_m + \overline{r}_m^B \sum_{j=1}^M [\boldsymbol{S}_{B,Q}^{B(X)}]_{k,mj} \overline{AP}_{mj}^- \\
[\widetilde{\boldsymbol{\Psi}}_Q^T]_{km} &= +[\widetilde{\boldsymbol{\kappa}}_{T,Q}]_{km} + \overline{r}_m^T \sum_{j=1}^M [\boldsymbol{S}_{T,Q}^{T(X)}]_{k,jm} \overline{AR}_{jm} - \overline{r}_m^B [\boldsymbol{S}_{T,Q}^{B(L)}]_{km} \overline{w\ell}_m - \overline{r}_m^B [\boldsymbol{S}_{T,Q}^{B(X)}]_{k,mj} \overline{AP}_{mj}^- \\
[\widetilde{\boldsymbol{\Psi}}_Q^\theta]_{km} &= +[\widetilde{\boldsymbol{\kappa}}_{\theta,Q}]_{km} + \overline{r}_m^T \sum_{j=1}^M [\boldsymbol{S}_{\theta,Q}^{T(X)}]_{k,jm} \overline{AR}_{jm} + \overline{r}_m^B [\boldsymbol{S}_{\theta,Q}^{B(L)}]_{km} \overline{w\ell}_m + \overline{r}_m^B \sum_{j=1}^M [\boldsymbol{S}_{\theta,Q}^{B(X)}]_{k,mj} \overline{AP}_{mj}^-
\end{aligned}
$$

Combining the structural elasticities above yields the respective elasticities in Equation (39).

□

Proof of Proposition 1. The proof of Proposition 1 follows directly from the derivations used in the proof of Lemma 12. Note that, $[\widetilde{\boldsymbol{\Psi}}_Q^B]_{km}$, captures the structural elasticity of sectoral output of sector k related to shocks to the risk premium of the bank interest rate faced by sector m to a first order approximation. The difference between the elasticities related to changes in the trade credit interest rates, $[\widetilde{\boldsymbol{\Psi}}_Q^T]_{km}$, and trade credit shares, $[\widetilde{\boldsymbol{\Psi}}_Q^\theta]_{km}$, captures the total effect of adjustment via the trade credit channel on output of sector k. Denote $[\Delta\widetilde{\boldsymbol{\Psi}}_Q^{\cdot}]_{km} = [\boldsymbol{S}_Q^{\cdot}]_{km} - [\boldsymbol{S}_Q^{\cdot}]_{km}$. If the adjustment of trade credit volumes and rates have a smoothing effect - e.g. crowd out the increase in the output wedge of sector k in response to a shock to the risk premium of sector m - then it holds that

$$0 \; > \; [\widetilde{\boldsymbol{\Psi}}_Q^T]_{km} - [\widetilde{\boldsymbol{\Psi}}_Q^\theta]_{km}$$

which implies that

$$\begin{aligned}
&\overline{r}_m^B \left\{ [\Delta\widetilde{\boldsymbol{\Psi}}_Q^{B(\kappa)}]_{km} + [\Delta\widetilde{\boldsymbol{\Psi}}_Q^{B(L)}]_{km}\overline{w\ell}_m + \sum_{j=1}^{M}[\Delta\widetilde{\boldsymbol{\Psi}}_Q^{B(X)}]_{k,mj}\overline{AP}_{mj}^{-} \right\} \\
&> [\Delta\widetilde{\boldsymbol{\Psi}}_Q^{T(\kappa)}]_{km} + \overline{r}_m^T \sum_{j=1}^{M}[\Delta\widetilde{\boldsymbol{\Psi}}_Q^{T(X)}]_{k,jm}\overline{AR}_{jm}
\end{aligned}$$

The right hand side is a function of the up-front financing needs of sector m and the left hand side is a function of the accounts receivable extended by sector m. Proposition 1 follows. □

D. Data and Calibration

D.1. Data Description

In Section D.1.1, I first describe the sample composition obtained from Compustat and used to calibrate the sector-to-sector equilibrium trade credit shares in model. I then discuss accounting issues related to the consistent assignment of interest rate expenditures on bank credit. Section D.1.2 discusses adjustments to the IO-tables made in the calibration of the model.

Table D.1: Sample Description

ID	Sector	Description	#Firms	RP(Y)	RP(R)	NL
1	11	Agriculture	8	0.15	0.06	0.16
2	211	Oil and Gas	24	0.96	0.62	0.57
3	212	Mining, except 211	14	0.55	0.28	0.10
4	213	Support for 212	9	0.56	0.41	0.03
5	22	Utilities	49	0.87	0.53	0.10
6	23	Construction	20	0.06	0.03	0.01
7	311T2	Food, Beverages and Tobacco	48	0.73	0.20	0.04
8	313T6	Textile, Apparel and Leather	38	1.97	0.62	0.07
9	321	Wood Products	8	0.20	0.06	0.06
10	322T3	Paper Products and Printing	23	0.78	0.29	0.10
11	324	Petroleum and Coal Products	12	1.40	0.38	0.06
12	325	Chemical Products	110	0.92	0.37	0.12
13	326	Plastics and Rubber Products	23	0.49	0.16	0.13
14	327	Nonmetallic Mineral Products	14	0.35	0.15	0.15
15	331	Primary Metals	16	0.68	0.17	0.16
16	332	Fabricated Metal Products	31	0.39	0.16	0.15
17	333	Machinery	78	0.98	0.36	0.06
18	334	Computer and electronic Products	157	1.33	0.74	0.09
19	335	Electrical Equipment and Components	30	0.43	0.18	0.12
20	3361MV	Motor Vehicles, Bodies and Parts	28	1.96	0.44	0.20
21	3364OT	Other Transportation Equipment	13	0.61	0.24	0.03
22	337	Furniture and Related Products	13	0.47	0.18	0.04
23	339	Misc Manufacturing	45	0.48	0.23	0.08
24	42	Wholesale Trade	79	0.32	0.22	0.06
25	441	Motor Vehicle and Parts Dealers	7	0.34	0.25	0.01
26	445	Food and Beverage Stores	7	0.90	0.61	0.00
27	452	General Merchandise Stores	15	3.39	2.24	0.00
28	4A0	Other Retail	9	0.86	0.39	0.01
29	481	Air Transport	4	1.20	0.65	0.02
30	482	Rail Transport	9	0.06	0.03	0.05
31	484	Truck Transport	6	0.89	0.47	0.07
32	486	Pipeline Transport	9	0.05	0.03	0.18
33	48A9	Other Transport and Warehousing	79	0.66	0.42	0.10
34	511	Publishing Industries	42	0.11	0.06	0.03
35	512	Motion Picture and Sound	7	0.36	0.22	0.13
36	513	Broadcasting & Telecommunications	41	0.47	0.24	0.10
37	514	Information Services	34	1.30	0.75	0.21
38	54	Professional & Technical Services	83	0.04	0.02	0.14
39	55	Management of Companies	-	-	-	0.00
40	56	Administrative & Waste services	42	0.12	0.08	0.09
41	62	Health Care & Social Assistance	33	0.03	0.02	0.00
42	71	Arts, Entertainment, Recreation	20	0.06	0.04	0.03
43	72	Accommodation & Food Services	42	0.13	0.07	0.01
44	81	Other Services except GOV	5	0.00	0.00	0.04
45	GOV	Government and Education	22	0.01	0.09	0.00
		Mean	31.95	0.65	0.31	0.09
		Stdv	35.35	0.65	0.36	0.09
		Min	4	0	0	0.00
		ID	(29)	(44)	(44)	(39)
		Max	157	3.39	2.24	0.57
		ID	(18)	(27)	(27)	(2)

Note: This table presents the NAICS (2007) IDs and descriptions of the sectors included in the calibration of the model. In addition, the table reports the average number of firms (#Firms) in each industry included in the sample from Compustat over the entire sample period 1997-2016. The representativeness of the sample for each sector is calculated as the share of total sales of firms in industry value added (RP(Y)) and in gross industry output (RP(R)) as reported by the BEA. The last column reports the average net-lending position of each sector based on Definition 1.

D.1.1. Trade Credit and Profit Decomposition

Trade Credit. *Data Composition.* The Compustat database is used to infer sectoral trade credit (shares) based on the balance sheet data on accounts receivables and payables of US-firms. A firm is included in the sample if all of the following criteria hold

(1) non-missing NAICS-classification

(2) headquarter in the US

(3) non-missing, non-zero and non-negative data on balance sheet items

(4) the (sequential) year-coverage is 2000-2007

In total 1,406 firms are included in the initial sample. The average number of firms in each sector and the representativeness of the firms for each industry are presented in Table D.1 across all years 1997-2016.

Sectoral Trade Credit Shares. The share of accounts payable in total input expenditures (θ_k^P) and the share of accounts receivable in total revenues (θ_s^R) are used to construct a proxy of inter-industry credit flows using the approach suggested in Altinoglu (2018). The inter-industry trade credit share from supplier s to customer k is constructed as a (sales) weighted average of the total trade credit shares shown in Equation (40).

Dealing with Missing Data and Domestic Non-Market Clearing. Since some industries are not or under-represented in the Compustat sample, it is possible that observations on industry trade credit share are missing. I account for missing observations as follows: (a) If a sector is missing all trade credit data, all trade credit shares are set to zero which implies that this sector is neither extending nor receiving and trade credit. (b) If the time series of trade credit shares of a sector contains some missing observations, I first identify the period with the highest number of consecutive non-missing observations. Using the first and last observation of this period, I use the median growth rate of trade credit shares in the sample to extrapolate the level of trade credit shares for the remaining observations.

As the model assumes a closed economy, all trade credit relations are between domestic firms. Therefore, I need to ensure market clearing for domestic trade credit as follows

$$\sum_{m=1}^{M} AP_{mt} = \sum_{m=1}^{M} AR_{mt} \tag{D.23}$$

I first calculate the implied level of total sectoral accounts payable and receivables using the sectoral trade credit shares derived from Compustat and the total intermediate expenditures and sales as recorded in the IO-tables. If Equation (D.23) does not hold, sectoral accounts receivable (shares) are adjusted by the share of exports (X_{kt}) in total sales (R_{kt}) for each sector.

Profit Decomposition. Bank interest rate expenditures, are recorded as part of the gross operating surplus in the IO-tables net of interest-income ($idit$). (see Horrowitz and Planting, 2009). I thus decompose the gross operating surplus - GOP - (π) into capital expenditures (dp), dividend payments ($ni + dv$) and bank interest rate expenditures ($xint$) using the shares of the respective counterparts in gross operating profits calculated from the income statements of the panel of US-firms from Compustat. From the income statement it follows that $\pi + idit = dp + ni + dv + xint = \Sigma$. Thus, total actual profits are a multiple of the observed profits π. The dividend, interest rate expenditure and capital shares for decomposing the GOP as recorded in IO-Tables are then given by

$$\left(1 - \frac{idit}{\Sigma}\right)^{-1} \pi = \Sigma \quad \text{such that} \quad shDV = \frac{ni + dv}{\Sigma}, \; shIR = \frac{xint}{\Sigma}, \; shK = \frac{dp}{\Sigma}$$

The level of dividends, interest payments and capital expenditures then follows directly from the GOP recorded in the IO-table.

D.1.2. Preparation of IO-Tables

The model is calibrated using the summary tables on "Use of Commodities by Industries After Redefinitions" provided by the BEA. In order to ensure an appropriate mapping of the model to the data, adjustments are made as described below.

Treatment of Used and Non-Comparable Imports. I exclude the rows of the IO-Matrix corresponding to expenditures on "Used Goods" and "Non-Comparable Imports" and deduct the same amount from total industry output. Any other negative intermediate expenditures entries are set to zero.

Treatment of FIRE. I follow BL(2017) and interpret the production function (1) as describing the technology at use related to physical production inputs rather than interest rates, insurance premia or rental rates. As in BL(2017), the expenditures on FIRE-services are treated as part of capital gains and not as intermediate production expenditures which implies a reassignment of the corresponding rows of the IO-tables to gross-operating profits. The purchases of the FIRE-sector are treated as part of final demand. In order to avoid double counting, the resulting share of capital gains attributed to FIRE-expenditures is treated as income accruing to foreign households and thus excluded from the calculation of GDP.

Inventories. Changes in inventories are recorded as part of final uses. However, the model is static and does not account for the accumulation of inventories. Therefore, as in BL(2017) I subtract changes in inventories from final uses and redistribute the dollar value supplied by sector i proportionally across i's intermediate customers using the sales share of each sector in i's total intermediate sales. Following the adjustment of intermediate sales for changes in inventories, I recalculate total intermediate expenditures and total industry output for each sector.

Final Demand, Imports and Exports. While the model is a closed economy without investment, sectors in the US-economy invest and engage in foreign trade. Two observations can be made: (1) The majority of commodities in the US are (a) both produced domestically and imported and (b) both used as intermediate inputs in production and consumed by final demand. (2) Total final uses (consumption, investment and exports) of most sectors exceed imports, which implies that the majority of commodities in the US are also produced domestically.
In order to take the data to the model, I treat investments and exports as part of domestic demand of the final good producer. In the calibration, I account for foreign trade (imports) in the form a intermediate sales residual in order to ensure market-clearing. Note that simply ignoring imports in the calibration of the model or assigning imports to final demand directly implies that good markets do not clear in equilibrium. The calibration ensures that the national accounting identity equalizing total value added and total final demand holds.

Interest Income, Taxes and Profits. Gross operating profits as recorded in the IO-tables include proprietor's and rental income, corporate profits, interest expenditures net of interest income, capital expenditures, etc. In order to map the IO-tables to the model, I follow the steps outlined below to obtain a separate measure of interest expenditures.

(1) *Negative Gross Operating Surplus.* Only a few sectors over the period 1997-2016 record negative profits in a few selected points in time (six observations). Since the model does not allow for negative profits, I set the gross operating surplus to

zero if a negative value was recorded. ($GOP_{1,kt}$)

(2) *Total Interest Income (1).* I then use the share of gross profits in total sales and the share of interest income in gross profits based on Compustat data to calculate a sectors interest income: $IIR = shIIR \cdot (shGPR \cdot R_{kt})$.

(3) *Gross Profits (1).* Gross profits ($GPR_{1,kt}$) are then calculated as the sum of the gross operating surplus adjusted for negative profits ($GOP_{1,kt}$) and the imputed interest income IIR_{kt}.

(4) *Winsorisation of Profit Ratio.* I calculate the ratio of gross profits ($GPR_{1,kt}$) to gross operating profits ($GOP_{1,kt}$) for the 90th-quantile and re-calculate the implied adjusted gross operating profits ($GOP_{2,kt}$). Using the share of interest income in gross profits based on Compustat data, I then re-calculate gross profits ($GPR_{2,kt}$) and interest income ($IIR_{2,kt}$).

(5) *Winsorisation of Cost to Profit Ratio.* Finally, I calculate and winsorize the ratio of operating costs ($w_t \ell_t + \sum_{s=1}^{M} p^e_{st} x_{ks,t}$) to gross profits ($GPR_{2,kt}$) for the 90th-quantile and recalculate implied gross profits ($GPR_{3,kt}$) and interest income ($IIR_{3,kt}$).

(6) *Adjustment of Taxes and Dividend.* In order to ensure that total value added of a sector is left unchanged, I reassign the imputed interest income for each sector by adding the interest income to the gross operating surplus of a sector and deducting it from taxes. Since the model does not account for taxes, I treat taxes as part of dividend payments to households. Due to the reassignment of interest income, tax-payments net of interest income and thus also total dividends can be negative.

Total Industry and Commodity Output. To ensure market clearing, the difference between total industry and total commodity output is added to final uses such that nominal output produced equals total sales. The sales residual is distributed between final demand (sum of consumption, investment and exports) and imports using the respective share in total final demand.

D.1.3. Bank Interest Rates

Risk Premium. The sectoral credit spreads derived in Gilchrist and Zakrajšek (2012) and provided to me by the authors are used as a baseline measure for the risk-premium. Additional adjustments are described below in order to ensure that consistency with the accounting of the IO-tables. The components of the risk-premium are calibrated as follows: The risk-free interest rate on bank credit, r_0^B, is set by calculating the time average (1997-2016) of the federal funds rate. The average leverage in the economy, $\overline{\theta}_0^D$ is calculated using the aggregate measures of the relevant balance sheet items for the sample of US-firms described above and taking the time mean. The exponent, μ, is estimated using a simple OLS-regression based on Equation (43) and is set to 1.1.

Adjustment of Bank Interest Rates. In order to ensure that the cost of bank-credit are consistent with the imputed interest rate expenditures for the extreme case that all intermediate input expenditures and labour costs need to be financed using bank-credit, I make the following adjustments:

(1) *Interest Rate Expenditures.* I first calculate three different measures of the bank interest rate and the maximal bank interest expenditures:

   (a) The bank interest rate implied by the IO-expenditure data ($r^B_{0,kt}$) is calculated using the interest expenditure share in gross profits based on Compustat data, the imputed gross profits and total operating costs

$$r^B_{0,kt} = (s^{\pi}_{r,kt} \cdot GPR_{3,kt}) : (w_t \ell_t + \sum_{s=1}^{M} p^e_{st} x_{ks,t})$$

   (b) The bank-interest rate imposed by the model ($r^B_{1,kt}$) is calculated using the GZ-spread based on which the maximal possible interest rate expenditures are derived using the total operating costs as recorded in the IO-tables.

   (c) As a third measure of the bank-interest rate ($r^B_{2,kt}$), I combine the level of the implied bank-interest rate by the IO-tables at the beginning of the observation period ($r^B_{0,k1}$) with the growth rate of the bank-interest rate implied by the GZ-spread ($r^B_{1,kt}$).

   If the interest rate expenditure share in gross profits implied by the GZ-based interest rate (b) is greater than one, then combined bank-interest rate measure (c) is used instead ($r^B_{3,kt}$), which represents a level adjustment of the imposed bank-interest rate in order to match the IO-tables.

(2) *Winsorisation of Interest Expenditure Share in Gross Profits.* In a final step, I winsorize the interest rate expenditure share in gross profits using the 90th-quantile and re-calculate the implied bank-interest rate ($r^B_{4,kt}$), the bank-interest rate spread and the maximum interest rate expenditure share in gross profits.

D.1.4. Labour Costs and Prices

Prices and Wages. Data on total hours worked and sectoral prices are provided by the Bureau of Labor Statistics (BLS). In particular, I combine the respective variables from the MFP- and the LPC-Database. Total hours worked are then used to infer an aggregate wage rate from total labor expenditures recorded in the IO-tables. The wage rate is chosen as the numeraire and all prices are normalized by the common wage rate.

Labour Expenditures and Hours Worked. Expenditures on non-productive labour input are proportional to the fraction of management (55) and administrative services (561) in $w\ell_k^Q = wL_k(1 - s_k^T)$ and $w\ell_k^T = wL_k s_k^T$.

D.2. Calibration

Algorithm 1 Calibration Steps

1: Load and Adjustment of Nominal IO-Tables and Credit Network
2: Calibration of Production Parameters
3: Calculate Steady State Shocks
4: Initial Guess of Intermediate Expenditure Shares Ω^X
5: **while** $|\Omega_t^X - \Omega_{t-1}^X| > \epsilon_\Omega$ **do**
6: Initial Guess of Quantity Shares ($w_{ck} = x_{ck}/q_k$)
7: **while do**$|w_{ck,t} - w_{ck,t-1}| > \epsilon_w$
8: Calculate Equilibrium Financial Variables
9: Calculate Equilibrium Prices and Quantities
10: Calculate Implied Productivity
11: Update Quantity Shares
12: **end while**
13: Update Intermediate Expenditure Shares
14: **end while**
15: Calculation of Parameters of Credit Management Cost Function

The iterative procedure outlined in Algorithm 1 is a rough sketch of the steps involved in calculated the steady state of the model economy. Based on this iterative procedure, the following production and financial parameters are derived: Table D.2 reports the time and cross-sectional average of the mean, the minimum and maximum as well as the standard deviation of the sectoral parameters. In addition, I split the sample of sectors based on their trade credit policy: a sector is counted as a net-lender if its net-lending position is above the median of the distribution of net-lending shares.

Table D.2: Calibrated Parameters

(a) Production Parameters

	Total (97-16)				
	VAR	All	NB	NL	p-Value
MEAN	α	0.338	0.297	0.381	0.060
	η	0.468	0.520	0.414	0.028
	χ	0.831	0.831	0.830	0.976
	Ω^F	0.022	0.034	0.010	0.031
MIN	α	0.263	0.237	0.291	0.147
	η	0.402	0.464	0.337	0.008
	χ	0.773	0.791	0.755	0.324
	Ω^F	0.019	0.030	0.008	0.034
MAX	α	0.406	0.350	0.464	0.024
	η	0.543	0.584	0.500	0.117
	χ	0.893	0.871	0.915	0.136
	Ω^F	0.026	0.039	0.012	0.032
STDV	α	0.041	0.034	0.049	0.017
	η	0.044	0.037	0.052	0.085
	χ	0.036	0.023	0.049	0.009
	Ω^F	0.002	0.003	0.001	0.081
	#OBS	45	23	22	0

(b) Financial Parameters

	Total (97-16)				
	VAR	All	NB	NL	p-Value
MEAN	κ^B	0.638	0.806	0.463	0.349
	κ_0^T	0.013	0.015	0.010	0.167
	κ_1^T	0.013	0.015	0.010	0.167
	$\overline{\theta}$	0.099	0.086	0.113	0.000
MIN	κ^B	0.462	0.565	0.354	0.458
	κ_0^T	0.001	0.002	-0.000	0.176
	κ_1^T	0.001	0.002	0.000	0.168
	$\overline{\theta}$	0.077	0.069	0.086	0.006
MAX	κ^B	0.852	1.083	0.611	0.324
	κ_0^T	0.036	0.035	0.038	0.725
	κ_1^T	0.036	0.035	0.038	0.722
	$\overline{\theta}$	0.127	0.110	0.143	0.000
STDV	κ^B	0.119	0.160	0.077	0.192
	κ_0^T	0.014	0.012	0.015	0.495
	κ_1^T	0.014	0.012	0.015	0.491
	$\overline{\theta}$	0.013	0.011	0.015	0.043
	#OBS	45	23	22	0

Note: This table reports the time-mean of the cross-sectional mean, the minimum, maximum and standard deviation of (a) the parameters of the production function and (b) the financial parameters. The first column reports the statistics for the entire sample. The second and third column report the same statistics for a subgroup of sectors based on the net-lending position Definition 1. The p-values for the differences in means between the two groups are reported in the last column.

The table reveals that across the entire sample period both sub-samples seem to differ significantly in their production but not in their credit management technology. In particular, the p-values for the differences in means between net-borrowing and net-lending sectors suggest that the two groups of sectors seem to differ in their capital, intermediate input and final demand share at a 5 and 10% significance level. Net-borrowers tend to have a lower capital (α) and composite intermediate input $(1 - \eta)$ input share while their final demand share is significantly higher. This is in line with the empirical observation that sectors which are further downstream and thus closer to the final consumer are sectors producing with a more labor intensive technology (e.g.

service industry).

D.2.1. Parameters of Credit Management Costs

The parameters of the credit management cost function are calibrated as follows. First, rearrange Equation (24) by taking the wage rate as a numeraire and replace $\kappa^T_{0,ks} = \overline{\kappa}^T_{0,k} = \overline{\kappa}^T_0 \forall k, s$ and $\kappa^T_{1,ks} = \overline{\kappa}^T_{1,k} = \overline{\kappa}^T_1 \forall k, s$. Then multiply by $p_s x_{ks}$ and take the sum over suppliers. Rearranging yields

$$\theta^P_k = \left[\overline{\theta}^S_k - (\overline{\theta}^S_k)^2 \frac{\overline{\kappa}^T_{0,k}}{\overline{\kappa}^T_{1,k}}\right] + \left[\frac{(\overline{\theta}^S_k)^2}{\overline{\kappa}^T_{1,k}}\right] p^E_k = \beta_{0,k} + \beta_{1,k} p^E_k \tag{D.24}$$

where

$$\theta^P_k = \frac{\sum_{s=1}^M \theta_{ks} p_s x_{ks}}{\sum_{s=1}^M p_s x_{ks}} \quad \text{and} \quad p^E_k = \underbrace{\left[\frac{\sum_{s=1}^M (p_s x_{ks})^2 \Delta_{ks}}{(\sum_{s=1}^M p_s x_{ks})^2}\right]}_{(1)} \underbrace{\left[\frac{\sum_{s=1}^M p_s x_{ks}}{p^V_k V_k}\right]}_{(2)} \frac{p^V_k}{(1 + r^B_k)} \tag{D.25}$$

As described in the main text, variable θ^P_k is simply the share of aggregate accounts payable in total intermediate cost of production excluding interest rate payments. The variable p^E_k can be interpreted as the discounted $(1 + r^B_k)$ marginal cost of producing one unit (p^V_k) multiplied by the (1) difference in the sector-specific credit expenditure herfindal index and (2) the share of intermediate expenditures excluding credit costs in total productive input expenditures. The data-counterparts are derived using the steady-state values of the corresponding variables derived as described above. The parameters are then calibrated using the estimated coefficients $\widehat{\beta}_{0,k}$ and $\widehat{\beta}_{1,k}$ by running a simple OLS regression of Equation (D.24).

The link-specific cost parameters are then derived as follows: First, multiply Equation (D.24) by θ_{ks}/θ^P_k. Then, derive the link-specific cost parameters by matching equations using the implied average cost parameters derived before.

$$\left[\frac{(\overline{\theta}^S_k)^2}{\overline{\kappa}^T_{1,k}}\right] = \left[\frac{(\overline{\theta}^S_k)^2}{\kappa^T_{1,ks}}\right] \implies \kappa^T_{1,ks} = \overline{\kappa}^T_{1,k} \frac{\theta^P_k}{\theta_{ks}} \tag{D.26}$$

$$\left[\overline{\theta}^S_k - (\overline{\theta}^S_k)^2 \frac{\overline{\kappa}^T_{0,k}}{\overline{\kappa}^T_{1,k}}\right] \frac{\theta_{ks}}{\theta^P_k} = \overline{\theta}^S_k - (\overline{\theta}^S_k)^2 \frac{\kappa^T_{0,ks}}{\kappa^T_{1,ks}} \implies \kappa^T_{0,ks} = \overline{\kappa}^T_{0,k} + \left(1 - \frac{\theta_{ks}}{\theta^P_k}\right) \frac{\theta^P_k}{\overline{\theta}^S_k} \frac{\overline{\kappa}^T_{1,k}}{\theta_{ks}} \tag{D.27}$$

Table D.3 reports the OLS-regressions results of Equation (D.24), the corresponding p-values and the implied cost-parameters for each year.

Table D.3: Calibrated Parameters κ_0^T, κ_1^T of Credit Management Costs Function

ID	β_0	p-val$_0$	β_1	p-val$_1$	R^2	NObs	κ_{0k}^T	κ_{1k}^T
1997	0.11**	(0.00)	-67.92	(0.26)	0.03	45	-0.68	1.66
1998	0.11**	(0.00)	-105.61	(0.15)	0.05	45	-0.60	1.03
1999	0.11**	(0.00)	-129.77*	(0.03)	0.10	45	-0.72	0.76
2000	0.11**	(0.00)	-88.62+	(0.09)	0.07	45	-0.74	1.27
2001	0.11**	(0.00)	-90.46*	(0.01)	0.13	45	-0.97	1.13
2002	0.10**	(0.00)	-71.87	(0.17)	0.04	45	-0.80	1.27
2003	0.11**	(0.00)	-150.48**	(0.00)	0.22	45	-0.82	0.66
2004	0.11**	(0.00)	-170.56**	(0.00)	0.19	45	-0.69	0.59
2005	0.11**	(0.00)	-209.99**	(0.00)	0.21	45	-0.57	0.49
2006	0.11**	(0.00)	-175.55**	(0.00)	0.25	45	-0.61	0.54
2007	0.11**	(0.00)	-200.36**	(0.00)	0.25	45	-0.65	0.48
2008	0.11**	(0.00)	-179.55**	(0.00)	0.28	45	-0.70	0.55
2009	0.09**	(0.00)	-95.78**	(0.00)	0.19	45	-0.99	0.76
2010	0.11**	(0.00)	-210.30**	(0.00)	0.32	45	-0.69	0.44
2011	0.12**	(0.00)	-322.90**	(0.00)	0.31	45	-0.51	0.32
2012	0.11**	(0.00)	-289.03**	(0.00)	0.35	45	-0.55	0.31
2013	0.11**	(0.00)	-278.74**	(0.00)	0.32	45	-0.57	0.35
2014	0.12**	(0.00)	-310.17**	(0.00)	0.22	45	-0.49	0.34
2015	0.11**	(0.00)	-308.06**	(0.00)	0.23	45	-0.45	0.33
2016	0.11**	(0.00)	-195.49*	(0.02)	0.13	45	-0.51	0.53

Note: This table presents the time series of the estimated coefficients of Equation (D.24), β_0 and β_1, based on a period-by-period OLS-regression. The table also records the corresponding p-values, the R^2 and the number of observations used in each regressions. The significance level is indicated by ** p<0.01, * p<0.05, + p<0.1. The last two columns record the time series of implied parameters derived as shown in Equation (D.26) and (D.27) multiplied by 1e4. The normalization by 1e4 is chosen as the wage rate (w = $\overline{w}$1e4) is used as the numeraire.

D.2.2. Parameters of Bank Interest Rate

The parameter governing the convexity of the risk-premium with respect to the combined default risk, μ, is calibrated by first estimating the following equation for each sector

$$\log(r_{kt}^Z) \;=\; \mu_0 + \mu_1 \log(\overline{\theta}_{0t}^D + \overline{\theta}_{kt}^C) + \epsilon_{kt} \tag{D.28}$$

where $\overline{\theta}_{kt}^C$ denotes the share of sectoral accounts receivables in total sales and $\overline{\theta}_{0t}^D$ denotes the aggregate leverage - the ratio of long-term debt and debt in current liabilities to total assets. The data-counterpart for the risk-premium is the sectoral credit spread calculated in Gilchrist and Zakrajšek (2012) and the data-counterpart for the aggregate leverage is calculated directly from the corresponding balance sheet items in Compustat. Table D.4 reports the OLS-regressions results of Equation (D.28) and the corresponding standard errors. The convexity parameter μ is then calibrated by calculating the sales-weighted (RW) average of the estimated coefficients of $\log(\widetilde{\theta}_{kt})$, where $\widetilde{\theta}_{kt} = \overline{\theta}^D_{0t} + \overline{\theta}^C_{kt}$ such that $\mu = 1.2$.

Table D.4: Calibrated Parameter μ of Risk-Premium

	11	21	22	23	31T33	42	44A5	48A9	51	54A6	62	71A2	81
$\log(\widetilde{\theta})$	0.17	1.13**	1.92	-1.79**	0.30	-0.23	2.09*	5.29**	3.15*	-1.12	0.11	0.46	0.29
	(0.62)	(0.37)	(1.23)	(0.44)	(0.86)	(0.96)	(0.83)	(1.53)	(1.45)	(1.09)	(0.40)	(1.16)	(1.46)
R^2	0.01	0.39	0.14	0.53	0.01	0.00	0.30	0.44	0.24	0.06	0.00	0.01	0.00
NObs	14	17	17	17	17	17	17	17	17	17	17	17	17
RW	(0.3)	(4.0)	(7.1)	(0.7)	(42.7)	(7.1)	(15.3)	(5.7)	(10.1)	(3.2)	(1.7)	(1.8)	(0.1)

Note: This table presents the results of an OLS regression of Equation D.28 for selected industries. The sales shares in total sales of each industry in percent is reported in row (*RW*). All regressions include a constant; Std.Errors in recorded in parentheses. ** $p<0.01$, * $p<0.05$, + $p<0.1$