The delegated Lucas tree[*]

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Abstract

We analyze the effects of the observed increased share of delegated capital for trading strategies and equilibrium prices.

We introduce delegation into a standard Lucas exchange economy, where in equilibrium some investors trade on their own account, but others decide to delegate trading to professional fund managers. Flow-performance incentive functions describe how much capital clients provide to funds at each date as a function of past performance. Convex flow-performance relations imply that the average fund outperforms the market in recessions and underperforms in expansions. When the share of capital that is delegated is low, all funds follow the same strategy. However, when the equilibrium share of delegated capital is high funds with identical incentives employ heterogeneous trading strategies. A group of managers borrow to take on a levered position on the stock. Thus, fund returns are dispersed in the cross-section and the outstanding amounts of borrowing and lending increase. The relation between the share of delegated capital and the Sharpe ratio typically follows an inverse U-shape pattern.

1 Introduction

Over the last 30 years there has been a gradual but profound change in the way money is invested in financial markets. While almost 50% of US equities were held directly in 1980, by 2007 this proportion decreased to around 20% (see French (2008)). What are the equilibrium implications of this shift? In particular, how does the increased presence of delegation affect trading strategies, and prices?[1]

To analyze the link between the incentives of financial institutions and asset prices, we introduce financial intermediaries into a Lucas exchange economy. Rather than study an optimal contracting problem, we rely on empirical regularities in flows and assume a convex relation between flows and performance relative to the market, as documented for example in Chevalier and Ellison (1997) Then we study how the risky and risk free asset are traded both by fund managers and by traders holding the assets directly (direct traders).

Based on the Jensen and Meckling (1976) risk-shifting argument that convex incentives induce gambling, naive intuition would suggest that in our model managers should leverage up, taking on more exposure to market risk than direct traders and, consequently, the presence of fund managers should lower the Sharpe ratio. Interestingly, this is not what we find. In equilibrium, the average manager has smaller exposure to market risk than direct traders. When the equilibrium share of delegated capital is small all managers follow the same strategy. However, when the equilibrium share is high a group of managers emerges that levers up, taking more exposure to the market risk than unity, and trading against the rest of the managers who hold a positive share of their capital in bonds. Thus, in equilibrium ex ante identical traders take positions against one another increasing open interest and leverage. Both the size of this latter group and the leverage of each member typically increases with the larger share of delegated capital. We connect this finding with the increased use of levered strategies and the large increase of the size of the repo market during the last decades before the financial crisis. Finally, the effect of delegation on the Sharpe ratio is non-monotonic. There is an inverse U-shape relation between the share of capital that is delegated and the Sharpe ratio of the market portfolio.

We study an exchange economy where the endowment process is represented by a Lucas tree paying a stochastic dividend each period. The dividend growth follows a two-state i.i.d. process with a larger chance for the high state. There are two financial assets: a stock which is a claim on the endowment process, and a riskless bond which is in zero net supply. The economy consists of two type of agents, both with log utility: investors and fund managers. Investors are the owners of the capital. Investors, arrive and die according to independent Poisson processes with constant intensity, while managers live forever. Newborn investors decide for life whether to be clients of managers or to trade directly in financial markets. Trading directly imposes on investors a utility

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[1]See, for example, the presidential address of Allen (2001) for an elaborate discussion on the importance of the role of financial intermediaries.

cost. This utility cost represents the cost of acquiring the knowledge to understand how capital markets work, as well as the utility cost imposed by making regular time consuming investment decisions. Investors can avoid this cost by becoming a client and delegating the determination of their portfolio to a fund. However, when they delegate they need to pay fund managers a fee each period that is determined by the fund. The fee is consumed by the fund manager.

Clients' allocate capital to funds to manage each period depending on funds' past relative performance, where the relation between last period's return compared to the market and new capital flow is described by each manager's *incentive function*. We interpret the incentive function as a short-cut for an unmodeled learning process by clients on managers' talent. Its empirical counterpart is the flow-performance relation. We are agnostic as to whether the learning process is rational or not.[2]

We approximate the convex relation between flows and excess returns by a function which is piece-wise linear in logs. The combination of log utility with incentive functions of this particular functional form is the key methodological contribution that allows us to derive analytical formulas for the trading pattern and asset prices under various incentive functions. This combination results in a locally concave, but globally non-concave portfolio problem for managers. The first property keeps the framework tractable, while the second property ensures that we do not lose the general insight connected to convex incentives.

We present a stationary equilibrium where the equilibrium share of delegated capital is constant. In this equilibrium, most objects are given by simple, closed form expressions. As the main focus of this paper is the effect of the increasing share of delegated capital for equilibrium strategies and prices, we construct a range of economies as follows. We fix all other parameters and vary only the cost of direct trading, in a way that the equilibrium share of delegated capital varies along the full range of $(0, 1)$. Then we compare strategies and prices across these economies.

We show that the combination of convex incentives with negatively skewed market returns lead the average fund to choose a smaller than one market-beta: implying that consistent with evidence in Moskowitz (2000), Kosowski (2006), Lynch and Wachter (2007), Kacperczyk, Van Nieuweburgh and Veldkamp (2010), and Glode (2010) the average fund overperforms the market in recessions and underperforms in expansions. Consider the case where financial markets are populated only by direct traders, and the first fund manager enters. She can decide whether to take a sufficiently contrarian position to overperform and get high capital flows in the recession, or a sufficiently levered position to get high capital flows in the expansion. Negative skewness implies a higher probability of an expansion than a recession. Consequently, the relative overperformance implied by her optimal contrarian position must be larger than the one implied by her optimal levered position. Because convex flows reward large overperformance disproportionately, she picks the

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[2]For example Berk and Green (2004) provides micro-foundation for convex flow-performance relationship in a setting with incomplete information about fund managers' talents.

contrarian position.

While the average fund always overperforms in recessions, the cross sectional distribution of fund returns depends on the equilibrium share of delegation in the economy. At low levels of delegation, all funds choose the same portfolio. However, as the share of delegation increases, there is a threshold above which fund managers follow heterogeneous strategies, even though funds are identical ex ante and all have the same incentive function. In particular, above the threshold as the share of delegation increases a group of decreasing size still follows a "contrarian strategy" of smaller than one market beta, while a group with increasing size follows a leveraged strategy by borrowing up and investing more than 100% of their assets under management in the stock. This is a consequence of the interaction of the shape of the flow-performance relationship and the larger share of total delegation. The idea is that when the market is dominated by fund managers, if each manager followed the same strategy, they could not beat the market in any states of the world. Thus, they could not profit from the convexity of the flow-performance relationship. Instead in equilibrium, the group of managers who leverage up beat the market and receive large capital inflow in the high state, while the other group beats the market and obtain large capital flows in the low state. Thus, there are gains from trade. The size of these two groups are determined in equilibrium so that prices make each manager indifferent between the two strategies.

Accounting for the fact that over the last three decades the share of delegation has increased considerably (Allen (2001) and French (2008)), this result is consistent with observations on the increased use of leveraged strategies across financial intermediaries in the last two decades before the 2007/2008 financial crisis.[3] Relatedly, a central contribution of the paper is to link increases in delegated portfolio management to increased amounts of borrowing and lending in equilibrium. As to lever up, managers have to borrow from the rest of the agents, the equilibrium is consistent with the observed large increase in the size of the repo market in the last decades before the 2008 financial crisis (Gorton-Metrick (2010)). Consistent with evidence in Kacperczyk, Van Nieuweburgh and Veldkamp (2010) on the return dispersion of mutual funds, we also show that the implied cross-sectional dispersion in returns among managers is typically larger in recessions than in booms.

We show that typically the Sharpe ratio follows an inverted U-pattern as the share of delegation increases. The idea is that when the share of delegation is low and all fund managers hold a contrarian portfolio, the Sharpe-ratio has to increase with the share of delegation to induce fund managers to hold more stocks. Otherwise, the market could not clear. However, when the share of delegation is sufficiently high, there is a new margin of adjustment: the increasing size of the group who follows the levered strategy. Thus, markets clear with a smaller Sharpe-ratio.

Using parameters implied by the data, we calculate a simple numerical example to investigate the magnitude of these effects. Because of the structure of our model, we can directly compare

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[3]See Adrian and Shin (2008) on the leverage in investment banks and Lo and Patel (2007) on the increased role of leveraged mutual funds and leveraged ETFs.

our results to the ones implied by the standard Lucas economy. We find that even small convexity leads to large effect on managers' strategies. Relatedly, the increasing share of delegated capital radically increases the lending and borrowing activity. Furthermore, under reasonable parameter values for the incentive function, delegation has the potential to significantly increase the Sharpe ratio relative to the case without delegation: in the example the Sharpe ratio is up to 2.5 times higher than in the Lucas economy.

To our knowledge, our paper is the first to study the effect of the interaction between the increasing share of delegated capital and nonconcave incentives on fund managers strategies and implied asset prices. We are also the first to show that although this interaction is consistent with a smaller-than-1 beta portfolio for the average manager, it also leads to levered portfolios for a small group of increasing size. Still, our paper is related to at least three main branches of the literature. First, it is related to papers that study the effects of delegated portfolio management on asset prices (e.g. Shleifer and Vishny (1997), Vayanos (2003), Dasgupta and Prat(2006)(2008), Vayanos and Woolley(2008), Malliaris and Yan (2010), Guerrieri and Kondor(2011), Cuoco and Kaniel (2011), Basak and Pavlova (2011)). Both the framework and the focus of all these papers differ significantly from ours. Among many others, studied questions in this literature include the effect of delegation on limited arbitrage, on trading volume, on price discovery, on procyclicality in premiums and on momentum. The closest to our exercise is He and Krishnamurthy (2008) who also studies the effect of delegation in a standard Lucas economy. However, in He and Krishnamurthy (2008) managers are not directly motivated by flows because they do not receive fees based on their capital under management. Its main focus is on the amplification of bad shocks through the incentive constraint of managers.

Second, starting with the seminal paper of Jensen and Meckling (1976), there is a large literature on the effect on nonconcave objectives on fund managers strategies either by taking incentives as given (e.g. Dow and Gorton (1997), Basak, Pavlova and Shapiro (2007), Basak and Makarov (2010), Carpenter (2000), Cuoco and Kaniel (2011), Ross (2004)) or by deriving them endogenously (Biais and Casamatta (2000), Cadenillas et al. (2007), Diamond (2001), Hellwig (2009), Ou-Yang (2003) and Palomino and Prat (2003), Makarov and Plantin (2010)). The starting point that nonconcave incentives induce gambling is the connection between our paper and this literature. While, the first group of papers focuses on optimal portfolios for given prices,[4] the second group focuses on optimal contracts to avoid risk-shifting. In contrast, we focus on the interaction of prices and portfolios under fixed contracts.

Third, our framework is also related to the literature on consumption based asset pricing with heterogeneous risk aversion (e.g. Dumas (1989), Wang (1996), Chan and Kogan (2002), Bhamra and Uppal (2007), Longstaff and Wang (2008)). Unlike in our work, in these papers identical agents follow identical strategies, less risk-averse agents always borrow from more risk averse agents which,

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[4]Cuoco and Kaniel (2011) endogenize prices as well.

typically, decreases the price of risk. This is true even when utility depends on consumption relative to others such as in Chan and Kogan (2002). The main reason for the different results is that this literature does not allow for convexities in incentives.

The structure of the paper is as follows. In the next section we present the general model. We discuss the general set up, our equilibrium concept and the main properties of the equilibrium. In Section 3, we present and discuss the derived implications. In Section 4, we present a simple calibrated example. Finally, we conclude.

2 The general model

In this section, we introduce professional fund managers into a standard Lucas exchange economy. Our main focus is effect of the increasing share of delegated asset management on the equilibrium strategies and asset prices. In what follows, we introduce our framework, define our equilibrium concept and present sufficient conditions for the existence of such an equilibrium and its basic properties.

2.1 The Economy

We consider a discrete-time, infinite-horizon exchange economy with complete financial markets and a single perishable consumption good. There is only one source of uncertainty and participants trade in financial securities to share risk.

The aggregate endowment process is described by the binomial tree

$$\delta_{t+1} = y_{t+1}\delta_t$$

where the growth process y_t has two i.i.d. states: $s_t = H, L$. The dividend growth is either high y_H or low y_L, with $y_H > y_L$. The probability of the high and the low states are $p > \frac{1}{2}$ and $1 - p$ respectively.[5] Investment opportunities are represented by a one period riskless bond and a risky stock. The riskless bond is in zero net supply. The stock is a claim to the dividend stream δ_t and is in unit supply. The price of the stock and the interest rate on the bond are q_t and $r_{f,t}$ respectively. The return on the stock is denoted by

$$R_{t+1} \equiv \frac{q_{t+1} + \delta_{t+1}}{q_t},$$

and return on a portfolio with portfolio weights of α in the stock and $1 - \alpha$ in the risk free bond is denoted by

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[5]We focus on $p > \frac{1}{2}$ because the consumption growth process is negatively skewed empirically; for example, Backus, Chernov, and Martin (2011) find that consumption growth skewness is -0.34 (-0.87) for 1986-2009 (1989-2009).

$$\rho_{t+1}(\alpha) \equiv \alpha(R_{t+1} - r_{f,t}) + r_{f,t}. \tag{1}$$

The economy is populated by investors and fund managers[6]. Investors own the stock, but, initially, only fund managers know how to trade assets.The mass of each group is normalized to one, all agents derive utility from inter-temporal consumption, and have *log* utility. At the beginning of each period $1 - \lambda$ fraction of investors die and the same fraction is born. We assume that the aggregate capital of those who died is inherited by newborn investors in equal shares. Each living investor in any given period belongs to one of three groups: *Newborn investors* (I), *direct traders* (D) and *clients* (C). Newborn investors can choose whether to trade directly, or delegate their trading decisions to fund managers (M). This decision is made once at birth and is irreversible. Trading directly imposes a one time utility cost, f, on investors but gives them the free choice over their consumption and portfolio decisions in every subsequent period. We think of f as the cost of acquiring the knowledge to understand how capital markets work. If they choose to trade directly, they belong to group of direct traders in all subsequent periods. If they choose to delegate, they will be assigned to a particular manager $(m \in M)$ randomly and for life. In this case they belong to the group of clients in all subsequent periods. A client doesn't suffer the utility cost she would bear if she traded directly, but gives up the flexibility to determine her consumption and stock to bond mix. As we will explain, her consumption-investment choice depends on the past performance of managers and is determined by an exogenously specified flow-performance relationship, while her portfolio is chosen by her fund manager for a fee.[7] Note that although there are four groups of agents in this economy: newly born investors, clients, direct traders and fund managers, financial assets are traded only by two of these groups: fund managers and direct traders. Figure 1 depicts the economy structure.

In what follows, we first describe the problem of each of the four groups in detail then present our specification for the flow-performance relationship.

We conjecture and later verify that we have to keep track of only two state variables to fully describe the aggregate state of the economy in period t. The first is the dividend shock realized at the end of the last period, $s_t = H, L$, while the second is the share of aggregate investment of

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[6]Conceptually, we think of fund managers as a group representing all type of institutional traders who actively participate in the equity market. That is, actively managed mutual funds, hedge funds, proprietary trading desks of investment banks, pension funds, etc. Still, when we confront our findings to empirical work, we often have to rely on observations about mutual funds only as the majority of empirical results are on this segment of the sector. Presumably, this is so because of data availability.

[7]It is apparent that in our model investors not "paying" the utility cost, f, delegate their trading decision by assumption: both trading and producing fruit from the tree requires a degree of sophistication that is obtained by bearing the utility costs f. This precludes clients from holding the tree passively, since their lack of sophistication implies that if they hold it passively it will not generate any fruit. A similar assumption is made in He and Krishnamurthy (2008).
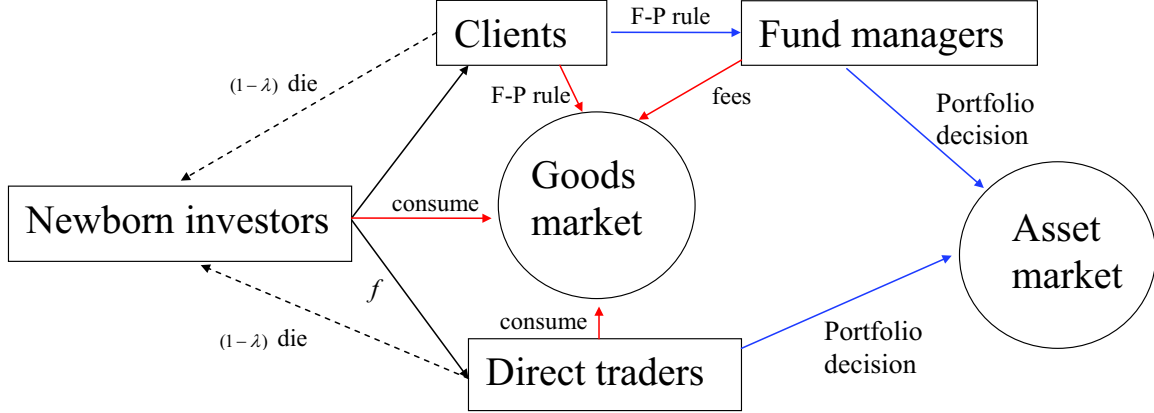
Figure 1: The figure exhibits the economy structure.

managers compared to total investment at the beginning of last period

$$\Omega_{t-1} \equiv \frac{\int_{m \in M} \left[w_{t-1}(m) - c_{t-1}(m) \right] dm}{\int_{i \in D} \left[w_{t-1}(i) - c_{t-1}(i) \right] di + \int_{m \in M} \left[w_{t-1}(m) - c_{t-1}(m) \right] dm}$$

where $c_{t-1}(m)$, $w_{t-1}(m)$ are the consumption and assets under management of a particular manager $m \in M$, and $c_{t-1}(i)$, $w_{t-1}(i)$ are the consumption and wealth level of a particular direct trader investor $i \in D$. With slight abuse of notation when we refer to a general direct trader or a general manager, we write w_{t-1}^D instead of $w_{t-1}(i)$, $i \in D$ and w_{t-1}^M instead of $w_{t-1}(m)$, $m \in M$. We follow the same convention for all variables. We refer to Ω_{t-1} as the share of delegated capital.

Fund Managers. In period, t, each manager with assets under management w_t^M chooses the fraction ψ_t^M she will receive as a fee. We assume the manager must consume her fee $\psi_t^M w_t^M$.[8] She then invests the remaining $(1 - \psi_t^M) w_t^M$ in a portfolio with α_t^M share in the stock and $\left(1 - \alpha_t^M\right)$ share in the bond. Her value function is given by

$$V^M \left(w_t^M, s_t, \Omega_{t-1} \right) = \max_{\psi_t^M, \alpha_t^M} \ln \psi_t^M w_t^M + \beta E \left(V^M \left(w_{t+1}^M, s_{t+1}, \Omega_t \right) \right) \tag{2}$$

$$s.t. \ w_{t+1}^M = \Gamma_t g \left(v_{t+1}^M \right) w_{t+1,-}^M \tag{3}$$

$$w_{t+1,-}^M \equiv \rho_{t+1} \left(\alpha_t^M \right) \left(1 - \psi_t^M \right) w_t^M \tag{4}$$

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[8]The assumption that managers cannot invest their fees is a major simplification allowing us to not keep track of fund managers private wealth. Note also that on one hand, we are allowing ψ_t to be conditional on any variable in the managers' information set in t. That is, we do not constrain our attention to proportional fees ex ante. On the other hand, our assumptions imply that fees are proportional in equilibrium, managers effectively maximize capital under management and fees do not play any role in the portfolio decision.

Note that assets under management at beginning of a period, w_{t+1}^M, are proportional to assets under management at the end of the previous period , $w_{t+1,-}^M$. This proportion depends on three quantities. First, the share of wealth each existing client delegates to the manager which depends on the past realized performance of this manager and given by $g(v_{t+1}^M)$ where

$$v_{t+1}^M \equiv \frac{\rho_{t+1}\left(\alpha_t^M\right)}{R_{t+1}} \tag{5}$$

is a fund's return relative to the market portfolio.[9] We specify the shape of this function below. Second, the total wealth of a manager's existing clients. Third, the total wealth of the fraction of newborn investors who decide to be clients and who are assigned to this particular manager. The second and third elements are combined into Γ_t, a state dependent scaling factor that is endogenously determined in equilibrium and which the manager takes as given. For simplicity, we refer to this variable as the size of the client-base. It impacts all funds similarly, and depends positively on the overall capital of clients in that state.

If more then one portfolio α_t^M solves (2)-(4), we will allow managers to mix between these portfolios. This will be useful, as sometimes the equilibrium portfolio profile requires a subset of managers to follow a different strategy than other managers, and we implement this by allowing mixed strategies.[10]

Clients. The utility going forward of an investor that decided to be a client, was matched with a particular manager, and has time t wealth of w_t is

$$V^C\left(w_t^C, v_t^M, s_t, \Omega_{t-1}\right) = \ln w_t^C\left(1 - g\left(v_t^M\right)\right) + \beta^I EV^C\left(w_{t+1}^C, v_{t+1}^M, s_{t+1}, \Omega_t\right) \tag{6}$$

$$s.t. \ w_{t+1}^C \equiv \rho_{t+1}\left(\alpha_t^M\right)\left(1 - \psi_t^M\right) g\left(v_t^M\right) w_t^C$$

where $\beta^I \equiv \lambda\beta$ is the effective discount factor of investors, and α_t^M and v_t^M are chosen portfolio and the relative return of the assigned manager in period t. Note that if the manager follows a mixed strategy than both α_{t+1}^M and v_{t+1}^M are random variables from the client's point of view. Instead of deriving the incentive function, $g(\cdot)$ from first principles, we take it exogenously in the spirit of Shleifer and Vishny (1997). Below, we motivate the form of this function by empirical observations. We think of this function as a reduced form description how a client matched to the manager decides how much she"trusts" the manager's abilities to outperform the market in the next period based on past performance.

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[9]In a previous version, we consider the possibility to allow the incentive function $g(\cdot)$ to depend non linearly on the fees ψ_t^M charged by the fund, but this change has very little effects on the result. Thus, we omit this treatment here.

[10]In Appendix B (available online) we argue that while we allow for mixed fund strategies in the equilibria presented, similar properties to the ones presented in the paper arise if we were to restrict to pure strategies. However, these equilibria are considerably less tractable.

Direct traders. Direct traders solve a standard asset allocation problem. Denoting by ψ_t^D the optimal fraction of time t wealth w_t a direct investor consumes, we have

$$V^D\left(w_t^D, s_t, \Omega_{t-1}\right) = \max_{\psi_t^D, \alpha_t^D} \ln \psi_t^D w_t^D + \beta^I E V^D\left(w_{t+1}, s_{t+1}, \Omega_t\right) \qquad (7)$$

$$s.t. \ \ w_{t+1}^D \equiv \rho_{t+1}\left(\alpha_t^D\right)\left(1 - \psi_t^D\right) w_t^D$$

Newborn investors. The expected lifetime utility of a newborn investor entering in period t with wealth w_t is given by

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\begin{aligned}
V^I\left(w_t^I, s_t, \Omega_{t-1}\right) \ &= \ \max_{\chi \in \{0,1\}, \psi_t^I, \alpha_t^I} \ln w_t^I \psi_t^I \\
&\quad + \chi \beta^I E V^C\left(w_{t+1}^C, s_{t+1}, \Omega_t\right) + (1-\chi)\beta^I\left(E V^D\left(w_{t+1}^D, s_{t+1}, \Omega_t\right) - f\right).
\end{aligned}
$$

$$
\begin{aligned}
s.t. \ \ w_{t+1}^C \ &\equiv \ \rho_{t+1}\left(\alpha_t^M\right)\left(1 - \psi_t^M\right)\left(1 - \psi_t^I\right) w_t^I \\
w_{t+1}^D \ &\equiv \ \rho_{t+1}\left(\alpha_t^I\right)\left(1 - \psi_t^I\right) w_t^I
\end{aligned}
$$

where χ is her decision whether to be a client or a direct trader, ψ_t^I is her consumption share, α_t^I is her first portfolio decision given that she chooses to be a direct trader.

Relative Performance Incentive Functions. Our key assumption is to model clients' share of delegated capital by a reduced form incentive function. The empirical counterpart of the incentive function is the flow-performance relationship. The incentive function $g\left(\cdot\right)$ describes how existing clients respond to the performance of a given manager. We assume it belongs to the following piece-wise constant elasticity class:[11]

$$g\left(v\right) \equiv \begin{cases} Z_B \, v^{n_B - 1} & if \ \ v < \kappa \\ Z_A \, v^{n_A - 1} & if \ \ v \geq \kappa \end{cases}. \qquad (8)$$

The function is parameterized by the kink $\kappa \geq 1$, the scalers $Z_A, Z_B > 0$ and the elasticity parameters, $n_A \geq n_B > 1$. The subscripts refer to the cases when the relative return is above (A) the kink, so managers are compensated at the higher-elasticity segment of the incentive function, and when the relative return is below (B) the kink, so managers are compensated at the low-elasticity segment of the incentive function. We assume that the g is continuous by imposing the restriction

$$Z_A = Z_B \kappa^{n_B - n_A}.$$

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[11] Allowing the incentive function to be a combination of more than two segments does not pose any conceptual difficulty for our method. However, as it does not add to the economic intuition either, we omit this treatment. Also, in equilibrium it must be that $g() < 1$, which we verify as part of the equilibrium existence proof.

For a more intuitive form, using (4) and (8) we have

$$\ln \frac{w_{t+1}^M}{w_{t+1,-}^M} = \ln \frac{w_{t+1}^M}{\rho_{t+1}\left(\alpha_t^M\right)\left(1-\psi_t^M\right)w_t^M} = \tag{9}$$

$$= \ln \Gamma_t Z_B + 1_{v_t \geq \kappa} \ln \kappa^{n_B - n_A} + \left[(n_B - 1)1_{v_t < \kappa} + (n_A - 1)1_{v_t \geq \kappa}\right]\left(\ln \rho_{t+1}\left(\alpha_t^M\right) - \ln R_{t+1}\right).$$

By choosing the appropriate parameters, this specification is a piecewise linear approximation of any convex relationship between log of capital flows and log of excess returns of funds. This is consistent with the well documented empirical convex flow-performance relation for a wide range of financial intermediaries.[12] We chose this particular approximation, because it both keeps our model analytically tractable and consistent with empirical specifications.[13]

2.2 The equilibrium

In this part, we show that under weak parameter restrictions, we can always find a competitive equilibrium where the share of delegated capital is constant over time, $\Omega_t = \Omega^*$. More formally, we are looking for a stationary competitive equilibrium defined as below.

Definition 1 *An Ω^* **equilibrium** is a price process q_t for the stock and $r_{f,t}$ for the bond, a relative investment by fund managers compared to all investment Ω^*, consumption and strategy profiles for newborn investors, direct investors, and managers such that*

1. *given the equilibrium prices*

   - *the initial consumption choice of newborn investors ψ_t^I and the decision on whether to become a direct trader or a client are optimal for each newborn investor,*
   - *fee choice ψ_t^M and trading strategies are optimal for each manager,*
   - *consumption choices ψ_t^D and trading strategies α_t^D are optimal for direct traders,*

2. *prices q_t, and $r_{f,t}$ clear both good and asset markets,*

3. *the relative investment by fund managers compared to all investment is constant overtime at the level $\Omega_t = \Omega^*$.*

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[12]There is a large empirical literature exploring the relationship between past performance and future fund flows. With the notable exception of Grossman, Ingersoll and Ross (2002), most papers find a positive relationship for varies type of financial intermediaries. Also, Chevalier and Ellison (1997), Sirri and Tufano (1998), and Chen et al. (2003) find that the relationship is convex for mutual funds, while Agarwal, Daniel and Naik (2003) finds similar convexity for hedge funds. Kaplan and Schoar (2004) finds a positive but concave relationship for private equity partnerships.

Anecdotal evidence suggests that the capital at the disposal of top traders at investment banks and hedge funds also increases significantly as response to their stellar performance (e.g. WSJ 09/06/06 A1 on Brian Hunter of Amaranth, and WSJ 02/06/09 A1 on Boaz Weinstein of Deutsche Bank). This should lead to similar incentives to our specification.

[13]In Section 4, we estimate the parameters of 9 on a sample of mutual fund flows and returns.

As the main focus of this paper is the interaction between the increasing share of delegation and the effect of a convex flow-performance relation to equilibrium strategies and prices, we construct a range of economies as follows. We fix all other parameters and change only f, the cost of trading directly, in a way that the implied equilibrium implies a different share of delegation, Ω^*, for each economy. Then we compare strategies and prices across these economies.[14]

Before highlighting the details, we discuss the methodology of equilibrium construction. The key is how to deal with the convex flow-performance relation. Convexity in incentives imply that our problem is globally non-concave, so that local conditions for the equilibrium will not be sufficient. However, the interaction of log utility and a piecewise constant-elasticity incentive function imply that the problem of the manager is locally concave almost everywhere in the portfolio choice α, even though it is globally non-concave. The dashed line on Figure 2 demonstrates this by depicting the expected utility of a manager for various αs in a particular case when all other traders hold the market. It is apparent that the curve can be divided to three segments in a way that the
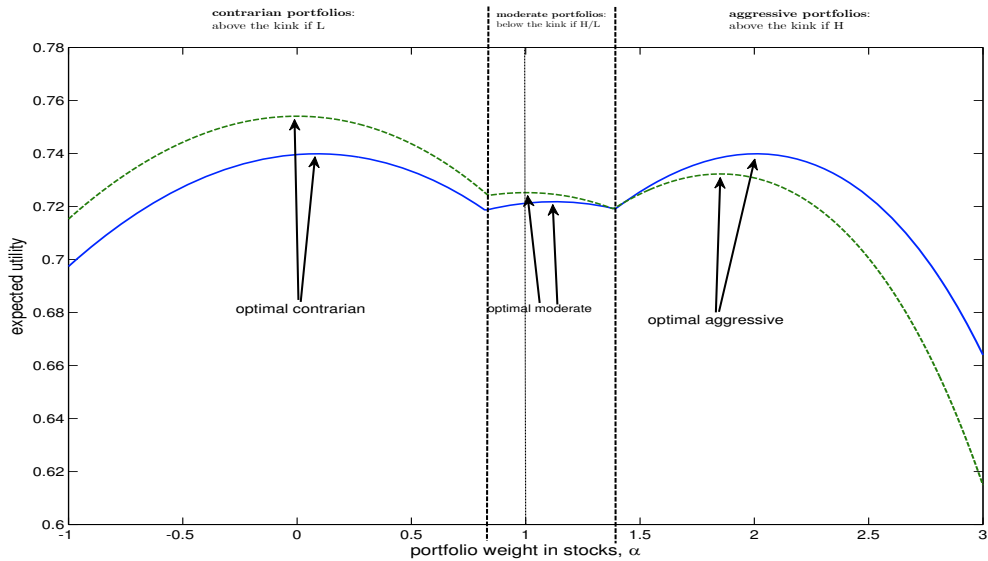

Figure 2: The graph plots the expected utility of a representative manager as function of her portfolio choice, α, for two different set of prices. The dashed line corresponds to the case when the invested capital share of managers, Ω^*, is zero. In this case all other traders hold the market. The solid line corresponds to the case when $\Omega^*=1$. The parameters are set to $\lambda = 0.5, \beta = 0.95, p = 0.7, y_H = 1.2, y_L = 0.8, Z_B = 0.3, n_A = 3,$ and $n_B = 2$.

curve is concave within each of these segments. Portfolios in a given segment differ from portfolios in other segments in which dividend state, if at all, the manager receives the extra capital flows

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[14]Formally, Ω^* is an equilibrium variable depending on f. Thus, we should define a function which gives an f for every Ω^*. Then, to analyze the effect of increasing Ω^*, we should change f along the values of this function. Instead, to keep things simple, we analyze "comparative statics" with respect to Ω^* allowing f to adjust in the background accordingly.

implied by the high elasticity segment of her incentive function. In particular, *Contrarian portfolios* have smaller than unity exposure to market risk; overperforming the market in the low state. This overperformance in the low state is sufficiently high to generate the extra capital flows implied by the high elasticity segment of the incentive function. *Moderate portfolios* are close to the market portfolio, they generate moderate over- or under-performance, and thus do not generate extra capital flows in any state. *Aggressive portfolios* have larger than unity exposure to market risk; overperforming the market in the high state. This overperformance is sufficiently high to generate the extra capital flows in the high state. Because of local-concavity, within each of these segments there is a single optimal portfolio. Consequently, for a given set of prices managers effectively compare three possible strategies: the locally optimal contrarian, moderate and aggressive portfolios. The relative ranking of these three choices depend on equilibrium prices.

Note that on the solid line of the figure the manager is indifferent between the optimal contrarian and optimal aggressive strategies. As will become apparent, this potential multiplicity is a key to getting heterogeneity of trading strategies in equilibrium. With globally concave objective functions there is always a unique optimum, and no heterogeneity arises.

Our treatment of convex incentive functions helps to reduce the construction of an Ω^* equilibrium to the following steps.

1. We fix a given Ω^* and conjecture an equilibrium profile of portfolios for managers and direct traders. In this profile each portfolio is one of the three types of locally optimal portfolios. We verify the conjecture by showing that the profile is indeed globally optimal under the set of relative prices consistent with this profile. Sometimes in equilibrium a group of managers have to hold a different portfolio than other managers. We implement such asymmetries by allowing managers to mix between portfolios. Importantly, the equilibrium strategy profile is independent from the utility cost f and the client base Γ_t.

2. By calculating the values of a client and a direct trader under the equilibrium strategies, we find the utility cost f of trading directly which implies that each newborn investor is indifferent whether to be a client or a direct trader, and verify that f is independent of the dividend state , $s_t = H, L$. Thus, any fraction of newborn investors choosing to be clients is consistent with the equilibrium strategies and prices for this f.

3. We pick the fractions of newborn investors choosing to be clients in a way that the implied total client base Γ_t gives exactly Ω^* as the share of delegated capital. This has to be true regardless of the dividend state, $s_t = H, L$. We show that this implies that the client-base $\Gamma_t = \Gamma_H, \Gamma_L$ depends only on the dividend state.

4. Finally, we calculate the equilibrium price of the assets implied by the consumption and portfolio decisions of each group of agents.

In the rest of this section, we characterize the Ω^* equilibrium by following the structure provided by the above steps. We show that our method gives simple analytical expressions for most equilibrium objects.

We proceed under the following two conjectures, which we validate at the end of Section 2.2.2.

Conjecture 1 *the value function of the manager has the form of*

$$V^M \left(w_t^M, s_t, \Omega_{t-1} \right) = \frac{1}{1-\beta} \ln w_t^M + \Lambda^M \left(s_t, \Omega_{t-1} \right). \tag{C1}$$

Conjecture 2 *under the manager's optimal trading strategy fund relative performance is never at the kink, i.e.*

$$\nu_{t+1}^M \neq \kappa \tag{C2}$$

Consequently, the locally optimal contrarian/aggressive/moderate portfolios are in the interior of the corresponding segments, as depicted in Figure 1.

2.2.1   Equilibrium portfolios

We start by finding the optimal consumption and portfolio decisions of direct traders and managers for fixed prices. The case of direct traders is standard. Given their log utility, the optimal consumption share is

$$\psi_t^D = \left(1 - \beta^I \right),$$

while the optimal share in stocks is given by the first order condition

$$p\frac{R_{t+1}(H) - r_{f,t}}{\alpha_t^D \left(R_{t+1}(H) - r_{f,t} \right) + r_{f,t}} = (1-p)\frac{r_{f,t} - R_{t+1}(L)}{\alpha_t^D \left(R_{t+1}(L) - r_{f,t} \right) + r_{f,t}}, \tag{10}$$

which implies a trading strategy of

$$\alpha_t^D = \frac{1-p}{1 - \frac{R_{t+1}(H)}{r_{f,t}}} + \frac{p}{1 - \frac{R_{t+1}(L)}{r_{f,t}}} \tag{11}$$

Now we consider the decision problem of a manager in period t.

To find the locally optimal portfolios we first introduce an *individual shape-adjusted probability,*

$$\xi_{lh} \equiv p\frac{n_h}{pn_h + (1-p)\, n_l}. \tag{12}$$

where the indices $l, h = A, B$ refer to whether for the given strategy the performance relative to the market has to be above (A) or below (B) the kink in the low state (l) and the high state (h), respectively.

ξ_{lh} is the probability of a high state adjusted to the relative elasticity of the incentive function in the two states. It is a change of measure that puts more weight on the states where the performance sensitivity is higher. For fixed parameters, ξ_{lh} depends only on whether the manager chooses a contrarian, moderate, or aggressive portfolio. For a contrarian portfolio $\xi_{lh} = \xi_{AB}$, as by definition it performs above the market in the low state and below the market in the high state. Similarly, for a moderate portfolio $\xi_{lh} = \xi_{BB} = p$, and for an aggressive portfolio $\xi_{lh} = \xi_{BA}$.

Subject to this change of measure the first order condition that identifies a manager's optimal share in stocks is similar to the one of a direct trader. To see this observe that a manager's optimization problem, given (2) and conjecture (1), is given by

$$\max_{\alpha_t^M, \psi_t^M} \ln \psi_t^M w_t^M + \tag{13}$$

$$+ \frac{\beta}{1-\beta} p \ln \Gamma_t Z_h \left(\frac{\rho_{t+1}\left(\alpha_t^M, H\right)}{R_{t+1}(H)} \right)^{n_h - 1} \rho_{t+1}\left(\alpha_t^M, H\right)(1 - \psi_t^M) w_t^M +$$

$$+ \frac{\beta}{1-\beta}(1-p) \ln \Gamma_t Z_l \left(\frac{\rho_{t+1}\left(\alpha_t^M, L\right)}{R_{t+1}(L)} \right)^{n_l - 1} \rho_{t+1}\left(\alpha_t^M, L\right)(1 - \psi_t^M) w_t^M$$

$$+ \beta \left(p\Lambda\left(H, \Omega_t\right) + (1-p)\Lambda\left(L, \Omega_t\right) \right),$$

It is easy to see the optimal fees are a constant proportion of capital under management,

$$\psi_t^M = (1 - \beta).$$

The first order condition with respect to the share in the stock α_t^M can be written as

$$\xi_{lh} \frac{R_{t+1}(H) - r_{f,t}}{\alpha_t^M \left(R_{t+1}(H) - r_{f,t} \right) + r_{f,t}} = (1 - \xi_{lh}) \frac{r_{f,t} - R_{t+1}(L)}{\alpha_t^M \left(R_{t+1}(L) - r_{f,t} \right) + r_{f,t}}. \tag{14}$$

Comparing this expression to (10), observe that the incentive function affects the problem only to the extent that it changes the weights of the marginal utilities in the two states. While the direct trader weights the marginal utility in the high state by p, its probability, the manager uses the individual shape-adjusted probability, ξ_{lh}.

We rewrite the first order condition, (14), as

$$\alpha_t^M = \alpha_{lh} = \frac{1 - \xi_{lh}}{1 - \frac{R_{t+1}(H)}{r_{f,t}}} + \frac{\xi_{lh}}{1 - \frac{R_{t+1}(L)}{r_{f,t}}} \tag{15}$$

and pick $lh = BA, BB, AB$ to get the locally optimal contrarian, moderate and aggressive portfolios, respectively.

With no kink in the incentive function, $n_A = n_B$, $\xi_{lh} = p$, (15) reduces to (11), implying

managers and direct traders follow the same strategy. Market clearing implies that this strategy must be that every one holds the market, and relative returns are always 1. This implies the following lemma.

Lemma 1 *When $n_A = n_B$, in equilibrium managers and direct traders all hold the market portfolio.*

$n_A > n_B$ implies that $\xi_{AB} < p < \xi_{BA}$. That is, a manager choosing the locally optimal contrarian (aggressive) portfolio acts as if she would distort downwards (upwards) the probability of the high state. When managers compare the three locally optimal portfolios they act as if deciding in which way to distort the probabilities.

The following proposition summarizes our findings.

Proposition 1 *For fund managers*

1. *the optimal consumption rule is given by*

$$\psi_t^M = (1 - \beta).\tag{16}$$

2. *for any given set of prices, the manager chooses among the three locally optimal portfolios:*

- *Contrarian:*

$$\alpha_{AB} = \frac{1 - \xi_{AB}}{1 - \frac{R_{t+1}(H)}{r_{f,t}}} + \frac{\xi_{AB}}{1 - \frac{R_{t+1}(L)}{r_{f,t}}}\tag{17}$$

- *Aggresive:*

$$\alpha_{BA} = \frac{1 - \xi_{BA}}{1 - \frac{R_{t+1}(H)}{r_{f,t}}} + \frac{\xi_{BA}}{1 - \frac{R_{t+1}(L)}{r_{f,t}}}\tag{18}$$

- *Moderate :*

$$\alpha_{BB} = \frac{1 - p}{1 - \frac{R_{t+1}(H)}{r_{f,t}}} + \frac{p}{1 - \frac{R_{t+1}(L)}{r_{f,t}}}\tag{19}$$

Which locally optimal portfolio is the globally optimal one? A convenient property of our structure is that to answer this question, we do not have to know the level of equilibrium prices. To see why, first observe that any set of prices clearing the asset market imply that relative returns take a simple form. To be more specific, let $\mu_{lh} = \mu_{AB}, \mu_{BB}, \mu_{BA}$ be the equilibrium fraction of managers whose realized portfolio is the locally optimal contrarian, moderate and aggressive portfolios, where, just as above, the index pair lh refers to whether the performance of the manager is below (B) or above (A) the kink, κ, after a low (l) and high (h) shock. Then, the *aggregate shape-adjusted probability of a high state* is

$$\tilde{\xi}(\Omega^*) \equiv \Omega^* (\mu_{AB}\xi_{AB} + \mu_{BA}\xi_{BA} + \mu_{BB}p) + (1 - \Omega^*) p,\tag{20}$$

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which is the weighted average of the individual shape adjusted probabilities. The next Lemma shows that relative returns generated by locally optimal portfolios are given by the proportion of individual shape adjusted probabilities to their aggregate counterpart.

Lemma 2 *For any set of prices for which the stock market clears, that is,*

$$\Omega^* \left(\mu_{AB} \alpha_{AB} + \mu_{BA} \alpha_{BA} + \mu_{BB} \alpha_{BB} \right) + (1 - \Omega^*) \alpha_t^D = 1 \tag{21}$$

the relative return implied by a locally optimal portfolio is

$$\nu_{t+1}^M \left(\alpha_{lh}, H \right) = \frac{\xi_{lh}}{\tilde{\xi} \left(\Omega^* \right)}, \tag{22}$$

in the high state and

$$\nu_{t+1}^M \left(\alpha_{lh}, L \right) = \frac{1 - \xi_{lh}}{1 - \tilde{\xi} \left(\Omega^* \right)} \tag{23}$$

in the low state where $lh = AB, BB, BA$ *for the locally optimal contrarian, moderate and aggressive portfolios, respectively.*

From (13), the difference between the value of choosing the optimal contrarian and the optimal aggressive strategy, for any given prices, is

$$\frac{\beta}{1 - \beta} \left[p \ln Z_B \left(\frac{\xi_{AB}}{\tilde{\xi} \left(\Omega^* \right)} \right)^{n_B} + (1 - p) \ln Z_A \left(\frac{1 - \xi_{AB}}{1 - \tilde{\xi} \left(\Omega^* \right)} \right)^{n_A} \right. \tag{24}$$
$$\left. - \left(p \ln Z_A \left(\frac{\xi_{BA}}{\tilde{\xi} \left(\Omega^* \right)} \right)^{n_A} - (1 - p) \ln Z_B \left(\frac{1 - \xi_{BA}}{1 - \tilde{\xi} \left(\Omega^* \right)} \right)^{n_B} \right) \right],$$

which is proportional to the expected log difference between the assets under management generated by relative returns of the two portfolios. Comparing other pairs of locally optimal portfolios gives similar expressions.

Thus, to figure out the equilibrium strategy profile of managers, we just have to use (25)-(23) to find fractions $\mu_{AB}, \mu_{BB}, \mu_{BA}$ such that $\mu_{AB} + \mu_{BB} + \mu_{BA} = 1$ and any positive μ_{lh} corresponds to a globally optimal portfolio. We show in the Appendix, that there are four different equilibria types depending on equilibrium fund managers portfolios:

Cont-Agg: some managers hold the locally optimal contrarian portfolio and others hold the locally optimal aggressive portfolio,

Cont-Mod: some managers hold the locally optimal contrarian portfolio and others hold the locally optimal moderate portfolio,

Cont: all managers hold the locally optimal contrarian portfolio.

Mod: all managers hold the locally optimal moderate portfolio..

The following proposition matches four subsets of the relevant parameter space to the four possible types of equilibria.

Theorem 1 *There are critical values $\hat{\kappa}_{high}, \hat{\kappa}_{low}, \hat{p}, \bar{p} \in \left(\frac{1}{2}, 1\right)$ and $\hat{\Omega} \in (0,1)$ that*

1. *if $\kappa > \hat{\kappa}_{high}$, there is a unique interior equilibrium and it is a Moderate (Mod) equilibrium where each agent holds the market: $\alpha^D = \alpha^M = 1$,*

2. *if $\hat{\kappa}_{low} < \kappa < \hat{\kappa}_{high}$, there is a unique interior equilibrium and its type depends on p as follows:*

	$p \in \left(\frac{1}{2}, \hat{p}\right)$	$p \in (\hat{p}, 1)$
$\Omega^* \leq \hat{\Omega}$	*Mod*	*Cont*
$\Omega^* > \hat{\Omega}$	*Mod*	*Cont − Mod*

.

3. *if $\kappa < \hat{\kappa}_{low}$, there is a unique interior equilibrium and its type depends on p as follows:*

	$p \in \left(\frac{1}{2}, \bar{p}\right)$	$p \in (\bar{p}, 1)$
$\Omega^* \leq \hat{\Omega}$	*Cont*	*Cont*
$\Omega^* > \hat{\Omega}$	*Cont − Agg*	*Cont − Mod*

$\hat{\kappa}_{high}, \hat{\kappa}_{low}$ *are functions of n_A, n_B only, while $\hat{p}, \bar{p}$ are functions of n_A, n_B, κ, and $\hat{\Omega}$ is a function of n_A, n_B, κ, p. These functions are given in the Appendix.*

The aggregate shape-adjusted probability $\tilde{\xi}\left(\Omega^*\right)$ is decreasing in Ω^* for $\Omega^* < \hat{\Omega}$, and constant otherwise.

From the theorem it is apparent that when reaching the high elasticity segment of the incentive function would require sufficiently high relative performance (high κ) then the moderate strategy is the global optimum, since the portfolio distortions required to achieve relative returns above κ in one of the states are too large and are suboptimal. Observe that in this case direct traders and managers follow the same strategy, which also implies that they all hold the market. This is why we sometimes refer to a moderate equilibrium as the indexed equilibrium. Outside of this range of parameters, managers always choose a contrarian strategy as long as their capital share is small (i.e., Ω^* is small). That is, they lend to direct traders, have a smaller-than-1 exposure to the market risk, and overperform the market only in the low state. From the point that the capital share of delegated management reaches a given threshold ($\Omega^* \geq \hat{\Omega}$), managers are indifferent between the contrarian portfolio and either the moderate or the aggressive portfolio. An example of equilibrium

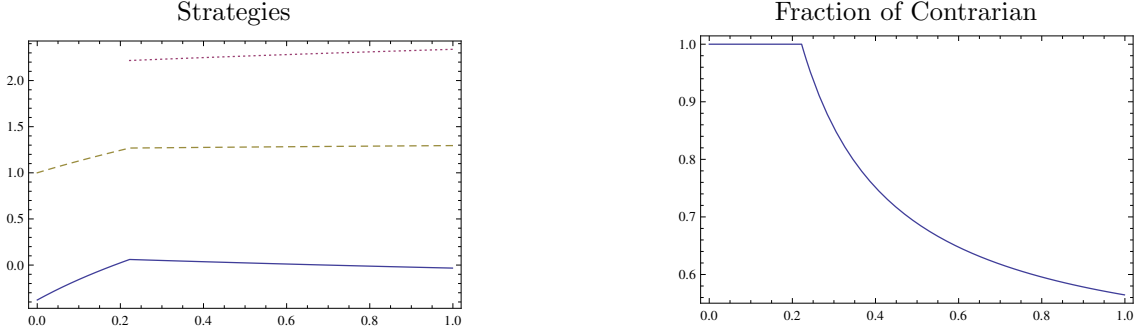| Strategies	Fraction of Contrarian

Figure 3: The graphs plot equilibrium strategies as a function of the share of delegation. The left panel contains portfolios of direct investors and funds: dashed line is of direct investors, and solid and dotted lines are of funds. The right panel shows the fraction of fund managers who are contrarian. Parameters are set to: $p = 0.8, y_H = 1.15, y_L = 0.85, n_A = 1.5, n_B = 1, \lambda = 0.5, Z_B = 0.1$.

strategies is shown in Figure 3. As shown in the left panel when the share of delegation is small managers follow a contrarian strategy. However, for larger shares of delegation some of managers follow an aggressive strategy, in this example. Given that managers are indifferent between the two strategies, they mix between the two globally optimal portfolios. By the law of large numbers, the mixing probabilities are given by μ_{lh} as they must be identical to the fraction of managers ending up with a given portfolio. As Ω^* increases, the mixing probabilities adjust in a way to keep managers indifferent between the two strategies, an shown on the right panel of the figure. The figure also shows that in the region $\Omega^* > \hat{\Omega}$ an increase in the share of delegation is associated with both the contrarian and aggressive strategies becoming more extreme (see left panel) and a decline in the fraction of managers picking the contrarian strategy (see right panel). We will show in Sections 3 and 4, where we further analyze the properties of the equilibrium strategies, that the later property always holds and the former is typical as well.

2.2.2 Newborn investors' decision and the client-base

Given their log utility, it is easy to see that the optimal consumption share of newborn investors is the same as that of the direct traders: $\psi_t^I = \psi_t^D = \left(1 - \beta^I\right)$

Relative returns in (22)-(23) directly imply the aggregate capital clients delegate to managers at the beginning of the period and the capital managers return to clients at the end of the period. For example, in the high state,

$$\Omega^* \mu_{AB}\left(\Omega^*\right) \frac{\xi_{AB}}{\tilde{\xi}\left(\Omega^*\right)},$$

$$\Upsilon_H = \Omega^* \left(\mu_{AB}\left(\Omega^*\right) \frac{\xi_{AB}}{\tilde{\xi}\left(\Omega^*\right)} + \left(1 - \mu_{AB}\left(\Omega^*\right)\right) \frac{\xi_2}{\tilde{\xi}\left(\Omega^*\right)}\right),$$

$$\Gamma_t \bar{g}_H = \Gamma_t \Omega^* \left(\mu_{AB}\left(\Omega^*\right) g\left(\frac{\xi_{AB}}{\tilde{\xi}\left(\Omega^*\right)}\right) \frac{\xi_{AB}}{\tilde{\xi}\left(\Omega^*\right)} + \left(1 - \mu_{AB}\left(\Omega^*\right)\right) g\left(\frac{\xi_2}{\tilde{\xi}\left(\Omega^*\right)}\right) \frac{\xi_2}{\tilde{\xi}\left(\Omega^*\right)}\right)$$

give the total return of managers following a contrarian strategy, the total share of capital returned to clients, and the total share of capital delegated to managers respectively, where $\xi_2 = \xi_{BA}, p, \xi_{AB}$ in a $Cont - Agg$, $Cont - Mod$, and $Cont$ equilibrium respectively. The total return of managers following a contrarian strategy is the product of the total share of invested capital by managers, the fraction holding the contrarian portfolio and the relative return corresponding to the contrarian strategy. The last two expressions follow the same logic. Note that from the total share $\Gamma_t \bar{g}_s$, $\lambda \bar{g}_s$ comes from those clients who survived from the previous period and the rest comes from newborn investors choosing to be clients. That is,

$$\Gamma_t \bar{g}_s = \lambda \bar{g}_s + (1 - \lambda) \beta^I \bar{\chi}_t \tag{25}$$

has to hold, where $\bar{\chi}_t$ is the aggregate share of newborn investors choosing to be clients. By allocating a given fraction of indifferent newborn investors to the group of clients, we can pick a Γ_t which keeps the share of delegated capital, Ω^* fixed. In particular,

$$\Omega^* = \frac{\beta \Gamma_t \bar{g}_s}{\beta \Gamma_t \bar{g}_s + \beta^I \lambda (1 - \Upsilon_s) + \beta^I (1 - \lambda)(1 - \bar{\chi}_t)} \tag{26}$$

has to hold in both states, $s = H, L$ where the numerator is the total invested capital share of managers while the denominator is the total invested capital share of all groups. In the denominator, the second term correspond to the invested share of aggregate capital of direct traders: $(1 - \Upsilon_s)$ is the wealth share of direct traders, of which a fraction λ survives and invests β^I share in the asset market. The third terms corresponds to the invested share of newborn investors deciding to be direct traders. It is easy to see that we can pick the client-base and the fraction of newborns deciding to be clients in a way that they both depend only on the dividend state; i..e., $\Gamma_t = \Gamma_H, \Gamma_L$ and $\bar{\chi}_t = \bar{\chi}_H, \bar{\chi}_L$.

The following Lemma summarizes the above.

Lemma 3 *Both the fraction of newborn investors choosing to delegate $\bar{\chi}_t$, and the client-base Γ_t depend only on the state and are*

$$\bar{\chi}_s = \bar{g}_s \frac{\Gamma_s - \lambda}{(1 - \lambda) \beta^I}$$

$$\Gamma_s = \Omega^* \frac{\beta^I (1 - \lambda \Upsilon_s) + \bar{g}_s \lambda}{\bar{g}_s (\beta (1 - \Omega^*) + \Omega^*)}$$

for $s = H, L$, where Υ_s is the total share of capital returned on aggregate to clients at the end of the previous period, and $\lambda \bar{g}_s$ is the share of capital delegated to managers coming from clients who survived from the previous period.

The left panel of Figure 4 shows that the fraction of newborn investors deciding to be clients

is larger in the high state than in the low state. The right panel shows the flows in the high and low state into contrarian and aggressive funds respectively. As expected, contrarian funds receive more flows in the low state versus the high state and aggressive funds receive more flows in the high state versus the low state. Furthermore, keeping in mind that in the probability of the high state is higher, on average aggressive funds receive more flows.
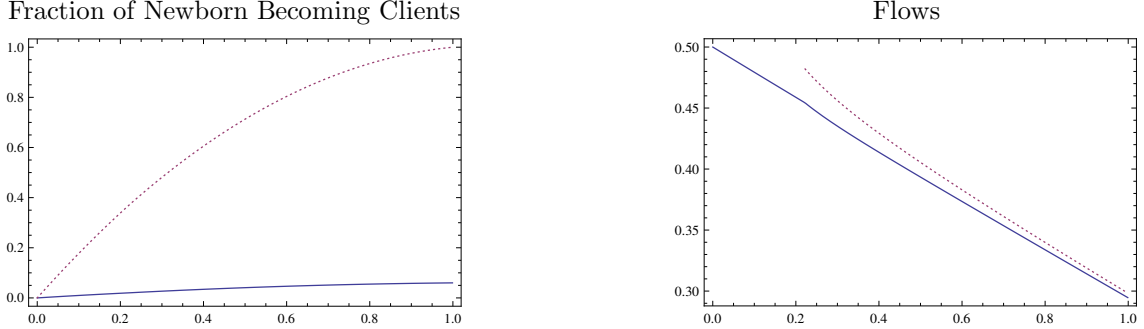

Figure 4: The graphs plot the fraction of new born investors deciding to be clients (left panel) and expected fund flows (right panel). In the left panel, the solid and dotted lines correspond to the low and high state respectively. In the right panel, the solid and dotted lines correspond to the expected flows to the contrarian and aggressive strategies respectively. Parameters are set to: $p = 0.8, y_H = 1.15, y_L = 0.85, n_A = 1.5, n_B = 1, \lambda = 0.5, Z_B = 0.1$.

The left panel of Figure 4 exhibits the fraction of newborn investors deciding to be clients in a Cont-Agg equilibrium. As shown in the figure, the fraction of newborn investors that decide to delegate is larger in the high state than in the low state. The left panel shows that expected flows to managers following the aggressive strategy is higher than flows to managers following the contrarian strategy. While the contrarian strategy has on average lower flows the flows are concentrated in the low state. In contrast, the aggressive strategy flows are concentrated in the high state and have a larger dispersion between the high and low state relative to the contrarian strategy(not shown).

Given the consumption and portfolio decision of direct traders, and the equilibrium strategies of managers, we can compare directly newborn traders value if they decide to be direct traders or clients. For a given Ω^*, we can find a cost of trading directly, f, that implies that newborn managers are indifferent which role to choose. Now we are ready to state the conditions under which a Ω^* equilibrium exists for any given $\Omega^* \in (0, 1)$.

Proposition 2 *For any set of other model parameters there is a $\hat{Z}$, $\hat{\lambda}$, and an interval $\left[\underline{f}, \bar{f}\right]$ such that if $Z_B < \hat{Z}, \lambda \leq \hat{\lambda}$ then*

1. *For any $f \in \left[\underline{f}, \bar{f}\right]$ there exists an Ω^* equilibrium for some $\Omega^* \in (0, 1)$,*

2. *for any $\Omega^* \in (0, 1)$ there is a corresponding $f \in \left[\underline{f}, \bar{f}\right]$ that with that choice there is an Ω^* equilibrium.*

We conclude by proving Conjectures1 and 2.

Lemma 4 *In equilibrium,*

1. *the value function of the manager has the form of (1),*

2. *under the manager's optimal trading strategy fund relative performance is never at the kink (i.e., $\nu_{t+1}^M \neq \kappa$).*

2.2.3 Equilibrium prices

Given all equilibrium actions, we can determine equilibrium prices by market clearing conditions. Instead of tracking the stock price q, and the stock price next period $q_{s'}$, it is more convenient to track the price-dividend ratio

$$\pi = \frac{q}{\delta}.$$

and the price dividend ratio next period

$$\pi_{s'} = \frac{q_{s'}}{\delta'}. \tag{27}$$

We start by briefly discussing the natural benchmark where the market is populated by direct traders only. For example, this is the case when the utility cost of direct trading is zero. It is simple to check that our model reduces to the standard Lucas economy where all traders hold the market and the price-dividend ratio and riskfree rate are constant:

$$\pi_H = \pi_L = \pi = \frac{\beta^I}{1 - \beta^I}, \tag{28}$$

$$r_f = \frac{1/\pi}{\frac{p}{y_H(1+\pi)} + \frac{1-p}{y_L(1+\pi)}}, \tag{29}$$

and the Sharpe ratio is constant as well and given by

$$S = \frac{p^{\frac{1}{2}}(1-p)^{\frac{1}{2}} \left\| \frac{y_H}{y_L} - 1 \right\|}{p + (1-p)\frac{y_H}{y_L}}. \tag{30}$$

Returning to our economy, taking the price-dividend ratios π_H, π_L and a strategy profile of portfolios (17)-(19) as given, and imposing the market clearing condition that all stock holdings have to sum up to 1 gives the implied interest rate. The equilibrium wealth level of all agents, their consumption share and the market clearing condition for the good market that requires that aggregate consumption has to be equal to the dividend gives the equilibrium price dividend ratios.

Proposition 3 *In state $s = H, L$ the interest rate is*

$$r_f = \frac{1/\pi_s}{\frac{\tilde{\xi}(\Omega^*)}{y_H(1+\pi_H(\Omega^*))} + \frac{1-\tilde{\xi}(\Omega^*)}{y_L(1+\pi_L(\Omega^*))}} \tag{31}$$

the price dividend ratio is

$$\pi_s = \frac{\beta^I (1 - \lambda \Upsilon_s) + \lambda \bar{g}_s - (1 - \beta) \Gamma_s \bar{g}_s}{1 - \beta^I (1 - \lambda \Upsilon_s) - \lambda \bar{g}_s + (1 - \beta) \Gamma_s \bar{g}_s}, \tag{32}$$

Figure 5 plots the price dividend ratio and the risk free rate relative to where all investors are direct traders. The presence of delegation decreases the price-dividend ratio, and typically increases the risk free rate. The reduction in the price dividend-ratio is in part a result of the presence of management fees. The presence of management fees that are consumed implies a reduction in investable capital and demand for equity. For markets to clear price-dividend ratios need to then decline. While the price-dividend ratio is typically counter cyclical, for example as shown in the figure, there are cases where it is pro-cyclical. The increase in the risk free rate is driven mostly by the reduction in the price dividend ratio: the price dividend ratios in the low and the high state are fairly close to each other, and from (29) it is straightforward to see that in an economy with only direct traders the risk free rate is decreasing in the price-dividend ratio.

Using Proposition 3 we can then compute the equilibrium Sharpe ratio

Lemma 5 *The Sharpe ratio is*

$$S(\Omega^*) = \frac{p^{\frac{1}{2}} (1-p)^{\frac{1}{2}} \left\| \frac{y_H}{y_L} X(\Omega^*) - 1 \right\|}{p + (1-p) \frac{y_H}{y_L} X(\Omega^*)}. \tag{33}$$

where $\frac{y_H}{y_L} X(\Omega^*)$ is the state price of the low state relative to the high state, and

$$X(\Omega^*) \equiv \frac{\frac{1-\tilde{\xi}(\Omega^*)}{1-p}}{\frac{\tilde{\xi}(\Omega^*)}{p}} \frac{1 + \pi_H}{1 + \pi_L}. \tag{34}$$

We find that typically as the share of delegation increases the Sharpe ratio follows an inverse U-shaped pattern, as shown for example in the left panel of Figure 6. It increases as long as $\Omega^* < \hat{\Omega}$, and decreases for $\Omega^* > \hat{\Omega}$. We find these observations robust to all the parameter variations we experimented with.[15] The basic intuition for this pattern is as follows. We have stated in Theorem 1 (and will explain in more detail below), when the price-effect of managers is small because their

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[15]A proof for the whole parameter space is not available. In the previous version of this paper we prove the statement for a subset of the parameter space. In particular, we show that the Sharpe ratio is increasing in the region $\Omega^* \le \hat{\Omega}$ for $n_a > n_b \ge 2$, it is monotonic in the region $\Omega^* > \hat{\Omega}$ is monotone and if Z_B is small then it is decreasing. The proof is available on request.
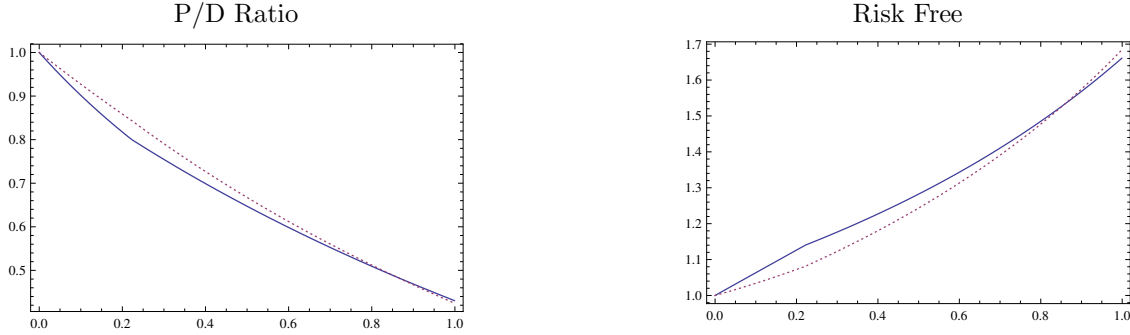
Figure 5: The graphs plot the equilibrium price-dividend ratio (left panel) and the risk free rate (right panel). The solid and dotted lines correspond to the low and the high state respectively. Quantities are normalized by their level in the Lucas economy. Parameters are set to: $p = 0.8, y_H = 1.15, y_L = 0.85, n_A = 1.5, n_B = 1, \lambda = 0.5, Z_B = 0.1$.
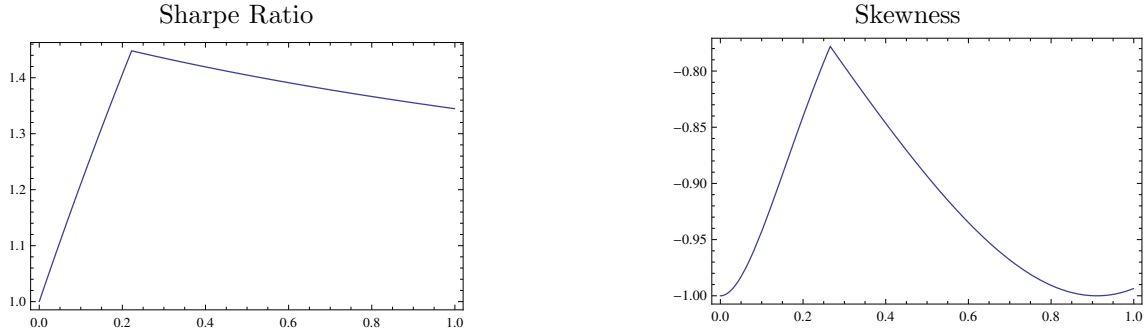

Figure 6: The graphs plot the equilibrium Sharpe ratio(left panel) and Skewness (right panel). Quantities are normalized by their level in the Lucas economy; for the skewness graph we normalize by the absolute level. Parameters are set to: $p = 0.8, y_H = 1.15, y_L = 0.85, n_A = 1.5, n_B = 1, \lambda = 0.5, Z_B = 0.1$.

capital share is low, managers prefer to follow a contrarian strategy. That is, they hold only a small amount of stocks relative to direct traders. As the share of delegation increases, for markets to clear they hold more stocks. As a compensation, the Sharpe-ratio has to increase. However, when $\Omega^* > \hat{\Omega}$ as the share of delegation increases, the group holding the contrarian portfolio decreases while an increasing group of managers hold a levered portfolio, thus the Sharpe-ratio decreases to provide adequate compensation for managers holding the levered portfolio.

The right panel of Figure 6 shows the skewness of equity returns. The intuition for the pattern is similar as before. When the share of delegation is low the skewness of market returns is close to that of the consumption growth process. Keeping in mind that contrarian managers strategies take advantage of negative skewness, in the region where all managers follow contrarian strategies an increase in the share of delegation increases skewness, as a result of the price impact of their trades. Above $\hat{\Omega}^*$ aggressive funds with aggressive trading strategies start to emerge, and the impact of

their trades more than offsets that of the contrarian funds leading to a decline in skewness.[16]

3 Implications

In this section, we further discuss the equilibrium and analyze its implications. We focus on the interaction of non-concave incentives and the increased level of delegation in financial markets. We contrast our findings with existing empirical work and present additional testable implications. We start by discussing in Section 3.1 implications connected to the distribution of relative returns and of strategies. Then, in Section 3.2 we focus on implications related to the gross amount of borrowing and lending.

3.1 Managers' excess log-return and heterogeneity in strategies

Proposition 1 and Theorem 1 describe the trading strategies in equilibrium. We can see immediately that when reaching the larger elasticity segment of the incentive function would be sub-optimal, because the kink κ is too large, then in equilibrium both direct traders and fund managers hold the market. Because in this equilibrium delegation has little effect, in the rest of the paper, we focus our attention to the segment of parameter space when the equilibrium is not of this type (i.e., $\kappa < \kappa_{low}$ or $\kappa_{low} < \kappa < \kappa_{high}$ and $p > \hat{p}$.).

For all remaining set of parameters, Theorem 1 implies that when share of delegation is low, $\Omega^* < \hat{\Omega}$, all fund managers follow a contrarian strategy in equilibrium. To see the intuition behind the equilibrium choice of managers, consider the first fund manager who enters a market which is populated only by direct traders, $\Omega^* \approx 0$. The manager has three choices. She can hold a moderate portfolio, but then she will never outperform the market sufficiently to get the extra capital flows in any of the states. Or she can hold the locally optimal aggressive portfolio leading to gains and extra capital flow in the high state and losses in the low state, or she can hold the locally optimal contrarian portfolio leading to gains and extra capital flow in the low state and losses in the high state. How do these two compare? Managers choose the contrarian portfolio because of the interaction of left skewed consumption growth ($p > \frac{1}{2}$) and convex flow-performance function. First, the fact that the high state has higher probability to occur implies that the size of the gain or loss compared to market return in the high state are small relative to the size of relative gain or loss compared to market return in the low state under any locally optimal strategy. For example, in the locally optimal contrarian strategy, large gains with small probability in the low state compensate

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[16]In the region $\Omega^* > \hat{\Omega}$ and increase in the share of delegation always implies an increase in the difference between the capital delegated to non-contrarian and contrarian managers. That is, $\Omega^*(1 - \mu_{AB}(\Omega^*)) - \Omega^* \mu_{AB}(\Omega^*)$ is increasing in Ω^*.

for small losses with large probability in the high state.[17]

Second, the fact that the flow-performance relationship is convex implies that capital-flow rewards for gains are larger than penalties for losses of similar magnitude. As a consequence of the two effects, the manager prefers the contrarian strategy, because the implied larger gain is rewarded more by the convex flow-performance relationship.

Note that our argument is the classic idea of risk-shifting, but with a slight twist. Risk shifting implies that agents with globally non-concave incentives might prefer to take on larger variance, that is, they gamble. However, in our case this not necessarily implies a levered position. Because managers have non-concave incentives in relative instead of absolute return, in this particular case, the contrarian strategy is the larger gamble. A similar point regarding funds increasing tracking error volatility in the presence of benchmarks has been made in Basak, Pavlova and Shapiro (2007), and Cuoco and Kaniel (2011).

As the share of delegation Ω^* increases, prices increasingly work against fund managers and they find the contrarian strategy less attractive. At some threshold $\hat{\Omega}$, managers become indifferent between the optimal contrarian strategy and, depending on the parameter values, either the optimal moderate strategy or the optimal aggressive strategy. For market clearing, as the market share of fund managers grows above this threshold a decreasing set of managers has to choose the contrarian strategy. Thus, the heterogeneity in strategies increases with Ω^* in this sense. The idea is simple. As managers start to dominate the market, the only way they can overperform the market in some state is if they bet against each other.

Consider now the relative return of the average manager as the share of delegation increases. We show that despite the increasing group of managers following an aggressive strategy when the share of delegation is large, the average manager remains contrarian for any $\Omega^* < 1$. Also, over the whole range of Ω^*, both managers' overperformance in the low state and underperformance in the high state becomes less extreme. For small share of delegation ($\Omega^* < \hat{\Omega}$) this is a consequence of the fact that as prices move against managers, each one chooses a portfolio which result in less extreme relative returns. For larger share of delegation ($\hat{\Omega} < \Omega^*$), the relative return of each individual manager is constant. However, as the proportion of managers choosing the aggressive portfolio increases, the relative return of the average manager has to increase in the high state and decrease in the low state. Given this monotonicity and the fact that at $\Omega^* = 1$ the average manager has to hold the market, the average manager must have a portfolio which overperforms in the low state

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[17]Formally, as the average relative returns under the two portfolios are equal,

$$p\frac{\xi_{AB}}{\tilde{\xi}(0)} + (1-p)\frac{1-\xi_{AB}}{1-\tilde{\xi}(0)} = p\frac{\xi_{BA}}{\tilde{\xi}(0)} + (1-p)\frac{1-\xi_{BA}}{1-\tilde{\xi}(0)} = 1,$$

$p > \frac{1}{2}$ implies that

$$\frac{\xi_{AB}}{\tilde{\xi}(0)} - 1 < 1 - \frac{1-\xi_{AB}}{1-\tilde{\xi}(0)} \quad \text{and} \quad \frac{\xi_{BA}}{\tilde{\xi}(0)} - 1 > 1 - \frac{1-\xi_{BA}}{1-\tilde{\xi}(0)}.$$

and underperforms in the high state for any $\Omega^* < 1$.

To translate our findings to testable implications, let us define some descriptive statistics. In particular, we consider the excess log return of the average fund manager,

$$\int_{m \in M} \ln \rho_{t+1} \left(\alpha_t^m, s_{t+1} \right) dm - \ln R_{t+1}(s_{t+1}),$$

the volatility of the excess log-return of a given fund manager is,

$$\sqrt{p \left(1 - p \right)} \left| \begin{array}{c} \left(\ln \rho_{t+1} \left(\alpha_t^M, H \right) - \ln R_{t+1}(H) \right) \\ - \left(\ln \rho_{t+1} \left(\alpha_t^M, L \right) - \ln R_{t+1}(L) \right) \end{array} \right|,$$

and the cross sectional dispersion across fund managers' excess log–returns in state s_{t+1},

$$\int_{n \in M} \left| \ln \rho_{t+1} \left(\alpha_t^n, s_{t+1} \right) - \int_{m \in M} \ln \rho_{t+1} \left(\alpha_t^m, s_{t+1} \right) dm \right| dn.$$

The intuition discussed above translates to the following statements.

Proposition 4   *1. For any $\Omega^* < 1$, the average fund's exposure to the market is always smaller than 1, so it overperforms the market in recessions and underperforms in booms.*

2. *For $\Omega^* > \hat{\Omega}$, funds follow heterogeneous strategies. In each period, a fraction of managers, $1 - \mu_{AB}\left(\Omega^* \right)$, levers up and invests more than 100% of their capital in stocks. This fraction increases in the share of delegated capital Ω^*.*

3. *For $\Omega^* > \hat{\Omega}$, fund managers' cross-sectional dispersion of log-returns is larger in the low state than in the high state when the equilibrium is Cont-Agg. When the equilibrium is Cont-Mod, this is also the case if and only if*

$$p > \frac{\sqrt{\frac{n_A}{n_B}}}{1 + \sqrt{\frac{n_B}{n_A}}}. \tag{35}$$

4. *As Ω^* increases, the excess log return of the average manager increases in the high state and decreases in the low state. That is, both the overperformance in the low state and the underperformance in the high state is less severe.*

5. *The volatility of the excess log-return of each manager is decreasing in the share of delegation as long as $\Omega^* < \hat{\Omega}$.*

Consistently with statement 1, Karceski (2002) finds that collectively equity funds CAPM beta is 0.95, and Kacperczyk, Sialem and Zheng (2005) find a market beta of 0.96 in a four factor model. Evidence also shows that mutual funds perform better in recessions than in booms (e.g., Moskowitz

(2000), and Glode (2010), Kacperczyk, Van Nieuweburgh and Veldkamp (2010), Kosowski (2006), Lynch and Wachter (2007)).[18]

Regarding statement 2, there is some evidence that the heterogeneity in strategies in the money management industry has been indeed increasing over the last decades. As argued by Adrian and Shin (2008), one sign of this is that the total balance sheet of investment banks[19], typically using leveraged strategies, was around 40% compared to bank holding companies in 1980 and increased over 160% by 2007. Indeed, by 2009, it has become a widely held view among policy makers that the excessive leverage of investment banks contributed to the financial crisis (see FSA (2009), FSB (2009)). We note that between 1995 and 2007 the size of the shadow banking sector increased relative to the delegated management industry. While mutual and pension fund assets increased by 323% and 127% respectively, broker-dealer and hedge fund assets increased by 444% and 775% respectively.[20]

Although we believe that our result has the potential to provide a simple and insightful explanation of the emergence of highly leveraged financial intermediaries over the last decade and their coexistence with more conservative institutions, we have to point out two caveats to this interpretation. First, our framework cannot distinguish between two possible interpretations of aggressive portfolio. An aggressive strategy can be interpreted as levered strategy, but it can be equally interpreted as a strategy of picking stocks with higher than 1 market-beta. Second, in our equilibrium there is no persistence in portfolios. That is, a manager who held an aggressive portfolio in one period, might hold a conservative portfolio in the next one. This does not map directly to the interpretation that managers holding different portfolios correspond to different type of financial intermediaries. However, we show in Appendix B (available online) that this is a technical issue. In particular, we argue that apart from the presented equlibrium with constant Ω^* and mixed strategies, there is also a class of equilibria with persistent portfolios and fluctuating Ω^*. Apart from these differences, all these equilibria have very similar properties. We analyze the one with constant Ω^* for analytical convenience.

If its conditions are satisfied, statement 3 is consistent with Kacperczyk, Van Nieuweburgh and Veldkamp (2010) who find that the dispersion in mutual funds return is larger in recessions. Interestingly, Kacperczyk, Van Nieuweburgh and Veldkamp (2010) present this result as an implication of optimal attention allocation across the business cycle by fund managers. Our model suggests that this result is consistent with a set-up where information does not play any explicit role. In-

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[18]Note, however, that Kosowski (2006), Lynch and Wachter (2007) and Glode (2010) find overperformance in recessions in terms of Jensen-alpha as opposed to in terms of total returns. Given that in our model funds cannot generate alpha, only the results in Moskowitz (2000), and Kacperczyk, Van Nieuweburgh and Veldkamp (2010) translate to our proposition one-to-one.

[19]Although mutual funds typically do not use leverage, interestingly, Lo and Patel (2007) notes a large increase of leveraged mutual funds and leveraged ETFs in the last decade before the crisis.

[20]Mutual fund, pension fund, and broker-dealer data are from the flow of funds, hedge fund data is from French (2008).

stead, it is driven by competition of managers for extra capital inflows and negative skewness of the consumption growth process. Given the negative skewness the high state is likely to occur most of the time. Because the high state occurs most of the time, the returns on the optimal contrarian and aggressive strategies will not deviate that much from the market in the high state. On the other hand, the low state occurs infrequently and this is where the distortions relative to the market will be large: the contrarian strategy will outperform by quite a bit and the aggressive strategy will underperform by quite a bit. As a consequence of this, the dispersion in returns in the low state between the contrarian and aggressive strategies will be large. Note also that while (35) tends to be satisfied when the consumption growth process is relatively skewed (large p), Theorem 1 shows that a Cont-Agg equilibrium typically arises when the consumption growth process p is close to half. Thus, we should expect to get larger dispersion in recessions for a wide range of parameters.[21]

Because of the lack of systematic evidence on the time-series pattern of managers' return volatility, relative returns and return dispersion we think of results 4 and 5 as testable predictions for the future.

3.1.1 Trading Volume

Our model can also speak to the link between heterogeneity and trading volume. In the region where funds follow heterogeneous strategies ($\Omega^* > \hat{\Omega}$), trading volume typically increases in the share of delegation; as seen, for example, in the middle panel of Figure 7.[22] Comparing the middle and left panels shows that the main driving force behind the increased trading volume is trading in between funds. In the region where fund trading strategies are heterogeneous the fraction of funds following a contrarian strategy endogenously decreases in the share of delegation, as shown in Proposition 4. In addition, in most cases the fraction of contrarian funds remains above 50% for all shares of delegation; implying increased heterogeneity in fund strategies as the share of delegation increases.[23] The fact that the cross sectional dispersion in fund returns is larger in the low state than in the high state (Proposition 4 part 3)suggests that trading volume be larger in the low state, as shown in the figure. The figure further suggests that the difference between trading volume in the low and high state increase in the share of delegation.[24]

Within our model part of the trading volume is due to funds randomly switching trading

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[21]Measuring dispersion as the ratio of relative returns implies a higher dispersion in the low state always for both $Cont - Agg$ and $Cont - Mod$ equilibria.

[22]In computing the trading volume component of direct traders we assumed that each new born investor is endowed with one share of stock. That is, we pool all assets of clients that die and equally distribute the shares across newborn investors. Also, to compute funds' trading volume we ignore the fact that a strict interpretation of the model requires capital to first leave funds at the end of the period before the re-allocation at the beginning of the subsequent period.

[23]In some cases for large shares of delegation the fraction of funds following a contrarian strategy is below 50%, implying an inverse U-shaped pattern in heterogeneity within the fund universe. However, typically trading volume between funds still increases in the share of delegation, as the increased share of funds in the economy outweighs the slight decrease in heterogeneity with funds.

[24]We find these results robust across parameter specifications.

strategies between periods. The right panel of the figure computes trading volume under a fictitious assumption that funds do not switch strategies. The qualitative patterns are similar to the ones shown in the second panel. Furthermore, comparing the middle and right panels shows that most of trading volume is due to funds need to accommodate flows, as opposed to funds switching strategy types.
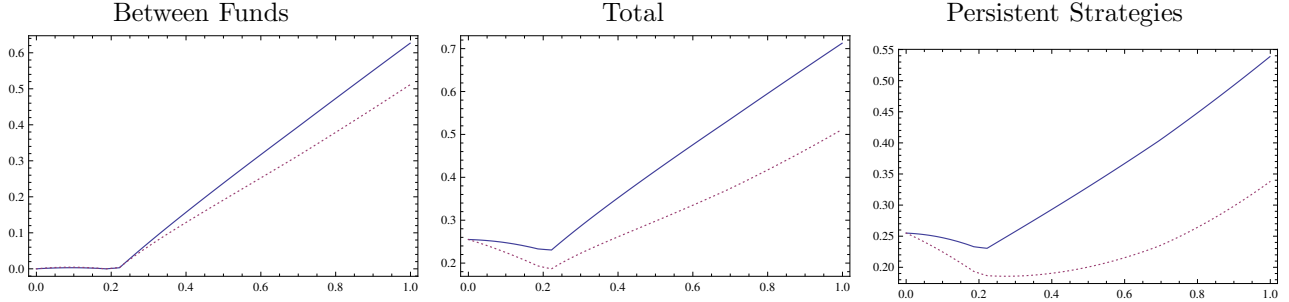

Figure 7: The graphs plot the equilibrium trading volume. The left panel shows the part of the volume that is due to trading between funds. The middle panel shows total volume: both trading between funds and between funds and direct traders. The right panel is a total volume measure under the fictitious assumption that funds trading strategies are persistent. The solid and dotted line correspond to the low and high state respectively. Parameters are set to: $p = 0.8, y_H = 1.15, y_L = 0.85, n_A = 1.5, n_B = 1, \lambda = 0.5, Z_B = 0.1$.

### 3.1.2 Exposure to market risk

In Proposition 4 we characterized the distribution of the average and individual excess returns as the share of delegation increases. However, the change in relative returns does not map one to one to the change in the exposure to market risk. This mapping also depends on the relative return of the stock and the bond, that is, the price of risk changes. We briefly discuss the change in agents' exposure to market risk and its relation to changes in the Sharpe ratio, a particular measure of the price of risk, as share of delegated capital increases.

We find that typically, as the share of delegation increases, the exposure of managers' holding a contrarian portfolio, α_{AB}, decreases for $\Omega^* > \hat{\Omega}$ and increases for $\Omega^* < \hat{\Omega}$, exposure of manager's holding an aggressive portfolio, α_{BA}, increases in the only relevant range $\Omega^* > \hat{\Omega}$, and direct trader's exposure to market risk, α^D, increases. Figure 8 and the left panel of Figure 9 illustrate this for a wide range of parameters.[25] We find all these observations robust to all the parameter variations we experimented with, with the only exception being the monotonicity of α_{AB} when $\Omega^* < \hat{\Omega}$.[26] However, analytically, we prove only the following weaker statement.

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[25]We elaborate on these figures in Section 4.

[26]Typically for high Z_B, $\frac{\partial \alpha_{AB}}{\partial \Omega^*}|_{\Omega^* < \hat{\Omega}} < 0$.
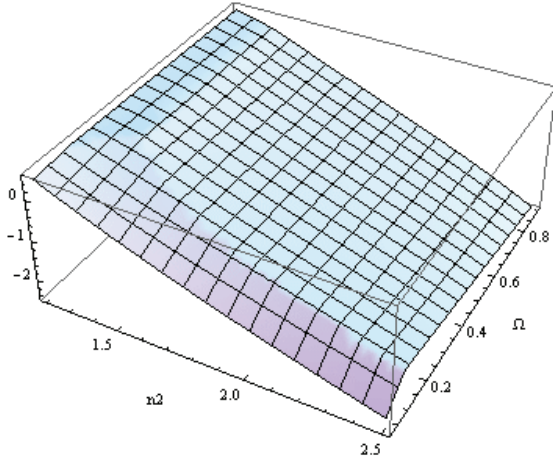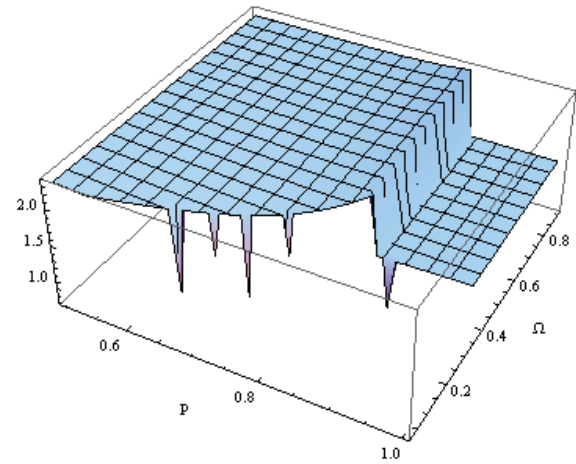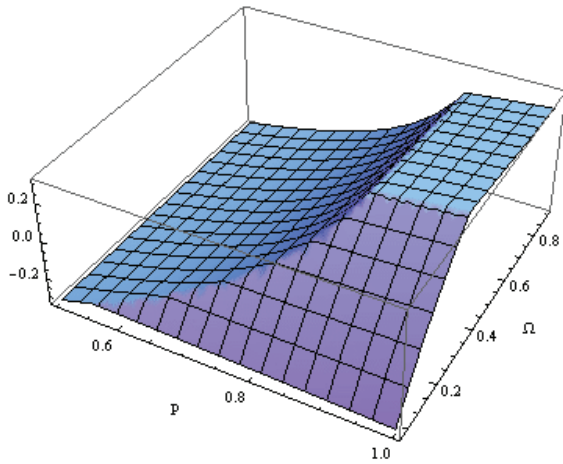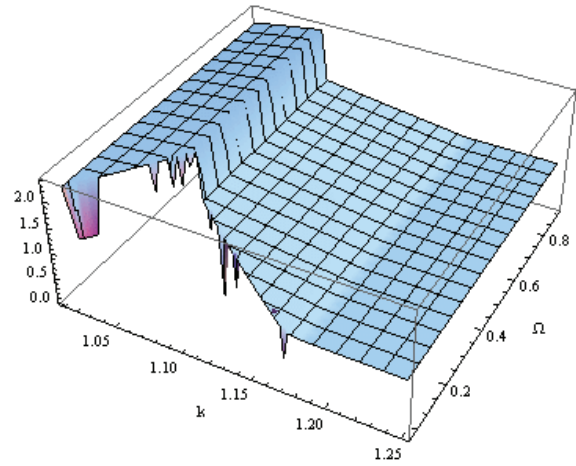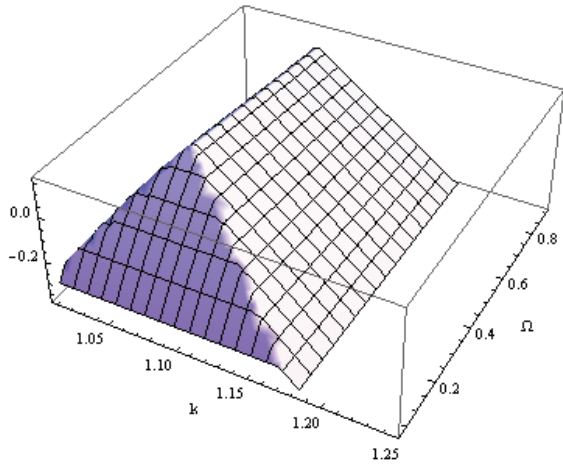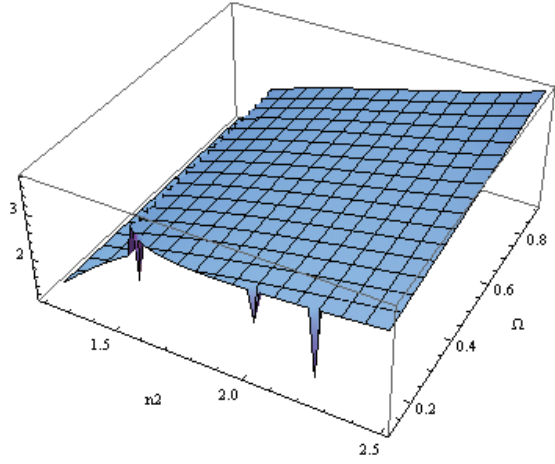
Figure 8: The graphs plot equilibrium stock position of managers following the contrarian strategy. In each of the graphs we vary the share of delegation Ω^* and one additional parameter. The Parameters are set to: $p = 0.8, y_H = 1.15, y_L = 0.85, n_A = 1.5, n_B = 1, \lambda = 0.5, Z_B = 0.1$.

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Figure 9: The graphs plot the trading strategy of Direct traders (right panels) and the Sharpe ratio(left panels). In each of the graphs we vary the share of delegation Ω^* and one additional parameter. Parameters are set to: $p = 0.8, y_H = 1.15, y_L = 0.85, n_A = 1.5, n_B = 1, \lambda = 0.5, Z_B = 0.1$.

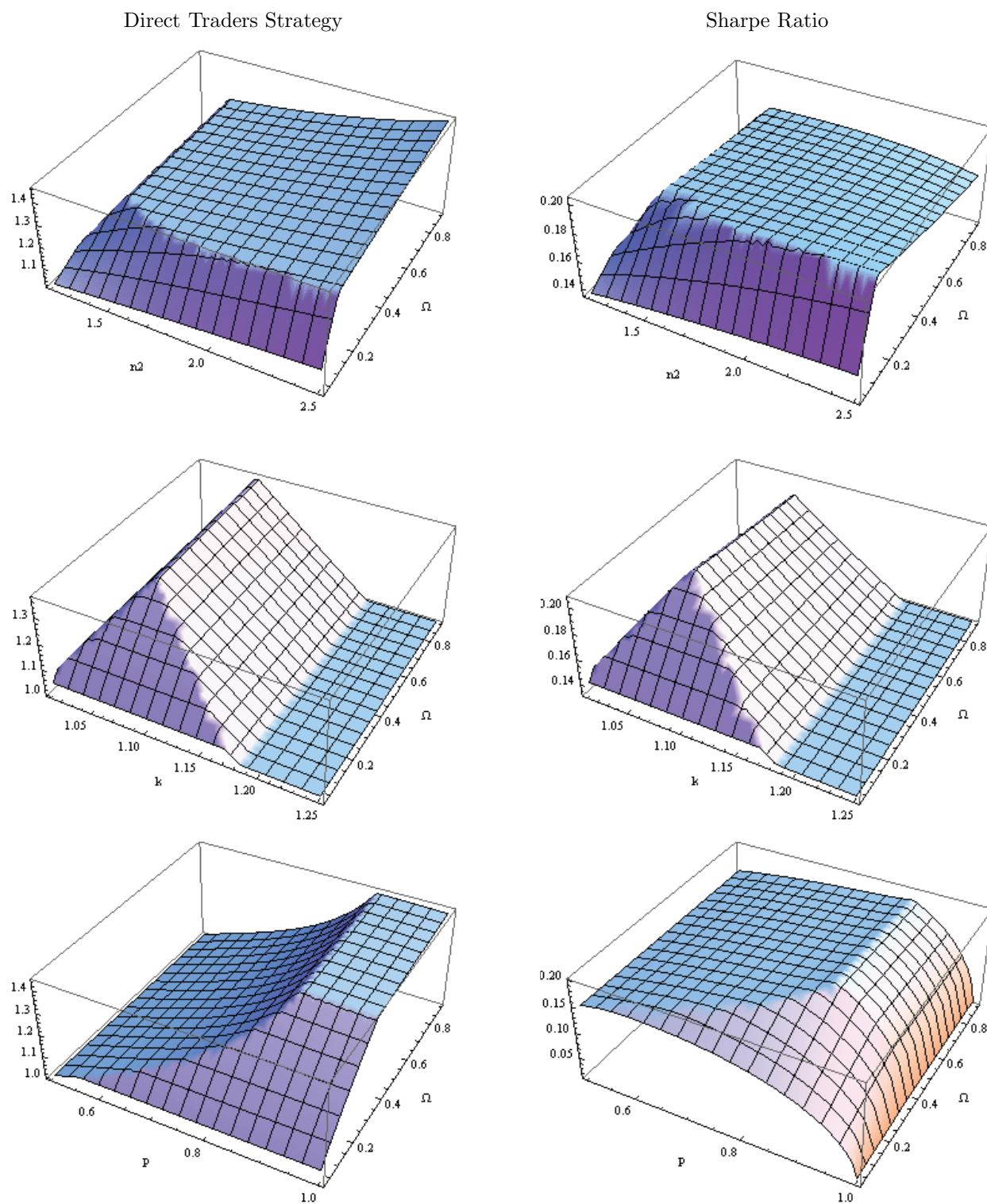**Proposition 5** *In the region $\Omega^* > \hat{\Omega}$, whenever the Sharpe ratio is decreasing in Ω^*, the exposure to market risk of direct traders and managers holding an aggressive portfolio, α^D, α_{BA} is increasing, while the exposure of managers holding a contrarian portfolio, α_{AB} is decreasing as Ω^* increases.*

3.2 Borrowing and lending, repo, derivative markets and gambling

As opposed to standard representative agent models, in our model traders typically do not hold the market portfolio. Agents buy or sell bonds in order to gain different exposure to market risk. In this section, we quantify the extent of this activity. We show that the gross amount of borrowing and lending compared to the size of the economy typically increases with an increase in the share of delegation.

Before we proceed to the formal results, it is useful to consider the empirical counterpart of our concepts. In our framework, buying or selling the risk-free asset is the only way agents can change their exposure to market risk. In reality, financial intermediaries use various instruments for this purpose. As repo agreements are one of the most frequently used tools for a large group of financial intermediaries to manage their leverage ratio (see Adrian and Shin, 2008), one possibility is to connect gross amount of borrowing and lending in our model with the size of repo markets. Alternatively, as most financial intermediaries would use derivatives like S&P futures and options to change their exposure to market risk, we can connect the amount of borrowing and lending risk-free bonds in our model to the open interest in derivative markets.

To measure gross amount of lending and borrowing positions, we use the fact that in any equilibrium, the only group of traders who lend are managers who follow a contrarian strategy. We define relative bond market size as the total long bond holding of this group compared to the value of the economy, $q_t + \delta_t$. Plugging in (31) into (17) and some simple algebra shows that this measure is

$$\Omega^* \mu_{AB}(\Omega^*)(1 - \alpha_{AB}) = \Omega^* \mu_{AB}(\Omega^*) \frac{1 - \frac{\xi_{AB}}{\tilde{\xi}(\Omega^*)}}{1 - \frac{1}{\tilde{\xi}(\Omega^*) + (1 - \tilde{\xi}(\Omega^*)) \frac{y_H(1+\pi_H)}{y_L(1+\pi_L)}}}. \tag{36}$$

The following Lemma describes the relationship between the portfolio of managers, relative bond market size and the Sharpe ratio whenever $\Omega^* > \hat{\Omega}$.

Lemma 6 *When the share of delegation is larger than $\hat{\Omega}$, whenever the Sharpe ratio is decreasing in Ω^*, amount of bond long positions relative to the size of the economy (36) increases as the share of delegation increases.*

Together with our observation that the Sharpe ratio is typically decreasing in the share of delegation when $\Omega^* > \hat{\Omega}$, the lemma implies that relative bond market size also increases with Ω^*. To interpret this result, note that risk-free bonds serve a double purpose in our economy. First, direct traders and managers have different incentives, which implies that they prefer to share risk.

As we saw before, this leads direct traders to hold a portfolio with a larger than one exposure to the market. We call the part of holdings explained by this motive as the risk-sharing amount of bond holdings. Second, when the share of delegated capital is sufficiently large managers start to trade against each other. By selling or buying bonds they increase or decrease their exposure to the market in order to beat the market at least in one of the states. We call this part the gambling share of bond holdings.

As direct traders hold bonds only because of risk-sharing motives, we can decompose the total size of the bond market by comparing (36) to the total bond holding of direct traders relative to the value of the economy; defining the gambling share as

$$(1-\Omega^*)\left|\frac{1-\frac{p}{\tilde{\xi}(\Omega^*)}}{1-\frac{1}{\tilde{\xi}(\Omega^*)+\left(1-\tilde{\xi}(\Omega^*)\right)\frac{y_H(1+\pi_H)}{y_L(1+\pi_L)}}}\right|. \tag{37}$$

The following lemma shows that the ratio of the gambling share, (37), to the total size of the credit market, (36) is increasing in the share of delegation whenever $\Omega^* > \hat{\Omega}$.

Lemma 7 *For $\Omega^* > \hat{\Omega}$ in both Cont-Mod and Cont-Agg equilibria direct traders fraction of total borrowing decreases in Ω^* at a rate proportional to $\frac{1}{(1-\Omega^*)^2}$, where the constant of proportionality is larger in the Cont-Agg equilibrium.*

As an example, Figure 10 plots both the total bond market size and size the risk-sharing share part (i.e., 1- gambling share) of the bond market.
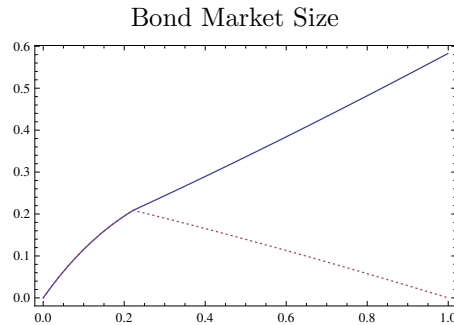
Bond Market Size

Figure 10: The graph plots the size of the bond market measured by the ratio of the value of the total long bond holdings relative to the value of the economy $q_t + \delta_t$ (solid line), and the non-gambling component of the bond market (dotted line). Parameters are set to: $p = 0.8, y_H = 1.15, y_L = 0.85, n_A = 1.5, n_B = 1, \lambda = 0.5, Z_B = 0.1$.

To complement our analytical results, we will argue in Section 4 that under reasonable parameter values the value of long bond holdings relative to the size of the economy monotonically increases

in the share of delegated capital for any Ω^*, and the implied increase in the borrowing and lending activity is quantitatively large. Furthermore, almost all of the increase is explained by gambling share. Thus, our model suggests that financial intermediaries increased competition for fund flows might explain the multiple fold increase of the repo market and derivative markets like S&P futures and options during the last decades before the financial crisis in 2007/2008.[27]

4 Numerical examples

In this section we present some simple calibrated examples to show that the magnitude of the effects we discuss, especially trading strategies distortions and the impact on bond markets, can be quantitatively large.

Before proceeding with the examples it is important to keep in mind that we have constructed our model to highlight the potential important role that delegated portfolio management has on the equilibrium size of bond markets, and the link between the size of these markets and the endogenous emergence of heterogeneous strategies within the money management industry. To obtain a parsimonious and tractable setup we have made three important assumptions. First, we use logarithmic utility. Second, we assume a piece-wise constant elasticity incentive function. The combination of the two is helpful in delivering a tractable model. Third, to allow us to focus on a stationary equilibrium with a constant share of delegation we impose a specific structure of periodically reborn investors choosing to be clients or direct traders. We conjecture that our insights paired with a more flexible model with habit formation, Epstein-Zin preferences or more complex consumption processes might be useful in the quantitative dimension, but such an exercise is outside of the scope of this paper.

We experiment with two sets of parameters for the consumption growth process (Table1) and two sets of parameters for the incentive function (Table 2). Then we conduct a sensitivity analysis with regards to the latter.

The difference between our two sets of consumption growth parameters is that the first is implied by the full post-war sample, 1946-2008, while the second one is implied by the second half of the full sample 1978-2008. We consider the moments from the shorter sample also to entertain the possibility that the distribution of the consumption growth process has changed over time. Using consumption growth data from Shiller's website we estimate the mean, standard deviation and skewness of consumption growth, and then solve for p, y_H, and y_L to match these three moments. It is apparent that the biggest impact of the change on the sample is on the skewness of the process. This implies a different value of p in our model.

For the incentive function we consider two specifications. The first is a minimal deviation from

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[27]See Gorton-Metrick (2010) for estimation on the change of the size of the repo market or institutional details on this market.

Table 1: Consumption Growth

Data		
Period	1946-2008	1978-2008
Mean	0.0216	0.0210
Standard deviation	0.0175	0.0154
Skewness	-0.220	-0.605

Model Estimated Parameters		
p	0.555	0.645
y_H	1.038	1.033
y_L	1.002	1

a constant elasticity incentive function. Constant elasticity incentives are a natural benchmark since as shown in Lemma 1 the combination of constant elasticity flows and log utility implies no heterogeneity in managers' strategies.[28] Moreover,

Lemma 8 *When $n_A = n_B$, the Sharpe ratio is identical to the one in the Lucas economy.*

The second specification is based on estimating the incentive function using data on mutual funds.[29] [30]

Using Equation (9) We can rewrite the flows as

$$
\begin{aligned}
FL_t &= \ln \frac{w_{t+1}^M}{\rho_{t+1}\left(\alpha_t^M\right)\left(1-\psi_t^M\right) w_t^M} = \\
&= \ln \Gamma_t Z_B + 1_{ExRet_t \geq \ln \kappa} \ln \kappa^{n_B - n_A} + \left[(n_B - 1) 1_{v_t < \ln \kappa} + (n_A - 1) 1_{ExRet_t \geq \ln \kappa}\right] ExRet_t \\
&= \ln \Gamma_t Z_B + (n_B - 1) ExRet_t + (n_A - n_B) 1_{ExRet_t \geq \ln \kappa} \left(ExRet_t - \ln \kappa\right)
\end{aligned}
$$

where $ExRet_t$ is the excess log return above the market:

$$
ExRet_t = \left(\ln \rho_t\left(\alpha_{t-1}^M\right) - \ln R_t\right).
$$

---

[28]The importance of considering the interaction of utility function and the incentives was also pointed out by Ross (2006).

[29]In our model managers should represent the whole financial intermediary sector including mutual funds, commercial banks, hedge funds, retirement funds etc. Our choice to use mutual fund data is based on data availability, and the fact that most empirical work on the estimation of flow-performance relationships is on mutual funds.

[30]We would like to thank Dong Lou for providing us with mutual fund performance and flow data and Eszter Nagy for providing research assistance.

Table 2: Incentive Function

	Minimal Deviation	Estimated from Fund Data
n_A	1.01	1.9
n_B	1	1.4
κ	1	1.05

To estimate $n_A, n_B \ln \kappa$ we therefore estimate the model

$$FL_t = \alpha_t + \beta_1 ExRet_t + \beta_2 1_{ExRet_t \geq \ln \kappa}(ExRet_t - \ln \kappa).$$

Our strategy is to run a large number of panel regressions with a different fix $\ln \kappa$ in each and search for the best fit. Details on the procedure and the results are available in an online appendix.

In all specifications we set the discount rate to $\beta = 0.98$, which implies a reasonable 2% annual management fee for managers. We set $\lambda = 0.5$ and $Z_B = 0.01$ to make sure that the equilibrium exists under all sets of parameters.

For the interpretation of the figures as implied time-series, note that the Ω^* values corresponding to the share of direct equity holdings in 1960, 1980 and 2007 would be $\Omega_{60}^* = 0.15, \Omega_{80}^* = 0.52, \Omega_{07}^* = 0.78$.

Consider first the minimal deviation scenario. As shown in the first row of Figure 11, even slight convexities lead to the emergence of heterogeneous fund strategies: 50% of managers hold 85% of their capital under management in stocks, while the other 50% hold 115%. Given these strategies, naturally the size of the bond market relative to total investment increases as the share of delegation increases. Even if, as shown in the top row of Figure 12 this increase seems small (from zero to 7%), considering that we deviate only slightly from linear incentives it is still a significant effect. The impact on the Sharpe ratio relative to the one in the Lucas economy is negligible. Why strategies react so strongly to little convexity? The reason is that for managers the cost of gambling is second order as they come from risk aversion, while the benefits in flows in the good state are first order because of the kink in the incentive function.

The second (third) row of Figure 11 display the strategies for the incentive function implied by the data, for the long (short) sample. It is apparent that the large convexity imply very large absolute positions in bonds. Focusing on the consumption process from the long sample, when the share of delegated capital is close to zero managers following the contrarian strategy invest 9 times their capital into the bond and short-sell the stock. They decrease this ratio to 7 as the share of delegated capital reaches $\hat{\Omega}$, and then increase it again to 9 as the share of delegated capital approaches one. Managers following the aggressive strategy exist in the market only if the share of delegation exceeds $\hat{\Omega}$. At $\Omega = \hat{\Omega}$ they borrow up to 10 times the size of their capital under
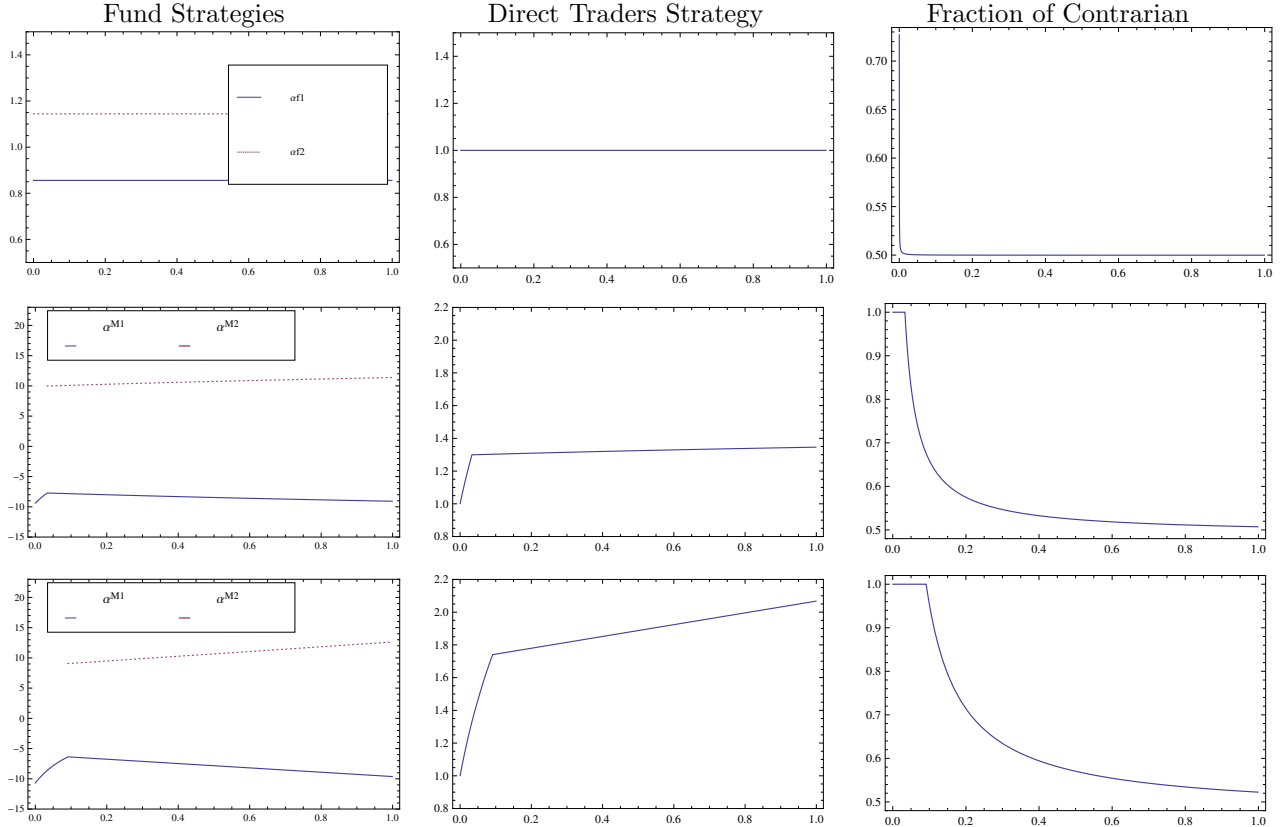
Figure 11: The graphs plot the equilibrium strategies as a function of the share of delegation. In each row the first panel plots funds' portfolios, the second direct traders' portfolio, and the third the fraction of fund managers' who are contrarian. The first row corresponds to a minimal deviation scenario where consumption parameters are taken from the full sample 1946-2008, i.e., $p = 0.555, y_H = 1.038, y_L = 0.002, n_A = 1.01, \kappa = 1 + 10^{-13}$ and $n_B = 1$. The second and third row corresponds to the parameters implied by the Chevalier-Ellison estimation, i.e., $\kappa = 1.05, n_A = 1.9,$ and $n_B = 1.4$. The second row uses the consumption parameters from the full sample, while the third row uses the consumption parameters from the shorter sample 1978-2008 ($p = 0.645, y_H = 1.033$ and $y_L = 1$). In each example $\beta = 0.98, \lambda = 0.5$ and $Z_B = 0.01$.

management to invest in stocks and increase this ratio to over 11 when they approach the point that only managers populate the market.

The right panel shows that the fraction of managers following the contrarian strategy decreases from 100% below $\hat{\Omega}$ to 50% when the share of delegation is close to 1. Using moments from the short sample, strategy patterns are similar but more extreme. However, aggressive managers start entering the market at a higher share of delegation and the rate at which they enter, as a function of the share of delegation, is slower. While these numbers are perhaps unrealistic at the industry level, they illustrate well the strengths of incentive to deviate from the market portfolio induced by convexities in the flow-performance relationship.

Corresponding to these extreme positions, the left panels of the bottom two rows of Figure 12
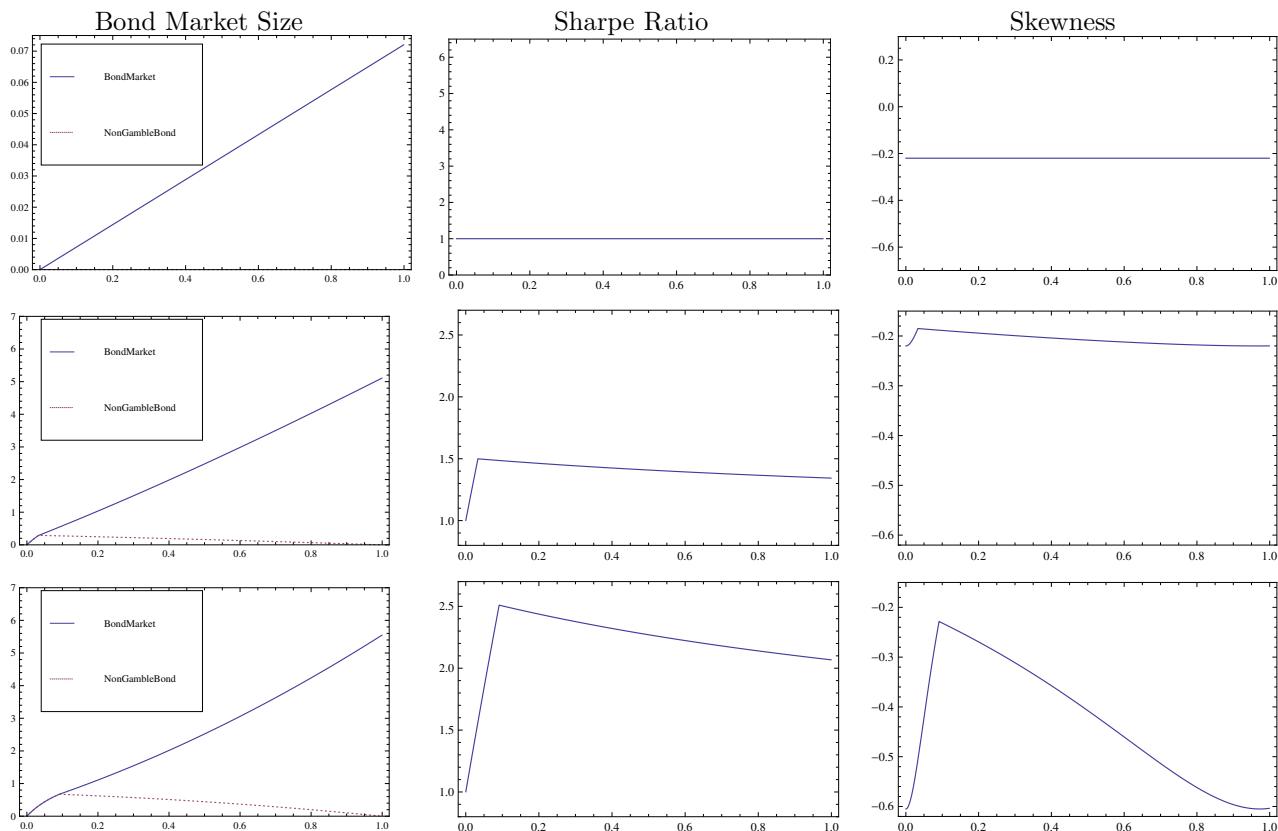
Figure 12: The graphs plot the size of the bond market measured by the ratio of the value of the total long bond holdings relative to the value of the economy $q_t + \delta_t$ (left panels), the Sharpe ratio (middle panels), and the equity return skewness (right panel) as a function of the share of delegation. The credit market graphs plot both the total size of the credit market (solid line), and the non-gambling component of the credit market (dotted line). The first row corresponds to a minimal deviation scenario where consumption parameters are taken from the full sample 1946-2008, i.e., $p = 0.555, y_H = 1.038, y_L = 0.002, n_A = 1.01, \kappa = 1 + 10^{-13}$ and $n_B = 1$. The second and third row corresponds to the parameters implied by the Chevalier-Ellison estimation, i.e., $\kappa = 1.05, n_A = 1.9,$ and $n_B = 1.4$. The second row uses the consumption parameters from the full sample, while the third row uses the consumption parameters from the shorter sample 1978-2008 ($p = 0.645, y_H = 1.033$ and $y_L = 1$). In each example $\beta = 0.98, \lambda = 0.5$ and $Z_B = 0.01$.

show that the size of the bond market increases considerably as the share of delegation increases.

Corresponding to these extreme positions, the left panels of the bottom two rows of Figure 12 show that the size of the bond market increases considerably as the share of delegation increases. The gross amount of long bond positions is around 100% of total net investment in the economy when the share of delegation is 25%, increases to about 2 times total net investment in the economy when the share of delegation is around 40%, and increases considerably as the share of delegation increases further. The initial small increase in the region $\Omega^* < \hat{\Omega}$ is due to non-gambling positions. Beyond $\hat{\Omega}$ managers start to utilize heterogeneous strategies, and gambling positions start to emerge as an important contributing factor that increases the size of bond markets as the share of delegation

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increases. The fact that the percentage of managers following the aggressive strategy increases from zero to 50% throughout this region combined with the fact that in this region both contrarian and aggressive fund strategies become more extreme as the share of delegation increases amplify the expansion of bond markets even further.

Interestingly, as is evident in the figure with these parameter values the effect of delegation on the Sharpe ratio relative to the level in the Lucas model is significant.[31] Considering the share of delegation in 1960, 1980 and 2007, our model suggests that the Sharpe ratio should have maxed between 1960 and 1980 and should have decreased since then.

Finally, we discuss the sensitivity of our results to variation in the parameters; for this we use Figures 8 and 9. [32] Consider first an increase in the convexity by increasing n_A. As expected, an increase in the convexity leads to more extreme strategies both for managers holding a contrarian portfolio and those holding the levered portfolio (first row of Figure 8). This also leads to a sharper increase in the amount of outstanding bonds as the share of delegation increases (not shown). The effect of increasing κ is a bit more subtle. On one hand, for large enough changes in κ, the system moves between different type of equilibria. This is why we see in the second rows there is a break around $\kappa = 1.08$, where system moves at that point from a Cont-Agg to a Cont-Mod equilibrium, because reaching the high-elasticity segment by an aggressive portfolio becomes too costly. A second break is around $\kappa = 1.18$ where the system switches between a Cont-Mod and a Mod equilibrium, since now even the contrarian portfolio strategy becomes too costly. On the other hand, note from (15) that the locally optimal portfolios are effected by κ only through prices. A higher κ increases the return on risk in a Cont-Agg equilibrium which leads to less extreme portfolios (Figure 8) and smaller increase in gross amount of bond borrowing and lending (not shown). Finally, making the consumption process more skewed by increasing p, leads to complex comparative statics (bottom rows of the three figures). It is so, because p effects portfolios both directly and indirectly through prices. Unreported plots show that a larger p typically decreases the amount of outstanding long bond positions. Also, whenever the aggressive portfolio is held in equilibrium it increases the Sharpe ratio (Figure 9 lower left panel) and makes the contrarian portfolio less extreme (Figure 11 lower left panel) , and has an opposite effect otherwise.

5   Conclusion

In this paper we have introduced delegation into a standard Lucas exchange economy, where in equilibrium some investors trade on their own account, but others (clients) decide to delegate trading in financial assets to funds. Flow-performance incentive functions describe how much

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[31]The state price of the low state relative to the high state increases by 2% (5%) between a share of delegation of 0 and $\Omega^* < \hat{\Omega}$ for the long (short) sample. It then declines to 1% (3.5%) in the long (short) sample as the share of delegation approaches one.

[32]the qualitative results are similar if we perturb parameters around the parameters used in this section.

capital fund clients provide to funds at each date as a function of past performance.

Given the significantly increased fraction of capital that is managed by delegated portfolio management intermediaries over the past 30 year, our analysis has focused on the interaction between the increased share of delegated capital and the empirically observed convex flow-performance relationship. We have been especially interested in the effects of this interaction on asset prices and on agents' optimal portfolios. The basic setup of our economy is intentionally close to the original Lucas model, allowing us a clear comparison of how delegation changes equilibrium dynamics in the Lucas economy.

When the share of delegated capital is low, all funds follow the same strategy. However, when the equilibrium share of delegated capital is high funds with identical incentives utilize heterogeneous trading strategies, trade among themselves, and fund returns are dispersed in the cross-section. As the share of delegated capital increases, so does the fraction of managers holding levered portfolios. Thus, the gross amount of borrowing and lending increases. We connect this fact to the sharpe increase in the size of repo markets and outstanding open interest in futures markets over the last decades. In addition, trading volume increases as well. Our model implies that with convex flow-performance relationship and negatively skewed consumption growth the average fund outperforms the market in recessions and underperforms in expansions; consistent with empirical evidence. We also show that in general there is an inverse U-shape relation between the share of capital that is delegated and the Sharpe ratio.

In our framework investor flows depend on excess returns relative to the market, and are not risk adjusted. First, this is similar to the setting in, for example, Chevalier and Ellison (1997). While evidence shows that institutional investors flows depend on risk adjusted returns, mutual fund investors are less sophisticated, and evidence suggests that their flow decisions do not fully account for risk (See for example Del Guerio and Tkac (2002) and Clifford, Fulkerson, Jordan, and Waldman (2011)). Second, while for a large fraction of the fund universe the market portfolio, proxied by the S&P500, is a natural benchmark to evaluate managers against, this is not the case for all funds; for example bond funds. However, our results on endogenous emergence of heterogeneous trading strategies can also be interpreted as applying to an asset class, instead of the whole market. Furthermore, our results on heterogeneity of strategies should hold in a multi asset setting where the flows to different sub groups of funds are dependent on performance that is evaluated relative to different benchmarks, as long as fund assets under management are sufficiently large.

Similar to the Lucas economy, our model has systematic risk, but no idiosyncratic risk. Consequently, the only way managers can beat the market in the high state is by levering up. Incorporating a source of idiosyncratic risk into the dynamic model is beyond the scope of the current paper. However, to understand the qualitative implications of introducing a source of idiosyncratic risk we have considered a stripped down two period example where we add the ability to take idiosyncratic

risk by entering into a zero net supply futures contract with a futures price of zero.[33] The futures is a derivative on a sunspot: whether the long or the short positions pay-off depends on a flip of a coin.

For direct traders it is obviously sub-optimal to take a position in the futures. When the share of delegation is small, and assuming that κ is not too large, all funds follow a contrarian strategy and also do not use the futures. The pure contrarian strategy dominates since to increase tracking error relative to the market it takes advantage of the negative skewness in returns: increasing tracking error by exposure to idiosyncratic risk is less efficient. As the size of the fund industry increases funds' trading begins to impact prices in equilibrium, reducing the attractiveness of the contrarian strategy and skewness becomes less negative. When the share of delegation becomes sufficiently large heterogeneity in fund strategies emerges. Some managers start using a less conservative yet still contrarian strategy combined with a position in the futures.[34] Such a strategy allows them in addition to being above the kink in the low state, although admittedly to a lesser extent than the pure contrarian strategy, to also be half of the time above the kink in the high state. As the share of delegation continues to grow more and more funds migrate to this strategy, and open interest in futures increases. At a higher share of delegation threshold some funds start following the aggressive strategy.[35] Keeping in mind that due to the negative skewness in returns the aggressive strategy considerably underperforms the market in the low state, the aggressive strategy does not add a position in the futures contract. The amount of exposure to idiosyncratic risk required in order sometimes be above the kink in the low state distorts the returns too much. As the share of delegation grows further the fraction of funds that follow the aggressive strategy increases, and the amount of borrowing and lending grows. Similar to our model, for large shares of delegation, in equilibrium there is always heterogeneity in strategies.

The incentive function we have focused on has two important features: incentives are sufficiently convex and flows depend on performance relative to the market. As we have shown, without sufficiently convex incentives all managers follow the same strategy. For our heterogeneity results to hold performance can be benchmarked relative to a subset of the market. Measuring absolute performance instead of relative performance is equivalent to measuring performance relative to the bond. While absolute performance need not unequivocally preclude ex-ante identical managers from following heterogeneous strategies, it does hinder on the potential emergence of such equilibria.

The fact that in our model managers are ex ante identical, in particular their flows are determined by the same flow-performance relation, allows us to highlight the central point that even ex ante identical managers find it optimal to follow heterogeneous strategies. In, reality different

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[33]Details available from the authors.

[34]If the absolute value of skewness is small this strategy may be the one that is already used when the share of delegation is zero.

[35]Similar to our model in intermediate levels of κ or if p is large instead of an aggressive strategy the moderate strategy is used.

financial intermediaries differ in their clientele, and consequently differ in the shape of the flow-performance relation they are exposed to. In parallel work, we are in the process of analyzing the implications of interaction of different intermediaries with differing incentive functions within the same economy.

Finally, our methodological contribution is to simplify the flow-performance relationship into a piece-wise constant elasticity function. The combination of log utility and piece-wise constant elasticity enables us to derive explicit expression for different model quantities. Arguably, we do not use our modeling framework to its full potential, because we impose a structure which implies a constant share of delegated capital for a given set of parameters. Although, we consider this framework a natural first step, our framework is well suited for the analysis of a truly dynamic structure where the share of delegated capital is a time-varying state variable. With such an extension we could investigate the changing role of different financial intermediaries over the business cycle. This is left for future work.

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A   Appendix

Proof of Lemma 1. The proof is shown in the main text.

Proof of Proposition 1. The proof is shown in the main text.

Proof of Lemma 2 First note that substituting (17)-(19) into (21) gives

$$\frac{\Omega^* \sum_{lh} \mu_{lh}\xi_{lh} + (1-\Omega^*)p}{1 - \frac{R_{t+1}(L)}{r_{f,t}}} + \frac{\Omega^* \sum_{lh} \mu_{lh}\left(1-\xi_{lh}\right) + (1-\Omega^*)p}{1 - \frac{R_{t+1}(H)}{r_{f,t}}} = \tag{38}$$

$$= \tilde{\xi}(\Omega^*) \frac{1}{1 - \frac{R_{t+1}(L)}{r_{f,t}}} + \left(1 - \tilde{\xi}(\Omega^*)\right)\frac{1}{1 - \frac{R_{t+1}(H)}{r_{f,t}}} = 1$$

We show the statement for a low shock. The proof for the high shock is analogous.
The return of a manager holding portfolio α_{lh} at the end of the period is

$$\rho_{t+1}\left(\alpha_{lh}, L\right) = \alpha_{lh}\left(R_{t+1} - r_{f,t}\right) + r_{f,t} = r_{f,t}\left((1-\xi_{lh})\frac{\left(\frac{R_{t+1}(L)}{r_{f,t}} - 1\right)}{1 - \frac{R_{t+1}(H)}{r_{f,t}}} + (1-\xi_{lh})\right) =$$

$$= r_{f,t}\left((1-\xi_{lh})\left(\frac{R_{t+1}(H) - R_{t+1}(L)}{R_{t+1}(H) - r_{f,t}}\right)\right)$$

where we used the definition of π_H, π_L, θ and α_{lh} and that (38) implies

$$\alpha_{lh} = 1 - \frac{1 - \frac{\xi_{lh}}{\tilde{\xi}(\Omega^*)}}{1 - \frac{r_{f,t}}{R_{t+1}(H)}}.$$

Rearranging (38) implies that

$$\left(\frac{R_{t+1}(L) - R_{t+1}(H)}{r_{f,t} - R_{t+1}(H)}\right) = \left(1 - \tilde{\xi}(\Omega^*)\right)\frac{R_{t+1}(L)}{r_{f,t}}$$

which gives

$$\rho_{t+1}\left(\alpha_{lh}, L\right) = r_{f,t}\left((1-\xi_{lh})\left(\frac{R_{t+1}(H) - R_{t+1}(L)}{R_{t+1}(H) - r_{f,t}}\right)\right) = \frac{(1-\xi_{lh})}{1 - \tilde{\xi}(\Omega)}R_{t+1}.$$

This gives (23).

Proof of Theorem 1. In this part, we show that the strategies described by Proposition 1 and Theorem 1 are optimal under Conjectures 1 and 2. In particular, we prove that whenever $\mu_{l_1 h_1} > 0$

for a given $lh = l_1 h_1$ in Theorem 1 then

$$V^{l_1 h_1}\left(w_t^M, s_t, \Omega^*\right) \geq V^{l_2 h_2}\left(w_t^M, s_t, \Omega^*\right) \tag{39}$$

for any $l_2 h_2$ with strict equality if $\mu_{l_2 h_2} > 0$. That is, deviation to an other locally-optimal port-folio from the equilibrium portfolios is suboptimal. First we introduce the analytical formulas for deviations from the prescribed equilibrium portfolios. Second, we show that condition (39) holds for $\Omega^* = 0$. Third, we show that condition (39) holds for $\Omega^* > 0$. Finally, we show that conditions (54)-(55) holds for any Ω^*.

We also show that in Theorem 1,

$$\hat{\kappa}_{high} \equiv \exp\left(\frac{\ln \frac{n_A}{n_B}}{\left(1 - \frac{n_B}{n_A}\right)} + 1\right) \tag{40}$$

$$\hat{\kappa}_{low} = \exp\left(\frac{n_B \ln n_B + n_A \ln n_A - (n_A + n_B) \ln \frac{n_B + n_A}{2}}{n_A - n_B}\right) \tag{41}$$

and $\hat{p}$ is given by the unique solution in $\left[\frac{1}{2}, 1\right]$ of

$$\Delta^{AB-BB}\left(\hat{p}\right) = 0 \tag{42}$$

where

$$\Delta^{l_1 h_1 - l_2 h_2}(p) \equiv p \ln \frac{Z_{h_1}}{Z_{h_2}} \frac{\left(\frac{\xi_{l_1 h_1}}{p}\right)^{n_{h_1}}}{\left(\frac{\xi_{l_2 h_2}}{p}\right)^{n_{h_2}}} + (1-p) \ln \frac{Z_{l_1}}{Z_{l_2}} \frac{\left(\frac{1-\xi_{l_1 h_1}}{1-p}\right)^{n_{l_1}}}{\left(\frac{1-\xi_{l_2 h_2}}{1-p}\right)^{n_{l_2}}},$$

while $\bar{p}$ is given by the unique solution in $\left[\frac{1}{2}, 1\right]$ of

$$\bar{p} \exp\left(\frac{\Delta^{BA-BB}(\bar{p})}{\bar{p}(n_A - n_B)}\right) + (1-\bar{p}) \exp\left(\frac{\Delta^{AB-BB}(\bar{p}_{BA-AB})}{(n_A - n_B)(1-\bar{p})}\right) = 1. \tag{43}$$

Useful expressions for comparing values of locally optimal portfolios

Define $\tilde{V}^{l_1 h_1 - l_2 h_2}(\Omega^*)$ as

$$\tilde{V}^{l_1 h_1 - l_2 h_2}(\Omega^*) \equiv \frac{1-\beta}{\beta}\left(V^{l_1 h_1}\left(w_t^M, s_t, \Omega^*\right) - V^{l_2 h_2}\left(w_t^M, s_t, \Omega^*\right)\right) = \qquad (44)$$

$$p \ln \frac{Z_{h_1}}{Z_{h_2}} \frac{\left(\frac{\xi_{l_1 h_1}}{\tilde{\xi}(\Omega^*)}\right)^{n_{h_1}}}{\left(\frac{\xi_{l_2 h_2}}{\tilde{\xi}(\Omega^*)}\right)^{n_{h_2}}} + (1-p) \ln \frac{Z_{l_1}}{Z_{l_2}} \frac{\left(\frac{1-\xi_{l_1 h_1}}{1-\tilde{\xi}(\Omega^*)}\right)^{n_{l_1}}}{\left(\frac{1-\xi_{l_2 h_2}^1}{1-\tilde{\xi}(\Omega^*)}\right)^{n_{l_2}}} =$$

$$= \Delta^{l_1 h_1 - l_2 h_2}(p) + p\left(n_{h_1} - n_{h_2}\right) \ln \frac{p}{\tilde{\xi}(\Omega^*)} + (1-p)\left(n_{l_1} - n_{l_2}\right) \ln \frac{1-p}{1-\tilde{\xi}(\Omega^*)},$$

the difference in the value of following the locally optimal $l_1 h_1$ and $l_2 h_2$ strategies.

Note also that both in a Cont-Mod and a Cont-Agg equilibrium, we can rewrite the second part of the above expression as

$$p\left(n_{h_1} - n_{h_2}\right) \ln \frac{p}{\tilde{\xi}(\Omega^*)} + (1-p)\left(n_{l_1} - n_{l_2}\right) \ln \frac{1-p}{1-\tilde{\xi}(\Omega^*)} = \qquad (45)$$

$$= \begin{cases} \left(n_{h_1} - n_{h_2}\right) p \ln\left(\frac{\Omega^* n_B}{(1-p)n_A + pn_B} + (1-\Omega^*)\right) & \\ \quad -\left(n_{l_1} - n_{l_2}\right)(1-p) \ln\left(\frac{n_A}{(1-p)n_A + pn_B}\Omega^* + (1-\Omega^*)\right) & for \quad \Omega^* < \hat{\Omega} \\ \left(n_{h_1} - n_{h_2}\right) p \ln\left(\frac{\hat{\Omega} n_B}{(1-p)n_A + pn_B} + \left(1-\hat{\Omega}\right)\right) & \\ \quad -\left(n_{l_1} - n_{l_2}\right)(1-p) \ln\left(\frac{n_A}{(1-p)n_A + pn_B}\hat{\Omega} + \left(1-\hat{\Omega}\right)\right) & otherwise \end{cases}$$

However, the value of $\hat{\Omega}$ depends on the type of the equilibrium. Denoting the type of the equilibrium in the subscript, $\hat{\Omega}_{Cont-Mod}$ and $\hat{\Omega}_{Cont-Agg}$ are defined as the solution of

$$\Delta^{AB-BB}(p) = (1-p)\left(n_A - n_B\right) \ln\left(\hat{\Omega}_{Cont-Mod} \frac{n_A}{(1-p)n_A + pn_B} + \left(1 - \hat{\Omega}_{Cont-Mod}\right)\right). \qquad (46)$$

and

$$\Delta^{AB-BA}(p) = \left(n_A - n_B\right)(1-p) \ln\left(\hat{\Omega}_{Cont-Agg} \frac{n_A}{(1-p)n_A + pn_B} + \left(1 - \hat{\Omega}_{Cont-Agg}\right)\right) \qquad (47)$$

$$- \left(n_A - n_B\right) p \ln\left(\frac{n_B \hat{\Omega}_{Cont-Agg}}{(1-p)n_A + pn_B} + \left(1 - \hat{\Omega}_{Cont-Agg}\right)\right), \qquad (48)$$

respectively.

Global optimality when $\Omega^* = 0$

In this part, we show that under the classification in Theorem 1, condition (39) holds at least when

$\Omega^* = 0$.Note that $\tilde{\xi}(0) = p$ by definition, so 44 implies that

$$\tilde{V}^{l_1 h_1 - l_2 h_2}(0) = \Delta^{l_1 h_1 - l_2 h_2}(p).$$

The following Lemmas characterize $\Delta^{l_1 h_1 - l_2 h_2}(p)$, thus, together with expressions (40)-(41), imply the result.

Lemma A.1 $\Delta^{BA-AB}(p) < 0.$

Proof. Consider that

$$\Delta^{BA-AB}(p) \equiv (n_A - n_B)((1-p) - p)\ln \kappa + \ln \frac{\left(\frac{n_B}{((1-p)n_B + pn_A)}\right)^{(1-p)n_B}\left(\frac{n_A}{((1-p)n_B + pn_A)}\right)^{pn_A}}{\left(\frac{n_A}{((1-p)n_A + pn_B)}\right)^{(1-p)n_A}\left(\frac{n_B}{((1-p)n_A + pn_B)}\right)^{pn_B}}.$$

Observe that

$$\Delta^{BA-AB}(1) = -(n_A - n_B)\ln \kappa < 0$$

$$\Delta^{BA-AB}\left(\frac{1}{2}\right) = 0.$$

and

$$\frac{\partial^2 \Delta^{BA-AB}(p)}{\partial^2 p} = (n_A - n_B)^3 \frac{2p - 1}{((1-p)n_A + pn_B)(n_B(1-p) + pn_A)} > 0.$$

Thus, there cannot be a maximum in the range $\left(\frac{1}{2}, 1\right)$. Thus, $\Delta^{BA-AB}(p) < 0$ for all p. ∎

Lemma A.2 $\Delta^{AB-BB}(p) < 0$ *for all* p, *if*

$$\kappa > \hat{\kappa}_{high}.$$

If

$$\hat{\kappa}_{low} > \kappa.$$

then $\Delta^{AB-BB}(p) > 0$ *for all* $p > \frac{1}{2}$. *If* $\hat{\kappa}_{low} < \kappa < \hat{\kappa}_{high}$ *then there is* $\hat{p} > \frac{1}{2}$ *that* $\Delta^{AB-BB}(p) < 0$ *iff* $p < \hat{p}$ *and* $\Delta^{AB-BB}(\hat{p}) = 0.$

Proof. Note that

$$\Delta^{AB-BB}(p) \equiv -(1-p)(n_A - n_B)\ln \kappa - (pn_B + (1-p)n_A)\ln(pn_B + (1-p)n_A) +$$
$$+ pn_B \ln n_B + (1-p)n_A \ln n_A.$$

As
$$\frac{\partial^2 \Delta^{AB-BB}(p)}{\partial^2 p} = -\frac{(n_B - n_A)^2}{(pn_B + (1-p)n_A)} < 0$$

this function does not have an interior minimum. Also, it is simple to check that $\Delta^{AB-BB}(1) = 0$, $\frac{\partial \Delta^{AB-BB}(p)}{\partial p}|_{p=1}$ is positive if and only if $\kappa > \hat{\kappa}_{high}$ and that $\Delta^{AB-BB}\left(\frac{1}{2}\right)$ is positive if and only if $\kappa < \hat{\kappa}_{low}$. These properties imply the statement. ∎

Lemma A.3 *If*
$$\kappa > \hat{\kappa}_{low}$$

then $\Delta^{BA-BB}(p) < 0$ *for all p. If* $\hat{\kappa}_{low} > \kappa$ *then* $\Delta^{BA-BB}(p) > 0$ *iff* $p < \hat{p}_{BA-BB}$ *and* $\hat{p}_{BA-BB} \in \left[\frac{1}{2}, 1\right]$.

    Proof. The proof is analogous to Lemma A.2, so it is omitted. ∎

    Global optimality when $\Omega^* > 0$
In this part, we prove that condition (39) holds for any $\Omega^* > 0$ under the classification in Theorem 1. We start with two Lemmas.

Lemma A.4 *If either* $\hat{\kappa}_{low} < \kappa < \hat{\kappa}_{high}$ *and* $p > \hat{p}$ *or* $\kappa < \hat{\kappa}_{low}$, *there is* $\hat{\Omega}_{Cont-Mod} \in (0,1)$ *which solves*

$$\Delta^{AB-BB}(p) = (1-p)(n_A - n_B)\ln\left(\hat{\Omega}_{Cont-Mod}\frac{n_A}{(1-p)n_A + pn_B} + \left(1 - \hat{\Omega}_{Cont-Mod}\right)\right)$$

    Proof. We have shown in Lemma A.2 that under the conditions of this Lemma $\Delta^{AB-BB}(p) > 0$. As the left hand side is zero for $\hat{\Omega} = 0$, we only have to prove that

$$\Delta^{AB-BB}(p) < (1-p)(n_A - n_B)\ln\left(\hat{\Omega}\frac{n_A}{(1-p)n_A + pn_B} + \left(1 - \hat{\Omega}\right)\right)|_{\hat{\Omega}=1}.$$

Substituting in the expression $\Delta^{AB-BB}(p)$ from Lemma A.2 shows that the inequality is equivalent to

$$0 < (1-p)(n_A - n_B)\ln\kappa + n_B\left(\ln(pn_B + (1-p)n_A) - p\ln n_B - (1-p)\ln n_A\right)$$

which holds by the concavity of the logarithmic function. ∎

Lemma A.5 *Consider the system in p and* Ω

$$\begin{aligned}
\Delta^{AB-BB}(p) &= (n_A - n_B)(1-p)\ln\left(\Omega\frac{n_A}{(1-p)n_A + pn_B} + (1-\Omega)\right) \\
\Delta^{BA-BB}(p) &= (n_A - n_B)p\ln\left(\frac{n_B\Omega}{(1-p)n_A + pn_B} + (1-\Omega)\right).
\end{aligned}$$

It has no solution if

$$\hat{\kappa}_{low} < \kappa \tag{49}$$

and it has a single solution $(\bar{p}, \bar{\Omega})$ in the range $\bar{p} \in \left(\frac{1}{2}, 1\right)$.

Proof. Note that the system is equivalent to

$$\exp\left(\frac{\Delta^{AB-BB}(p)}{(n_A - n_B)(1-p)}\right) \equiv \left(\Omega\frac{n_A}{(1-p)n_A + pn_B} + (1 - \Omega)\right)$$
$$\exp\left(\frac{\Delta^{BA-BB}(p)}{(n_A - n_B)p}\right) = \left(\frac{n_B\Omega}{(1-p)n_A + pn_B} + (1 - \Omega)\right),$$

hence, any solution of the system has to satisfy

$$\tilde{\Pi}(p) \equiv (1-p)\exp\left(\frac{\Delta^{AB-BB}(p)}{(n_A - n_B)(1-p)}\right) + p\exp\left(\frac{\Delta^{BA-BB}(p)}{(n_A - n_B)p}\right) = 1.$$

From

$$\frac{\Delta^{BA-BB}(p)}{p(n_A - n_B)} = -\ln\kappa - \frac{n_A}{n_A - n_B}\ln\frac{((1-p)n_B + pn_A)}{n_A}$$
$$-\frac{(1-p)}{p}\frac{n_B}{n_A - n_B}\ln\frac{((1-p)n_B + pn_A)}{n_B}$$
$$\frac{\Delta^{AB-BB}(p)}{(1-p)(n_A - n_B)} = -\ln\kappa - \frac{n_A}{n_A - n_B}\ln\frac{(pn_B + (1-p)n_A)}{n_A}$$
$$-\frac{p}{1-p}\frac{n_B}{n_A - n_B}\ln\frac{pn_B + (1-p)n_A}{n_B}$$

observe that this function is symmetric in the sense that if

$$\Pi(p) \equiv p\exp\left(\frac{\Delta^{BA-BB}(p)}{p(n_A - n_B)}\right)$$

then

$$\tilde{\Pi}(p) = \Pi(p) + \Pi(1-p)$$

which implies

$$\tilde{\Pi}(p) = \tilde{\Pi}(1-p).$$

Also

$$\begin{aligned}
\frac{\partial \Pi(p)}{\partial p} &= e^{\frac{\Delta_{BA-BB}(p)}{p(n_A - n_B)}} \left(1 + p \frac{\partial \left(\frac{\Delta_{BA-BB}(p)}{p(n_A - n_B)} \right)}{\partial p} \right) = \\
&= e^{\frac{\Delta_{BA-BB}(p)}{p(n_A - n_B)}} \frac{1}{p} \frac{n_B}{n_A - n_B} \ln \frac{((1-p) n_B + p n_A)}{n_B} > 0.
\end{aligned}$$

and

$$\lim_{p \to 0} \tilde{\Pi}(p) = \lim_{p \to 1} \tilde{\Pi}(p) = \frac{1}{\kappa} < 1.$$

Thus, $\tilde{\Pi}(p)$ is increasing for $p < \frac{1}{2}$ and decreasing for $p > \frac{1}{2}$ and its maximum is at $p = \frac{1}{2}$. If $\hat{\kappa}_{low} < \kappa$ holds, then

$$\tilde{\Pi}\left(\frac{1}{2} \right) = 2\Pi\left(\frac{1}{2} \right) < 1,$$

which implies that $\tilde{\Pi}(p) = 1$ does not have a solution. However, if $\hat{\kappa}_{low} > \kappa$ holds, then $\tilde{\Pi}(p) = 1$ has two solutions. If we denote the first by $\bar{p} > \frac{1}{2}$ then the second one is $(1 - \bar{p})$. Note that $n_A > n_B$ implies that a given p' can be the part of the solution of our system only if $\Delta^{BA-BB}(p') < 0$ and $\Delta^{AB-BB}(p') > 0$. Also, by Lemmas A.2- A.3, this is possible only if $p' > \frac{1}{2}$. Thus, the only relevant solution is $(\bar{p}, \bar{\Omega})$ where $\bar{\Omega}$ solves

$$\Delta^{BA-BB}(\bar{p}) = (n_A - n_B) \bar{p} \ln \left(\frac{n_B \bar{\Omega}}{(1-p) n_A + p n_B} + (1 - \bar{\Omega}) \right).$$

■

To see that Theorem 1 holds, first note from (44)-(45) that for $l_1 h_1 = BA, AB$ $\tilde{V}^{l_1 h_1 - BB}(\Omega^*)$ is monotonically decreasing for any $\Omega^* < \hat{\Omega}$ and constant for $\Omega^* > \hat{\Omega}$ regardless of the type of equilibrium. The first implies that if the moderate portfolio dominates an other locally optimal portfolio at $\Omega = 0$, then it dominates it for any Ω. This monotonicity together with Lemmas A.3 and A.2 imply that if either $\kappa > \hat{\kappa}_{high}$ or $\hat{\kappa}_{low} < \kappa < \hat{\kappa}_{high}$ and $p \in \left(\frac{1}{2}, \hat{p} \right)$, then

$$\begin{aligned}
\tilde{V}^{AB-BB}(\Omega^*) &< 0 \\
\tilde{V}^{BA-BB}(\Omega^*) &< 0
\end{aligned}$$

for all Ω^*. Thus, the locally optimal moderate portfolio is globally optimal.

Also, for $l_2 h_2 = BB, BA$ $\tilde{V}^{AB-l_2 h_2}(\Omega^*)$ is monotonically decreasing for any $\Omega^* < \hat{\Omega}$ and constant for $\Omega^* > \hat{\Omega}$ regardless of the type of the equilibrium. Thus, in all remaining cases of Theorem 1 it is sufficient to show that $\hat{\Omega}_{Cont-Mod}$ always exist in the range $(0, 1)$ and whenever both $\hat{\Omega}_{Cont-Mod} < (>)\hat{\Omega}_{Cont-Agg}$ exist and Theorem 1 describes a Cont-Mod (Cont-Agg) equilibrium then $\hat{\Omega}_{Cont-Mod} < (>)\hat{\Omega}_{Cont-Agg}$.

For example, for a Cont-Agg equlibrium, we need that at $\Omega = 0$, $\tilde{V}^{AB-l_2h_2}(0) = \Delta^{AB-l_2h_2}(p) > 0$, for $l_2h_2 = BB, BA$ so managers prefer the contrarian portfolio initially and as we increase Ω, we reach a Ω' where managers are indifferent between the contrarian and aggressive portfolios before we were to reach a Ω'' where they are indifferent between the contrarian and moderate portfolios, i.e., $\hat{\Omega}_{Cont-Mod} > \hat{\Omega}_{Cont-Agg}$.

The existence of $\hat{\Omega}_{Cont-Mod}$ under the relevant parameter restrictions is ensured by Lemma A.4.

To compare $\hat{\Omega}_{Cont-Mod}$ and $\hat{\Omega}_{Cont-Agg}$ consider expression

$$\Delta^{AB-BB}(p) = (1-p)(n_A - n_B) \ln \left(\Omega \frac{n_A}{(1-p)n_A + pn_B} + (1-\Omega) \right). \tag{50}$$

as an implicit function giving a p for any given Ω whenever $\hat{\Omega}_{Cont-Mod}$ exists. Let us call this function $\mathbf{p}_1(\Omega)$. By definition, in a Cont-Mod equilibrium, $p = \mathbf{p}_1\left(\hat{\Omega}_{Cont-Mod}\right)$. Similarly,

$$\begin{aligned} \Delta^{AB-BA}(p) &= (n_A - n_B)(1-p) \ln \left(\Omega \frac{n_A}{(1-p)n_A + pn_B} + (1-\Omega) \right) \\ &\quad - (n_A - n_B) p \ln \left(\frac{n_B \Omega}{(1-p)n_A + pn_B} + (1-\Omega) \right), \end{aligned} \tag{51}$$

determine a function $\mathbf{p}_2(\Omega)$ which gives a p for any given Ω, whenever $\hat{\Omega}_{Cont-Agg}$ exists. By definition, in a Cont-Agg equilibrium $p = \mathbf{p}_2\left(\hat{\Omega}_{Cont-Agg}\right)$. Note that the system in Lemma A.5 and (50)-(51) are equivalent, because subtracting (50) from (51) gives the second equation in Lemma A.5. Thus, if $\hat{\kappa}_{low} < \kappa < \hat{\kappa}_{high}$ then Lemmas A.2-A.3 imply

$$\mathbf{p}_2(0) < \frac{1}{2} < \mathbf{p}_1(0)$$

and Lemma A.5 ensures that the functions $\mathbf{p}_1(\Omega), \mathbf{p}_2(\Omega)$ do not cross in the space $[0,1] X \left[\frac{1}{2}, 1\right]$. That is, $\hat{\Omega}_{Cont-Mod} < \hat{\Omega}_{Cont-Agg}$ for all possible p. This implies a Cont-Mod equilibrium.

If $\hat{\kappa}_{low} > \kappa$ then Lemmas A.2-A.3 imply that

$$\mathbf{p}_1(0) < \frac{1}{2} < \mathbf{p}_2(0)$$

and Lemma A.5 ensures that the functions $\mathbf{p}_1(\Omega), \mathbf{p}_2(\Omega)$ cross exactly once in the space $[0,1] X \left[\frac{1}{2}, 1\right]$. The intersection is given by the pair $(\bar{p}, \bar{\Omega})$. Thus, whenever $\frac{1}{2} < p < \bar{p}$, $\hat{\Omega}_{Cont-Agg} < \hat{\Omega}_{Cont-Mod}$ while the relationship reverses if $p > \bar{p}$. This concludes the proof.

Proof of Lemma 3. The main steps for the derivation of expressions of **Lemma 3** are given in the main text. The expression for $\bar{\chi}_s$ is a direct consequence of (25), and the expression for Γ_s is a

direct consequence of (26)

Proof of Proposition 2.

To complete the proof of Proposition 2 we have to find thresholds $\hat{Z}$ and $\hat{\lambda}$. Threshold $\hat{Z}$ comes from the requirement that the delegated share of capital for any of the managers following equilibrium strategies in any of the states always have to be smaller than 1, i.e.

$$g_{1H} \equiv g\left(\frac{\xi_{AB}}{\tilde{\xi}(\Omega^*)}\right), g_{1L} \equiv g\left(\frac{1-\xi_{AB}}{1-\tilde{\xi}(\Omega^*)}\right), g_{2H} \equiv g\left(\frac{\xi_2}{\tilde{\xi}(\Omega^*)}\right), g_{2L} \equiv g\left(\frac{1-\xi_2}{1-\tilde{\xi}(\Omega^*)}\right) < 1$$

where $\xi_2 = \xi_{BA}, p$ in a $Cont - Agg$ equilibrium and a $Cont - Mod$ equilibrium respectively. As all these expressions are proportional to Z_B such $\hat{Z}$ clearly exists. While the threshold $\hat{\lambda}$ comes from the requirement that $\bar{\chi}_s^j$ is between zero and 1. Such $\hat{\lambda}$ exists by the following arguments. For any other parameters $\Omega^* = 1$ implies $\bar{\chi}_s = 1$, and $\Omega^* = 0$ implies $\bar{\chi}_s = 0$ by simple substitution, while for any $\Omega^* \in (0,1)$

$$\lim_{\lambda \to 0} \bar{\chi}_s = \lim_{\lambda \to 0} \bar{g}_s \frac{\Gamma_s - \lambda}{(1-\lambda)\beta^I} = \frac{\Omega^*}{\beta(1-\Omega^*) + \Omega^*} \in (0,1).$$

Thus, sufficiently low λ pushes $\bar{\chi}_s$ into $[0,1]$ for any Ω^* by continuity.

To conclude the proof, we only have to verify that for any Ω^*, there is an f which makes investors indifferent between being direct traders or clients and that this relationship is continuous. Conjecture that the value functions of direct traders and clients have the form of

$$V^D(w, s_{t-1}) = \frac{1}{1-\beta^I} \ln w + \Lambda_{s_{t-1}}^D$$

and

$$V^C(w, \upsilon_{t-1}, s) = \frac{1}{1-\beta^I} \ln w + \Lambda_{\upsilon_{t-1}s, s_{t-1}}^C$$

where υ_{t-1} is the relative return of the manager given the followed strategy and the state. For the direct trader, substituting direct traders' optimal choices into problem (7) immediately validates the conjecture. For clients, we use Lemma 2 for the implied relative returns of equilibrium strategies, and the fact that managers follow mixed strategies. Thus, from the point of view of the client, all manager is expected to follow the same mixed strategy regardless of the followed strategy of the previous period. Then ,straightforward, but tedious algebra gives explicit expressions for $\Lambda_{\upsilon_{t-1}s, s_{t-1}}^C$ which validates the conjecture. we omit these steps here and give only the final expression for the expected difference between the equilibrium values of being a client or a manager

$$\left(EV^D - EV^C\right)\left(1 - \beta^I\right)^2 = -\ln\beta + \beta^I \ln\beta^I + \left(1 - \beta^I\right)\ln(1 - \beta^I)$$

$$- \mu \left(\begin{array}{c} p\left(\left(1 - \beta^I\right)\ln\left(1 - g_{1H}\right) + \beta^I \ln g_{1H} + \beta^I \ln\frac{\xi_1}{p}\right) + \\ + (1 - p)\left(\left(1 - \beta^I\right)\ln\left(1 - g_{1L}\right) + \beta^I \ln g_{1L} + \beta^I \ln\frac{(1-\xi_1)}{1-p}\right) \end{array} \right)$$

$$- (1 - \mu)\left(\begin{array}{c} p\left(\left(1 - \beta^I\right)\ln\left(1 - g_{2H}\right) + \beta^I \ln g_{2H} + \beta^I \ln\frac{\xi_2}{p}\right) \\ + (1 - p)\left(\left(1 - \beta^I\right)\ln\left(1 - g_{2L}\right) + \beta^I \ln g_{2L} + \beta^I \ln\frac{(1-\xi_2)}{1-p}\right) \end{array} \right) \quad (52)$$

Picking $f = \left(EV^D - EV^C\right)$ satisfies our conditions.

Proof of Lemma 4. For the first part, substituting back α_{lh} and ψ^M into the value function and using Lemma 2 implies that our conjecture is correct with the choice of function $\Lambda^{lh}\left(s_t, \Omega^*\right)$ solving

$$\Lambda^{lh}\left(s_t, \Omega^*\right) = \ln\left(1 - \beta\right) + \quad (53)$$

$$+ \beta\frac{1}{1 - \beta}p\ln Z_h\left(\frac{\xi_{lh}}{\tilde{\xi}\left(\Omega^*\right)}\right)^{n_h}\frac{1}{\pi_{s_t}}y_H\left(1 + \pi_H\right)\beta +$$

$$+ \beta\frac{1}{1 - \beta}\left(1 - p\right)\ln Z_l\left(\frac{1 - \xi_{lh}}{1 - \tilde{\xi}\left(\Omega^*\right)}\right)^{n_l}\frac{1}{\pi_{s_t}}y_L\left(1 + \pi_L\right)\beta$$

$$+ \beta\left(p\Lambda^{lh}\left(H, \Omega^*\right) + (1 - p)\Lambda^{lh}\left(L, \Omega^*\right)\right)$$

which has the conjectured form.

For the second part, the Lemmas below prove that whenever $\mu_{l_1 h_1} > 0$ for a given $lh = l_1 h_1$ in Theorem 1 then

$$\frac{\xi_{l_1 h_1}}{\tilde{\xi}\left(\Omega^*\right)} > (<)\kappa \quad (54)$$

if $h_1 = A(B)$ and

$$\frac{1 - \xi_{l_1 h_1}}{1 - \tilde{\xi}\left(\Omega^*\right)} > (<)\kappa \quad (55)$$

if $l_1 = A(B)$.

Lemma A.6 *Suppose that* $\Omega^* > \hat{\Omega}$. *Then*

1. $\tilde{V}^{BA-BB}\left(\Omega^*\right) > 0$ *implies*

$$\frac{\xi_{BA}}{\xi_{BA}\left(\hat{\Omega}\right)} = \frac{\frac{n_A}{pn_A + (1-p)n_B}}{\hat{\Omega}\frac{n_A}{pn_A + (1-p)n_B} + \left(1 - \hat{\Omega}\right)} > \kappa.$$

2. $\tilde{V}^{AB-BB}(\Omega^*) > 0$ *implies*

$$\frac{1 - \xi_{AB}}{1 - \xi_{AB}(\Omega^*)} > \kappa.$$

Proof. As the proofs of the two parts are analogous, we prove only the first part.

$$0 < \tilde{V}^{BA-BB}(\Omega^*) =$$

$$= \tilde{V}^{BA-BB}(0) - p(n_A - n_B)\ln\left(\hat{\Omega}\frac{pn_A}{pn_A + (1-p)n_B} + p\left(1 - \hat{\Omega}\right)\right) =$$

$$= (n_A - n_B)p\ln\frac{\overline{\frac{n_A}{((1-p)n_B + pn_A)}}}{\kappa} + n_B\ln\frac{n_A^p n_B^{(1-p)}}{((1-p)n_B + pn_A)}$$

$$- p(n_A - n_B)\ln\left(\hat{\Omega}\frac{n_A}{pn_A + (1-p)n_B} + \left(1 - \hat{\Omega}\right)\right) =$$

$$= (n_A - n_B)p\ln\frac{\overline{\frac{n_A}{((1-p)n_B + pn_A)}}}{\kappa\left(\hat{\Omega}\frac{n_A}{pn_A + (1-p)n_B} + \left(1 - \hat{\Omega}\right)\right)} +$$

$$+ n_B\ln\frac{n_A^p n_B^{(1-p)}}{((1-p)n_B + pn_A)}$$

As $\frac{n_A^p n_B^{(1-p)}}{((1-p)n_B + pn_A)} < 1$ because of the inequality of arithmetic and geometric means,

$$\frac{\overline{\frac{n_A}{((1-p)n_B + pn_A)}}}{\left(\hat{\Omega}\frac{n_A}{((1-p)n_B + pn_A)} + \left(1 - \hat{\Omega}\right)\right)} > \kappa$$

must hold. ∎

Proof of Proposition 3. Rearranging (38) and plugging in the definitions for r_f, we get (31). We get the expression for the price-dividend ratio by the market clearing condition for the good market

$$\delta_t = \left(\left(1 - \lambda\right)\left(1 - \beta^I\right) + \lambda\left(\tilde{\Upsilon}_s - \bar{g}_s\right) + \lambda\left(1 - \tilde{\Upsilon}_s\right)\left(1 - \beta^I\right) + (1 - \beta)\Gamma_s\bar{g}_s\right)(\delta_t + q_t)$$

where the terms in the bracket on the right hand side are the consumption share of newborns, the consumption share of old clients, the consumption share of old direct traders and the consumption share of managers respectively. Simple algebra gives π_H and π_L.

Proof of Proposition 4. Note that Lemma 2, Proposition 1, and Theorem 1 together imply that

the average funds' excess log-return is

$$
\int \ln \rho_{t+1}\left(\alpha_t^m, s_{t+1}\right) dm - \ln R_{t+1}(s_{t+1}) =
$$

$$
= \left\{
\begin{array}{ll}
\ln \frac{\xi_{AB}}{\tilde{\xi}(\Omega^*)} & if \quad s_{t+1} = H \text{ and } \Omega^* < \hat{\Omega} \\
\ln \frac{\mu_{AB}(\Omega^*)\xi_{AB}+(1-\mu_{AB}(\Omega^*))\xi_2}{\bar{\xi}} & if \quad s_{t+1} = H \text{ and } \Omega^* \geq \hat{\Omega} \\
\ln \frac{1-\xi_{AB}}{1-\tilde{\xi}(\Omega^*)} & if \quad s_{t+1} = L \text{ and } \Omega^* < \hat{\Omega} \\
\ln \frac{1-\mu_{AB}(\Omega^*)\xi_{AB}-(1-\mu_{AB}(\Omega^*))\xi_2}{1-\bar{\xi}} & if \quad s_{t+1} = L \text{ and } \Omega^* \geq \hat{\Omega}
\end{array}
\right\}
$$

where $\xi_2 = \xi_{BA}$ in a $Cont - Agg$ equilibrium and $\xi_2 = p$ in a $Cont - Mod$ equilibrium. This implies that the volatility of the average funds' excess log return is

$$
p\left(1-p\right) \left\{
\begin{array}{ll}
\left(\ln \frac{\xi_{AB}}{\tilde{\xi}(\Omega^*)} - \ln \frac{1-\xi_{AB}}{1-\tilde{\xi}(\Omega^*)}\right)^2 & if \quad \Omega^* < \hat{\Omega} \\
\mu_{AB}\left(\Omega^*\right)\left(\ln \frac{1-\bar{\xi}}{\bar{\xi}}\frac{\xi_{AB}}{1-\xi_{AB}}\right)^2 + \left(1-\mu_{AB}\left(\Omega^*\right)\right)\left(\ln \frac{\bar{\xi}}{1-\bar{\xi}}\frac{1-\xi_2}{\xi_2}\right)^2 & if \quad \Omega^* \geq \hat{\Omega}
\end{array}
\right\}
$$

while the cross-sectional dispersion of managers excess log-return is proportional to

$$
\left\{
\begin{array}{ll}
\mu_{AB}\left(\Omega^*\right)\left(1-\mu_{AB}\left(\Omega^*\right)\right)\ln \frac{\xi_2}{\xi_{AB}} & if \quad s_{t+1} = H \text{ and } \Omega^* \geq \hat{\Omega} \\
\mu_{AB}\left(\Omega^*\right)\left(1-\mu_{AB}\left(\Omega^*\right)\right)\ln \frac{1-\xi_{AB}}{1-\xi_2} & if \quad s_{t+1} = L \text{ and } \Omega^* \geq \hat{\Omega}
\end{array}
\right\}
$$

and 0 otherwise.

Combining the definition of the aggregate shape adjusted probability (Equation 20), with the fact that in equilibrium managers use at most two distinct strategies, and the fact that for $\Omega^* > \hat{\Omega}$ $\tilde{\xi}\left(\Omega^*\right) = \bar{\xi}$ yield that

$$
\mu_{AB}\left(\Omega^*\right) = \left\{
\begin{array}{ll}
1 & if \quad \Omega^* \leq \hat{\Omega} \\
\frac{\bar{\xi}-\xi_2}{\xi_{AB}-\xi_2} + \left(\frac{1}{\Omega^*}-1\right)\frac{\bar{\xi}-p}{\xi_{AB}-\xi_2} & if \quad \Omega^* > \hat{\Omega}
\end{array}
\right\}
\tag{56}
$$

Statement 1 and 4 comes directly from the facts that $\xi_{AB} < \tilde{\xi}\left(\Omega^*\right)$ and $\frac{\partial \mu_{AB}(\Omega^*)}{\partial \Omega^*} < 0$ and $\frac{1-\mu_{AB}(1)\xi_{AB}-(1-\mu_{AB}(1))\xi_2}{1-\bar{\xi}} = 1$. **Statement 2** is direct consequence of Proposition 1 and Theorem 1 and that $\frac{\partial \mu_{AB}(\Omega^*)}{\partial \Omega^*} < 0$. For **Statement 3**, it is sufficient that

$$
\frac{\xi_2}{\xi_{AB}} < \frac{1-\xi_{AB}}{1-\xi_2}.
$$

In a Cont-Mod equilibrium this is equivalent to

$$
\frac{\left(1-p\right)n_A + pn_B}{n_B} < \frac{n_A}{\left(1-p\right)n_A + pn_B}
$$

or

$$p > \frac{\sqrt{\frac{n_A}{n_B}}}{\sqrt{\frac{n_A}{n_B}} + 1}.$$

Substituting for Cont-Agg equilibrium shows that the condition always holds. **Statement 5** is a consequence of $\frac{\partial \tilde{\xi}(\Omega^*)}{\partial \Omega^*} < 0$.

Proof of Lemma 5. Observe that reading (31) as $\pi E(\phi_s) = \frac{\pi}{R}$ where ϕ_s is the state price, one can see that

$$\phi_H = \frac{\tilde{\xi}(\Omega^*)}{p} \frac{1}{\frac{1}{\pi} y_H (1 + \pi_H(\Omega_t^*))}$$

$$\phi_L = \frac{\left(1 - \tilde{\xi}(\Omega^*)\right)}{1 - p} \frac{1}{\frac{1}{\pi} y_L (1 + \pi_L(\Omega_t^*))} = .$$

By definition, $X(\Omega^*) = \frac{\phi_L}{\phi_H}$ which gives our decomposition..Also, the Sharpe ratio is

$$S(\Omega^*) = \frac{\sqrt{Var(\phi_s)}}{E(\phi_s)} = \frac{p^{\frac{1}{2}}(1-p)^{\frac{1}{2}} \left\| \frac{\left(1 - \tilde{\xi}(\Omega^*)\right)}{1-p} y_H (1 + \pi_H(\Omega^*)) - \frac{\tilde{\xi}(\Omega^*)}{p} y_L (1 + \pi_L(\Omega^*)) \right\|}{\tilde{\xi}(\Omega^*) y_L (1 + \pi_L(\Omega^*)) + \left(1 - \tilde{\xi}(\Omega^*)\right) y_H (1 + \pi_H(\Omega^*))}$$

$$= \frac{p^{\frac{1}{2}}(1-p)^{\frac{1}{2}} \|y_H X(\Omega^*) - y_L\|}{p y_L + (1-p) y_H X(\Omega^*)}.$$

Proof of Proposition 5. Given that $\tilde{\xi}(\Omega^*)$ is constant in the region $\Omega^* > \hat{\Omega}$, expression (34) combined with the fact that the Sharpe ratio is monotone in $X(\Omega^*)$ shows that in this region the Sharpe ratio is monotone in $\frac{y_H(1+\pi_H)}{y_L(1+\pi_L)}$. Rewriting the equilibrium strategies as

$$\alpha_{lh} = 1 - \frac{1 - \frac{\xi_{lh}}{\xi}}{1 - \frac{1}{\tilde{\xi}(\Omega^*) + \left(\tilde{\xi}(\Omega^*)\right) \frac{y_H(1+\pi_H)}{y_L(1+\pi_L)}}}$$

gives the result.

Proof of Lemma 6. We already showed in the proof of Proposition 5 that $(1 - \alpha_{AB})$ is increasing in Ω^* in the region $\Omega^* > \hat{\Omega}$ whenever the Sharpe ratio is decreasing. Note also that using (56) in a Cont-Mod equilibrium

$$\frac{\partial \mu_{AB}(\Omega^*) \Omega^*}{\partial \Omega^*} = 0$$

while in a Cont-Agg equilibrium

$$\frac{\partial \mu_{AB}(\Omega^*)\,\Omega^*}{\partial \Omega^*} = \frac{\xi_{BA} - p}{\xi_{BA} - \xi_{AB}} > 0.$$

Putting together these two points gives the result.

Proof of Lemma 7. Dividing (36) by (37), using the facts that $p > \tilde{\xi}(\Omega^*) > \xi_{AB}$ and that for $\Omega^* > \hat{\Omega}$ $\tilde{\xi}(\Omega^*) = \bar{\xi}$, and simplifying gives

$$\frac{\Omega^* \mu_{AB}(\Omega^*)}{1 - \Omega^*} \frac{\bar{\xi} - \xi_{AB}}{p - \bar{\xi}}$$

Plugging in (56) for μ_{AB} gives

$$\frac{1}{1 - \Omega^*}\left(\frac{p - \bar{\xi}}{p - \xi_{AB}}\right)\frac{\bar{\xi} - \xi_{AB}}{p - \bar{\xi}}$$

for Cont-Mod and

$$\frac{1}{1 - \Omega^*}\left(\frac{\xi_{BA} - \bar{\xi}}{\xi_{BA} - \xi_{AB}} - (1 - \Omega^*)\frac{\xi_{BA} - p}{\xi_{BA} - \xi_{AB}}\right)\frac{\bar{\xi} - \xi_{AB}}{p - \bar{\xi}}$$

for Cont-Agg.

The result follows by taking a derivative with respect to Ω^* and noting that

$$\frac{\xi_{BA} - \bar{\xi}}{\xi_{BA} - \xi_{AB}} > \frac{p - \bar{\xi}}{p - \xi_{AB}} > 0$$

Proof of Lemma 8. $n_A = n_B \Rightarrow \xi_{lh} = p \Rightarrow \tilde{\xi}(\Omega^*) = p$. The fact that direct traders hold the market portfolio (Lemma 1) and have log utility implies that $\pi_H = \pi_L$. Combining these two implies that $X(\Omega^*) = 1$, in Equation (34), which implies that the Sharpe ratio (Equation 33)coincides with the one in the Lucas economy.