Priceless Consumption*

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ABSTRACT

This paper studies the type of consumption that, despite its economic relevance, is unaccounted for in aggregate consumption measures and thus ignored in applications of economic models that rely on consumption data. We denote this latent consumption *priceless*, given that it cannot be bought or sold for any amount of money. Although not directly observable, priceless consumption can be identified through its heterogeneous effects across different consumption categories. We propose a structural estimation methodology to recover a priceless consumption series from the joint dynamics of non-durables, durables, and services consumption. The estimation results show that: i) priceless consumption is economically significant, its value has increased from around 26% of marketable consumption expenditures ($2,900 per capita in 2012 U.S. Dollars) in 1960 to about 48 % ($19,100 per capita) in 2018; ii) it is quite volatile, about nine times more volatile than measured consumption; and iii) it is strongly procyclical, the difference in growth rate of PC in expansion and recessions is 16.5%. As an application, we show that using a total consumption measure that includes priceless consumption leads to a significant improvement of the fit of the standard consumption-based CAPM with power utility. In particular, the model can match the observed equity premium with a low coefficient of relative risk aversion between 6.5 and 8.2.

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The starting point of this paper is the fact that not everything with consumption value is accounted for in aggregate consumption measures. We denote *priceless consumption* (PC) all non-pecuniary drivers of utility that are left out of aggregate consumption measures but that affect the composition of aggregate consumption.[1] This paper proposes a methodology to estimate the aggregate shadow value of PC from its effects on the composition of aggregate consumption. The results of our estimation methodology show that PC represents a significant portion of aggregate consumption and that its relative importance has been increasing over time. The paper also finds that the dynamics of the PC provide information that is priced in financial assets yet that is absent from the dynamics of aggregate measurable consumption.

The central idea of the paper is that we can recover the dynamics of the latent PC through its effect on the consumption of marketable goods. To illustrate this idea, consider the fact that one could learn something about the amount of sunshine enjoyed by individuals over a given month by observing changes in their relative expenditure shares of running shoes, which complement sunny weather, and umbrellas, which substitute sunny weather. We embed this idea into a model based on an endowment economy where agents derive utility from the consumption of differentiated goods, each produced by a *Lucas tree*. One of the trees provides the PC good, which differs from the other goods in that it is *non-marketable* (i.e., it cannot be traded or otherwise transferred between agents) and is thus latent. The solution to the representative agent's optimal consumption decision shows the conditions in which the parameters of the model can be identified without the use of any information about PC. An implication of this finding is that the quantity and shadow price of the latent

[1] Illustrative examples of non-pecuniary drivers of utility considered in the literature are (the satisfaction derived from): the quality of non-working time (Becker (1965)), education (Lazear (1977) and Eckstein and Wolpin (1999)), work (Akerlof (1982) and Hagedorn and Manovskii (2008)), good health (Jack and Suri (2014)), social connections (Ambrus, Mobius, and Szeidl (2014)), religious activities (Azzi and Ehrenberg (1975)), and charitable donations (Benabou and Tirole (2011)).

PC good can, in principle, be fully recovered solely from observable data.

The mapping of the model to the data requires the definition of a partition of the universe of observable aggregate consumption into components (i.e., *baskets*) that represent the marketable goods in the model. The model shows that the dynamics of PC can be recovered from the data as long as the partition of measured aggregate consumption used satisfies two requirements. The first requirement is that the partition must contain at least three consumption baskets.[2] The second requirement for the recoverability of PC is that the consumption baskets resulting from the partition must have strictly different elasticities of substitution (EOS) with PC. For simplicity and to avoid parameter proliferation, we partition aggregate marketable consumption into durable goods, nondurable goods, and services. The chosen partition satisfies the two requirements discussed above. Although we cannot determine ex ante whether the chosen partition of the aggregate marketable consumption leads to consumption types with different EOS with PC, we check ex post that this is the case once we recover PC from the data with the estimated model.

We assume that the consumption aggregator is parameterized by a transcendental logarithmic (translog) function (Christensen, Jorgenson, and Lau, 1975). The key feature of the chosen aggregator, which is absent from the constant elasticity of substitution (CES) class of aggregators, is that it allows for variation in EOS across different consumption goods. The Cobb-Douglas is the most commonly used and the most restrictive member of the CES class. The Cobb-Douglas aggregator implies that any two goods have unit EOS thus ruling out the existence of strict substitutes (i.e., EOS>1) or strictly complements (i.e., EOS<1) in the economy. In this case, the prices and expenditure shares of any marketable consumption good would be unaffected by the dynamics of PC. An CES aggregator that is more general than the Cobb-Douglas aggregator does allow for EOS values different than one.

[2]We elaborate on this requirement in Section 1.2.

However, even the more general CES aggregator is overly restrictive for our methodology since it implies that every pair of goods in the economy has the same EOS value. In this case, the *relative* prices and *relative* expenditure shares between observable goods would be uninformative about the dynamics of PC.

The solution of the model gives rise to a demand system that we can use to recover the latent PC series from the observed consumption expenditures (prices and quantities). We estimate the demand system as follows. We measure quantity, price, and expenditure share of nondurables consumption, durables consumption, and services consumption, using data from the Bureau of Economic Analysis (BEA) from 1959 to 2018. We estimate the parameters of the translog consumption aggregator by minimizing the sum of the squared errors between the estimated and the observed shares of the marketable consumption good types. We then use the estimated parameter values and the data on the marketable consumption bundles to produce a shadow consumption expenditure series of the PC by inverting the demand system and solve for the unobserved PC.

The empirical results can be summarized as follows. We find that PC and durable goods are complements, PC and nondurable goods are substitutes, and PC and services consumption are slight substitutes (all relative to the Cobb-Douglas case). Thus, the estimation results confirm that the three marketable good have different EOS values relative to the PC series, a necessary condition to properly identify the PC series.

More important, the estimation results reveal that the shadow expenditure on PC is large, and has increased over time. While in the earlier part of the sample the shadow expenditure in PC represented about 26% of measured consumption in 1959 and represents about 48% of the expenditure on marketable consumption in 2018.[3] Translating these shares into real 2012 U.S. Dollars, the PC per capita has increased from about $2,900 in 1959 to $19,100 in

[3]In terms of shares of the *true* total consumption expenditure, which includes both measured and latent consumption, the figures are 20% in 1959 and 32% in 2018.

2018. In addition, the shadow PC expenditure is significantly more volatile, about nine times larger, than the expenditure on total aggregate marketable consumption. Because utility is ultimately determined by total consumption, which includes PC, our results suggest that aggregate accounting measures on consumption understate consumer's well-being, especially in more recent years.

The main departure point of the model is to consider the existence of an entire class of consumption goods (i.e., PC) that is latent and thus ignored in aggregate measures of consumption. The PC series that we retrieve from the data with the estimated model greatly improves the performance of the canonical consumption-based CAPM with power utility. Another important departure point of the model is in the choice of consumption aggregator. We show in the paper that the constant elasticity of substitution (CES) class, which is the most commonly used in the literature (e.g., Yogo (2006)), is overly restrictive and cannot be used to retrieve information about latent consumption from the data. Moreover, a CES consumption aggregator is unable to explain the joint *dynamics* of durable goods, nondurable goods, and services. Our results show that the translog class of consumption aggregator addresses the limitation of the CES at explaining consumption dynamics. In particular, we show that the mean absolute error (M.A.E.) between empirical and model-implied expenditure shares scaled by the empirical of the three goods decreases from an average of 19% across the goods with the CES aggregator to 3.5% with the translog aggregator.

The estimated PC allows us to construct a more comprehensive measure of aggregate consumption. A natural question is whether this broader measure of consumption captures systematic risk in the economy better than the standard measures based on aggregate marketable consumption only. To investigate this question, we use the estimated PC series to test the ability of the standard consumption-based CAPM (as in Breeden, Gibbons, and Litzenberger (1989)) with CRRA utility to match the observed equity risk premium, and also the cross section of the Fama and French (1993) twenty-five double sorted portfolios on size and book to market. We denote the application of the consumption-based model estimated with the *true* total aggregate consumption, which combines both aggregate marketable consumption and PC, as the *True Consumption* CAPM or simply TC-CAPM. While the C-CAPM estimated with measured annual consumption growth data requires a (coefficient of relative) risk aversion of over 20 to explain the equity risk premium, the same model estimated with *true* total consumption (i.e. the TC-CAPM) requires a risk aversion coefficient of around eight. In a cross-sectional test, we show that the risk aversion required is even lower at 6.5. This result is consistent with the hypothesis that the PC growth contains information that is priced in stocks and that is possibly unrelated to the information contained by aggregate marketable consumption growth.

Related Literature

This paper is related to the vast economic and finance literature that relies on consumption data, a central macroeconomic variable in many models in this literature. More specifically, our analysis is closely related to the consumption-based approach to asset pricing. Many papers in this literature use aggregate measures of total consumption data or consumption by broad type (e.g., durables and non-durables).[4] We argue in this paper that useful information about the *true* total consumption, which includes PC, is left out when we aggregate consumption data. We present support for this statement by showing that our estimated PC series contains information relevant for asset pricing. The paper most closely related to ours is Savov (2011) who uses aggregate garbage as a proxy for total aggregate consumption. Our motivation and hence empirical approach is quite different, however. Savov (2011) work addresses the measurement and statistical issues in NIPA expenditures by using data on garbage production as an alternative measure of aggregate consumption.

[4]A very short list of illustrative examples of this vast literature are Hansen and Singleton (1982), Jagannathan and Wang (1996), Lettau and Ludvigson (2001), Bansal and Yaron (2004).

We differ from this approach in that we focus on studying a broad consumption category that, irrespective of the existence of the well-known statistical and measurement challenges associated with the construction of the national accounts, is entirely *absent* from aggregate consumption measures.

This paper also contributes to the strand of the asset pricing literature that studies heterogeneous goods. Examples of this literature are Ait-Sahalia, Parker, and Yogo (2004) (luxury goods vs basic consumption), Yogo (2006) (durables and non-durables), Piazzesi, Schneider, and Tuzel (2007) (housing and ex-housing consumption), Binsbergen (2016) (several narrowly defined goods).[5] Our paper also considers the asset pricing implications of preferences that imply imperfect substitutability across heterogeneous goods. The main departure point from this literature is to consider that a significant fraction of the true aggregate consumption is nonmarketable and thus unaccounted for in aggregate measures. In addition, we use the dynamics of heterogeneous marketable goods to extract information about the latent portion of aggregate consumption and not directly in our asset pricing tests.

1 Model

1.1 General Setup

The model is based on an endowment economy populated by a large number of agents. In what follows, we omit subscripts to indicate the identity of the agent since agents are identical. Every period, agents are endowed with a basket consisting of goods from four productive technologies (i.e., Lucas trees): a durable good (E_t^{D}), a non-durable good (E_t^{N}), a "service good" (E_t^{S}), and a latent good (E_t^{L}). The latent good is non-marketable, which

[5] A less directly related yet important strand of the literature studies production-based asset pricing models that consider the *production* of heterogeneous goods. A couple of illustrative examples from this literature are Gomes, Kogan, and Yogo (2009) and Gorodnichenko and Weber (2016).

implies that individual agents cannot trade it and thus cannot adjust their consumption. With the exception of the durable good, all other goods, including the latent good, are perishable and cannot be stored for later use. The durable good is the numeraire in the economy, so that all values discussed are expressed in terms of units of that good. The stock of the durable good held by an agent follows the law of motion given by

$$Q_{t+1}^{\mathrm{D}} = (1-\delta)Q_{\mathrm{t}}^{\mathrm{D}} + E_t^{\mathrm{D}}, \tag{1}$$

where $0 < \delta \leq 1$ is the annual depreciation rate of durable goods.

Agents' preferences are represented by a time-separable utility U over a consumption aggregator defined over the vector of quantities consumed at time t of the four goods, $\mathbf{Q}_t \equiv \{Q_t^{\mathrm{D}}, Q_t^{\mathrm{N}}, Q_t^{\mathrm{S}}, Q_t^{\mathrm{L}}\}$.[6] Although complete markets are not ruled out in the model, in what follows we will only explicitly consider the existence of a zero coupon risk-free bond given that the solution to the demand system is not affected by the existence of additional financial assets.

The agent's maximization problem is given by

$$\max_{\{\mathbf{Q}_s, A_s\}_{\mathrm{s=t}}^{\infty}} \sum_{s=0}^{\infty} \mathbb{E}_t \left[\beta^s U[F[\mathbf{Q}_{t+s}]]\right], \tag{2a}$$

subject to the law of motion in Equation (1) and to the constraints

$$\frac{A_t}{R_t^f} \leq A_{t-1} - C_t^{\mathrm{DNS}} - \Delta_t^{\mathrm{L}} P_t^{\mathrm{L}}, \tag{2b}$$

$$Q_t^{\mathrm{L}} \leq E_t^{\mathrm{L}} + \Delta_t^{\mathrm{L}}, \tag{2c}$$

[6] Although we refer to a component Q_t^{D} as the time-t quantity *consumed* of the durable good, this terminology should be qualified: Consistent with the law of motion in Equation (1), the quantity Q_t^{D} is not used up every period and instead provides a stream of utility to the agent. The utility stream that arises from the durable good is proportional to the stock held and not to the periodic endowment received of the good.

where U is the agent's utility over total consumption represented by the consumption aggregator F, A_t is the time-t allocation in a zero coupon risk free bond with price $\frac{1}{R_t^f}$ that matures and pays one unit of the durable good at time $t+1$, $0 < \beta \leq 1$ is a constant that represents the agents' impatience rate,

$$C_t^{\text{DNS}} \equiv E_t^{\text{D}} + E_t^{\text{N}} P_t^{\text{N}} + E^{\text{S}} P_t^{\text{S}}, \tag{2d}$$

is the time-t expenditure in the marketable consumption goods, Δ_t^{L}, and P_t^{L} are the shadow quantity traded and the shadow price of the latent good.[7]

1.2 General Solution

Agents' rationality implies that the inequalities in Equations (2b) and (2c) are binding, so that the optimal aggregate consumption of the non-durable, service, and latent goods are given by $Q_t^{\text{N}} = E_t^{\text{N}}$, $Q_t^{\text{S}} = E_t^{\text{S}}$, and $Q_t^{\text{L}} = E_t^{\text{L}}$. Applying the Bellman principle, we can express the maximization problem in Equation (2a), which involves decisions over an infinite series of consumption quantities and risk-free bond allocations into a recursive maximization problem that only involves decisions over the current consumption and bond investment allocation, as given by

$$V_t = \max_{\{\mathbf{Q}_t, A_t\}} U[F[\mathbf{Q}_t]] + \beta \mathbb{E}_t[V_{t+1}], \tag{3}$$

subject to the law of motion in Equation (1) and to the constraints in Equation (2b)–(2c). The Lagrangian function associated with the maximization problem in Equation (3) is given

[7]Note that the optimization problem in Equation (2a) includes the shadow quantity traded Δ_t^{L} and the shadow price P^{L} even though the latent good is non-marketable. The shadow price P^{L} is defined as the price which would the agent to optimally refrain from trading the latent good (i.e., $\Delta^{\text{L}} \to 0$) even if doing so were possible.

by

$$\mathcal{L}_t = U[F[\mathbf{Q}_t]] + \beta \mathbb{E}_t[V_{t+1}] + \lambda_t \left(A_{t-1} - C_t^{\text{DNS}} - \Delta_t^{\text{L}} P^{\text{L}} - \frac{A_t}{R_t^f} \right). \tag{4}$$

The first order conditions of the Lagrangian function in Equation (4) w.r.t. E_t^{D}, Q_t^{N}, Q_t^{S}, Δ_t^{L}, and A_t are given by

$$\frac{\partial \mathcal{L}_t}{\partial D_t} = U_{\text{F}}[F[\mathbf{Q}_t]] F_{\text{D}}[\mathbf{Q}_t] + \mathbb{E}_t \left[\beta \left(\frac{\partial V_{t+1}}{\partial E_t^{\text{D}}} \right) \right] - \lambda_t = 0, \tag{5a}$$

$$\frac{\partial \mathcal{L}_t}{\partial N_t} = U_{\text{F}}[F[\mathbf{Q}_t]] F_{\text{N}}[\mathbf{Q}_t] - \lambda_t P_t^N = 0, \tag{5b}$$

$$\frac{\partial \mathcal{L}_t}{\partial S_t} = U_{\text{F}}[F[\mathbf{Q}_t]] F_{\text{S}}[\mathbf{Q}_t] - \lambda_t P_t^S = 0, \tag{5c}$$

$$\frac{\partial \mathcal{L}_t}{\partial \Delta_t^{\text{L}}} = U_{\text{F}}[F[\mathbf{Q}_t]] F_{\text{L}}[\mathbf{Q}_t] - \lambda_t P_t^{\text{L}} = 0, \tag{5d}$$

$$\frac{\partial \mathcal{L}_t}{\partial A_t} = \mathbb{E}_t \left[\beta \left(\frac{\partial V_{t+1}}{\partial A_t} \right) \right] - \frac{\lambda_t}{R_t^f} = 0, \tag{5e}$$

where the first condition follows from the fact that $\frac{\partial Q_t^{\text{D}}}{\partial E_t^{\text{D}}} = 1$. The envelope conditions of the Lagrangian function in Equation (4) w.r.t. Q_{t-1}^{D} and A_{t-1} are given by

$$\frac{\partial V_t}{\partial Q_{t-1}^{\text{D}}} = (1-\delta)\lambda_t, \tag{6a}$$

$$\frac{\partial V_t}{\partial A_{t-1}} = \lambda_t. \tag{6b}$$

The first order conditions and the envelope conditions in Equations (5) and (6) jointly imply that

$$P_{\mathrm{t}}^{\mathrm{N}} = \left(\frac{F_{\mathrm{N}}[\mathbf{Q}_t]}{F_{\mathrm{D}}[\mathbf{Q}_t]}\right)\Upsilon_t, \tag{7a}$$

$$P_{\mathrm{t}}^{\mathrm{S}} = \left(\frac{F_{\mathrm{S}}[\mathbf{Q}_t]}{F_{\mathrm{D}}[\mathbf{Q}_t]}\right)\Upsilon_t, \tag{7b}$$

$$P_t^{\mathrm{L}} = \left(\frac{F_{\mathrm{L}}[\mathbf{Q}_t]}{F_{\mathrm{D}}[\mathbf{Q}_t]}\right)\Upsilon_t, \tag{7c}$$

where

$$\Upsilon_t = 1 - \frac{1-\delta}{R_{\mathrm{t}}^{\mathrm{f}}}. \tag{7d}$$

We define the expenditure share as the ratio of the time-t value consumed (i.e., actual expenditure for marketable goods or shadow expenditure for the latent good) over the consumption expenditure on *marketable* consumption goods C_t^{DNS}. Equations (2d) and (7) jointly define the optimal expenditure share in each of the goods is given by

$$S_t^{\mathrm{D}} \equiv \frac{C_t^{\mathrm{D}}}{C_t^{\mathrm{DNS}}} = \frac{E_t^{\mathrm{D}}}{E_t^{\mathrm{D}} + Q_t^{\mathrm{N}}P_t^{\mathrm{N}} + Q_t^{\mathrm{S}}P_t^{\mathrm{S}}}, \tag{8a}$$

$$S_t^{\mathrm{N}} \equiv \frac{C_t^{\mathrm{N}}}{C_t^{\mathrm{DNS}}} = \frac{Q_t^{\mathrm{N}}P_t^{\mathrm{N}}}{E_t^{\mathrm{D}} + Q_t^{\mathrm{N}}P_t^{\mathrm{N}} + Q_t^{\mathrm{S}}P_t^{\mathrm{S}}}, \tag{8b}$$

$$S_t^{\mathrm{S}} \equiv \frac{C_t^{\mathrm{S}}}{C_t^{\mathrm{DNS}}} = \frac{Q_t^{\mathrm{S}}P_t^{\mathrm{S}}}{E_t^{\mathrm{D}} + Q_t^{\mathrm{N}}P_t^{\mathrm{N}} + Q_t^{\mathrm{S}}P_t^{\mathrm{S}}}, \tag{8c}$$

$$S_t^{\mathrm{L}} \equiv \frac{C_t^{\mathrm{L}}}{C_t^{\mathrm{DNS}}} = \frac{Q_t^{\mathrm{L}}P_t^{\mathrm{L}}}{E_t^{\mathrm{D}} + Q_t^{\mathrm{N}}P_t^{\mathrm{N}} + Q_t^{\mathrm{S}}P_t^{\mathrm{S}}}. \tag{8d}$$

The goal of the estimation presented in Section 2 is to recover the quantity consumed Q_t^{L} and the (shadow) price P_t^{L} and expenditure share S_t^{L} of latent good from observable series in the data: expenditure shares, quantities, and prices of the marketable goods (i.e., the durable, nondurable, and service goods), and the short term interest rate.

We use three marketable goods in the model because this is the minimum number of goods required in the estimation methodology presented in Section 2. To see the intuition for the requirement, note that the last equality in Equation (8) cannot be used in the estimation because the share S_t^{L} is unobservable. Moreover, one out of the first three equalities in Equation (8) is redundant, since, by construction, $S_t^{\mathrm{D}} + S_t^{\mathrm{N}} + S_t^{\mathrm{S}} = 1$. In order to produce an expression that can be estimated directly from the data, we combine the remaining two equalities into a single one that does not involve any unobservable variable. This last step would be impossible without at least three observable goods.

Before we present our estimation methodology, we must first specify the functional forms of the consumption aggregator F that allow for a parametric representation of the general solution for prices in Equation (7) and expenditure shares in Equation (8).

1.3 Parametric Solution

In this section, we specify the functional form for the consumption aggregator F, and thus the functional forms for the model-implied prices and expenditure shares in Equations (7) and (8) used to recover information about latent consumption.

1.3.1 Consumption Aggregator: CES Case

To motivate our chosen functional form for the consumption aggregator F, which we present in the next section, we start by discussing why the commonly used constant elasticity of substitution (CES) functional form, which includes the Cobb-Douglas form, cannot be used to recover the quantity consumed of the latent good Q^{L} from Equation (8). Let F^{CES} be a CES consumption aggregator over the four goods, as given by:

$$F^{\mathrm{CES}}[\mathbf{Q}_t] = (\alpha_{\mathrm{D}}(Q_t^{\mathrm{D}})^{\rho} + \alpha_{\mathrm{N}}(Q_t^{\mathrm{N}})^{\rho} + \alpha_{\mathrm{S}}(Q_t^{\mathrm{S}})^{\rho} + (1 - \alpha_{\mathrm{D}} - \alpha_{\mathrm{N}} - \alpha_{\mathrm{S}})(Q_t^{\mathrm{L}})^{\rho})^{\frac{1}{\rho}}\,, \tag{9}$$

where $\frac{1}{1-\rho} > 0$ is the elasticity of substitution between the goods. When $F = F^{\text{CES}}$, the model-implied prices from Equation (7) become

$$P_t^{\text{N,CES}} = \frac{\alpha_{\text{N}}}{\alpha_{\text{D}}} \left(\frac{Q_t^{\text{N}}}{Q_t^{\text{D}}} \right)^{\frac{1-\rho}{\rho}} \Upsilon_t, \tag{10a}$$

$$P_t^{\text{S,CES}} = \frac{\alpha_{\text{S}}}{\alpha_{\text{D}}} \left(\frac{Q_t^{\text{S}}}{Q_t^{\text{D}}} \right)^{\frac{1-\rho}{\rho}} \Upsilon_t, \tag{10b}$$

$$P_t^{\text{L,CES}} = \frac{\alpha_{\text{L}}}{\alpha_{\text{D}}} \left(\frac{Q_t^{\text{L}}}{Q_t^{\text{D}}} \right)^{\frac{1-\rho}{\rho}} \Upsilon_t. \tag{10c}$$

The equation above suggests that Equation (8) cannot be inverted to recover Q_t^{L} when the aggregator F has the CES form. The reason for this result is that, in the *CES* case, the elasticity of substitution is constant both across time and across goods. A constant elasticity of substitution across goods implies that prices and expenditure shares of the marketable goods are not directly affected by changes in the quantity consumed of the latent good.[8]

1.3.2 Consumption Aggregator: Translog Case

This subsection presents the translog functional form for the consumption aggregator F used in the estimation of the model.[9] The functional form employed is formalized in the assumption below:

Assumption 1. The intra-temporal consumption aggregator F is defined by the transcendental

[8]Strictly speaking, changes in the quantity consumed of the latent consumption good could still have subtle effects on observable prices and expenditure shares through their effect on interest rates. However, such indirect effects are unlikely to be significant enough to allow for the recovery of the latent series and would require additional structure to the model (e.g., the functional form for the utility function U) and additional series from the data (e.g., stock returns).

[9]See Christensen, Jorgenson, and Lau (1973) and Christensen et al. (1975) for seminal discussions of the transcendental logarithmic preferences.

logarithmic function given by

$$F[\mathbf{Q}_t] \equiv \text{Exp}\left[\text{Log}[\mathbf{Q}_t] \times \mathbf{a} + \frac{1}{2}\,\text{Log}[\mathbf{Q}_t] \times \mathbf{B} \times \text{Log}[\mathbf{Q}_t]'\right], \tag{11a}$$

where

$$\mathbf{a} \equiv \begin{bmatrix} a_{\text{D}} \\ a_{\text{N}} \\ a_{\text{S}} \\ a_{\text{L}} \end{bmatrix} \in [0,1]^{4\times 1}, \quad \textit{and} \quad \mathbf{B} \equiv \begin{bmatrix} b_{\text{DD}} & b_{\text{DN}} & b_{\text{DS}} & b_{\text{DL}} \\ b_{\text{ND}} & b_{\text{NN}} & b_{\text{NS}} & b_{\text{NL}} \\ b_{\text{SD}} & b_{\text{SN}} & b_{\text{SS}} & b_{\text{SL}} \\ b_{\text{LD}} & b_{\text{LN}} & b_{\text{LS}} & b_{\text{LL}} \end{bmatrix} \in \mathbb{R}^{4\times 4}. \tag{11b}$$

Assumption 1 implies the following functional form for the model-implied prices in the system of Equations (7):

$$P^{\text{N}}[\mathbf{Q}_t] = \frac{Q_t^{\text{D}}}{Q_t^{\text{N}}}\left(\frac{2a_{\text{D}} + (b_{\text{ND}} + b_{\text{DN}})\,q_t^{\text{D}} + 2b_{\text{NN}}q_t^{\text{N}} + (b_{\text{NS}} + b_{\text{SN}})\,q_t^{\text{S}} + (b_{\text{NL}} + b_{\text{LN}})\,q_t^{\text{L}}}{2a_{\text{D}} + 2b_{\text{DD}}q_t^{\text{D}} + (b_{\text{DN}} + b_{\text{ND}})\,q_t^{\text{N}} + (b_{\text{SD}} + b_{\text{DS}})\,q_t^{\text{S}} + (b_{\text{DL}} + b_{\text{LD}})\,q_t^{\text{L}}}\right)\Upsilon_t, \tag{12a}$$

$$P^{\text{S}}[\mathbf{Q}_t] = \frac{Q_t^{\text{D}}}{Q_t^{\text{S}}}\left(\frac{2a_{\text{S}} + (b_{\text{SD}} + b_{\text{DS}})\,q_t^{\text{D}} + (b_{\text{NS}} + b_{\text{SN}})\,q_t^{\text{N}} + 2b_{\text{SS}}q_t^{\text{S}} + (b_{\text{SL}} + b_{\text{LS}})\,q_t^{\text{L}}}{2a_{\text{D}} + 2b_{\text{DD}}q_t^{\text{D}} + (b_{\text{DN}} + b_{\text{ND}})\,q_t^{\text{N}} + (b_{\text{SD}} + b_{\text{DS}})\,q_t^{\text{S}} + (b_{\text{DL}} + b_{\text{LD}})\,q_t^{\text{L}}}\right)\Upsilon_t, \tag{12b}$$

where $q_t^{\text{D}} \equiv \text{Log}[Q_t^{\text{D}}]$, $q_t^{\text{N}} \equiv \text{Log}[Q_t^{\text{N}}]$, $q_t^{\text{S}} \equiv \text{Log}[Q_t^{\text{S}}]$, and $q_t^{\text{L}} \equiv \text{Log}[Q_t^{\text{L}}]$.

Equation (12) shows that the translog-form for the intra-temporal consumption aggregator F across goods implies that the prices and expenditure shares of the marketable goods respond to changes in q^{L}. Conversely, the Equation (12) suggests that, when the consumption aggregator has the translog form, the series of q^{L} could be recovered by inverting Equation (8). In the next section, we present the methodology used in the estimation of the demand system.

The unrestricted translog aggregator presented in Assumption 1 contains 20 free parameters. To reduce the number of free parameters in the estimation, we follow the literature and impose two standard parameter restrictions to the translog function. The first parameter restriction, which is formalized in Assumption 2 below, is to impose symmetry in the matrix $\mathbf{B}$.

Assumption 2. The parameter matrix $\mathbf{B}$ *in Equation* (11a) *is symmetric around its main diagonal.*

The second parameter restriction is the normalization of the vector α and the matrix $\mathbf{B}$, as formalized in Assumption 3 below.

Assumption 3. The vector α *is normalized as follows:*

$$\sum_{\mathrm{i}\in\{\mathrm{D,N,S,L}\}} a_{\mathrm{i}} = 1. \tag{13a}$$

The parameter matrix $\mathbf{B}$ *is normalized as follows:*

$$\sum_{\mathrm{i}\in\{\mathrm{D,N,S,L}\}} b_{\mathrm{k,i}} = 0, \quad \forall k \in \{D, N, S, L\}. \tag{13b}$$

Assumption 3 implies that the consumption aggregator F is homothetic over the four consumption goods, which implies that preferences should *not* appear homothetic solely over the marketable (i.e., non-latent) goods, which is an implication of the CES aggregator class that has been rejected by the empirical literature (e.g., Eichenbaum and Hansen, 1990).

We implement Assumptions 2 and 3 by restricting the coefficient vector α and the coefficient matrix $\mathbf{B}$ as follows:

$$a_{\mathrm{L}} = 1 - a_{\mathrm{D}} - a_{\mathrm{N}} - a_{\mathrm{S}}. \tag{14a}$$

$$\mathbf{B} = \begin{bmatrix} b_{\text{DD}} & b_{\text{DN}} = b_{\text{ND}} & b_{\text{DS}} = b_{\text{SD}} & b_{\text{DL}} = -b_{\text{D}} \\ b_{\text{ND}} & b_{\text{NN}} & b_{\text{NS}} = b_{\text{SN}} & b_{\text{NL}} = -b_{\text{N}} \\ b_{\text{SD}} & b_{\text{SN}} & b_{\text{SS}} & b_{\text{SL}} = -b_{\text{S}} \\ b_{\text{LD}} = -b_{\text{D}} & b_{\text{LN}} = -b_{\text{N}} & b_{\text{LS}} = -b_{\text{S}} & b_{\text{LL}} = b_{\text{D}} + b_{\text{N}} + b_{\text{S}} \end{bmatrix}, \tag{14b}$$

where

$$b_{\text{D}} = b_{\text{DD}} + b_{\text{ND}} + b_{\text{SD}}, \tag{14c}$$

$$b_{\text{N}} = b_{\text{ND}} + b_{\text{NN}} + b_{\text{SN}}, \tag{14d}$$

$$b_{\text{S}} = b_{\text{SD}} + b_{\text{SN}} + b_{\text{SS}}. \tag{14e}$$

Assumptions 2 and 3 effectively reduce the number of free parameters from 20 to 9. The parameter restriction implies that prices in Equation (12), P^{N} and P^{S} can be expressed as the functions of q_t^{L} given by

$$P^{\text{N}}[\mathbf{Q}_t] = \frac{Q_t^{\text{D}}}{Q_t^{\text{N}}} \left(\frac{a_{\text{N}} + b_{\text{ND}} q_t^{\text{D}} + b_{\text{NN}} q_t^{\text{N}} + b_{\text{SN}} q_t^{\text{S}} - q_t^{\text{L}} b_{\text{N}}}{a_{\text{D}} + b_{\text{DD}} q_t^{\text{D}} + b_{\text{ND}} q_t^{\text{N}} + b_{\text{SD}} q_t^{\text{S}} - q_t^{\text{L}} b_{\text{D}}} \right) \Upsilon_t, \tag{15a}$$

$$P^{\text{S}}[\mathbf{Q}_t] = \frac{Q_t^{\text{D}}}{Q_t^{\text{S}}} \left(\frac{a_{\text{S}} + b_{\text{SD}} q_t^{\text{D}} + b_{\text{SN}} q_t^{\text{N}} + b_{\text{SS}} q_t^{\text{S}} - q_t^{\text{L}} b_{\text{S}}}{a_{\text{D}} + b_{\text{DD}} q_t^{\text{D}} + b_{\text{ND}} q_t^{\text{N}} + b_{\text{SD}} q_t^{\text{S}} - q_t^{\text{L}} b_{\text{D}}} \right) \Upsilon_t. \tag{15b}$$

2 Measuring Priceless Consumption

This section presents the methodology to recover from the data the series of quantity Q_t^{L} and (shadow) price P_t^{L} and expenditure share S_t^{L} of the latent consumption.

2.1 Estimation Methodology

In what follows, we use the symbol $\widehat{\ }$ to differentiate empirical and estimated variables from the variables from the model. Setting the model implied prices from Equation (15a) to their corresponding observable prices (i.e., $P^{\mathrm{S}}[\hat{\mathbf{Q}}_t] = \hat{P}^{\mathrm{S}}$), solving each of the resulting equalities for q_t^L, and then equating the results, generates the expression that underlies our first moment condition:

$$\frac{\hat{S}_t^{\mathrm{N}}\hat{\Phi}_t^{\mathrm{D}} - \hat{S}_t^{\mathrm{D}}\hat{\Phi}_t^{\mathrm{N}}\hat{\Upsilon}_t}{\hat{S}_t^{\mathrm{N}}b_{\mathrm{D}} - \hat{S}_t^{\mathrm{D}}b_{\mathrm{N}}\hat{\Upsilon}_t} = \frac{\hat{S}_t^{\mathrm{S}}\hat{\Phi}_t^{\mathrm{D}} - \hat{S}_t^{\mathrm{D}}\hat{\Phi}_t^{\mathrm{S}}\hat{\Upsilon}_t}{\hat{S}_t^{\mathrm{S}}b_{\mathrm{D}} - \hat{S}_t^{\mathrm{D}}b_{\mathrm{S}}\hat{\Upsilon}_t}, \tag{16a}$$

where $\hat{\Upsilon}_t = 1 - (1-\delta)(\hat{R}_{\mathrm{t}}^{\mathrm{f}})^{-1}$ is the sample analog of Υ_t from Equation (7d) and

$$\hat{\Phi}_t^{\mathrm{D}} = a_{\mathrm{D}} + b_{\mathrm{DD}}\hat{q}_t^{\mathrm{D}} + b_{\mathrm{ND}}\hat{q}_t^{\mathrm{N}} + b_{\mathrm{SD}}\hat{q}_t^{\mathrm{S}}, \tag{16b}$$

$$\hat{\Phi}_t^{\mathrm{N}} = a_{\mathrm{N}} + b_{\mathrm{ND}}\hat{q}_t^{\mathrm{D}} + b_{\mathrm{NN}}\hat{q}_t^{\mathrm{N}} + b_{\mathrm{SN}}\hat{q}_t^{\mathrm{S}}, \tag{16c}$$

$$\hat{\Phi}_t^{\mathrm{S}} = a_{\mathrm{S}} + b_{\mathrm{SD}}\hat{q}_t^{\mathrm{D}} + b_{\mathrm{NS}}\hat{q}_t^{\mathrm{N}} + b_{\mathrm{SS}}\hat{q}_t^{\mathrm{S}}. \tag{16d}$$

Let $\theta = \{a_{\mathrm{D}}, a_{\mathrm{N}}, a_{\mathrm{S}}, b_{\mathrm{DD}}, b_{\mathrm{ND}}, b_{\mathrm{NN}}, b_{\mathrm{SD}}, b_{\mathrm{SN}}, b_{\mathrm{SS}}\}$ be the vector of the 9 parameters from the translog consumption aggregator to be estimated, $\hat{X}_t = \{\hat{Q}_t^{\mathrm{N}}, \hat{Q}_t^{\mathrm{D}}, \hat{Q}_t^{\mathrm{S}}, \hat{S}_t^{\mathrm{D}}, \hat{S}_t^{\mathrm{N}}, \hat{S}_t^{\mathrm{S}}, \hat{E}_t^{\mathrm{D}}, \hat{P}_t^{\mathrm{N}}, \hat{P}_t^{\mathrm{S}}, \hat{\Upsilon}_t\}$ the tuple of empirical series used in the estimation, and $S^{\mathrm{D}}[\hat{X}_t|\theta]$ and $S^{\mathrm{N}}[\hat{X}_t|\theta]$ the solutions to the equality in Equation (16a) for the expenditure share of the durable good and non-durable good, respectively. The double objective of the estimation is to find the parameter vector $\hat{\theta}$ that minimizes the averages of the squared deviations between the model-implied expenditure share of the durable good and non-durable good, as given by

$$\frac{1}{T}\sum_{t=1}^{T}\left(\hat{S}^{\mathrm{D}} - S^{\mathrm{D}}[\hat{X}_t|\theta]\right)^2, \tag{17a}$$

$$\frac{1}{T}\sum_{t=1}^{T}\left(\hat{S}^{\mathrm{N}} - S^{\mathrm{N}}[\hat{X}_t|\theta]\right)^2, \tag{17b}$$

where T denotes the number of time-periods in the sample used. The first-order conditions of the minimization problem in Equation (17) are

$$g\left[\hat{X}_t|\theta\right] = \{g^{\mathrm{D}}\left[\hat{X}_t|\theta\right], g^{\mathrm{N}}\left[\hat{X}_t|\theta\right]\}, \tag{18a}$$

where

$$g^{\mathrm{D}}\left[\hat{X}_t|\theta\right] = -\frac{2}{T}\sum_{t=1}^{T}\left(\hat{S}^{\mathrm{D}} - S^{\mathrm{D}}[\hat{X}_t|\theta]\right)\frac{\partial S^{\mathrm{D}}[\hat{X}_t|\theta]}{\partial\theta} = 0, \tag{18b}$$

$$g^{\mathrm{N}}\left[\hat{X}_t|\theta\right] = -\frac{2}{T}\sum_{t=1}^{T}\left(\hat{S}^{\mathrm{N}} - S^{\mathrm{N}}[\hat{X}_t|\theta]\right)\frac{\partial S^{\mathrm{N}}[\hat{X}_t|\theta]}{\partial\theta} = 0. \tag{18c}$$

Equation (18a) represents the 18 moment conditions from the two objectives in Equation (17) and the nine parameters in the vector θ. We use the 18 moment conditions to estimate the model using the generalized method of moments (GMM). Specifically, the procedure is implemented through the search of the minimum of the GMM loss function given by

$$\hat{\theta} = \mathrm{argmin}_{\theta}\left(g\left[\hat{X}_t|\theta\right] W g\left[\hat{X}_t|\theta\right]'\right), \tag{19}$$

where W is an 18×18 identity matrix.

2.2 Data

We obtain all the marketable consumption data from the Bureau of Economic Analysis (BEA). The data is based on series from the following BEA tables: *Real Personal Consumption Expenditures by Type of Product, Quantity Indexes* (BEA Table 2.4.3U) and *Price Indexes for Personal Consumption Expenditures by Type of Product* (BEA Table 2.4.4U). The data is at the annual frequency and spans the period 1959 to 2018. Within each of these table, we use data on durable goods (series code *DDUR*), nondurable goods (series code *DNDG*), and services (series code *DSER*). All expenditures series are scaled by the total U.S. population series produced by the U.S. Census Bureau and retrieved from the Federal Reserve Bank of St. Louis (FRED, series code *POP*). We construct series of quantities purchased of the durable ($\hat{E}_t^{\mathrm{D}}$) , nondurable ($\hat{E}_t^{\mathrm{N}}$), and service goods ($\hat{E}_t^{\mathrm{S}}$) as the ratios of expenditures and prices of the respective good. The quantities consumed of the nondurable and service goods in a given year equal the quantities purchased by agents of these goods (i.e., $\hat{Q}_t^{\mathrm{N}} = \hat{E}_t^{\mathrm{N}}$ and $\hat{Q}_t^{\mathrm{S}} = \hat{E}_t^{\mathrm{S}}$). Since the durable good is non perishable, we have that the quantity purchased ($\hat{E}_t^{\mathrm{D}}$) is different than the stock ($\hat{Q}_t^{\mathrm{D}}$) of this good. We use the perpetual inventory method to construct the series for the stock quantity of the durable good. Specifically, we set the stock of durables in 1959 to

$$\hat{Q}_{1959}^{\mathrm{D}} = \frac{\hat{E}_{1959}^{\mathrm{D}}}{\hat{\delta} + \text{Mean}\left[\frac{\Delta\hat{E}_t^{\mathrm{D}}}{\hat{E}_{t-1}^{\mathrm{D}}}\right] - \text{Mean}\left[\frac{\Delta\hat{P}_t^{\mathrm{D}}}{\hat{P}_{t-1}^{\mathrm{D}}}\right](1-\hat{\delta})}, \tag{20}$$

where $\hat{\delta} = 17.27\%$ is the annualized quarterly depreciation rate for durable goods from Gomes et al. (2009), Mean $\left[\frac{\Delta\hat{E}_t^{\mathrm{D}}}{\hat{E}_{t-1}^{\mathrm{D}}}\right] = 1.7\%$ is the average percentage growth in the real expenditure in durables and Mean $\left[\frac{\Delta\hat{P}_t^{\mathrm{D}}}{\hat{P}_{t-1}^{\mathrm{D}}}\right] = -2.5$ is the average percentage growth in the price of the durable good from 1959 to 2018. The stock of durables for the years 1960 onwards are then constructed iteratively using the law of motion in Equation (1).

Since the durable good is the numeraire in the model, we construct two sets of series of the observable goods: one using durables as the numeraire good and one in real 2012 U.S. Dollars. To construct the series based on durables as the numeraire good, we scale all prices and expenditures series with the price index series for the durable good and multiply the resulting series with the durable price index value of 2012. The risk free rate in Equation (7d), which represents the rate of a bond that pays a fixed number of units of the numeraire good, is not observable in the data. We approximate this rate with the nominal Treasury Bill rate from Kenneth French's data library deflated by the change in the price index of the durable good over the year. The error from this approximation should be small given that the price growth of the durable good has a low covariance with aggregate consumption growth and is highly predictable. For instance, as shown in Table 1, the annual price growth for the durable good has low volatility (1.41%), is almost uncorrelated (0.04) with the aggregate marketable consumption expenditure growth, and is highly autocorrelated (0.67). We use the Personal Consumption Expenditures price index to construct the series based on real 2012 U.S. Dollars. We construct series of aggregate expenditure in marketable goods by summing the expenditures of durable, nondurable, and service goods. Finally, we construct expenditure shares series by dividing the expenditures of a given good by the aggregate expenditure in marketable goods. Figure 1 plots the time series of the (log) quantity and expenditure shares of the three marketable consumption goods. The growth of the expenditure share in service goods relative to nondurable goods is consistent with the document rise of the service sector in the U.S. economy (e.g., Buera and Kaboski (2012)).

<< Figure 1 here >>

Panels A and B of Table 1 report the summary statistics of the growth in real expenditures (denoted Δy) and prices (denoted Δp) of the three marketable consumption bundles. In addition, Panel A reports the summary statistics of total real expenditure in nondurables plus services consumption (denoted Δc_{NS}), which is the standard measure of consumption used in the baseline consumption-based asset pricing model, and in durables plus nondurables plus services consumption (denoted Δc_{DNS}). Finally, for comparison, the table also reports the summary statistics of the garbage proxy consumption measure of Savov (2011) (denoted Δc_G).

As expected, the real expenditure on durable consumption is the most volatile component of marketable consumption (5.41% versus less than 2% for the other two components). The consumption bundles have high positive correlation with the aggregate (with and without durables) marketable consumption (the correlation with marketable consumption growth ranges from 54% for nondurables consumption to 91% for durables consumption). The pairwise correlations between the three consumption bundles reveal that the expenditure on durable consumption and services is highly correlated, with a correlation of 79%. The correlation between the expenditure on nondurables consumption and the other consumption items is significantly lower, 27% with durable consumption and 20% for services. This lack of imperfect correlation between the three real consumption expenditure series is important for a proper identification of the latent PC series (as noted, we need the different series to respond different to PC).

Panel C of Table 1 reports the summary statistics of the expenditure shares in the three marketable consumption series. On average, services has the highest expenditure share, 59%, while durable consumption has the lowest, 13%. The autocorrelation is quite high because the expenditure shares in the three consumption bundles exhibit clear trends, specially the services and non durables consumption. Figure 1 shows that while the expenditure share on services has steadily risen over the sample period from about 44% to 65%, the expenditure share on nondurables consumption has steadily decreased from about 40% to 20%. The expenditure share on durables consumption has remained stable over time.

<< Table 1 here >>

2.3 Parameter Estimates and Model Fit

We now examine whether our proposed demand system can well explain the observed consumption dynamics. In the next section, we use the estimated parameters to recover the latent consumption series from the data.

Table 2 reports the point estimates and associated standard errors of the parameters of the translog consumption aggregator from Equation (11a).

<< Table 2 here >>

Figure 2 shows the performance of the estimated model at explaining expenditure shares of the three marketable consumption goods. The model does a decent job matching the dynamics of expenditure shares of these goods. The model-implied expenditure shares is able to not only track the level and the trend, but also the higher frequency movements of the expenditure shares in the data. To quantify the performance of the model in matching the expenditure shares, we use the goodness-of-fit measure from Belo, Gala, Salomao, and Vitorino (2019) defined as the average mean absolute error scaled by the expenditure share from the data. The values of the goodness-of-fit measure are 5.5% for durable goods, 2.6% for nondurable goods, and 2.3% for durable goods. As we discuss later, the translog aggregator, which underlies our model, has a significantly greater than the more commonly used constant-elasticity-of-substitution (CES) aggregator.

<< Figure 2 here >>

2.4 Recovering the Priceless Consumption Series from the Data

We now use the vector of estimated parameters $\hat{\theta}$ to recover the model-implied log-quantity of the latent consumption series using the equation:

$$q^{\mathrm{L}}[\hat{X}_t|\hat{\theta}] = \frac{\hat{S}_t^{\mathrm{N}}\left(\hat{a}_{\mathrm{S}} + \hat{b}_{\mathrm{SD}}\hat{q}_t^{\mathrm{D}} + \hat{b}_{\mathrm{SN}}\hat{q}_t^{\mathrm{N}} + \hat{b}_{\mathrm{SS}}\hat{q}_t^{\mathrm{S}}\right) - \hat{S}_t^{\mathrm{S}}\left(\hat{a}_{\mathrm{N}} + \hat{b}_{\mathrm{ND}}\hat{q}_t^{\mathrm{D}} + \hat{b}_{\mathrm{NN}}\hat{q}_t^{\mathrm{N}} + \hat{b}_{\mathrm{SN}}\hat{q}_t^{\mathrm{S}}\right)}{\hat{S}_t^{\mathrm{N}}\hat{b}_{\mathrm{S}} - \hat{S}_t^{\mathrm{S}}\hat{b}_{\mathrm{N}}}. \tag{21}$$

The expression above is obtained by replacing the prices P_t^{N} and P_t^{S} from Equation (15) into the expenditure shares S_t^{N} and S_t^{S} in Equation (7), combining the resulting two resulting expressions to eliminate Υ_t, and then solving the final equality for q_t^{L}. The shadow price of the latent good is given by

$$P^{\mathrm{L}}[\hat{X}_t|\hat{\theta}] = \frac{\hat{Q}_t^{\mathrm{D}}}{\hat{Q}_t^{\mathrm{L}}}\left(\frac{\hat{a}_{\mathrm{L}} + \hat{b}_{\mathrm{LD}}\hat{q}_t^{\mathrm{D}} + \hat{b}_{\mathrm{LL}}q^{\mathrm{L}}[\hat{X}_t|\hat{\theta}] - \hat{b}_{\mathrm{N}}\hat{q}_t^{\mathrm{N}} - \hat{b}_{\mathrm{S}}\hat{q}_t^{\mathrm{S}}}{\hat{a}_{\mathrm{D}} + \hat{b}_{\mathrm{DD}}\hat{q}_t^{\mathrm{D}} + \hat{b}_{\mathrm{LD}}q^{\mathrm{L}}[\hat{X}_t|\hat{\theta}] + \hat{b}_{\mathrm{ND}}\hat{q}_t^{\mathrm{N}} + \hat{b}_{\mathrm{SD}}\hat{q}_t^{\mathrm{S}}}\right)\hat{\Upsilon}_t, \tag{22}$$

where $q^{\mathrm{L}}[\hat{X}_t|\hat{\theta}]$ is the estimated log quantity of the latent good from Equation (21). Finally, the shadow expenditure share in the latent good is given by

$$S^{\mathrm{L}}[\hat{X}_t|\hat{\theta}] = \frac{C^{\mathrm{L}}[\hat{X}_t|\hat{\theta}]}{C_t^{\mathrm{DNS}}}, \tag{23}$$

where $C^{\mathrm{L}}[\hat{X}_t|\hat{\theta}] = Q^{\mathrm{L}}[\hat{X}_t|\hat{\theta}]P^{\mathrm{L}}[\hat{X}_t|\hat{\theta}]$ is the shadow expenditure in the latent good, and $Q^{\mathrm{L}}[\hat{X}_t|\hat{\theta}] \equiv \mathrm{Exp}\left[q^{\mathrm{L}}[\hat{X}_t|\hat{\theta}]\right]$.

With the estimated parameter, we now show the properties of the model-implied shadow expenditure share of PC, S^{L}. Figure 4 plot the quantity (Panel A), shadow price (Panel B), and the shadow consumption expenditure share (Panel C) of the priceless consumption good. The shadow expenditure share S^{L} is significant and ranges from 10% to over 50% of the observable consumption expenditure.[10]

[10]Recall that S^{L} is defined as a shadow share of the (observable) marketable consumption expenditure (as

<< *Figure 4 here* >>

2.5 Implied Elasticity of Substitution

To help interpret the parameter estimates, we compute the implied elasticity of substitution (henceforth EOS) between the marketable consumption goods, and also between the marketable consumption goods and the priceless consumption (which we examine later). As noted, with the translog aggregator, the EOS varies both across goods and over time, in contrast with a CES aggregator. This analysis is motivated by the fact that, as discussed before, a necessary condition for the identification of PC is that the there is dispersion in EOS between the marketable goods and PC.

We compute the $EOS[C_a, C_b]$ between two consumption goods C_a and C_b in a standard was as follows:

$$EOS[C_a, C_b] \equiv \frac{F_{\mathrm{a}}[C_a, C_b] F_{\mathrm{b}}[C_a, C_b]}{F[C_a, C_b] F_{\mathrm{ab}}[C_a, C_b]}, \tag{24}$$

where F_{x} is the partial derivative of the aggregator F with respect to the consumption type C_x. As a reference, the case $EOS < 1$ implies that the two goods are complements, the case $EOS > 1$ implies that the two goods are substitutes, and the case $EOS = 1$ represents the knife-edge case in which the goods are neither complements nor substitutes, as in a Cobb-Douglas consumption aggregator.

Panel A of Figure 3 shows that PC and durables consumption are complements, PC and nondurables consumption are substitutes, and PC and services consumption are slight substitutes. Thus, the estimation results confirm that the three marketable good have different EOS values relative to the PC series, a necessary condition to properly identify the PC series.

opposed to the model-implied total consumption expenditure), which implies that the consumption shares across goods C^{D}, C^{N}, C^{S}, and C^{L} do not sum to one.

Panel B of Figure 3 shows the model implied elasticities of substitution values from Equation (24) across the marketable consumption goods. The figure confirms that there is dispersion in the EOS between the marketable consumption bundles. In particular, the figure shows that durables strongly substitutes services while they complement nondurable goods. The documented complementarity between durables and nondurables is consistent with the findings of Yogo (2006) and Gomes et al. (2009).[11]

<< Figure 3 here >>

Figure 5 plots the time series of aggregate consumption by adjusting the aggregate marketable consumption expenditure C_{DNS} with the model implied shadow expenditure share of PC, S^{L} (denoted C_{DNSL}). Panel A presents the aggregate series in log U.S. Dollars and shows the importance of PC good in the aggregate U.S. consumption. In the first half of the sample (i.e., between 1960 and 1988), the gap between the true total consumption C_{DNSL} and the (observed) marketable consumption C_{DNS} ranges between a minimum of \$359 billion in 1975 (10% of observable consumption) and a maximum of \$1.7 trillion in 1986 (33% of observable consumption), with an average of \$920 billion (25% of the aggregate consumption).[12] In the second half of the sample (i.e., between 1989 and 2018), the gap ranges between a minimum of \$1.4 billion in 1992 (22% of observable consumption) and a maximum of \$6.3 trillion in 2018 (48% of observable consumption), with an average of \$3.4 trillion (36% of observable consumption).

Panel B represent per capita figures and show a similar overall upward trend in the shadow share of PC in the U.S. economy. In the first half of the sample, the gap between the per capita true total consumption C_{DNSL} and the (observed) per capita marketable

[11]Note that the definition of the goods in this paper are different in Yogo (2006) and Gomes et al. (2009). For instance, these papers combine expenditures in nondurable goods and services to define nondurables while this paper treats them as separate goods.

[12]All monetary values presented are in real 2012 U.S. Dollars.

consumption C_{DNS} ranges between a minimum of \$1,660 in 1975 and a maximum of \$7,190 in 1986, with an average of \$4,230. In the second half of the sample, the gap ranges between a minimum of \$5,250 in 1993 and a maximum of \$19,150 in 2018, with an average of \$11,520.

<< Figure 5 here >>

The shadow value of PC is not only significant but also volatile. Panel A in Figure 6 documents the volatility of total aggregate consumption growth ΔC_{DNSL} and that of the aggregate marketable consumption growth ΔC_{DNS}. Panel B documents the volatility of the shadow expenditure in the PC. The plots suggest different volatility levels of ΔC_{DNSL} and ΔC_{DNS}. This finding is consistent with that presented in Table1, that the PC growth is in fact significantly more volatile than marketable consumption growth.

<< Figure 6 here >>

Table 4 reports the mean and the standard deviation of the variables presented in Table 1 conditional on the business cycle. Specifically, we report the statistics for the sample years that experienced recessions and all other years, which we label *expansions*.[13] Overall, the table shows that the mean and volatility of the latent consumption series are significantly affected by the state of the economy. In particular, the growth in the shadow expenditure of the latent consumption good has a mean of 7.45% and a standard deviation of 12.42% during expansions and a mean of -8.80% and a standard deviation of 25.64% during recessions. The table suggests that most of these differences are due to the effect of the business cycle on the price of the latent good, although the effect of the business cycle on the quantity growth of this good has the same sign and thus reinforces the effect.

[13]We include a year in our working sample in the *Recession* column if any month within that year was classified as a recession by the NBER's Business Cycle Dating Committee and in the *Expansions* column otherwise.

<< Table 4 here >>

2.6 Alternative Specifications

Our empirical specification departs from previous work in that we consider a translog consumption aggregator, which is significantly more flexible than the more standard CES aggregator used in, for example, Yogo (2006)).

Here, we show that the additional flexibility given by the translog aggregator is important for the ability of the demand system to explain the dynamics of expenditure shares of the marketable goods. Figure 7 shows the fit of expenditure shares of the three marketable consumption goods when we use a CES consumption aggregator. The ability of the demand system to explain the share dynamics deteriorates significantly. In particular, the CES aggregator is unable to capture the low frequency dynamics of the expenditure shares in durable goods and in services. The poor performance of the estimated CES aggregator is even starker when we compare the average values of the goodness-of-fit measure (i.e., the average mean absolute error scaled by the expenditure share from the data) from Figure 7 (19.0%) with those from the translog aggregator (3.5%), which are presented in Figure 2.

<< Figure 7 here >>

3 Asset Pricing Implications

We argue in this paper that the aggregate measures of consumption reported in National Economic Accounts are incomplete in that they ignore an entire consumption category, which is intrinsically latent (the PC). The analysis of the recovered latent series shows that the shadow expenditure growth of the latent consumption good is significantly more volatile than those of the marketable consumption goods while having a relatively low correlation with these. In face of these facts, a natural question is whether our proposed *true* aggregate consumption measure, which incorporates priceless consumption to the aggregate marketable consumption expenditure measure, helps recover information about the marginal utility of the representative agent and by extension helps price assets in the economy. We answer this question by using the canonical consumption-based model framework started by Breeden (1979).

Specifically, we assume that the preferences of the representative agent are described by a power utility function (CRRA), as given by

$$U[C_t] = \frac{C_t^{1-\gamma}}{1-\gamma}, \tag{25}$$

where $\gamma > 1$ is the coefficient of relative risk aversion (RRA).

The representative agent's power utility over true aggregate consumption implies the standard Euler equation given by:

$$\mathbb{E}\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} R^{\mathrm{j}}_{t+1}\right] = 1. \tag{26}$$

where R^{j} is the gross return on asset j.

We test the performance of the baseline model by following the procedure in Savov (2011). Specifically, we set the coefficient β to 0.95 and estimate the implied coefficient of RRA by minimizing the pricing errors of Equation (26) in explaining the returns of the market portfolio, and also the Fama-French 25 size and book to market portfolios.[14]

We test two specifications of the model which vary in the consumption series used. In

[14]The series of the risk free rate, market factor return, and returns of the 25 portfolios of stocks double sorted on market value and book-to-market ratios used in this section are obtained from Kenneth French's data library.

the baseline case, we abstract away from the distinction between durables and the other types of goods and assume for simplicity that the utility function in Equation (25) is defined over the consumption of all goods (i.e., the *true* consumption expenditure, which includes expenditures in the durable good). In an alternative specification, we assume that the utility function is defined over the true aggregate consumption *net* of expenditures in the durable good (consistent with the standard applications of the consumption-based model which exclude durable consumption from the analysis). For comparison, we also test the standard model using three additional definitions of aggregate consumption expenditures: (i) aggregate consumption defined over the three marketable consumption goods (i.e., including the durable good but excluding the latent good), (ii) aggregate consumption defined over the consumption of services and nondurable goods (i.e., excluding both the durable and latent goods), and (iii) aggregate consumption proxied by the municipal solid waste (i.e., "garbage") measure from Savov (2011).

<< Table 5 here >>

The results of the tests are presented in Panel A of Table 5. The results generally confirm the documented high coefficient of RRA implied by power utility preferences using standard NIPA measures of aggregate consumption, which are around 22 (column 3) when durable consumption is included and around 32 (column 4) when durable consumption is excluded from the measure. The table also perfectly replicates the original estimate of RRA from Savov (2011) (around 17, column 5) with the garbage-based proxy for aggregate consumption growth is used. The new insight from the table and one of the main contributions of our paper is to show that the aggregate consumption measure proposed in this paper leads to a relatively low implied coefficient of RRA (around 8, columns 1 and 2). Despite the lower estimate, the standard errors presented suggest that we cannot reject the hypothesis that our RRA estimates are statistically different from that in the garbage-based model. The panel also presents the risk-free rates implied by the five models, and confirms the failure of this class of model to match both the equity risk premium and the risk free rate. Although still unrealistically high (around 20%), the implied risk free rates of the proposed models (columns 1 and 2) are significantly lower than that of those (columns 3 and 4) based on standard consumption measures (58.5% and 99%) and consistent with that based on the garbage series to proxy for consumption (17%).

Panel B of Table 5 presents the results of a similar exercise, but based on the returns of the 25 portfolios of stocks double sorted on market value and book-to-market ratios from Ken French's data library. We use a single-stage GMM procedure with an equally weighted weighting matrix in the estimation. The estimates of RRA are generally consistent with those presented in Panel A. Interesting, the point estimates using the broad consumption measure that includes PC (columns 1 and 2) are even lower (around 6.5) than that of the successful garbage-based model of Savov (2011) (around 22 column 5).

Also following Savov (2011), we further extend the analysis presented in Panel A of Table 5 by investigating the effects of the different values for the RRA coefficient on the pricing error and on the risk free rate implied by models reported in columns 1, 3, and 5 of the table. This is reported in Figure 8. The figure confirms the documented extent of the failure of models with power utility estimated with traditional measures of aggregate consumption to match both the risk free rate and the rate equity premium. The figure shows that the results of tests of the C-CAPM based on our proposed measures of the true aggregate consumption are generally consistent with those based on the garbage series as a proxy for aggregate consumption growth. Overall, the results in Table 8 are consistent with the validity and economic significance of the latent component of consumption recovered in our proposed methodology.

<< Figure 8 here >>

4 Conclusion

We show in this paper that the dynamics of unobservable consumption are relevant for economic models that rely on the dynamics of marginal utility over consumption, which are exemplified in the paper by the consumption-based CAPM. The motivation for the study is the fact that aggregate consumption measures can only account for goods and services for which prices and quantities can be observed. We denote by priceless consumption (PC) the universe of consumption types that are unaccounted for in aggregate consumption measures. We propose a structural estimation methodology to recover the dynamics of PC from its effects on durables, non durables and services consumption. Our estimation results suggest that PC is a large and volatile component of total consumption. Finally, the paper presents evidence that the dynamics PC is consistent with the theory behind the standard consumption-based CCAPM. When we measure total consumption accounting for PC, the empirical fit of the model improves significantly. In particular, the model is able to match the observed with equity risk premium with a low relative risk aversion between 6.5 and 8.2.

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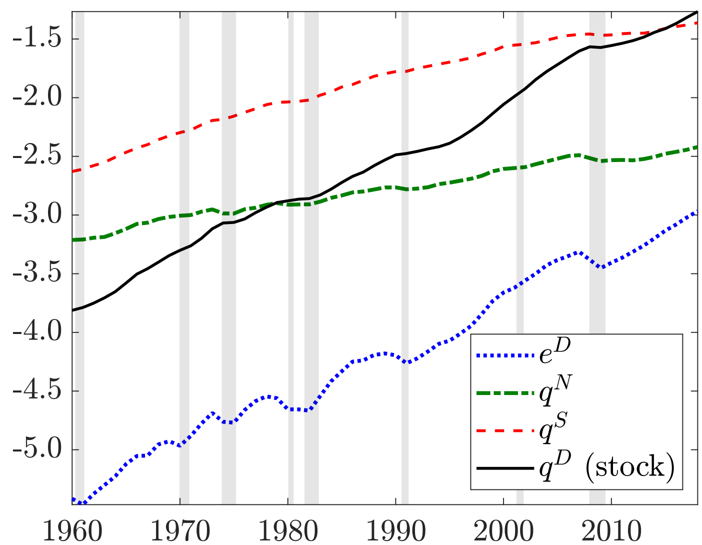
Figure 1
Consumption Quantities and Expenditure Shares of Marketable Consumption Goods

Panel A shows the time series of log quantities q of the durable (D), nondurable (N), and services (S) marketable consumption goods. Panel B shows the consumption expenditure shares S of the marketable consumption goods. The sample is at the annual frequency and covers the period from 1959 to 2018.

Panel A: Log Quantities



Panel B: Expenditure Shares

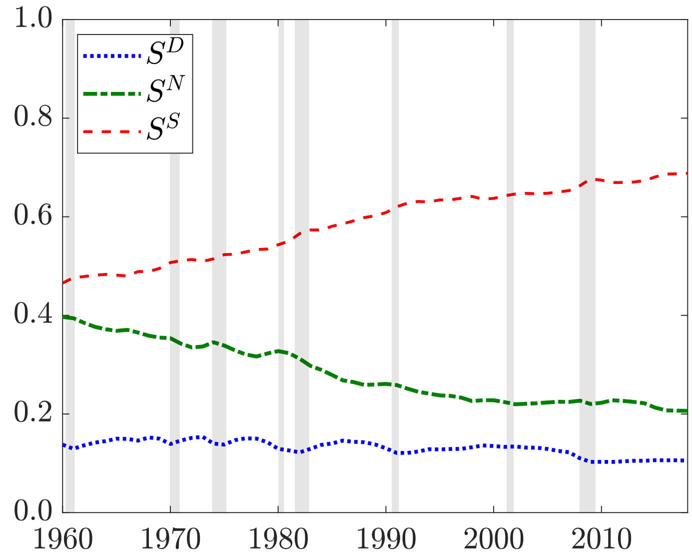
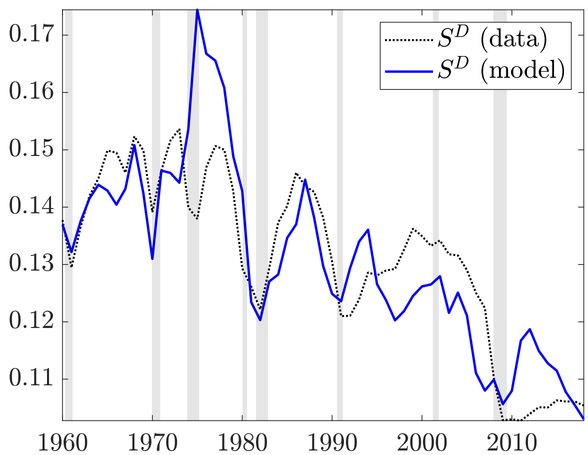


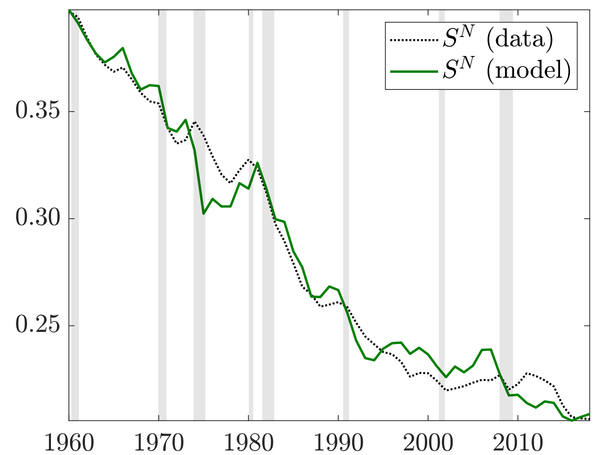
Figure 2
Estimation: Fit of Expenditure Shares of Marketable Consumption Goods

This figure plots the time series of the expenditure shares of the durable goods (D), nondurable goods (N), and services (S) from the data and from the estimated model. *M.A.E.* is the mean absolute error between the expenditure shares in the data and from the estimated model. The sample is at the annual frequency and covers the period from 1959 to 2018.

Panel A: Durable Goods
$\left({}^{M.A.E.}/_{\text{Mean}\,[S^{\text{D}}]} = 5.5\%\right)$



Panel B: Nondurable Goods
$\left({}^{M.A.E.}/_{\text{Mean}\,[S^{\text{N}}]} = 2.6\%\right)$



Panel C: Service Goods
$\left({}^{M.A.E.}/_{\text{Mean}\,[S^{\text{S}}]} = 2.3\%\right)$

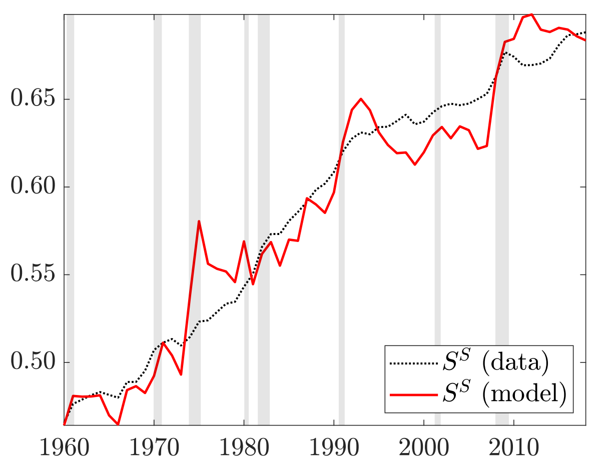


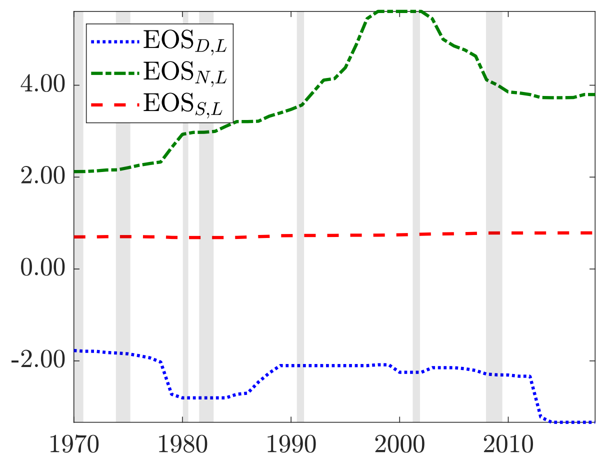
Figure 3
Estimation: Model-Implied Elasticities of Substitution Between Consumption Goods

This plot shows the 10-year moving median of the model-implied elasticity of substitution between the latent consumption good and the marketable consumption goods (i.e., the durable, nondurable, and service goods). The elasticity of substitution between any two goods of types a and b is standard and is defined by

$$EOS_{\mathrm{a,b}}[\mathbf{Q}_t] \equiv \frac{U_{\mathrm{a}}[\mathbf{Q}_t]U_{\mathrm{b}}[\mathbf{Q}_t]}{U[\mathbf{Q}_t]U_{\mathrm{ab}}[\mathbf{Q}_t]},$$

where U_{x} denotes the partial derivative of the utility function U with respect to Q_x (i.e., the quantity consumed of consumption type x). An $EOS = 1$ means that the two goods are Cobb-Douglas substitutes, an $EOS < 1$ means that the two goods are complements, and an $EOS > 1$ means that the two goods are substitutes. The sample is at the annual frequency and covers the period from 1959 to 2018.

Panel A: Elasticities of Substitution Between Latent and Marketable Goods



Panel B: Elasticities of Substitution Between Marketable Goods

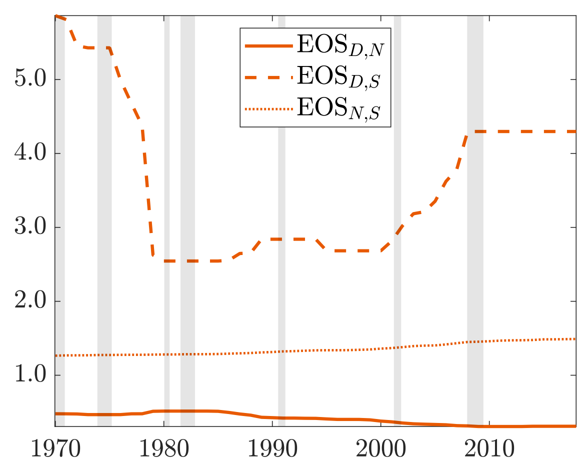


Figure 4
Estimation: Recovered Latent Good Series

This figure shows the model-implied log quantity, shadow price, and shadow expenditure share of the latent consumption good. The sample is at the annual frequency and covers the period from 1959 to 2018.

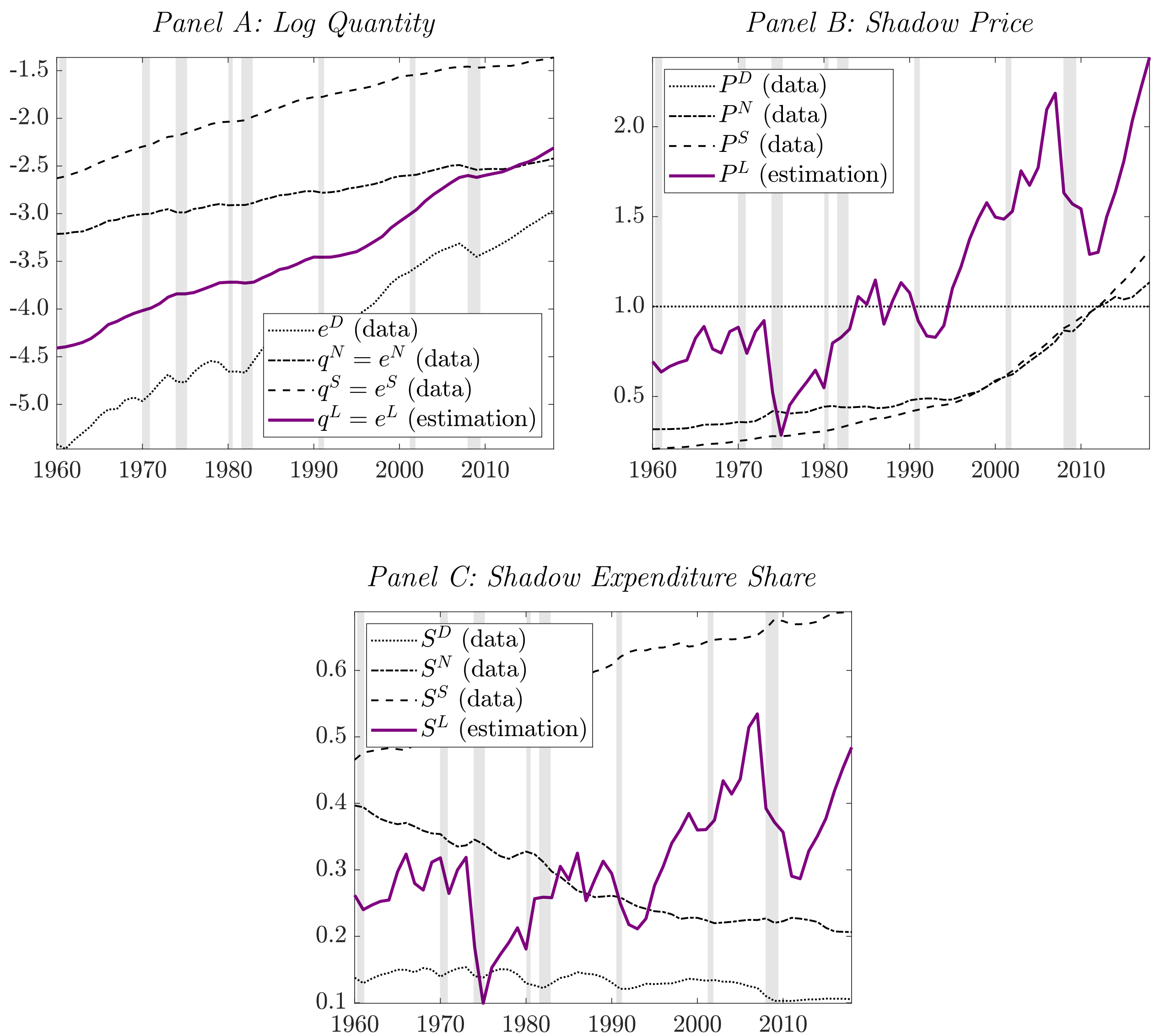
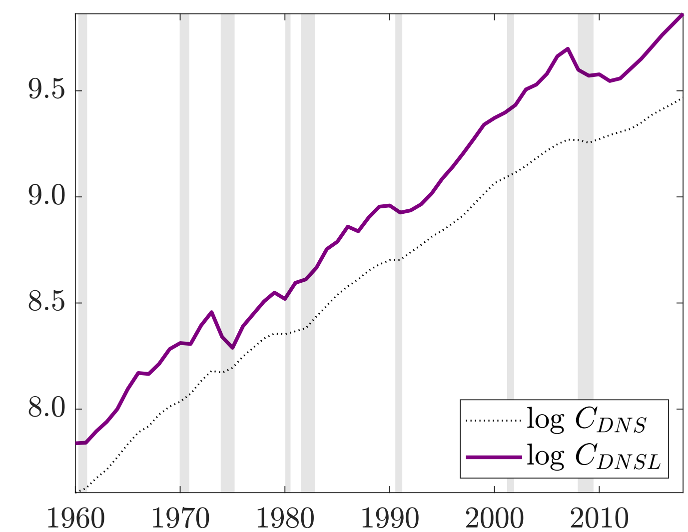


Figure 5
Marketable Consumption and Shadow Aggregate Consumption

Panel A shows (the logarithm of) the measured aggregate expenditure in marketable consumption goods C_{DNS} and the model-implied true aggregate consumption, $C_{\text{DNSL}} = C_{\text{DNS}}(1+S_L)$, which includes the shadow expenditure in the latent consumption good. Panel B shows the marketable (C_{DNS}) and non-marketable (C_{L}) components of C_{DNSL} on a per-capita basis. Values are expressed in 2012 U.S. Dollars using the PCE index. The sample is at the annual frequency and covers the period from 1959 to 2018.

Panel A: Consumption Expenditures (in Log of Trillions of 2012 U.S. Dollars)



Panel B: Per-Capita Consumption Expenditures (in Thousands of 2012 U.S. Dollars)

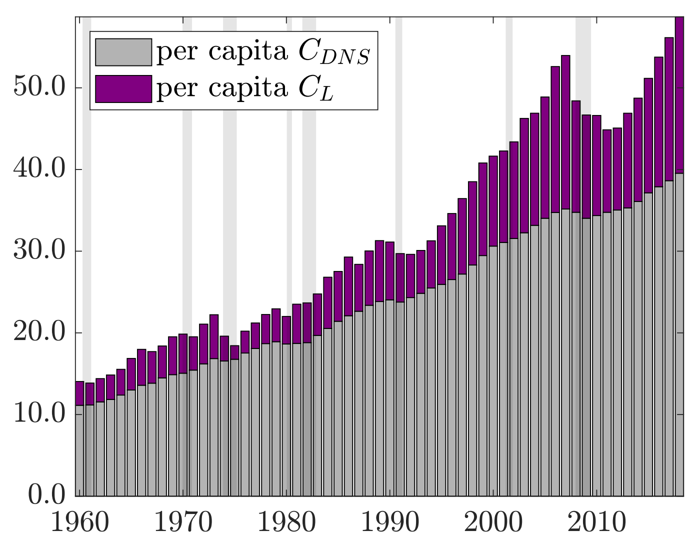
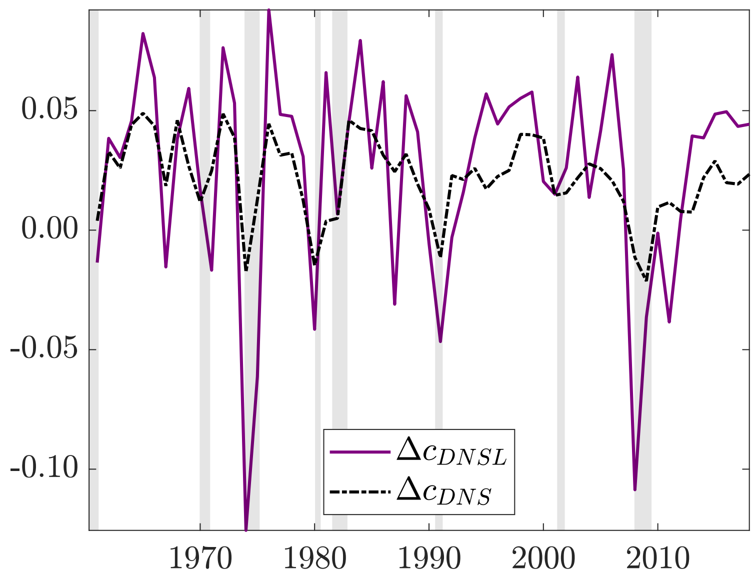


Figure 6
Volatility of Consumption Growth

The figure shows the time series of aggregate consumption expenditure growth $\Delta c_t \equiv \text{Log}[{}^{C_t}/{}_{C_{t-1}}]$. Panel A shows the marketable consumption expenditure (C_{DNS}) growth series and the shadow total consumption expenditure ($C_{\text{DNSL}} = C_{\text{DNS}}(1+S_{\text{L}})$) growth series. Panel B presents the latent consumption expenditure ($C_{\text{L}} = C_{\text{DNS}}S_{\text{L}}$) growth series. The sample is at the annual frequency and covers the period from 1959 to 2018.

Panel A: Marketable Consumption Expenditure vs Shadow True Expenditure Growth



Panel B: Shadow Latent Consumption Expenditure Growth

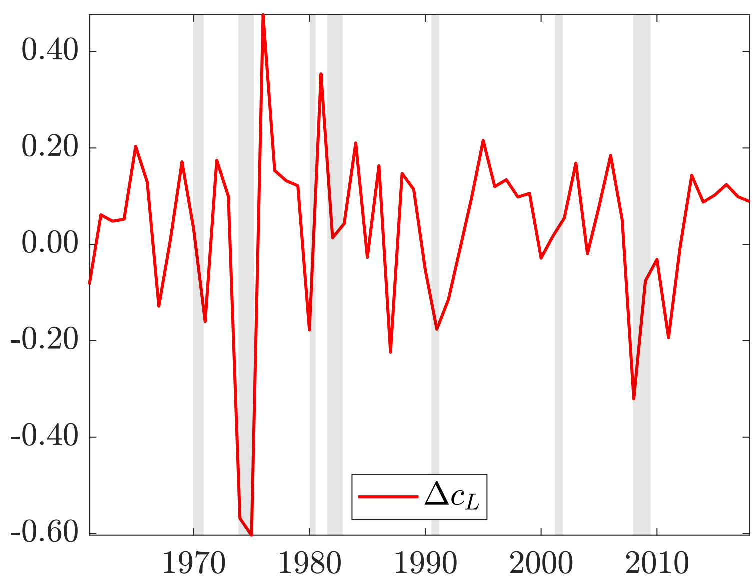
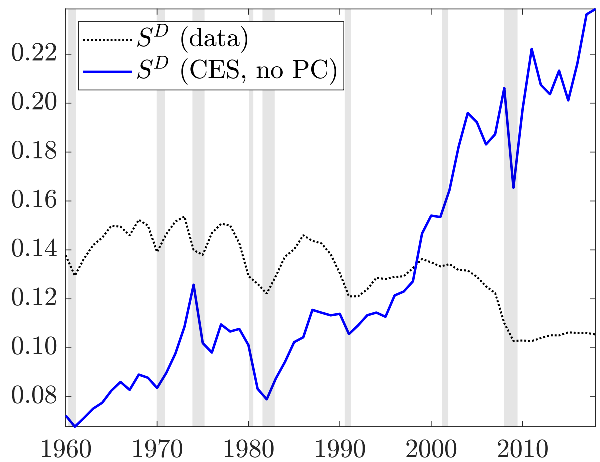


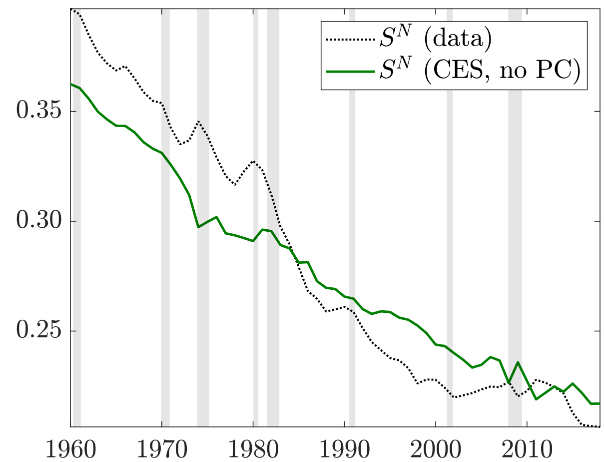
Figure 7
Performance of CES Consumption Aggregator with no Latent Consumption

This figure plots the time series of the expenditure shares of the durable goods (D), nondurable goods (N), and services (S) from the data and from the estimation of a constant elasticity of substitution (CES) consumption aggregator $F_t^{\text{CES}} = (a_{\text{D}}(Q_t^{\text{D}})^\rho + a_{\text{N}}(Q_t^{\text{N}})^\rho + (1 - a_{\text{D}} - a_{\text{N}})(Q_t^{\text{S}})^\rho)^{1/\rho}$. *M.A.E.* is the mean absolute error between the expenditure shares in the data and from the estimated model. The sample is at the annual frequency and covers the period from 1959 to 2018.

Panel A: Durable Goods
$\left({}^{M.A.E.}/_{\text{Mean}\,[S^{\text{D}}]} = 39.7\%\right)$



Panel B: Nondurable Goods
$\left({}^{M.A.E.}/_{\text{Mean}\,[S^{\text{N}}]} = 6.3\%\right)$



Panel C: Service Goods
$\left({}^{M.A.E.}/_{\text{Mean}\,[S^{\text{S}}]} = 11.1\%\right)$

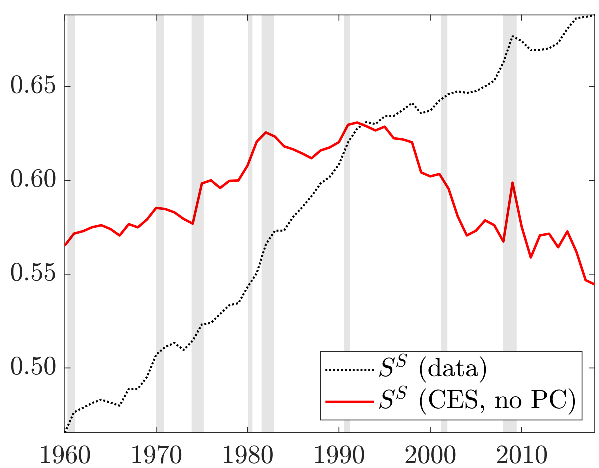


Figure 8
Estimation: Pricing Errors

This figure shows the pricing errors of the equity premium and the risk free rate for different coefficients of relative risk aversion.

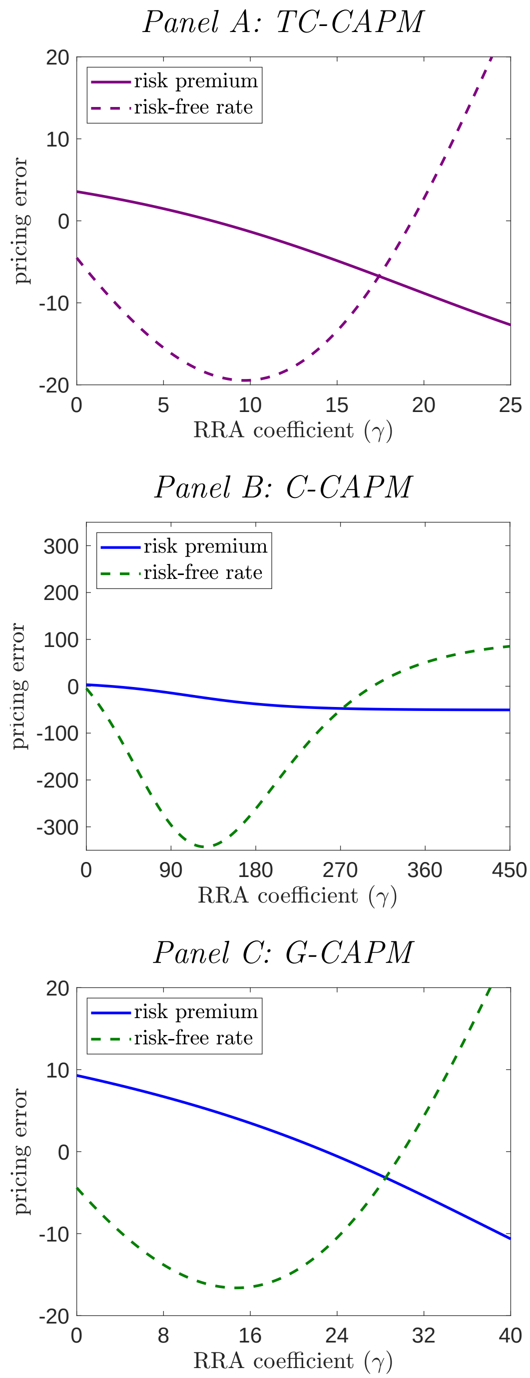


Table 1
Summary Statistics of Consumption Data

This table presents summary statistics of the series related to consumption from the data and from the estimated model presented in Section 1. The variables c, p, and S represent log-expenditures, log-prices, and expenditure shares. The subscripts in the variables denote the goods considered in the series: D = durables, N = nondurables, S = services, $L = PC$ = latent (i.e., *the priceless consumption goods*), and G = the proxy for aggregate consumption based on municipal solid waste (i.e., *garbage*) data from Savov (2011). Panels A, B, and C present the statistics of expenditure growth, price growth, and expenditure shares of the marketable consumption goods (i.e., D, N, and S), respectively. Panel D presents the statistics of the latent consumption series recovered by the model estimation. The sample is at the annual frequency and covers the period 1959 to 2018, except for the series of garbage growth Δc_{G}, which covers the period 1960 to 2006.

Series	Mean	Std	Auto Corr	Correlations Δc_{DNS}	Δc_{NS}	Δc_{D}	Δc_{N}	Δc_{S}	Δc_{G}
Panel A: Expenditures of Marketable Goods									
Δc_{DNS}	2.19	1.68	0.33	1.00	0.95	0.91	0.54	0.88	0.51
Δc_{NS}	2.25	1.24	0.32	0.95	1.00	0.74	0.68	0.85	0.54
Δc_{D}	1.72	5.41	0.30	0.91	0.74	1.00	0.28	0.79	0.40
Δc_{N}	1.06	1.89	0.36	0.54	0.68	0.28	1.00	0.21	0.45
Δc_{S}	2.86	1.46	0.39	0.88	0.85	0.79	0.21	1.00	0.40
Δc_{G}	1.47	2.88	-0.15	0.51	0.54	0.40	0.45	0.40	1.00
Panel B: Prices of Marketable Goods									
Δp_{D}	-2.50	1.41	0.67	0.04	-0.07	0.18	-0.39	0.25	0.01
Δp_{N}	-0.31	1.55	0.43	-0.24	-0.06	-0.45	0.60	-0.49	0.04
Δp_{S}	0.68	0.65	0.44	0.32	0.18	0.45	-0.42	0.52	0.00
Panel C: Expenditure Shares of Marketable Goods									
S_{D}	0.13	0.02	0.83	0.52	0.55	0.40	0.44	0.48	0.39
S_{N}	0.28	0.06	1.00	0.06	0.07	0.04	0.00	0.22	0.16
S_{S}	0.59	0.07	1.00	-0.14	-0.15	-0.09	-0.07	-0.27	-0.20
Panel D: Recovered Latent Consumption Series									
Δc_{L}	3.25	18.07	0.03	0.50	0.45	0.47	0.30	0.38	0.48
Δc_{DNSL}	2.47	4.41	0.08	0.71	0.67	0.65	0.43	0.57	0.46
Δp_{L}	2.13	17.49	0.03	0.45	0.40	0.43	0.24	0.35	0.46
S_{L}	0.31	0.09	0.88	0.15	0.21	0.03	0.37	-0.02	0.01

Table 2
Estimated Parameters of the Translog Consumption Aggregator

This table presents the point estimates and standard errors of the vector α (Panel A) and the matrix $\mathbf{B}$ (Panel B) of the estimation of the translog consumption aggregator from Equation (11a):

$$F[\mathbf{Q}_t] \equiv \mathrm{Exp}\left[\mathrm{Log}[\mathbf{Q}_t] \times \mathbf{a} + \frac{1}{2}\,\mathrm{Log}[\mathbf{Q}_t] \times \mathbf{B} \times \mathrm{Log}[\mathbf{Q}_t]'\right],$$

Panel A: Vector **a**				
	a_{D}	a_{N}	a_{S}	a_{L}
	0.10	0.48	0.04	0.38
	(0.01)	(0.02)	(0.10)	(0.13)

Panel B: Matrix **B**				
	$b_{\_\mathrm{D}}$	$b_{\_\mathrm{N}}$	$b_{\_\mathrm{S}}$	$b_{\_\mathrm{L}}$
$b_{\mathrm{D}\_}$	-1.32	-1.24	1.28	1.28
	(0.04)	(0.03)	(0.00)	(0.12)
$b_{\mathrm{N}\_}$	-1.24	-0.87	0.91	1.21
	(0.03)	(0.03)	(0.00)	(0.06)
$b_{\mathrm{S}\_}$	1.28	0.91	-0.85	-1.34
	(0.00)	(0.00)	(0.05)	(0.05)
$b_{\mathrm{L}\_}$	1.28	1.21	-1.34	-1.15
	(0.12)	(0.06)	(0.05)	(0.16)

Table 3
Summary Statistics of Risk Factors

This table presents summary statistics of the growth rate in aggregate expenditure and the growth rate and expenditure share across the marketable consumption goods. The sample is at the annual frequency and covers the period from 1960 to 2018

				Correlations				
Series	Mean	Std	Auto Corr	$r^f_{nominal}$	$r^f_{real(PCE)}$	$r^f_{real(P_D)}$	$r^f_{real(P_N)}$	R_{market}
$r^f_{nominal}$	4.68	3.22	0.83	1.00	0.48	0.34	0.32	-0.16
$r^f_{real(PCE)}$	1.34	2.14	0.73	0.48	1.00	0.82	0.87	0.02
$r^f_{real(P_D)}$	3.85	2.40	0.67	0.34	0.82	1.00	0.58	-0.11
$r^f_{real(P_N)}$	1.63	3.06	0.67	0.32	0.87	0.58	1.00	0.19
R_{market}	7.06	17.52	-0.08	-0.16	0.02	-0.11	0.19	1.00
Δc_{DNS}	2.22	1.68	0.36	-0.36	0.02	-0.16	0.28	0.46
Δc_{NS}	2.27	1.25	0.32	-0.36	0.03	-0.12	0.24	0.42
Δc_{L}	3.45	18.16	0.03	-0.18	-0.07	-0.27	0.20	0.48
Δc_{DNSL}	2.53	4.42	0.08	-0.28	-0.07	-0.24	0.21	0.47
Δc_{G}	1.46	2.91	-0.15	-0.19	0.00	-0.21	0.23	0.59

Table 4
Latent Consumption and the Business Cycle

This table reports the mean and the standard deviation of the growth rates of consumption expenditures and the shadow expenditure shares of *PC* over expansions and recessions. All numbers are in percentage points. The sample is at the annual frequency and covers the period from 1960 to 2018. We include a given year in the *Recessions* sample if any month within that year is classified as a recession by the NBER's Business Cycle Dating Committee.

	Expansions		Recessions	
Series	Mean	Std	Mean	Std
Δc_{DNS}	2.80	1.17	0.41	1.69
Δc_{NS}	2.63	0.98	1.16	1.29
Δc_{D}	3.91	3.46	-4.55	5.16
Δc_{N}	1.48	1.59	-0.15	2.19
Δc_{S}	3.22	1.30	1.84	1.47
Δc_{G}	2.08	2.16	-0.14	3.88
Δp_{D}	-2.50	1.40	-2.50	1.48
Δp_{N}	-0.47	1.37	0.17	1.94
Δp_{S}	0.75	0.52	0.46	0.91
S_{D}	0.13	0.02	0.13	0.01
S_{N}	0.27	0.06	0.30	0.06
S_{S}	0.60	0.07	0.57	0.07
Δc_{L}	7.45	12.42	-8.80	25.64
Δc_{DNSL}	3.78	2.93	-1.31	5.74
Δp_{L}	5.83	12.07	-8.48	25.41
S_{L}	0.31	0.08	0.29	0.10
Δq_{L}	4.12	2.09	2.19	2.68

Table 5
Estimates of Relative Risk Aversion

This table presents the estimation of the coefficient of relative risk aversion (RRA) across different CRRA-based models with parameter β set to 0.95. The estimations in Panel A are based on the market factor and the estimations in Panel are based on the 25 portfolios of stocks double sorted by size and B/M. *TC-CAPM* denotes the consumption-based CAPM estimated with our proposed series of *true* measure of aggregate consumption growth, which includes both marketable and latent consumption. *C-CAPM* denotes the consumption-based CAPM estimated with consumption growth from the NIPA tables. *G-CAPM* denotes the consumption-based CAPM estimated with the proxy for aggregate consumption growth from Savov (2011) based on municipal solid waste (i.e., *garbage*). All test portfolios data are from Kenneth French's website, except for those in the estimation of the G-CAPM, in which portfolio data used was provided by Alexi Savov. The implied net risk-free rate, r^f, and the Root Mean Square Error (*RMSE*) are expressed in percentages. The sample is at the annual frequency and covers the period from 1959 to 2018 in models 1–4 and from 1960 to 2006 in model 5.

	Model				
	TC-CAPM		C-CAPM		G-CAPM
	Δc_{DNSL}	Δc_{NSL}	Δc_{DNS}	Δc_{NS}	Δc_{G}
	1	2	3	4	5
Panel A: Market Factor					
RRA (γ)	8.21	8.20	21.86	31.64	17.32
	(5.72)	(5.70)	(16.17)	(23.79)	(8.68)
Implied r^f	20.00	20.20	58.14	97.98	17.19
Panel B: 25 Size-B/M Portfolios					
RRA (γ)	6.51	6.49	19.11	29.95	22.31
	(5.26)	(5.26)	(14.87)	(22.17)	(9.41)
Implied r^f	18.48	18.69	51.48	92.21	13.65
RMSE	2.66	2.70	2.61	2.73	3.85