JOB MARKET PAPER:
Measuring Agency Costs over the Business Cycle*

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ABSTRACT

This paper investigates the effects of manager-shareholder agency conflicts on corporate policies in a structural model with intertemporal macroeconomic risk. In the model, a firm consists of assets in place and a growth option, and is run by a self-interested manager who receives part of the firm's free cash flows as private benefits. Fitting the model, parameter estimates imply substantial agency costs due to managerial diversion at initiation (around 3%), and higher agency costs for growth firms than for value firms (3.45% vs. 1.77%). Further, aggregate dynamic agency costs are strongly procyclical (on average, 2.31% in boom and 0.95% in recession periods). The reason for the latter observation is that, in times of recession, firms profit from managerial underleverage, which increases the distance to costly default. Finally, the model also generates predictions regarding default and investment rates, as well as on the intertemporal pattern of investment.

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1. Introduction

Macroeconomic conditions matter in powerful ways for corporate credit risk, because both a firm's default probability and the loss given default typically increase during economic recessions. Hence, market frictions such as the tax benefits of debt and bankruptcy costs exhibit regime-dependency, a feature inherited by corporate policies because they are determined by trading off the effects of market frictions (Hackbarth, Miao, and Morellec, 2006). Consequently, macroeconomic conditions have important implications for corporate securities, firm value, leverage, credit spreads, and default and investment behavior. Additionally, a crucial determinant of corporate policies is conflicts of interest between claimholders, in particular between managers and shareholders (see, for example, Stulz, 1990). Manager-shareholder agency conflicts result in sizeable agency costs in both level and variation across firms (Morellec, Nikolov, and Schürhoff, 2012a). Further, these agency conflicts are successfully used to explain a number of empirical regularities such as the underleverage puzzle (Morellec, 2004), cash balances (Nikolov and Whited, 2011), or the dynamics of leverage (Morellec, Nikolov, and Schürhoff, 2012a). Despite the qualitative and quantitative importance of both macroeconomic conditions and manager-shareholder agency conflicts, little is known on how they interact, on the consequences of their interaction for corporate policies, and on the resulting implications. In particular, how large are the resulting agency costs in the cross section, and how do they vary over the business cycle? What are the impacts on default and investment behavior?

The purpose of this paper is, therefore, to address the question of how macroeconomic regimes impact manager-shareholder agency conflicts, and how the regimes thereby influence managers' corporate policies and their implications. To do so, I develop a structural tradeoff model with intertemporal macroeconomic risk, explicitly taking into account manager-shareholder agency conflicts.[1] Changing macroeconomic conditions imply time variations in the risk free rate. Further, I assume that the stochastic discount factor prices both firm-specific shocks and economy-wide shocks. Market frictions are introduced by incorporating taxes and bankruptcy costs in case of default. Firms are heterogenous in their asset composition, a feature included by modeling both assets in place and expansion options. Each firm is run by a manager who controls financing and investment decisions, while shareholders decide about default. Agency conflicts arise because managers divert part of the free cash flow to equity as private benefits and exercise control rights on financing and investment in their own best interest. In this framework, I investigate manager-selected investment and financing policies and the implied effects on the loss in firm value. Further, I analyze the impacts on default and investment rates as well as the timing of investment.

[1]Surprisingly, existing structural tradeoff models typically include only one of these two crucial features. For models on manager-shareholder agency conflicts, but without macroeconomic conditions, see, for example, Stulz (1990), John and John (1993), Hart and Moore (1995), Zwiebel (1996), Morellec (2004), Malmendier and Tate (2005), Hackbarth (2008), or Lambrecht and Myers (forthcoming). Corporate models with macroeconomic conditions, but not taking into account agency conflicts, are, for example, Hackbarth, Miao, and Morellec (2006), Bhamra, Kuehn, and Strebulaev (2010a), Bhamra, Kuehn, and Strebulaev (2010b), Chen (2010), or Arnold, Wagner, and Westermann (forthcoming).

In standard tradeoff models and in the absence of agency conflicts, shareholders select leverage by balancing tax benefits of debt against bankruptcy costs (Mello and Parsons, 1992). When the value of the aggregate shock shifts between two different states (boom and recession), tax benefits of debt are larger in boom, and bankruptcy costs are larger in recession due to higher default probability and larger loss given default (Hackbarth, Miao, and Morellec, 2006). Hence, financing decisions depend on the current state of the economy. Similarly, by way of asset substitution, investment decisions correspond to a risk transfer between equityholders and bondholders (Jensen and Meckling, 1976). Because default risk varies over the business cycle, investment policies depend on macroeconomic conditions as well. In the presence of agency conflicts, managerial decisions reflect not only the impact of market frictions, but also take into account managers' private benefits. As is well known, managers choose lower debt levels and invest more aggressively due to the disciplining effect of debt and the increase in expected value of future private benefits upon investment (see, e.g., Morellec, 2004). Additionally and importantly, the regime-dependency of both the costs of debt and the expected value of future cash flow renders manager-selected financing and investment policies sensitive to macroeconomic conditions.

These distortions in managerial policies have important effects on the value of the firm. Due to the tradeoff mechanisms explained above, managerial policies, and, hence, the loss in firm value depend explicitly on macroeconomic conditions and on the importance of investment opportunities. This paper quantifies agency costs stemming from manager-shareholder agency conflicts depending on a firm's asset composition in different economic regimes. Agency costs, reported as the percentage loss in firm value compared to the first-best scenario in which firm value maximizing strategies are employed, are substantial and weakly procyclical. In boom [recession], agency costs rise from 1.78% [1.73%] for a value firm, to 3.04% [2.91%] for an average firm, and to 3.45% [3.41%] for a growth firm. I show that total agency costs for all firms are mainly driven by managers' desire to underleverage (between 84% for a growth firm and 100% for a value firm). The procyclicality of agency costs stems from two sources. First, and importantly, the loss in tax benefits due to lower debt levels chosen by the manager is larger in boom. Second, firms are more prone to invest when economic conditions are favorable, increasing the probability of suboptimal investment in boom.

In a dynamic aggregate economy, I find that agency costs remain substantial (on average, 1.77% of the first-best firm value) and become strongly procyclical (on average, 2.31% in boom and 0.95% in recessions). The strong procyclicality can be explained by the fact that the managerial tendency to underleverage reduces default risk, particularly so in recessions, when both default risk and the loss given default are more prevalent. Interestingly, for firms close to default, the total impact on firm value because of reduced default risk is positive, such that these firms enjoy agency benefits. Similarly, Hackbarth (2008) finds a possible positive role of manager-shareholder agency conflicts by way of investigating the effects of managerial traits for single firms, even without the need to appeal to optimal incentive contracts. Surprisingly, investigation of the time series of agency costs reveals that agency benefits may persist even in the aggregate economy at some points in time when taking into account changing macroeconomic conditions.

Further, comparing default and investment rates in the aggregate economy to an economy in which first-best policies are applied yields that the presence of self-interested managers strongly decreases the aggregate default rate (by approximately 60%) and slightly increases the aggregate investment rate (by approximately 12%). Finally, manager-shareholder agency conflicts generate predictions regarding the intertemporal pattern of investment. Compared to a first-best economy, the investment hazard function is decreased for short and intermediate horizons up to approximately eight years, but increased for longer horizons.[2] In particular, the non-monotone effect on the investment hazard function implies that it is important to take into account the severity of manager-shareholder agency conflicts when investigating empirical hazards.

This study relates to different strands of literature. First, it belongs to the field of research that investigates manager-shareholder conflicts, their impact on firms' financing and investment decisions, and the implications for the value of the firm.[3] The closest paper is Morellec, Nikolov, and Schürhoff (2012a), who use a dynamic tradeoff model with manager shareholder agency conflicts to investigate the impact on the dynamics and the cross section of leverage. The modeling of self-interested managers is analogous in my model, but there are three important differences between this paper and Morellec, Nikolov, and Schürhoff (2012a). First, the authors do not consider macroeconomic risk. Second, they do not account for the heterogeneity in the asset base of firms', i.e., they do not consider investment. And, third, to investigate the dynamics of leverage, Morellec, Nikolov, and Schürhoff (2012a) allow for a dynamic capital structure, while I allow for refinancing only at the time of investment. Most importantly, by introducing macroeconomic risk, I am able to analyze the evolution of agency costs in different economic regimes, and to derive new predictions for aggregate default and investment behavior of firms. Two closely related papers by Levy and Hennessy (2007) and Chen and Manso (2010) also investigate agency costs and macroeconomic

[2]The investment hazard function is the probability that a firm invests at a certain time after initiation given it has not invested yet.

[3]This stream of literature builds on early work by Jensen and Meckling (1976), who investigate formally the impact of agency conflicts on the cost of equity and debt, and Jensen (1986), who discovers the disciplining effect of debt by reducing the free cash flow. The theoretical models of Harris and Raviv (1990), Stulz (1990), and Hart and Moore (1995) assume that managers and shareholders disagree about investment decisions, which is also a feature of my model. Chang (1993) and John and John (1993) are among the first to investigate optimal compensation contracts to reduce agency costs. However, optimal contracts are beyond the scope of this paper. Zwiebel (1996) builds a model of dynamic capital structure in the presence of agency conflict, which he uses to derive implications for the frequency, level, and maturity structure of debt. In the model of Parrino, Poteshman, and Weisbach (2005), managers are risk averse, resulting in a reluctancy to invest in risky projects. In a real options model with both investment and disinvestment, Lambrecht and Myers (2008) shows that firms with weaker investor protection choose higher debt levels, contrary to the results of Morellec (2004). For further and more recent models in the field of manager shareholder agency conflicts, see footnote 1. Early empirical work is, for example, presented by Agrawal and Mandelker (1987), who document a positive relationship between managerial security holdings and changes in financial leverage. The authors conclude that the findings are consistent with the view that executive security holdings reduce agency conflicts, which is a property in my model as well. Similarly, Amihud, Lev, and Travlos (1990) present evidence consistent with the hypothesis that managers value control. For a survey on early models of agency problems as well as early empirical evidence, see Harris and Raviv (1991). Jung, Kim, and Stulz (1996) present strong support of the agency model with respect to a firm's financing decisions. Importantly, Berger, Ofek, and Yermack (1997) document that entrenched managers choose lower debt levels, consistent with the results in my model. On the contrary, the empirical study of Graham and Harvey (2001) find only little evidence of relations between managerial discretion and free cash flow or asset substitution.

regimes. Levy and Hennessy (2007) propose a general equilibrium model in discrete time to analyze the relation between financial flexibility and cyclical variation in leverage. In their model, single-period financial contracts are issued, and the authors show that no managerial diversion takes place in equilibrium. This study differs in that it considers long-term financial contracts, and it investigates agency costs over the business cycle stemming from exogenously given managerial diversion. Finally, Chen and Manso (2010) find that the agency costs of debt overhang are substantially higher in the presence of macroeconomic regimes, and they quantify the costs of debt overhang depending on the value of a firm's growth option. On the contrary, this study focuses on the implications of manager-shareholder conflicts, and not on the debt overhang problem.

Second, this paper relates to the macroeconomic literature that investigates macroeconomic agency costs defined as the loss in aggregate productivity. Traditionally, this literature emphasizes countercyclical agency costs (see, for example, Bernanke and Gertler, 1986 or Eisfeldt and Rampini, 2008). Here, the focus is on corporate agency costs, i.e., the loss in firm value due to suboptimal managerial behavior, which implies that the results are not directly comparable to the ones obtained in the macroeconomic literature.

Finally, this study belongs to the field of structural corporate finance. In detail, the proposed model is in the spirit of Mello and Parsons (1992), as extended by Hackbarth, Miao, and Morellec (2006) for macroeconomic regimes. Manager-shareholder agency conflicts are introduced by way of assuming private benefits, as in La Porta, de Silanes, Shleifer, and Vishny (2002) or Morellec, Nikolov, and Schürhoff (2012a). Further, investment opportunities are modeled as in Arnold, Wagner, and Westermann (forthcoming), and the stochastic discount factor is implied by the work of Bhamra, Kuehn, and Strebulaev (2010b) or Chen (2010).

The paper proceeds as follows. Section 2 presents and solves the model. Section 3 quantifies and decomposes agency costs for firms with different asset composition ratios. In Section 4, I investigate the evolution of agency costs in the aggregate economy, and the implications for investment and default rates as well as the intertemporal pattern of investment. Finally, Section 5 concludes.

2. The model

I consider agency conflicts between managers and shareholders within the framework of a structural model for financing and investment decisions of firms with assets in place and investment opportunities. The economy is subject to intertemporal macroeconomic shocks. The structural tradeoff model is similar to Arnold, Wagner, and Westermann (forthcoming), and, additionally, agency conflicts are introduced as in Morellec, Nikolov, and Schürhoff (2012a). I first describe the economy, then the firms, and, finally, I turn to manager shareholder agency conflicts.

2.1. Assumptions

I start by specifying a probability space $\left(\Omega, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P}\right)$, in which $\mathbb{P}$ is the physical probability measure. In the following, the presented processes are adapted to this probability space. Assets are continuously traded in complete and arbitrage-free markets. The risk neutral probability measure, denoted by $\mathbb{Q}$, is implied by the stochastic discount factor. In the analysis, this setup is used to investigate default and investment rates under the historical measure.

The economy. The economy includes a large number N of infinitely lived firms, a large number of identical infinitely lived households, and a government serving as a tax authority. I assume that there are two different macroeconomic states, namely boom (B) and recession (R). Formally, I define a time-homogeneous Markov chain I_t with state space $\{B, R\}$ and generator $Q := \begin{bmatrix} -\lambda_B & \lambda_B \\ \lambda_R & -\lambda_R \end{bmatrix}$, in which $\lambda_i \in (0, 1)$ denotes the rate of leaving state i. The realization of the Markov chain I_t at time t, i.e., boom or recession, constitutes an economy wide state variable at time t. In the main analysis, I consider $\lambda_B < \lambda_R$, as in Hackbarth, Miao, and Morellec (2006).[4]

Following Chen and Manso (2010), I specify an exogenous stochastic discount factor, which is determined by the regime-dependent risk free rate, and the risk prices for firm-level shocks and regime shifts, respectively. Chen (2010) and Bhamra, Kuehn, and Strebulaev (2010b) show that this pricing kernel is the solution of a representative agent problem, who has the continuous-time analog of Epstein-Zin-Weil preferences (Epstein and Zin, 1989 and Weil, 1990), given that the expected growth rate and volatility of aggregate output is regime-dependent.[5]

The firm. A firm n consists of assets in place and a growth option. At each time, assets in place generate a nominal cash flow stream X_t^n. For the sake of a parsimonious exposition of the model, I suppress the firm dependence on the cash flow. The cash flow X_t of the firm constitutes the firm-specific state variable and follows a regime dependent Brownian motion under the physical measure $\mathbb{P}$,

$$\frac{\mathrm{d}X_t}{X_t} = \mu_i \mathrm{d}t + \sigma_i \mathrm{d}Z_t, \tag{1}$$

in which μ_i and σ_i are the regime-dependent drift and volatility, respectively, and Z_t is a Brownian motion under $\mathbb{P}$. As in Chen (2010), the drift and the volatility of the nominal cash flow process

[4] The following properties hold: First, the probability that the chain stays in state i longer than some time $t \geq 0$ is given by $e^{-\lambda_i t}$. Second, the probability that the regime shifts from i to j during an infinitesimal time interval Δt is given by $\lambda_i \Delta t$. Third, the expected duration of regime i is $\frac{1}{\lambda_i}$, and the expected fraction of time spent in that regime is $\frac{\lambda_j}{\lambda_i + \lambda_j}$.

[5] Technical details of the derivation and the resulting stochastic discount factor can be found in Appendix A.1. It is also important to highlight the main limitations of this approach in my framework. First, I assume that aggregate output is given exogenously, in particular, I abstract away from the impacts of firm-specific default and investment on aggregate output. This assumption may be justified by considering a large number of firms in the economy, such that each firm's contribution to aggregate output is minor. Second, the model ignores the impact of agency conflicts on the state-price density. While this feedback effect is certainly important, solving the corresponding model is beyond the scope of this work.

are determined by the dynamics of the real cash flow process and a stochastic price index. The real cash flow process, in turn, depends on the realization of aggregate consumption and a firm specific idiosyncratic component. Details on the setup, the derivation of the cash flow dynamics, and the derivation of risk neutral parameters are presented in Appendix A.1. Because the part of volatility which is connected to the evolution of aggregate consumption is smaller in boom than in recession (Ang and Bekaert, 2004), I obtain that the total volatility, σ_i is also smaller in boom, i.e., $\sigma_B < \sigma_R$. Following Bhamra, Kuehn, and Strebulaev (2010b), I assume that the regime-dependent drift is higher in boom than in recession, i.e., $\mu_B > \mu_R$. Formally, the state variable in the model is given by the vector (X_t, I_t) in which the first component corresponds to the firm-specific cash flow level realization, and the second component to the economy-wide realization of the economic regime.

An investment opportunity of the firm is modeled as an American call option on the cash flows, analogous to Arnold, Wagner, and Westermann (forthcoming). Specifically, at any time $\bar{t}$, a firm can pay exercise costs K to achieve an additional future cash flow of $(s-1)\,X_t$ for some factor $s > 1$ for all $t \geq \bar{t}$. After option exercise, the firm consists of only invested assets. Intuitively, the increased cash flows can be attributed either to a larger asset base, or, equivalently, to a higher productivity of existing assets. The exercise of the growth option is irreversible. As in Morellec and Wang (2004), financing of the exercise price K takes place by issuing a mix of additional equity and debt. To obtain a closed-form solution of the model, I assume that at the time of investment, first, debt is called at market value, and, second, new debt with coupon c_n is issued.[6] This assumption is similar to Goldstein, Ju, and Leland (2001), who suppose that upon refinancing debt is first called at par to acquire a scaling property of the model. Fixed financing of the investment opportunity (e.g., debt or equity only) introduces distortions in option exercise policies. Further, Hackbarth and Mauer (2012) show that it is, in general, suboptimal to separate investment and financing decisions.

The firm is financed by issuing equity and debt. To facilitate the analysis, I present the case of infinite maturity debt. Once debt has been issued, a firm pays a total coupon c_o at each moment in time until investment. After investment, the total coupon is given by c_n. Subsequently to paying the coupon, the firm pays corporate taxes at a constant rate τ. Full offsets of corporate losses are allowed. I abstract away from cash holdings. Hence, after paying debt service and taxes, the free cash flow is given by $(1-\tau)(X-c)$, in which $c = c_o$ (before investment) or $c = c_n$ (after investment). In the model, a firm has an incentive to issue debt because it can shield part of its cash flows from taxation. Following the standard in the literature, I assume that firms finance coupons by shareholders' injection of funds. At any time, shareholders have the option to default on their debt obligations. Default is triggered when shareholders are no longer willing to inject additional equity capital to meet net debt service requirements (Leland, 1998). If default occurs, the firm is immediately liquidated and bondholders receive the unlevered asset value and growth option less default costs, reflecting the absolute priority of debt claims. The default costs in regime i are

[6] The firm's motivation to do so may be justified by existing debt covenants concerning investment and/or financing.

assumed to be a fraction $1 - \alpha_i$ of the unlevered value of the assets in place and the growth option at default, with $\alpha_i \in (0, 1]$. I suppose that recovery rates are lower in recession, i.e., $\alpha_R < \alpha_B$. This assumption is consistent with the literature introducing search frictions for corporate bonds in structural models, because liquidity tends to dry out in recession resulting in larger search cost (He and Milbradt, 2012).

The manager. Agency conflicts are introduced by assuming that a firm is run by a self-interested manager. Before investment, the manager diverts a fraction ϕ of the firm's free cash flow as private benefits (as in La Porta, de Silanes, Shleifer, and Vishny, 2002, Lambrecht and Myers, 2008, Albuquerque and Wang, 2008, and Morellec, Nikolov, and Schürhoff, 2012a). Examples for managerial private benefits include perquisites, excessive salary, transfer pricing, or employing relatives and friends who are not qualified.[7] The fraction of free cash flow that the manager diverts, ϕ, is assumed to be exogenous and captures the severity of manager shareholder agency conflicts in the model. Because manager receives a fraction ϕ of free cash flow, i.e., $\phi(1-\tau)(X_t - c_o)$, equityholders get only a fraction $(1-\phi)$ of free cash flow, i.e., $(1-\phi)(1-\tau)(X_t - c_o)$. Further, as in Nikolov and Whited (2011) or Morellec, Nikolov, and Schürhoff (2012a), managers own a fraction $\psi > 0$ of the firm's equity. Hence, the total cash flow to the manager is given by the sum of his equity share and managerial rents, i,e., $\psi(1-\phi)(1-\tau)(X_t - c_o) + \phi(1-\tau)(X_t - c_o) = (\psi - \psi\phi + \phi)(1-\tau)(X_t - c_o)$. In the extreme, when private benefits are zero, i.e., $\phi = 0$, no diversion takes place, and, hence, there is no agency conflict between managers and shareholders about corporate policies. In the analysis, I consider fixed values of ϕ and ψ based on the empirical results of Morellec, Nikolov, and Schürhoff (2012a), and then investigate the magnitude and dynamics of agency costs for firms with growth options over the business cycle.

Upon investment, the exercise price K of the option is financed by issuing a mix of equity and debt. Because new equity is issued, equityholders' claim is diluted. Denote N^{old} $[N^{new}]$ the number of old [new] shares issued, such that N^{new} times the value of equity at investment is equal to the exercise price K. Existing equityholders claim declines to $\frac{N^{old}}{N^{new}+N^{old}}$. Similarly, the manager stills owns a fraction ψ of the old equity, i.e., the manager does not receive a fraction of newly issued equity. Hence, this assumption implies that after investment, the manager's equity share declines to $\psi\frac{N^{old}}{N^{new}+N^{old}}$.

In the model, agency costs arise due the allocation of control rights within the firm. Specifically, I presume that the manager controls investment and capital structure decisions, whereas shareholders decide about default. When making financial and investment decisions, the manager acts in is own interest to maximize the present value of total cash flows from managerial rents and equity stake. Managers' control rights on investment policies are the standard in the literature, see, e.g., Zwiebel (1996), Morellec (2004) or Nikolov and Whited (2011). Simultaneously with the manager choosing his investment decision, equityholders select the default policy that maximizes equity value (for a

[7] For evidence of private benefits of control, see Barclay and Holderness (1989) or La Porta, de Silanes, Shleifer, and Vishny (2000). For a catalog of legal and illegal forms of managerial tunneling, see Johnson, La Porta, de Silanes, and Shleifer (2000)

discussion, see Morellec, 2004). Managers' control rights on capital structure decisions are in line with Morellec (2004), Hackbarth, Miao, and Morellec (2006) or Morellec, Nikolov, and Schürhoff (2012a). In particular, in my model, the manager chooses his preferred coupon at the two points in time at which debt is issued: upon investment and at initiation. Upon investment, the manager chooses the coupon of the new debt that is issued to finance part of the exercise price.[8] At initiation, the manager chooses the coupon that maximizes his objective function, anticipating his own investment policy, equityholders' default policy, as well as his preferred financing policy at the time of investment. This specification of the model gives rise to three sources of agency costs, namely, through suboptimal investment, through suboptimal leverage, and through interaction effects between the two.

2.2. Model solution

I solve the model by backward induction. I first present the value functions after investment. Subsequently, I show the value functions before investment and the capital structure chosen by the manager. Finally, I define agency costs in my model.

2.2.1. Value functions and capital structure after investment

Suppose that $[\hat{D}_B, \hat{D}_R]$ are the default boundaries after investment in boom and recession, respectively, and recall that c_n is the coupon to be paid after investment. I present the case that the default boundary in boom is lower than then one in recession, i.e., $\hat{D}_B < \hat{D}_R$.[9] The solutions for the value functions after investment, i.e., the value of corporate debt, $\hat{d}_i\left(X; c_n\right)$, the tax shield, $\hat{t}_i\left(X; c_n\right)$, bankruptcy costs, $\hat{b}_i\left(X; c_n\right)$, and managerial compensation, $\hat{n}_i\left(X; c_n\right)$, are presented in Appendix A.2. Technically, the solution is similar to Hackbarth, Miao, and Morellec (2006). The firm value, $\hat{v}_i\left(X; c_n\right)$, consists of the value of assets in place and the tax shield, $\hat{t}_i\left(X; c_n\right)$, minus default costs, $\hat{b}_i\left(X; c_n\right)$, and managerial rents, $\phi\hat{n}_i\left(X; c_n\right)$. Given a cash flow X, the value of assets in place is given by $(1-\tau)\,Xy_i$, in which y_i is the price-cash flow ratio in regime i, see eq. (A-18) in Appendix A.1. Hence, the firm value can be written as

$$\hat{v}_i\left(X; c_n\right) = (1-\tau)\,Xy_i + \hat{t}_i\left(X; c_n\right) - \hat{b}_i\left(X; c_n\right) - \phi\hat{n}_i\left(X; c_n\right). \tag{2}$$

[8]Since the manager controls the investment decision, this setup implies that the manager can issue equity to finance a suboptimal investment decision from the point of view of shareholders. To justify this assumption, I suppose that it is costly for shareholders to act collectively, and, hence, they cannot directly influence decisions taken by managers (Hackbarth, 2008). Alternatively, Morellec (2004) takes into account the market for corporate takeover, presuming that the incumbent manager has specific skills in administering the firm's assets, and control challenges are costly. As a consequence, the manager has some discretion over policy choices.

[9]Optimal default boundaries for reasonable parameter values satisfy this inequality. Further, also Hackbarth, Miao, and Morellec (2006), Bhamra, Kuehn, and Strebulaev (2010a), Chen (2010), or Arnold, Wagner, and Westermann (forthcoming) find lower default boundaries in boom than in recession.

The value of equity, $\hat{e}_i(X; c_n)$, is calculated as firm value minus the value of debt, $\hat{d}_i(X; c_n)$:

$$\hat{e}_i(X; c_n) = \hat{v}_i(X; c_n) - \hat{d}_i(X; c_n) \tag{3}$$

Once debt has been issued, equity holders select the default policy that maximizes the value of equity ex post. Value matching requires that the value of equity at the time of default be zero:

$$\begin{cases} \hat{e}_B(\hat{D}_B; c_n) &= 0 \\ \hat{e}_R(\hat{D}_R; c_n) &= 0. \end{cases} \tag{4}$$

Hence, the optimal default policy $\left[\hat{D}^*_B, \hat{D}^*_R\right]$ is determined by equating the first derivative of the equity value to zero at the corresponding default boundary:

$$\begin{cases} \hat{e}'_B(\hat{D}^*_B; c_n) &= 0 \\ \hat{e}'_R(\hat{D}^*_R; c_n) &= 0. \end{cases} \tag{5}$$

The problem is solved numerically.

Upon investment, existing debt is first called at market value. Next, the new capital structure is chosen and new total debt is issued. Because the issue proceeds of both new equity and total debt accrue to shareholders, shareholders' objective function at the time of investment is given by the firm value. To determine the capital structure, the manager selects the coupon level $c^*_{n,i}$ that maximizes the ex ante value of his claims in regime i. Hence, the manager solves

$$c^*_{n,i} := \text{argmax}_{c_n} \left(\psi \hat{v}_i(X; c_n) + \phi \hat{n}_i(X; c_n)\right). \tag{6}$$

At the time of investment, a scaling property holds: Conditional on the current state, the manager-selected coupon, the default boundaries, the value of total debt, equity, bankruptcy costs, the tax shield, and managerial compensation are all homogenous of degree one in cash flows.[10] This scaling property is based on the scaling property of Fischer, Heinkel, and Zechner (1989) and Goldstein, Ju, and Leland (2001) for the case of only one regime, and extended by Hackbarth, Miao, and Morellec (2006), Bhamra, Kuehn, and Strebulaev (2010a), Bhamra, Kuehn, and Strebulaev (2010b) and Chen (2010) for regime-switching models. In the next section, I exploit the scaling property of corporate securities at the time of investment when calculating the value of corporate securities before investment.

[10] In my model, the firm structure is different before and after investment. Before investment, the firm has the investment opportunity, and the possibility to recover part of the investment opportunity value in case of bankruptcy. After investment, the firm consists of only invested assets. Hence, the scaling property does not imply that value functions after investment can be expressed as the product of a factor times the corresponding value function before investment. As discussed in Goldstein, Ju, and Leland (2001), this property is fulfilled only in models in which the firm structure is not changed, e.g., in models of dynamic refinancing.

2.2.2. Value functions, corporate policies, and capital structure before investment

Consider a set of default and investment boundaries, $[D_B, D_R, X_B, X_R]$. I present the case in which default and investment boundaries are lower in boom than in recession for both default and investment, i.e., $D_B < D_R$ and $X_B < X_R$. Optimal policies fulfil these inequalities for reasonable parameter values.[11] Recall that the coupon before investment is denoted by c_o. The value functions of the growth option, $G_i(X; c_o)$, corporate debt, $d_i(X; c_o)$, the tax shield, $t_i(X; c_o)$, bankruptcy costs, $b_i(X; c_o)$, and future cash flows, $n_i(X; c_o)$, are presented in Appendix A.3. For the value of debt, the tax shield, and bankruptcy costs, the solutions are similar to Arnold, Wagner, and Westermann (forthcoming). However, an important difference is that I assume now that investment is financed by a mix of debt and equity chosen by the manager.

The total firm value $v_i(X; c_o)$ for a given level of cash flow X in regime $i = B, R$ is given by the value of assets in place, $(1-\tau) X y_i$, plus the value of the expansion option, $G_i(X)$, and the value of tax benefits from debt, $t_i(X; c_o)$, less the value of default costs, $b_i(X; c_o)$, and the present value of managerial rents, $\phi n_i(X; c_o)$, i.e.,

$$v_i(X; c_o) = (1-\tau) y_i X + G_i(X) + t_i(X; c_o) - b_i(X; c_o) - \phi n_i(X; c_o). \quad (7)$$

Denote the equity value in regime i by $e_i(X; c_o)$, $i = B, R$. Because the total firm value equals the sum of debt and equity values the latter can be written as

$$\begin{aligned} e_i(X; c_o) &= v_i(X; c_o) - d_i(X; c_o) \\ &= y_i X + G_i(X) + t_i(X; c_o) - b_i(X; c_o) - \phi n_i(X; c_o) - d_i(X; c_o). \end{aligned} \quad (8)$$

To determine the default and investment policies chosen by shareholders and managers, respectively, I derive the value matching conditions of equity and manager's objective function. The smooth-pasting conditions are then implied by the value matching conditions. Consider first the value of equity. At default, the value of equity is zero, reflecting the absolute priority of debt claims. Upon exercise, the growth option is financed by issuing a mix of additional equity and debt. Hence, equity satisfies the following value matching conditions at default and option exercise:

$$\begin{cases} e_B(D_B; c_o) = 0 \\ e_R(D_R; c_o) = 0 \\ e_B(X_B; c_o) = \hat{e}_B\left(sX_B; c^*_{n,B}\right) - \left(K - \hat{d}_B\left(sX_B; c^*_{n,B} - c_o\right)\right) \\ e_R(X_R; c_o) = \hat{e}_R\left(sX_R; c^*_{n,R}\right) - \left(K - \hat{d}_R\left(sX_R; c^*_{n,R} - c_o\right)\right). \end{cases} \quad (9)$$

Here, the two terms in brackets in the last two lines correspond to the amount of new equity issued to finance the exercise of the growth option. Equivalently, using the definitions of equity and firm

[11] Chen and Manso (2010) and Arnold, Wagner, and Westermann (forthcoming) also find these relations to hold.

value and simplifying, the value matching conditions at option exercise in the last two lines of eq. (9) may be written as

$$e_i\left(X_i;c_o\right)=\left(1-\tau\right)sX_iy_i+\hat{t}_i\left(sX_i;c^*_{n,i}\right)-\hat{b}_i\left(sX_i;c^*_{n,i}\right)-\phi\hat{n}_i\left(sX_i;c^*_{n,i}\right)-\hat{d}_i\left(sX_i;c_o\right)-K. \quad (10)$$

Next, consider the manager's objective function, which is denoted by $m_i\left(X\right)$ for any value of cash flow X in regime i. The objective function is given as the sum of manager's equity stakes, given by a fraction ψ of equity, and private benefits, determined as a fraction ϕ of the present value of future cash flow, i.e.,

$$m_i\left(X;c_o\right)=\psi e_i\left(X;c_o\right)+\phi n_i\left(X;c_o\right). \quad (11)$$

Using this definition of the manager's objective function (11) and the boundary conditions for equity at exercise (10), it follows that the value matching condition for manager's objective function at exercise is given by

$$\begin{aligned}\psi e_i\left(X_i;c_o\right)+\phi n_i\left(X_i;c_o\right) &= \psi\left(\left(1-\tau\right)sX_iy_i+\hat{t}_i\left(sX_i;c^*_{n,i}\right)\right.\\ &\quad\left.-\hat{b}_i\left(sX_i;c^*_{n,i}\right)-\phi\hat{n}_i\left(sX_i;c^*_{n,i}\right)-\hat{d}_i\left(sX_i;c_o\right)-K\right)\\ &\quad+\phi\hat{n}_i\left(sX_i;c^*_{n,i}\right).\end{aligned} \quad (12)$$

I denote the default policy chosen by shareholders simultaneously chosen with manager's investment boundaries by $D^{sb;*}_i$, and the option exercise policy chosen by the manager by X^*_i. The smooth pasting conditions that determine these policies are given by the derivatives of the corresponding value matching conditions. In detail, the value matching conditions of equity at default (first two lines of eq. (9)) imply that the default policy that maximizes the equity value is determined by postulating that the first derivative of the equity value be zero at the default boundary in each regime. Simultaneously, the manager equates the first derivatives of both sides of the value-matching condition for his objective function, eq. (12), to find the investment policies that maximize his objective function. Thus, these four optimality conditions translate into smooth-pasting conditions at the respective boundaries:

$$\begin{cases}\begin{aligned}e'_B\left(D^{sb;*}_B;c_o\right) &= 0\\ e'_R\left(D^{sb;*}_R;c_o\right) &= 0\\ \psi e'_B\left(X^*_B;c_o\right)+\phi n'_B\left(X^*_B;c_o\right) &= \psi\left(\left(1-\tau\right)sy_B+\hat{t}'_B\left(sX_B;c^*_{n,B}\right)\right.\\ &\quad\left.-\hat{b}'_B\left(sX_B;c^*_{n,B}\right)-\phi\hat{n}'_B\left(sX_B;c^*_{n,B}\right)-\hat{d}'_B\left(sX_B;c_o\right)\right)\\ &\quad+\phi\hat{n}'_B\left(sX_B;c^*_{n,B}\right)\\ \psi e'_R\left(X^*_R;c_o\right)+\phi n'_R\left(X^*_R;c_o\right) &= \psi\left(\left(1-\tau\right)sy_R+\hat{t}'_R\left(sX_R;c^*_{n,R}\right)\right.\\ &\quad\left.-\hat{b}'_R\left(sX_R;c^*_{n,R}\right)-\phi\hat{n}'_R\left(sX_R;c^*_{n,R}\right)-\hat{d}'_R\left(sX_R;c_o\right)\right)\\ &\quad+\phi\hat{n}'_R\left(sX_R;c^*_{n,R}\right).\end{aligned}\end{cases} \quad (13)$$

I solve this system numerically.

Next, the manager determines his preferred coupon level by maximizing the value of his objective function ex ante, taking default and investment policies as given. At the time of issue, the value of equity equals firm value. Hence, the manager solves:

$$c_{o,i}^{*} = argmax_{c_o} \left(\psi v_i \left(X; c_o \right) + \phi n_i \left(X; c_o \right) \right). \tag{14}$$

The corresponding firm value is denoted by v_i^*, which is, expressing the dependency on all controls:

$$v_i^* = v_i \left(X; c_{o,i}^*, D_B^{sb;*}, D_R^{sb;*}, X_B^*, X_R^*, c_{n,B}^*, c_{n,R}^* \right) \tag{15}$$

2.3. Agency costs

To define agency costs in my framework, I consider the first-best solution as a benchmark. The first-best solution is characterized by firm-value maximizing investment and financial policies. Agency costs are then calculated in the second and third best case. In the second best case, shareholders control financial and investment decisions, whereas in the third best solution the manager has control rights over financial and investment policies. In the following, I start by explaining the first-best benchmark. Next, I present the second best solution and the definition of agency costs. Finally, I define agency costs in the third best case.

First-best solution. Investment and financial policies are chosen to maximize firm value. As before, the policies are determined by backward induction. First, I show how the first-best capital structure at option exercise is determined. Next, I explain how to find the first-best investment boundaries, while equity holders still control the default decision. Finally, I present the optimal first-best capital structure.

At exercise, the firm value maximizing coupon solves

$$c_{n,i}^{fb} := \text{argmax}_{c_n} \hat{v}_i \left(X; c_n \right). \tag{16}$$

In standard structural models, the firm value maximizing capital structure is determined by trading off tax benefits of debt against bankruptcy costs (Leland, 1998). In my model, additionally, the realized regime affects both the tax shield and the bankruptcy costs. Furthermore, equityholders face an additional incentive to issue debt, namely, to reduce the free cash flow from which the manager diverts (Jensen, 1986). Hence, the optimal coupon also depends on the realized regime at investment as well as the presence of the manager shareholder agency conflicts. Next, I denote the first-best option exercise boundaries in boom and recession by X_B^{fb} and X_R^{fb}, respectively. The default boundaries chosen by shareholders, but, simultaneously, taking into account the optimal first-best investment boundaries are denoted by $D_B^{sb;fb}$ and $D_R^{sb;fb}$ in boom and recession, respectively. Value matching of the equity and firm value at default and option exercise, respectively, requires:

$$\begin{cases} e_B\left(D_B^{sb;fb};c_o\right) &= 0 \\ e_R\left(D_R^{sb;fb};c_o\right) &= 0 \\ v_B\left(X_B^{fb};c_o\right) &= \hat{v}_B\left(sX_B;c_{n,B}^{fb}\right) \\ v_R\left(X_R^{fb};c_o\right) &= \hat{v}_R\left(sX_R;c_{n,R}^{fb}\right). \end{cases} \tag{17}$$

Hence, the firm-value maximizing investment policy is determined by solving

$$\begin{cases} e'_B\left(D_B^{sb;fb};c_o\right) &= 0 \\ e'_R\left(D_R^{fb};c_o\right) &= 0 \\ v'_B\left(X_B^{fb};c_o\right) &= \hat{v}'_B\left(sX_B^{fb};c_{n,B}^{fb}\right) \\ v'_R\left(X_R^{fb};c_o\right) &= \hat{v}'_R\left(sX_R^{fb};c_{n,R}^{fb}\right) \end{cases} \tag{18}$$

The last two equations of the system (18) postulate smoothness of the firm value at the exercise boundaries. The first-best investment boundaries are determined by trading off the additional realization of interest tax shield earned on debt financing and the decrease in bankruptcy costs, against the exercise price K of the option and the increase in the expected value of managerial benefits.

The first-best capital structure is determined by the coupon that maximizes the firm value, given first-best default and investment policies:

$$c_{o,i}^{fb} = argmax_{c_o}\left(v_i\left(X;c_o\right)\right). \tag{19}$$

Finally, the first-best firm value v_i^{fb}, with explicitly stating all controls, corresponds to

$$v_i^{fb} = v_i\left(X;c_{o,i}^{fb},D_B^{sb;fb},D_R^{sb;fb},X_B^{fb},X_R^{fb},c_{n,B}^{fb},c_{n,R}^{fb}\right) \tag{20}$$

Second best solution. Investment and financial policies are selected to maximize equity.

At option exercise, after existing debt is called, equity holders maximize the ex ante value of equity, i.e., the value of the firm. Therefore, the coupon chosen by equity holders is equal to the first-best optimal coupon after exercise, i.e., $c_{n,i}^{sb} = c_{n,i}^{fb}$, for $i = B, R$, in which $c_{n,i}^{fb}$ is as determined by equation (16).[12] In particular, there is no stockholder-conflict over the financial policy choice at option exercise. I denote the shareholder's optimal exercise boundaries in boom and recession by X_B^{sb} and X_R^{sb}, respectively. The shareholder-selected default boundaries in boom and recession, which are chosen simultaneously with the exercise boundaries, are denoted by $D_B^{sb;sb}$ and $D_R^{sb;sb}$,

[12]Hence, the assumption that debt is called upon investment is equivalent to assuming that shareholders can commit to first-best financing of the option exercise price K, see also Hackbarth and Mauer (2012).

respectively. The value matching conditions for equity are stated in (9), leading to the smooth-pasting conditions:

$$\left\{\begin{array}{rcl} e_B'\left(D_B^{sb;sb};c_o\right) & = & 0 \\ e_R'\left(D_R^{sb;sb};c_o\right) & = & 0 \\ e_B'\left(X_B^{sb};c_o\right) & = & \hat{e}_B'\left(sX_B;c_{n,B}^{sb}\right)+\hat{d}_B'\left(sX_B;c_{n,B}^{*}-c_o\right) \\ e_R'\left(X_R^{sb};c_o\right) & = & \hat{e}_R'\left(sX_R;c_{n,R}^{sb}\right)+\hat{d}_R'\left(sX_R;c_{n,R}^{*}-c_o\right). \end{array}\right. \tag{21}$$

The difference between first-best and second best investment boundaries depends strongly on the financing of the option. If investment is financed by issuing additional equity, shareholders have an incentive to underinvest relative to the first-best policy due to risk shifting or asset substitution (cf. Jensen, 1986). However, if part of the option is exercised by issuing additional debt, shareholders have an incentive to overinvest because they transfer the increased risk of bankruptcy (compared to the first-best solution) to new bondholders (cf. Mauer and Sarkar, 2005). Less valuable growth options induce a higher incentive to adjust the capital structure and to issue additional debt, because they are exercised at larger values of the firm's cash flow. Therefore, shareholders' desire to overinvest is inversely related to the value of the growth option.

Finally, because at initiation shareholders maximize the ex-ante value of their claims, i.e., the value of the firm, the initial coupon is chosen according to

$$c_{o,i}^{sb} = argmax_{c_o}\left(v_i\left(X;c_o\right)\right). \tag{22}$$

The realizations of the objective function v_i are lower in the second best as in the first-best case, because of equity value maximizing investment boundaries in the second best solution. Hence, the second best coupon differs from the first-best coupon, even though the functional form of the objective function v is identical. In detail, to mitigate the risk shifting effects of suboptimal investment, the coupon at initiation is slightly lower than the first-best coupon if shareholders overinvest and vice versa. The second best firm value is given by

$$v_i^{sb} = v_i\left(X;c_{o,i}^{sb},D_B^{sb;sb},D_R^{sb;sb},X_B^{sb},X_R^{sb},c_{n,B}^{sb},c_{n,R}^{sb}\right). \tag{23}$$

I focus on agency costs due to control rights on financial and investment policies. Default policies are always chosen by equity holders. Therefore, when defining agency costs, I explicitly show the dependence of the value function on financial and investment controls, but I omit default boundaries as chosen by shareholders in the value functions before and after investment. Agency costs AC_i^{sb} in the second best case are the difference between the hypothetical first-best firm value and the second best firm value, expressed as a percentage of the first-best firm value:

$$AC_i^{sb}(X) = 100\left(1 - \frac{v_i\left(X \left| c_{o,i}^{sb}, X_B^{sb}, X_R^{sb}, c_{n,B}^{fb}, c_{n,R}^{fb}\right.\right)}{v_i\left(X \left| c_{o,i}^{fb}, X_B^{fb}, X_R^{fb}, c_{n,B}^{fb}, c_{n,R}^{fb}\right.\right)}\right) = 100\left(1 - \frac{v_i^{sb}(X)}{v_i^{fb}(X)}\right). \tag{24}$$

To understand the mechanics of agency costs in my model, I decompose agency costs into three sources: Investment induced agency costs (i.e., due to suboptimal investment), leverage induced agency costs (i.e., due to suboptimal financial policies), and agency costs due to interaction effects between suboptimal investment and financial policies. I start with investment induced agency costs, denoted by $IAC_i^{sb}(X)$. I define investment induced agency costs in the second best case as the loss in firm value relative to the firm value when shareholders choose the investment policy, and first-best financial policies are chosen.[13]

$$IAC_i^{sb}(X) = 100\left(1 - \frac{v_i\left(X \left| c_{o,i}^{fb}, X_B^{sb}, X_R^{sb}, c_{n,B}^{fb}, c_{n,R}^{fb}\right.\right)}{v_i\left(X \left| c_{o,i}^{fb}, X_B^{fb}, X_R^{fb}, c_{n,B}^{fb}, c_{n,R}^{fb}\right.\right)}\right). \tag{25}$$

Analogously, I define leverage induced agency costs, denoted by $LAC_i^{sb}(X)$. Leverage induced agency costs are defined as the loss in firm value relative to the firm value when shareholders choose financial policies, and financial policies are chosen such that the firm value is maximized:

$$LAC_i^{sb}(X) = 100\left(1 - \frac{v_i\left(X \left| c_{o,i}^{sb}, X_B^{fb}, X_R^{fb}, c_{n,B}^{sb}, c_{n,R}^{sb}\right.\right)}{v_i\left(X \left| c_{o,i}^{fb}, X_B^{fb}, X_R^{fb}, c_{n,B}^{fb}, c_{n,R}^{fb}\right.\right)}\right). \tag{26}$$

Upon investment and after existing debt is called, shareholders maximize the value of the firm, and, hence, the second best coupon at investment is equal to the first-best coupon at investment, i.e., $c_{n,i}^{fb} = c_{n,i}^{sb}$ for $i = B, R$. To calculate leverage induced agency costs according to (26), note that, at initiation, shareholders maximize firm value given first-best investment policies. Therefore, at initiation, the second best coupon in formula (26) is equal to the first-best coupon, i.e., $c_{o,i}^{fb} = c_{o,i}^{sb}$ for $i = B, R$. Consequently, in the second best case, leverage induced agency costs are zero:

$$LAC_i^{sb}(X) = 100\left(1 - \frac{v_i\left(X \left| c_{o,i}^{fb}, X_B^{fb}, X_R^{fb}, c_{n,B}^{fb}, c_{n,R}^{fb}\right.\right)}{v_i\left(X \left| c_{o,i}^{fb}, X_B^{fb}, X_R^{fb}, c_{n,B}^{fb}, c_{n,R}^{fb}\right.\right)}\right) = 0. \tag{27}$$

[13] Because shareholder choose the default policy simultaneous to choosing the investment boundaries, there is also an interaction effect between the two. However, the magnitude is negligible. Therefore, in the following, I do not discuss the interaction effect between default and investment.

Finally, interaction agency costs $SAC_i^{sb}(X)$ are given by the part of total agency costs that are not explained by direct investment and leverage induced agency costs due, i.e.,

$$SAC_i^{sb}(X) = AC_i^{sb}(X) - IAC_i^{sb}(X) - LAC_i^{sb}(X). \tag{28}$$

Because in the second best case leverage induced agency costs are zero, eq. (28) simplifies to

$$SAC_i^{sb}(X) = AC_i^{sb}(X) - IAC_i^{sb}(X). \tag{29}$$

At initiation, shareholders choose the coupon to maximize the ex ante value of their claims, i.e., firm value, whereas shareholders exercise the growth option to maximize the ex post value of their claims, i.e., the value of equity. Hence, shareholders choose the initial coupon to mitigate the negative effect on firm value due to their equity value maximizing investment policy. Therefore, interaction costs SAC_i^{sb} are negative in the second best case. This effect is particularly pronounced in boom, because the increase in bankruptcy costs due to a suboptimal coupon is smaller in boom than in recession. Hence, in boom, it is less costly to adjust the coupon to partially offset shareholders' suboptimal investment decision.

Third best solution. The manager chooses investment and financial policies to maximize the sum of his private benefits and his equity stake. Eqs. (6), (13), and (14) describe the manager's problems to find his preferred capital structure after investment, the investment boundaries he selects, and his chosen capital structure before investment, respectively. Analogously to Leland (1998) and Childs and Mauer (2008), I define agency costs AC_i^{tb} as the difference between the hypothetical first-best firm value and the firm value with agency conflicts, expressed as a percentage of the first-best firm value:

$$AC_i^{tb}(X) = 100\left(1 - \frac{v_i\left(X; c_{o,i}^*, X_B^*, X_R^*, c_{n,B}^*, c_{n,R}^*\right)}{v_i\left(X; c_{o,i}^{fb}, X_B^{fb}, X_R^{fb}, c_{o,B}^{fb}, c_{o,R}^{fb}\right)}\right) = 100\left(1 - \frac{v_i^*(X)}{v_i^{fb}(X)}\right). \tag{30}$$

Analogous to the analysis of agency costs in the second best case, I decompose total agency costs $AC_i^{tb}(X)$ into investment induced agency costs, $IAC_i^{tb}(X)$, leverage induced agency costs, $LAC_i^{tb}(X)$, and agency costs due to interaction effects between the two, $SAC_i^{tb}(X)$. Agency costs due to suboptimal investment, $IAC_i^{tb}(X)$ in the third best case, are defined as the loss in firm value relative to firm value when the manager selects the investment policy and financial policies are chosen to maximize firm value:

$$IAC_i^{tb}(X) = 100\left(1 - \frac{v_i\left(X; c_{o,i}^{fb}, X_B^*, X_R^*, c_{n,B}^{fb}, c_{n,R}^{fb}\right)}{v_i\left(X; c_{o,i}^{fb}, X_B^{fb}, X_R^{fb}, c_{n,B}^{fb}, c_{n,R}^{fb}\right)}\right). \tag{31}$$

In the model, the manager overinvests to increase his private benefits. This result is in line with the literature, see, for example, Morellec (2004), Malmendier and Tate (2005), and Hackbarth (2008). Definition (31) allows to quantify the costs of managerial desire to overinvest on the firm value.

Next, I define leverage induced agency costs, denoted by $LAC_i^{tb}(X)$, as the loss in firm value relative to the firm value when the manager chooses financial policies, and investment policies are chosen to maximize firm value:

$$LAC_i^{tb}(X) = 100\left(1 - \frac{v_i\left(X; c_{o,i}^*, X_B^{fb}, X_R^{fb}, c_{n,B}^*, c_{n,R}^*\right)}{v_i\left(X; c_{o,i}^{fb}, X_B^{fb}, X_R^{fb}, c_{n,B}^{fb}, c_{n,R}^{fb}\right)}\right). \tag{32}$$

In the model, due to agency conflicts between the manager and shareholders, the private benefits $\psi\hat{n}_i(X)$ distort the capital structure decision of the manager [see eqs. (14) and (6)]. In particular, the manager chooses a lower coupon than the firm-value maximizing one. The manager has two incentives to do so: First, by choosing a lower coupon, the manager increases the value of the free cash flow (cf. Morellec, Nikolov, and Schürhoff, 2012a). Second, the manager induces shareholder to defer default, since the required funds to inject are lower. The deferred default decision increases the expected value of future cash flows, and, hence, the manager's private benefits. Typically, the increase of private benefits due to a lower coupon strongly outweighs the reduction in firm value in the manager's objective function. Thus, the manager chooses a lower coupon than the firm value maximizing one, i.e., he underleverages. Underleverage is also in line with the theoretical literature (see, e.g., Morellec, 2004; Morellec and Wang, 2004; Morellec, Nikolov, and Schürhoff, 2012a), as well as empirically observed (Berger, Ofek, and Yermack, 1997). Definition (32) allows to measure the loss in firm value due to the managerial desire to underleverage.

Finally, I define interaction agency costs in the third best case, SAC_i^{tb}, as the part of total agency costs that is not explained by direct investment or leverage induced agency costs:

$$SAC_i^{tb}(X) = AC_i^{tb}(X) - IAC_i^{tb}(X) - LAC_i^{tb}(X). \tag{33}$$

In the third best case, interaction agency costs stem from two sources. First, because the manager chooses his preferred capital structure at the time of investment, he has an incentive to overinvest even more than if the first-best capital structure at investment was selected. Second, overinvestment induces the manager to choose a lower coupon at initiation than if firm value maximizing investment boundaries were employed. The decrease in firm value requires a lower coupon to reach the manager's preferred leverage level. In conclusion, because both suboptimal investment and financial policies reinforce each other, interaction agency costs are positive in the third best case.

3. Quantitative results

The previous section presented the qualitative properties of second and third best model solutions. In this section, I quantify total agency costs for firms with different portions of growth options in their asset composition depending on the current economic regime. Further, decomposing agency costs into investment induced agency costs, leverage induced agency costs, and interaction effects permits the quantification of the components of total agency costs. Subsection 3.1 presents the choice of parameters, and Subsection 3.2 quantifies agency costs for firms with different asset composition ratios in boom and recession at the time of debt issue.

3.1. Choice of parameters

I fix the fraction of equity owned by the manager as $\psi = 0.0747$ and the fraction of private benefits as $\phi = 0.01$. The fraction of equity owned by the manager ψ and the fraction of free cash flow diverted as private benefits ϕ are the average values reported in Morellec, Nikolov, and Schürhoff (2012a). The remaining parameters are chosen as in Arnold, Wagner, and Westermann (forthcoming), presented in Table I.

3.2. Quantifying agency costs for value and growth firms

This subsection quantifies agency costs for firms with heterogenous asset compositions in economic boom and recession. While the qualitative impact of agency conflicts is well-documented in the literature, the quantitative effect with respect to a firm's asset composition ratio has not been studied extensively, in particular not in the presence of varying macroeconomic conditions. A firm's *asset composition ratio* is defined as the value of the firm divided by the value of invested assets. As in Arnold, Wagner, and Westermann (forthcoming), this measure captures the relative importance of a firm's investment opportunities in the value of its assets. The direct empirical analogon is Tobin's q.

The results are presented in Tables II, III, and IV. Each table shows the investment policy and the resulting asset composition ratio (Panel A), the financial policies and the resulting leverage (Panel B), the value functions of interest (Panel C), and agency costs and their decomposition (Panel D) for the first-best solution (rows one and two), the second best solution (rows three and four), and the third best solution (rows five and six), in boom and recession, respectively. Table II corresponds to a firm with only invested assets, Table III to a (roughly) average firm with a scale parameter of $s = 2.7$, and Table IV displays the result for a growth firm with scale parameter $s = 3.5$.

I start the analysis with the second best solution, i.e., equityholders select investment and financing policies. Table III presents the results for a firm with only invested assets. Shareholders select the financing policies that maximize the ex ante value of their claims, i.e., the firm value. Hence, in the absence of an expansion option, the first-best solution is identical to the first-best solution, reflecting the notion that there is no bondholder shareholder conflict on financing policies for firms with only invested assets. Comparing first and second best solutions in Table III (columns 1 and 3, and columns 2 and 4, respectively), yields, hence, that these two solutions are identical. Next, Tables III and IV show the results for an average firm with an option scale parameter of 2.7 and for a growth firm with an option scale parameter of 3.5, respectively. The first observations from the first two rows of Panel A is that shareholders overinvest slightly compared to the first-best solution. For example, for an average firm initiated in boom, shareholders invest at a cash flow value of 1.62 (1.76) in boom (recession), whereas the firm value maximizing strategy invests at 1.65 (1.79). This result is driven by the effective financing of the option exercise at investment. If the option is financed by additional equity, equityholders engage in asset substitution and underinvest (Mauer and Ott, 2000, Moyen, 2002 or Titman and Tsyplakov, 2002). However, as documented by Mauer and Sarkar (2005) or Hackbarth and Mauer (2012), if the option exercise is financed by additional debt, equityholders have an incentive to overinvest. The reason is that equityholders transfer the increased default risk by early option exercise to bondholders, while enjoying the additional tax benefit due to increased cash flows. In my model, the exercise of the option is financed by a mix of equity and debt. Consistent with the literature, I find that equityholders' incentive to overinvest is stronger the larger is the percentage of debt financing of the exercise price. Because more valuable growth options are exercised at lower values of cash flow, and because the new coupon at exercise is linear in cash flow, the percentage of debt financing of the exercise price is decreasing in the asset composition ratio. For example, the first-best solution of an average firm which exercises its growth option at the exercise boundary in boom uses 125.47% debt financing of the option exercise price (i.e., equityholders receive a debt financed dividend, as in Hackbarth and Mauer, 2012). The first-best percentage of debt financing of the option exercise price declines to 105.75% at the exercise boundary in boom for a growth firm (Panel B, row four). Hence, as a result, I find that equityholders' incentive to overinvest is weaker the larger is the asset composition ratio of a firm. However, these results are driven by the assumption that upon investment, debt is called at market value, which is, in the second best solution, equivalent to assuming that equityholders can commit to value-maximizing financing of the investment opportunity. Finally, the initial coupon c_o in the second best case is slightly higher than in the first-best case (for example, 0.6517 in the first-best case and 0.6596 in the second best case for an average firm initiated in boom, see Panel B, row one). The reason is that shareholders select the initial coupon to maximize firm value, and, hence, mitigate the effects of overinvestment. Interestingly, for both an average firm and a growth firm, the second best equity value is slightly below the first-best equity value (Panel C, row two). The reason is that shareholders cannot credibly commit to a firm value maximizing investment policy, once debt is issued. The main finding for the second best solution is that agency costs are larger for an average firm than for a growth firm due to a stronger incentive to overinvest. Because the coupon selected by equityholders is chosen to maximize firm value, one implication is that this coupon decreases equityholders' incentive to overinvest. Consequently, the interaction costs of investment and leverage induced agency costs are slightly negative. For example, total agency costs for an average firm initiated in boom are decreased by -0.1064% due to this interaction, as documented in Panel D, row four. Importantly, absolute values of agency costs are negligibly small in both boom and recession. The agency costs for an average firm initiated in boom are 0.0090% of firm value, and for a growth firm initiated in boom agency costs are 0.0016% (Panel D, row one). This result conforms to Childs, Mauer, and Ott (2005), who find that stockholder-bondholder agency costs are negligibly small when the firm has financial flexibility. However, these results depend on the assumption that equityholders can commit to value-maximizing financing of the investment opportunity.

Next, I investigate the third best solution, i.e., when the manager controls investment and financing decisions (columns five and six of Tables II, III, and IV). I start by considering a firm with only invested assets (Table II). In row one of Panel B, I find that the manager selected coupon is significantly lower than the first-best coupon in both boom and recession. The implied first-best leverage is 46.97% (45.75%) in boom (recession), and 21.82% (21.25%) in boom (recession) for the third best solution (Panel B, row two). As in Morellec, Nikolov, and Schürhoff (2012a), managers choose a lower coupon to increase net cash flows of which they divert private benefits. Empirical evidence of underleverage can be found in Berger, Ofek, and Yermack (1997), while further theoretical work is, for example, Morellec (2004) or Morellec and Wang (2004).

Tables III and IV present the results for an average firm and a growth firm, respectively. Because the manager diverts private benefits from cash flows, he has an incentive to overinvest in the option. For an average firm initiated in boom, the manager selects investment policies of 1.51 and 1.62 in boom and recession, respectively, while firm value maximizing policies correspond to 1.65 and 1.79 in boom and recession, respectively (Panel A, rows one and two, Table III). Managerial overinvestment is also documented in the literature, see, e.g., Morellec (2004), Malmendier and Tate (2005) or Hackbarth (2008). Simultaneously, as explained in the previous paragraph, the manager underleverages to increase the net cash flow from which he diverts private benefits. Both manager shareholder agency conflicts and the presence of the investment opportunity decrease optimal leverage (see Morellec (2004) for the effects of agency conflicts on leverage, and Arnold, Wagner, and Westermann (forthcoming) for the implications of growth options on optimal leverage). Hence, the resulting optimal leverage in a model with both growth options and agency conflicts is significantly decreased. For example, the optimal leverage for a firm initiated in boom decreases from 36.71% (first-best) to 15.05% (third best) for an average firm, and from 30.21% to 11.13% for a growth firm (Panel B, row two).

Overinvestment and underleverage are not independent from each other. In particular, the two create interaction agency costs by two channels. First, the fact that manager decreases the firm's leverage when exercising the option, constitutes a further incentive to overinvest, even compared to his desire to overinvest when firm value maximizing financing is chosen. Second, overinvestment induces the manager to underleverage even further at initiation in order to counterbalance the increased default risk stemming from overinvestment. These two mechanisms give rise to positive and important interaction effects in the third best case.

I now turn to the quantification of agency costs depending on the asset composition ratio and on the macroeconomic regime. First, agency costs are strongly increasing in the asset composition ratio. Total agency costs for a firm with only invested assets are 1.78% (1.73%), 3.04% (2.92%) for an average firm, and 3.45% (3.41%) for a growth firm initiated in boom (recession), see row one of Panel D. The reason is that for valuable growth options, the manager's desire to overinvest and underleverage, as well as their interactions, are stronger. The percentage of investment induced agency costs is increasing in the asset composition ratio: 0.00% (0.00%) for a firm with only invested assets, 7.66% (6.55%) for an average firm, and 10.07% (10.26%) for a growth firm in boom (recession), row two of Panel D. Further, also interaction costs are strongly increasing in the asset composition ratio, with 0.00% (0.00%) for a firm with only invested assets, 3.10% (3.14%) for an average firm, and 5.35% (4.81%) for a growth firm in boom (recession), row four of Panel D. Consequently, while the absolute value of underleverage costs is increasing in the asset composition ratio, the relative percentage of underinvestment costs in total agency costs is decreasing from 100.00% (100.00%) for a firm with only invested assets, 89.23% (90.30%) for an average firm, and 84.58% (84.93%) for a growth firm in boom (recession), row three of Panel D. Second, agency costs are slightly procyclical at initiation. For example, for an average firm, agency costs are 3.04% in boom, but only 2.92% in recession (Panel D, row one). There are two reasons for the procyclicality of agency costs. The first reason is given by the fact that in boom the firm is closer to the investment threshold, and, hence, the probability of suboptimal investment increases. Therefore, investment induced agency costs are larger in boom. The second reason is due to the loss in tax benefits because of the lower coupon chosen by the manager. The loss in tax benefits is larger in boom, where economic conditions are more favorable, and, in particular, the drift of the cash flow is positive.

To conclude, I find that in this framework shareholder bondholders agency costs are negligibly small, but manager shareholder agency costs are substantial. Even for an average firm, manager shareholder agency costs are about 3% of first-best firm value in both boom and recession. Further, I find that these agency costs are strongly increasing in the asset composition ratio and weakly procyclical.

4. Aggregate dynamics of agency costs

In this section, implications of manager-shareholder agency conflicts on the level of the aggregate economy are investigated. First, the dynamic properties of agency costs generated by the model-implied economy are analyzed. Second, default and investment rates are considered. Finally, the impact of agency conflicts on investment hazards is presented.

4.1. Time series of agency costs in the aggregate economy

The previous Section 3 presents agency costs at the time of initiation. However, as noted by Strebulaev (2007), it can be substantially misleading to draw empirical implications from the results at the time of issuance. As time evolves, a firm's cash flow deviates from the initial one. This deviation changes the value of the firm, which, in turn, affects agency costs. Because the impact of cash flow variations on firm value is non-linear, the impact of cash flow variation on agency costs is non-linear as well. Therefore, when making predictions for the cross section of firms, it is crucial to take into account the time evolution. For example, Bhamra, Kuehn, and Strebulaev (2010a) show that while leverage is procyclical at initiation, aggregate leverage is countercyclical in the aggregate economy. Because both leverage and agency costs depend on the value of the firm, the dynamic behavior of aggregate agency costs is also expected to deviate substantially from agency costs at the time of issue. Hence, this section investigates the effects of aggregate agency costs in the aggregate economy. In the following, I describe the simulation approach and how to measure agency costs in the resulting aggregate economy. Finally, I present the results.

Simulation. The simulation is conducted similarly to Arnold, Wagner, and Westermann (forthcoming), as inspired by Bhamra, Kuehn, and Strebulaev (2010a). I generate a dynamic economy consisting of 10,000 identical firms with infinite debt maturity. Initially, each firm's cash flow is $X = 1$, and the option scale parameter is assumed to be $s = 2.90$ if the firm's initial regime is boom, and $s = 3.06$ otherwise. These choices of s imply an asset composition ratio of 1.5 in both states at initiation, given manager-selected leverage. Firms receive the same macroeconomic and inflation shocks, but experience different idiosyncratic shocks. Each firm observes its current cash flow as well as the current regime on a monthly basis. If the current cash flow is below the corresponding default threshold (as chosen by equityholders), the firm defaults immediately; if the current cash flow is above the corresponding manager-selected option exercise boundary, the firm exercises its expansion option; otherwise, the firm takes no action.

Firms have a growth option, which can only be exercised once. To maintain a balanced sample of firms with active growth options, I exogenously introduce new firms. In particular, I substitute each defaulted or exercised firm by a new firm whose growth option is still alive. New firms have initial cash flow of $X = 1$, and an option scale parameter s according to the current regime as described above.

To ensure convergence to the long-run steady state, I first simulate the economy consisting of 10,000 firms for 100 years. The starting period for the reported results is the final period of the first 100 years of simulation. Next, I simulate the model for 100 years and present the aggregate dynamics.

Dynamic agency costs. In the dynamic economy, I define agency costs at each time as the aggregate loss in firm value due to the presence of the managers at this time. To this end, I consider a hypothetical economy consisting of firms employing firm-value maximizing investment boundaries and optimal first-best leverage at initiation. For each firm in the aggregate economy, there exists a corresponding firm in the hypothetical economy, which is hit by the same shock at each point in time. Then, at each point in time, I compare the firm value of each firm in the aggregate economy to the firm value of the corresponding firm in the hypothetical first-best economy. Consistent with the general formula (30), in a dynamic setting, agency costs $AC_i^{n,t}$ for firm n at time t are defined as

$$AC_i^{n,t} = 100 \left(1 - \frac{v_i \left(X_t^n \middle| c_n^*, X_B^*, X_R^*, c_{o,B}^*, c_{o,R}^* \right)}{v_i \left(X_t^n \middle| c_n^{fb}, X_B^{fb}, X_R^{fb}, c_{o,B}^{fb}, c_{o,B}^{fb} \right)} \right), \tag{34}$$

in which X_t^n is the value of the cash flow for firm n at time t, and the superscripts * and ^{fb} refer to manager-selected and first-best policies, respectively. First-best option exercise and default boundaries are higher than manager-selected option exercise and default boundaries. For the computation of agency costs, I consider only firms that are still active both in the first-best and in the manager-controlled case. In particular, once a firm's cash flow falls below its default threshold of the first-best solution, this firm is excluded from the calculation of agency costs for the rest of its firm life. Once a manager-controlled firm exercises its expansion option, it is substituted immediately both in the aggregate and in the first-best economy. Finally, total aggregate agency costs are value-weighted:

$$AC_i^{agg,t} = \frac{1}{\sum_{n=1}^{N} e_i^n \left(X_t^n \right)} \sum_{n=1}^{N} e_i^n \left(X_t^n \right) AC_i^{n,t}. \tag{35}$$

The value weighting of agency costs avoids that the results are driven by firms close to default, for which the relative loss in firm value is large because absolute values are small.

Importantly, dynamic agency costs can be negative, both at the firm level and at the aggregate level. On the contrary, at initiation, the first-best firm value is always greater than the firm value with agency conflicts, and, hence, agency costs are always positive. In the dynamic economy, firms deviate from their optimal leverage due to changes in cash flows. Because the effect of credit risk on firm value is convex in the distance to default, the deviation from optimal leverage in the firm's time series can impact the firm value of the first-best firm more severely than the firm value of the manager-controlled firm. In this case, the firm value of the manager-controlled firm exceeds the firm value of the first-best firm. Hence, agency costs as defined by (34) can be negative for firms that are not at the time of issue.

To understand the sources of agency costs, I decompose agency costs at each point in time for each firm into investment induced agency costs, $IAC_i^{n,t}$, leverage induced agency costs, $LAC_i^{n,t}$ and interaction effects, $SAC_i^{n,t}$. The formulas are analog to the general definitions (25), (26), and (28), respectively:

$$IAC_i^{n,t} = 100\left(1 - \frac{v_i\left(X_t^n \middle| c_{o,i}^{fb}, X_B^*, X_R^*, c_{n,B}^{fb}, c_{n,R}^{fb}\right)}{v_i\left(X_t^n \middle| c_{o,i}^{fb}, X_B^{fb}, X_R^{fb}, c_{n,B}^{fb}, c_{n,R}^{fb}\right)}\right), \tag{36}$$

$$LAC_i^{n,t} = 100\left(1 - \frac{v_i\left(X_t^n \middle| c_{o,i}^{*}, X_B^{fb}, X_R^{fb}, c_{n,B}^{*}, c_{n,R}^{*}\right)}{v_i\left(X_t^n \middle| c_{o,i}^{fb}, X_B^{fb}, X_R^{fb}, c_{n,B}^{fb}, c_{n,R}^{fb}\right)}\right), \tag{37}$$

and

$$SAC_i^{n,t} = AC_i^{n,t} - IAC_i^{n,t} - LAC_i^{n,t}. \tag{38}$$

Aggregate agency costs are obtained by value-weighting, analogous to Eq. (35).

Results. Table V presents the time-series mean, standard deviation and 10%, 25%, 50%, 75%, and 90% Quantiles of aggregate total agency costs, investment induced agency costs, leverage induced agency costs, and interaction agency costs. Panel A shows the overall results, while Panels B and C present the results in boom and recession, respectively. To illustrate the dynamics and time-series properties, I also display aggregate value-weighted agency costs of the simulated economy. To start, Figure 1 presents the time-series of value-weighted aggregate total agency costs, in which shaded areas correspond to recessions. The moments and statistics for the total aggregate

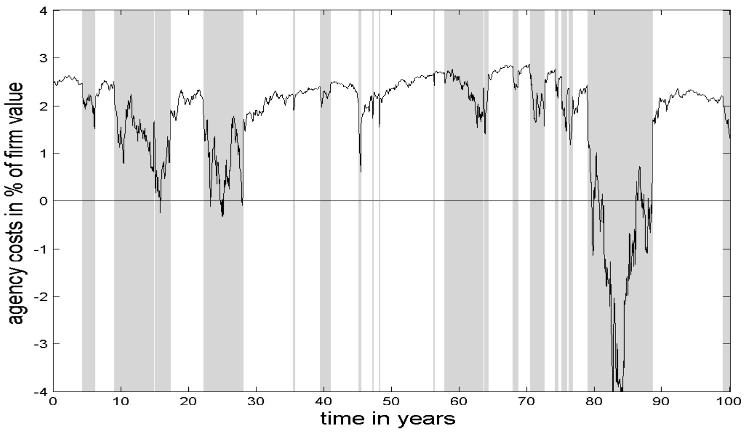

Figure 1. *Time series of value-weighted agency costs.* The solid line shows the aggregate value-weighted agency costs of the simulated economy. The shaded areas represent times of recession. Standard parameters from Table I are used.

agency costs are shown in row one, Panels A, B, and C of Table V. The overall time-series average of aggregate agency costs is substantial: 1.77% of the first-best firm value with a standard deviation of 1.18 (Panel A). Further, the level as well as the dynamics of agency costs are fundamentally different in boom and recession. I provide the following three novel results concerning the evolution of agency costs over the business cycle. First, agency costs in boom are significantly higher than in recession (on average, 2.31% in boom vs. 0.95% in recession, Panels B and C). Consistent with this observation, when the economy switches from boom to recession [from recession to boom], agency costs decrease [increase] drastically. Second, the volatility of the time series of agency costs is much larger in recession (0.27 in boom vs. 1.51 in recession, Panels B and C). Third, inspection of the quantiles suggests that while the distribution of agency costs is approximately symmetric in boom, it is strongly negatively skewed (Panels B and C).

The procyclical property of simulated agency costs may seem surprising at the first glance, given the fact that the macroeconomic literature typically emphasizes countercyclical agency costs (e.g., Bernanke and Gertler, 1986, Rampini, 2004, or Eisfeldt and Rampini, 2008). However, the macroeconomic literature measures agency costs as costs due to a loss in in productivity ('macroeconomic agency costs'), whereas the corporate finance literature measures agency costs as a loss in firm value ('corporate agency costs'). In the framework of a structural model, the firm has to be run by a manager, and the manager is partially entrenched. I calculate agency costs due to the choice of suboptimal controls. Because macroeconomic models typically abstract away from the possibility of debt financing and the resulting heterogeneity of firms (e.g., Carlstrom and Fuerst, 1998, or Eisfeldt and Rampini, 2008), corporate agency costs are unaccounted for in the macroeconomic literature. Similarly, my structural corporate model does not speak to macroeconomic agency costs as defined above. Managers are entrenched, but a firm's productivity is not affected by the specific manager, nor by his effort, nor by his skill. Further, based on the assumption of a large economy in which a single firm's contribution to aggregate output is negligible, I presume that the aggregate output process is given exogenously (see Eq. (A-1) in Appendix A.1). To the best of my knowledge, the literature lacks a model which can address macroeconomic and corporate agency costs simultaneously over the business cycle. Therefore, the relation and interaction between macroeconomic and corporate agency costs is left unexplained. However, I do not exclude the possibility that countercyclical macroeconomic agency costs can be consistent with procyclical corporate agency costs. For example, high macroeconomic agency costs in recession due to low productivity are not conflicting with low corporate agency costs on the firm level due to underleverage.

Next, I investigate the intuition driving my three main results by decomposing agency costs into investment induced agency costs, leverage induced agency costs, and interaction agency costs. The moments and quantiles of investment agency costs, leverage agency costs, and interaction agency costs are presented in row two, three, and four, respectively, of Panels A, B, and C of Table V. The following Figures 2, 3, and 4 depict the time-serious evolution in the simulated economy of the three types of agency costs.

Inspection of the figures suggests that the level of aggregate total agency costs is mainly driven by leverage induced agency costs, while the impact of investment induced and interaction agency costs are much smaller. Indeed, Table V shows that average aggregate leverage induced agency

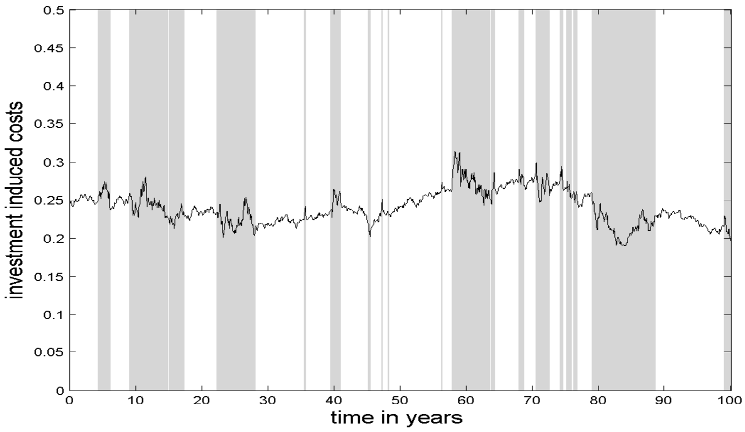

Figure 2. *Time Series of value-weighted investment induced agency costs.* The solid line shows the investment induced agency costs in the simulated economy. The shaded areas represent times of recession. Standard parameters from Table I are used.

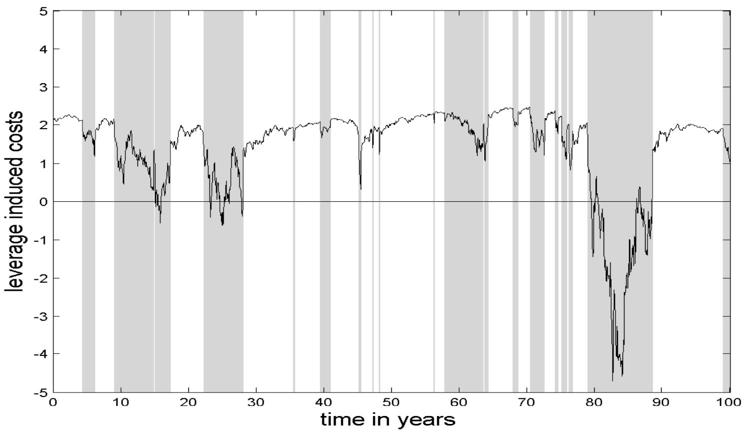

Figure 3. *Time series of value-weighted investment induced agency costs.* The solid line shows the leverage induced agency costs in the simulated economy. The shaded areas represent times of recession. Standard parameters from Table I are used.

costs are 1.44%, whereas aggregate investment induced and interaction agency costs are only 0.24% and 0.0045%, respectively. Similarly to the properties of agency costs at initiation (see Section 3), manager's control rights over financial policies explain a large part of total agency costs over time. Next, Figure 3 suggests that the significant difference in mean and standard deviation of total agency costs in boom and recession is also driven by leverage induced agency costs. Leverage induced agency costs in boom have a mean and standard deviation of 1.98% and 0.26, respectively, (Panel B, row three), whereas leverage induced agency costs in recession have a mean and standard deviation of 0.61% and 1.48, respectively (Panel C, row three). On the contrary, the mean of investment induced [interaction] agency costs in boom is not significantly different from the mean in recession (0.24% vs. 0.24% [0.0045% vs. 0.0056%] in boom and recession with standard deviations

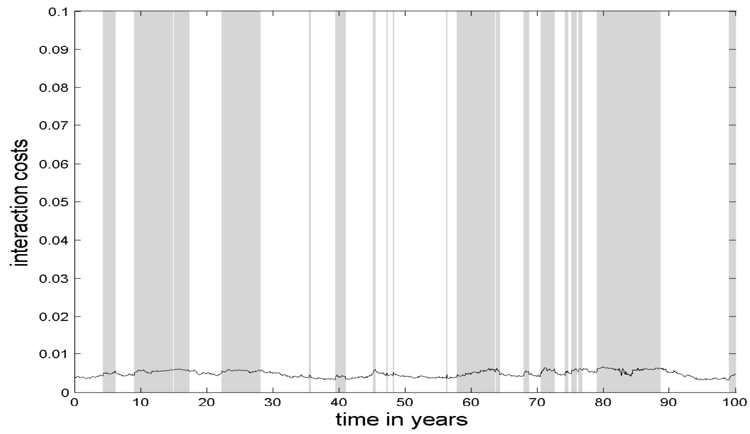

Figure 4. *Time series of value-weighted agency costs due to interaction effects.* The solid line shows the agency costs due to interaction effects between suboptimal investment and financing decisions in the simulated economy. The shaded areas represent times of recession. Standard parameters from Table I are used.

of 0.02 vs. 0.03 [0.0007 vs. 0.0006], respectively). Overinvestment is more costly in recessions, when cash flows are lower. However, firms are closer to their exercise boundaries in boom, leading to an increased probability of investment in boom. Because investment induced agency costs in the aggregate economy are approximately constant across regime, I conclude that these two effects seem to cancel each other out, at least in the cross section. Further, Figure 3 shows that the strong decrease [increase] in agency costs upon a regime switch from boom to recession [recession to boom] is governed by a strong decrease [increase] in leverage induced agency costs upon switching. To understand these effects, note that leverage induced agency costs are driven by the distance to default and bankruptcy costs. In recession, a firm's distance to default is larger if its leverage is lower, and default is less costly for firms with lower leverage because of lower default boundaries. Hence, leverage induced agency costs are smaller in recession. In particular, leverage induced agency costs can be negative in recession: For example, in Fig. 3, around year 85, even aggregate leverage induced agency costs are negative. That is, on average, around year 85, a firm enjoys agency benefits stemming from the larger distance to default. When the economy switches from recession to boom, firms' distance to default increases, since default boundaries are lower in boom than in recession. However, this effect is weaker for manager-controlled firms, because their default boundaries are lower due to lower leverage. Consequently, the first-best firm value increases more than the one of the manager-controlled firm given a regime switch to boom, and, hence, agency costs increase. The analogous reasoning holds for a regime switch from recession to boom. Finally, also the distributional properties of total agency costs, i.e., the higher volatility in recession as well as the skewness of the regime-dependent distributions, are inherited from the distributional properties of leverage induced agency costs. Because the firm value is more sensitive to changes in cash flow when the firm is closer to default, leverage induced agency costs are more volatile in recession. The negative skewness of the distribution of leverage induced agency costs can be explained similarly. During economic recessions, it is more likely that a number of firms is closed to default, benefiting from managers' incentive to underleverage. These benefits increase more strongly the closer the firm is to default, because the probability of actual default increases more strongly. Hence, there are more extreme realizations of leverage induced agency costs for low (negative) values. On the contrary, in boom, firms are, in general, more far away from default, and, hence, the change in default probability does not have the strong systematic asymmetric effect on agency costs. Therefore, the distribution of leverage induced agency costs in boom is approximately symmetric.

4.2. Default and investment rates

Because managers control investment and financing decisions, these control rights have implications for default and investment rates, as well as for the timing of investment. In this subsection, I first explain the simulation approach to investigate the implied changes in default and investment rates in the aggregate economy. Next, I present the implications for default and investment rates. Finally, I show the effects on the timing of investment projects.

Simulation. To investigate the impact of manager-shareholder conflicts on default and investment, I compare the aggregate economy to a hypothetical first-best economy in which firms invest according to the firm-value maximizing policies and also choose the firm-value maximizing capital structure. The aggregate economy, in which managers control default and investment decisions, is designed as described in the previous Subsection 4.1. The first-best economy is hit by the same realization of shocks as the aggregate economy, but the operating strategies correspond to the value maximizing policies. Importantly, whenever a firm defaults or expands in any economy, it is immediately replaced by a new firm. The growth option of a new firm is still intact. This assumption of immediate replacement in the first-best economy is different from the assumption for the simulation in the previous Subsection 4.1.[14] Finally, default and investment rates are calculated as the fraction of firms that default and invest, respectively, relative to the total number of firms populating the economy during the time window used in the simulation.[15]

Implied default and investment rates. First, I compare the default and investment rates of the aggregate economy in which firms are run by self-interested managers to the default and investment rates of the hypothetical first-best economy. Table VI presents the change in default and investment rates in the aggregate economy compared to the first-best economy. Panel A shows the overall

[14]Previously, to maintain corresponding firms in both economies at each time, firms are always replaced simultaneously in both economies, e.g., after default in both economies is triggered. Because I now compare investment and default rates, it is not important to refer to corresponding firms as required when calculating agency costs.

[15]Because manager-selected default thresholds are lower than first-best default thresholds, more firms are replaced after default in the first-best economy. However, due to the fact that manager-selected investment thresholds are lower than first-best thresholds, more firms are replaced after investment in the aggregate economy. In total, the simulation reveals that more firms are replaced in the first-best economy. By normalizing with the total number of firms populating the economy (instead of normalizing with the number of firms at each time, i.e., 10,000), I control for the differences in the number of firms that are replaced in the first-best vs. the aggregate economy.

results, and Panel B and C present the results in boom and recession, respectively. Comparing the aggregate economy to the first-best economy, I find that the total default rate decreases by -59.02%, and the investment rate increases by +12.53%. While the signs of the changes are as implied by overinvestment and underleverage, the magnitudes of the changes are striking. The important decrease in the default rate is due to the strong managerial desire to underleverage. Table VI shows that the decrease in default rates is, surprisingly, slightly lower in boom (-56.20% in boom vs. -59.64% in recession), while the increase in the investment rate is slightly stronger in recessions (+11.98% in boom vs. +14.61% in recession). In summary, the results indicate that agency conflicts have important implication for default and investment rates, particularly in times of economic recession.

4.3. The intertemporal pattern of investment

Second, I investigate the effect of manager-shareholder conflicts on the timing of investment. The simulation is identical to the one in the previous Subsection 4.2. Row two of Panel A in Table VI show that the investment rate increases by more than 12% overall due to the presence of manager-shareholder agency conflicts. Rows two of Panels B and C document that the increase in boom is 11.98%, and the increase in recession 14.61%. Interestingly, the magnitude of the change of more than 12% indicates that the presence of manager-shareholder conflicts has important implications for the intertemporal pattern of investment. To investigate this conjecture, I calculate and analyze the simulation-implied distribution of the event investment as well as investment hazard rates.

For the analysis, I consider only the subsample of firms that exercise their option, and neglect firms that default. The restriction to this subsample is necessary because the time span between initiation and investment for a firm that defaults is not defined, because default excludes investment in the simulation. This selection method is consistent with the sample construction in the empirical literature, see, e.g., Whited (2006), or Morellec, Valta, and Zhdanov (2012b). Next, for all firms in my subsample, I calculate the spell, i.e, the time span between initiation and investment. Formally, I define the random variable T measuring the spell between firm's n initiation and investment. The cumulative probability function F^* of the random variable T is given by

$$F^*(t) = P(T \leq t) \ \forall \ t \geq 0. \tag{39}$$

I calculate the simulation-implied cumulative distribution of manager-controlled firms of the random variable T, $F^*(t)$, by counting firms that invest, i.e.,

$$F^*(t) = \frac{\#\text{firms that invest at s, } s \leq t}{\#\text{firms that invest}}. \tag{40}$$

Analogously, I calculate the cumulative distribution implied by the first-best economy, F^{fb} by counting the firms that invest in the first-best economy. The left panel in Figure 5 shows the two

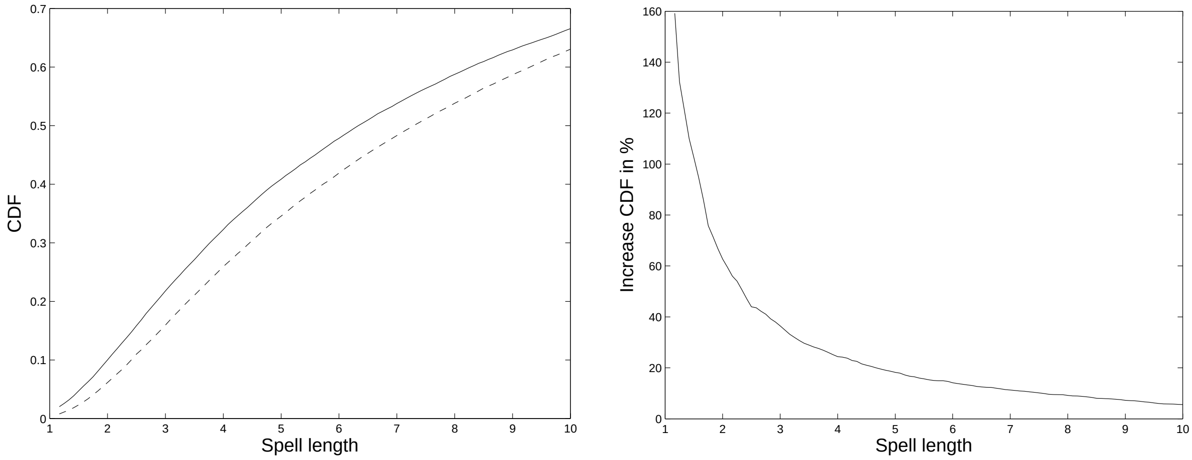

Figure 5. *Cumulative distribution functions of the event investment and relative changes.* The left panel shows the cumulative distribution functions of the event that investment takes place before time t. The solid line corresponds to the cumulative distribution function resulting from the aggregate economy, the dotted line shows the cumulative distribution function resulting from the hypothetical first-best economy. The right panel presents the increase of the cumulative distribution function of the aggregate economy compared to the cumulative distribution function of the hypothetical first-best economy. Standard parameters from Table I are used.

cumulative distributions of manager-controlled firms (solid line) and first-best firms (dashed line). I observe that the general shape of the cumulative distribution functions is comparable to Figs. 1-6 in Whited (2006). To analyze the impact of the presence of manager-shareholder conflicts, the right panel of Figure 5 presents the increase in the cumulative distribution functions of the manager-controlled economy compared to the first-best economy, i.e.,

$$\Delta_F(t) = 100\left(\frac{F^*(t)}{F^{fb}(t)} - 1\right). \tag{41}$$

The right panel in Figure 5 shows the increase of the cumulative distribution function, $\Delta_F(t)$. The smaller is the length of the spell, the bigger is the increase in the cumulative distribution function for a manager-controlled firm compared to a first-best firm. For longer spells, the increase of the cumulative distribution function gradually decreases. For spells around one year, the cumulative distribution function of the manager-controlled firm is more than 160% of the cumulative distribution function for the first-best firm; for spells around ten years, the cumulative distribution functions are close to each other. An increase in the cumulative distribution function can also stem from varying model parameters such as productivity or depreciation, as documented in Whited (2006). My model offers an alternative explication of observed patterns of investment based on manager-shareholder agency conflicts.

Finally, I consider investment hazard rates. The impact of the presence of managers on the cumulative distribution function suggests that their presence may also have an important effect on investment hazard rates. To this end, analogous to Meyer (1990), Leary and Roberts (2005), and Akdogu and MacKay (2008), I define the investment hazard rate as the probability that a manager-controlled firm will invest in the next time period, given it has not invested yet. Then the investment hazard function is defined as

$$h^{*}(t) = \lim_{\delta \to 0} \frac{P(t \leq T < t + \delta \,|T \geq t)}{\delta}. \tag{42}$$

Because I simulate at the monthly frequently, I consider monthly hazard rates, i.e., $\delta = 1$ month. I follow Leary and Roberts (2005) for the intuition that $h^{*}(t)\delta$ is (approximately) the probability that a manager-controlled firm will invest in the next δ units of time, given it has not invested until time t. For example, the hazard function at date $t = 36$ corresponds to the probability that a manager-controlled firm will invest in the next month ($\delta = 1$), conditional on not having invested during the first three years (36 months) after initiation. To calculate the simulation-implied investment hazard rates, I consider a naive approach using the identity

$$h^{*}(t) = \frac{f^{*}(t)}{1 - F^{*}(t)}, \tag{43}$$

in which f^{*} is the probability density function of the random variable T, and F^{*} is the corresponding cumulated probability function as defined above in Eq. (39). Analogous to the calculation of the simulation-implied cumulative distribution function (Eq. (40)), I approximate the simulation-implied probability density function using by counting firms that invest during a certain month after initiation:

$$f^{*}(t) = \frac{\#\text{firms that invest during month s+1, } s+1=t}{\#\text{firms that invest}}. \tag{44}$$

For the sake of presentation, I smooth the resulting empirical probability density function using a moving average approach using a time span of 25 months. Next, I define the investment hazard rate of the first-best economy, $h^{fb}(t)$, and the first-best simulation implied probability density function $f^{fb}(t)$ analogously to the definitions in the aggregate economy. The left panel of Figure 6 shows the simulation-implied investment hazard rates of the aggregate economy (black solid line) and the first-best economy (red dashed line). For example, after 36 months (3 years), the probability for a firm in the aggregate economy to invest in the next month given it has not yet invested is 0.0117, whereas the analogous probability is only 0.0097 for a firm in the first-best economy. For both economies, the hazard rate seems to be increasing for about the first three years and then slightly decreasing. Interestingly, during the first eight years, hazard rates are higher in the aggregate economy than in the first-best economy. However, after the first eight years, when hazard rates start to decline, the implied investment hazards in the first-best economy are higher than the implied investment hazards of the aggregate economy. In summary, both hazard functions are hump shaped. However, the hump is more prevalent in the hazard function implied by the aggregate economy, and the hazard function is less strongly decreasing in the case of the first-best economy. These findings are also illustrated in the right panel of Figure 6, which demonstrates that the increase in the hazard rates implied by the aggregate economy compared to the first-best economy is largest for small spells. Importantly, the increase in hazard rates is declining, and turns

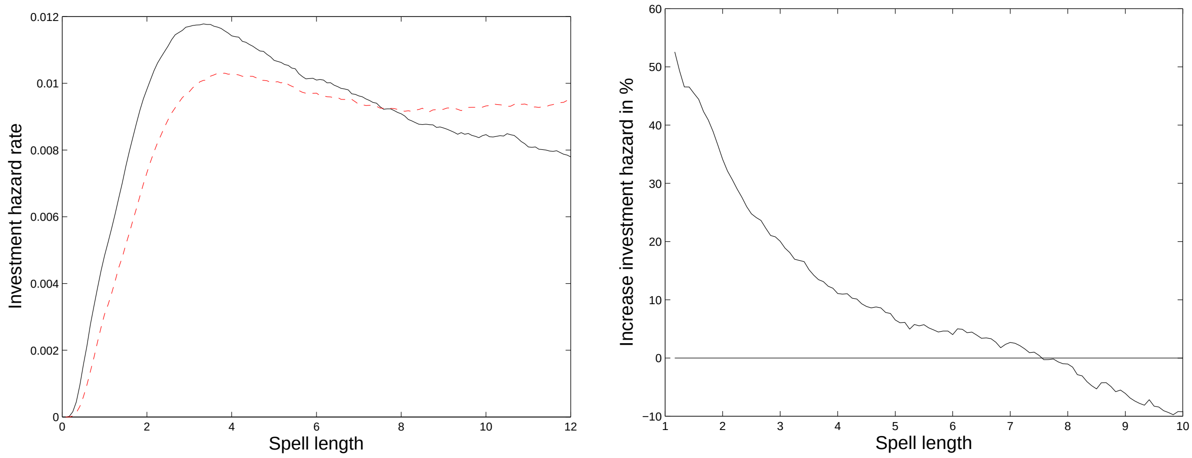

Figure 6. *Investment hazard rates and relative changes.* The left graph shows the simulated investment hazard rates, defined as the probability to invest in the next instant given the firm has not invested yet. The solid black line corresponds to the investment hazard rates resulting from the aggregate economy, the dotted red line shows the hazard rates resulting from the hypothetical first-best economy. The right graph presents the increase in hazard rates of the aggregate economy compared to the hazard rates of the hypothetical first-best economy. Standard parameters from Table I are used.

negative for spells longer than approximately eight years. Intuitively, manager-controlled firms have a higher probability of reaching the investment threshold sooner, because their boundaries are lower. Hence, the hazard rate for a manager-controlled firm is larger than for a first-best firm for short spells. As time evolves, the probability to reach the first-best thresholds increases, driving up the hazard rate implied by the first-best economy. Further, the probability of a default in the aggregate economy is lower, because default boundaries are lower. Hence, on average, cash flows in the aggregate economy are also lower than in the first-best economy, and have a greater distance to the investment boundaries. Therefore, the hazard rates implied by the aggregate economy are decreasing more strongly than the hazard rates implied by the first-best economy. I conclude that manager-shareholder agency conflict affect the intertemporal pattern of investment positively in the short and intermediate horizon, whereas the impact of manager-shareholder agency conflicts on the intertemporal pattern of investment in the long term is negative.

These results complement the findings by Akdogu and MacKay (2008), who present evidence that competition increases hazard rates. I show that the presence of manager-shareholder agency conflicts is an important determinant of hazard rates. Importantly, the relation between agency conflicts and investment hazard rates are non-trivial and non-monotone: The presence of manager shareholder agency conflicts increases the hazard rate only in the short to medium term. There are three main implications of this result: First, it is essential to control for systematic differences in the severity of agency conflicts when empirically analyzing hazard rates. Second, the analysis of empirical hazard rates might possibly shed light on the implied severities of manager-shareholder agency conflicts, when controlling for other factors influencing the timing of investment (e.g., productivity, adjustment costs, firm size, Whited, 2006 or competition, Akdogu and MacKay, 2008, or credit rationings as well as long-term incentives plans). Third, when investigating empirical hazard rates it is important to take into account the complete distribution over time. As the results above show, it can be misleading to draw conclusion on the general shape of the hazard function based on too few hazard rates.

5. Conclusion

This paper quantifies the costs of manager-shareholder agency conflicts in the presence of macroeconomic risk and investigates their evolution and implications using a dynamic approach. To do so, I develop a structural tradeoff model with intertemporal macroeconomic risk, explicitly taking into account manager-shareholder agency conflicts. Firms are heterogenous in their asset composition, a feature included by modeling both assets in place and investment opportunities. In the model, each firm is run by a manager who controls financing and investment decisions, while shareholders decide about default. Agency conflicts arise because managers divert part of the free cash flow to equity as private benefits and exercise control rights on financing and investment in their own best interest. In this framework, I investigate manager-selected investment and financing policies and the implied effects on the loss in firm value. I find that, at initiation, agency costs are substantial, increasing in the asset composition ratio, and slightly procyclical. In a dynamic aggregate economy, agency costs remain of substantial magnitude and are strongly procyclical. In recessions, when default is particularly likely and costly, firms benefit from the larger distance to default due to managerial underleverage. Further, the presence of agency conflicts strongly decreases the default rate, and slightly increases the investment rate. Finally, I show that manager-shareholder agency conflicts have important implications for the intertemporal pattern of investment. In detail, the investment hazard decreases in the short and medium term, and increases in the long term.

I contribute to the literature by providing a first analysis of the interaction between manager-shareholder agency conflicts and macroeconomic conditions, given that firms are heterogeneous in their asset composition. My results also raise two new research questions, which are directly connected to this paper. First, what are the implications of agency conflicts and macroeconomic regimes for the cross section of firms? In particular, to address the cross section, it is important to take into account that the severity of manager-shareholder agency conflicts might be systematically different for firms with different asset compositions. Further, business cycle variations in managerial diversion should be measured and taken into account. Second, is it possible to offer a compensation contract to the manager that reduces agency costs, and what are the implications in a dynamic setting? In particular, a management compensation contract including a regime-dependent component may allow to better align managers' and shareholders' interests in different regimes.

6. Tables

Table I
Baseline Parameter Choice

This table describes the baseline scenario. Panel A presents the parameters of managerial compensation. Panel B contains the annualized parameters of a typical BBB-rated S&P 500 firm. Panels C and D show the parameter choice for the expansion option and the macro economy, respectively. The asset composition ratio (ACR) is the value of the firm, divided by the value of the invested assets.

parameter	boom	recession
Panel A. Managerial characteristics		
managerial ownership ψ	0.0747	
fraction of managerial diversion of cash flow ϕ	0.01	
Panel B. Firm characteristics		
initial value of cash flows (X)	8	
tax advantage of debt and man. rents (τ)	0.15	
nominal cash flow growth rate (μ_i)	0.0782	-0.0401
systematic cash flow volatility ($\sigma_i^{X,C}$)	0.0834	0.1334
idiosyncratic cash flow volatility ($\sigma^{X,id}$)	0.168	
recovery rate (α_i)	0.7	0.5
Panel C. Expansion option parameters of a typical firm (ACR=1.5)		
exercise price (K)	31	31
scale parameter if initiated in boom (s)	2.90	
scale parameter if initiated in recession (s)	3.06	
Panel D. Economy		
rate of leaving regime i (λ_i)	0.2718	0.4928
consumption growth rate (θ_i)	0.042	0.0141
consumption growth volatility (σ_i^C)	0.0094	0.0114
expected inflation fate (π)	0.0342	
systematic price index volatility ($\sigma^{P,C}$)	-0.00035	
idiosyncratic price index volatility ($\sigma^{P,id}$)	0.0132	
rate of time preference (ρ)	0.015	
relative risk aversion (γ)	10	
elasticity of intertemporal substitution (Φ)	1.5	

Table II
Firm with only invested assets: Investment and financial policies, value functions and agency costs.

This table presents investment and financial policies, value functions, and agency costs for cash flow $X_0 = 1$. ‘First-best’ presents firm value maximizing investment and financial policies, ‘Second best’ corresponds to shareholders’ optimal investment and financial policies, and ‘Third best’ presents optimal investment and financial policies from the manager’s point of view. The asset composition ratio is defined as firm value divided by invested assets, and leverage is calculated as debt value divided by firm value.

Firm with only invested assets

	First-best (firm value)		Second best (equityholders)		Third best (manager)	
	boom	recession	boom	recession	boom	recession
Panel A. Investment policy						
investment boundary boom X_B	–	–	–	–	–	–
investment boundary recession X_R	–	–	–	–	–	–
asset composition ratio	1.0402	1.0389	1.0402	1.0389	1.0217	1.0208
Panel B. Financial policy						
coupon before investment c_o	0.5933	0.4926	0.5933	0.4926	0.2444	0.2029
leverage at initiation	0.4697	0.4575	0.4697	0.4575	0.2182	0.2125
coupon factor at investment c_n^i	–	–	–	–	–	–
% of debt financing of exercise price K at X_i	–	–	–	–	–	–
Panel C. Value functions						
value of debt d_i	7.6825	6.5313	7.6825	6.5313	3.5061	2.9813
value of equity e_i	8.6742	7.7448	8.6742	7.7448	12.5594	11.0473
value of the firm v_i	16.3567	14.2761	16.3567	14.2761	16.0655	14.0286
manager’s objective function m_i	1.3095	1.1447	1.3095	1.1447	1.3270	1.1595
Panel D. Agency costs						
total agency costs AC_i in % of firm value	–	–	0.0000	0.0000	1.7800	1.7338
investment induced agency costs IAC_i in % of AC_i	–	–	–	–	0.0000	0.0000
leverage induced agency costs LAC_i in % of AC_i	–	–	–	–	100.0000	100.0000
interaction agency costs SAC_i in % of AC_i	–	–	–	–	0.0000	0.0000

Table III

Average firm: Investment and financial policies, value functions and agency costs.

This table presents investment and financial policies, value functions, and agency costs for cash flow $X_0 = 1$. 'First-best' presents firm value maximizing investment and financial policies, 'Second best' corresponds to shareholders' optimal investment and financial policies, and 'Third best' presents optimal investment and financial policies from the manager's point of view. The asset composition ratio is defined as firm value divided by invested assets, and leverage is calculated as debt value divided by firm value.

Average firm

	First-best (firm value)		Second best (equityholders)		Third best (manager)	
	boom	recession	boom	recession	boom	recession
Panel A. Investment policy						
investment boundary boom X_B	1.6517	1.6440	1.6207	1.6123	1.5075	1.5068
investment boundary recession X_R	1.7879	1.7801	1.7595	1.7485	1.6175	1.6165
asset composition ratio	1.4545	1.3925	1.4544	1.3924	1.4103	1.3519
Panel B. Financial policy						
coupon before investment c_o	0.6517	0.5440	0.6596	0.5441	0.2330	0.1969
leverage at initiation	0.3671	0.3717	0.3676	0.3717	0.1505	0.1555
coupon factor at investment c_n^i	0.5933	0.4926	0.5933	0.4926	0.2444	0.2029
% of debt financing of exercise price K at X_i	125.4711	116.8878	122.5460	114.3916	52.9303	48.7630
Panel C. Value functions						
value of debt d_i	8.3970	7.1135	8.4056	7.1113	3.3379	2.8883
value of equity e_i	14.4743	12.0227	14.4636	12.0231	18.8374	15.6897
value of the firm v_i	22.8712	19.1362	22.8692	19.1345	22.1753	18.5780
manager's objective function m_i	1.8408	1.5387	1.8418	1.5394	1.8829	1.5723
Panel D. Agency costs						
total agency costs AC_i in % of firm value	–	–	0.0090	0.0088	3.0429	2.9169
investment induced agency costs IAC_i in % of AC_i	–	–	100.1064	100.0016	7.6621	6.5574
leverage induced agency costs LAC_i in % of AC_i	–	–	0.0000	0.0000	89.2321	90.2985
interaction agency costs SAC_i in % of AC_i	–	–	-0.1064	-0.0016	3.1058	3.1440

Table IV

Growth firm: Investment and financial policies, value functions and agency costs.

This table presents investment and financial policies, value functions, and agency costs for cash flow $X_0 = 1$. 'First-best' presents firm value maximizing investment and financial policies, 'Second best' corresponds to shareholders' optimal investment and financial policies, and 'Third best' presents optimal investment and financial policies from the manager's point of view. The asset composition ratio is defined as firm value divided by invested assets, and leverage is calculated as debt value divided by firm value.

Growth firm

	First-best (firm value)		Second best (equityholders)		Third best (manager)	
	boom	recession	boom	recession	boom	recession
Panel A. Investment policy						
investment boundary boom X_B	1.1303	1.1265	1.1225	1.1136	1.0269	1.0262
investment boundary recession X_R	1.2203	1.2172	1.2199	1.2082	1.1018	1.1009
asset composition ratio	1.9172	1.7887	1.9172	1.7886	1.8510	1.7276
Panel B. Financial policy						
coupon before investment c_o	0.7169	0.5911	0.7203	0.5935	0.2262	0.1919
leverage at initiation	0.3021	0.3125	0.3034	0.3135	0.1113	0.1185
coupon factor at investment c_n^i	0.5933	0.4926	0.5933	0.4926	0.2444	0.2029
% of debt financing of exercise price K at X_i	105.7540	98.9482	104.6691	97.9217	45.7915	42.1933
Panel C. Value functions						
value of debt d_i	9.1081	7.6803	9.1456	7.7055	3.2389	2.8129
value of equity e_i	21.0386	16.8999	21.0006	16.8736	25.8671	20.9279
value of the firm v_i	30.1467	24.5802	30.1463	24.5791	29.1060	23.7408
manager's objective function m_i	2.4601	1.9997	2.4607	2.0007	2.5237	2.0580
Panel D. Agency costs						
total agency costs AC_i in % of firm value	–	–	0.0016	0.0043	3.4522	3.4149
investment induced agency costs IAC_i in % of AC_i	–	–	102.2999	100.7743	10.0653	10.2601
leverage induced agency costs LAC_i in % of AC_i	–	–	0.0000	0.0000	84.5801	84.9275
interaction agency costs SAC_i in % of AC_i	–	–	-2.2999	-0.7743	5.3546	4.8124

Table V

Impact of agency conflicts on agency costs in the aggregate economy.

This table presents time series statistics of total agency costs, investment induced agency costs, leverage induced agency costs, and interaction agency costs in the aggregate economy. Panel A shows the overall economy. Panel B and C contain the statistics in boom and recession only, respectively.

	Moments		Quantiles				
	mean	std	10%	25%	median	75%	90%
Panel A. Overall							
total agency costs in % of firm value	1.7732	1.1824	0.3559	1.6953	2.1847	2.4328	2.6141
investment induced agency costs in % of firm value	0.2402	0.0217	0.2144	0.2250	0.2368	0.2553	0.2709
leverage induced agency costs in % of firm value	1.4377	1.1657	0.0468	1.3568	1.8561	2.0782	2.2539
interaction agency costs in % of firm value	0.0049	0.0008	0.0038	0.0042	0.0049	0.0057	0.0061
Panel B. Boom							
total agency costs in % of firm value	2.3149	0.2731	1.9079	2.1565	2.3076	2.5101	2.6861
investment induced agency costs in % of firm value	0.2396	0.0175	0.2180	0.2265	0.2364	0.2524	0.2657
leverage induced agency costs in % of firm value	1.9801	0.2557	1.5829	1.8413	1.9837	2.1630	2.3156
interaction agency costs in % of firm value	0.0045	0.0007	0.0036	0.0039	0.0044	0.0049	0.0054
Panel C. Recession							
total agency costs in % of firm value	0.9492	1.5125	-1.2073	0.3317	1.3986	2.0524	2.4132
investment induced agency costs in % of firm value	0.2410	0.0270	0.2083	0.2198	0.2384	0.2612	0.2782
leverage induced agency costs in % of firm value	0.6126	1.4832	-1.5104	0.0326	1.0731	1.6974	2.0327
interaction agency costs in % of firm value	0.0056	0.0006	0.0047	0.0053	0.0057	0.0060	0.0062

Table VI
Impact of agency conflicts on default and investment rates in the aggregate economy.

This table presents the change in default and investment rates in the aggregate economy. Panel A shows the overall changes. Panel B and C contain the changes in boom and recession only, respectively.

	Moments	
	mean	std
Panel A. Overall		
Δ default rate in %	-59.0179	-50.2112
Δ investment rate in %	+12.5253	+15.3220
Panel B. Boom		
Δ default rate in %	-56.1963	-36.7068
Δ investment rate in %	+11.9841	+15.8699
Panel C. Recession		
Δ default rate in %	-59.6420	-48.7147
Δ investment rate in %	+14.6140	+17.0124

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A. Appendix

A.1. The model

The stochastic discount factor can be derived as follows (cf. Bhamra, Kuehn, and Strebulaev, 2010b or Chen, 2010). Aggregate output C_t follows a regime-switching geometric Brownian motion:

$$\frac{\mathrm{d}C_t}{C_t} = \theta_i \mathrm{d}t + \sigma_i^C \mathrm{d}W_t^C, \ i = B, R, \tag{A-1}$$

in which W_t^C is a Brownian motion independent of the Markov chain, and θ_i, σ_i^C are the regime-dependent growth-rates and volatilities of the aggregate output. In equilibrium, aggregate consumption equals aggregate output. Hence, the above specification gives rise to uncertainty about the future moments of consumption growth.

To incorporate the impact of the intertemporal distribution of consumption risk on the representative household's utility, I assume the continuous-time analog of Epstein-Zin-Weil preferences (Epstein and Zin, 1989; Weil, 1990), which are of stochastic differential utility type (Duffie and Epstein, 1992a,b). Specifically, the utility index U_t over a consumption process C_s solves

$$U_t = \mathbb{E}^{\mathbb{P}}\left[\int_t^\infty \frac{\rho}{1-\delta} \frac{C_s^{1-\delta} - \left((1-\gamma)U_s\right)^{\frac{1-\delta}{1-\gamma}}}{\left((1-\gamma)U_s\right)^{\frac{1-\delta}{1-\gamma}} - 1} \mathrm{d}s \,|\mathcal{F}_t\right], \tag{A-2}$$

in which ρ is the rate of time preference, γ determines the coefficient of relative risk aversion for a timeless gamble, and $\Phi := \frac{1}{\delta}$ is the elasticity of intertemporal substitution for deterministic consumption paths. Solving the Bellman equation associated with the consumption problem of the representative agent, it can be shown that the stochastic discount factor sdf_t follows the dynamics (see Bhamra, Kuehn, and Strebulaev, 2010b; Chen, 2010)

$$\frac{\mathrm{d}sdf_t}{sdf_t} = -r_i \mathrm{d}t - \eta_i \mathrm{d}W_t^C + \left(e^{\kappa_i} - 1\right) \mathrm{d}M_t, \tag{A-3}$$

with M_t being the compensated process associated with the Markov chain, and

$$r_i = \bar{r}_i + \lambda_i \left[\frac{\gamma-\delta}{\gamma-1}\left(w^{-\frac{\gamma-1}{\gamma-\delta}} - 1\right) - \left(w^{-1} - 1\right)\right], \tag{A-4}$$

$$\eta_i = \gamma \sigma_i^C, \tag{A-5}$$

$$\kappa_i = (\delta - \gamma) \log\left(\frac{h_j}{h_i}\right). \tag{A-6}$$

The parameters h_B, h_R solve

$$0 = \rho\frac{1-\gamma}{1-\delta} h_i^{\delta-\gamma} + \left((1-\gamma)\theta_i - \frac{1}{2}\gamma(1-\gamma)\left(\sigma_i^C\right)^2 - \rho\frac{1-\gamma}{1-\delta}\right) h_i^{1-\gamma} + \lambda_i\left(h_j^{1-\gamma} - h_i^{1-\gamma}\right). \tag{A-7}$$

r_i are the regime-dependent real risk-free interest rates, composed of the interest rate if the economy stayed in regime i forever, $\bar{r}_i$, and the adjustment for possible regime switches as shown by the second term. η_i are the risk prices for systematic Brownian shocks affecting aggregate output, and κ_i is the relative jump size of the discount factor when the Markov chain leaves state i (and, consequently, $\kappa_j = \frac{1}{\kappa_i}$). The no-jump part of the interest rate, $\bar{r}_i$, is given by

$$\bar{r}_i = \rho + \delta\theta_i - \frac{1}{2}\gamma\left(1+\delta\right)\left(\sigma_i^C\right)^2, \tag{A-8}$$

and

$$w := e^{\kappa_R} = e^{-\kappa_B} \tag{A-9}$$

measures the size of the jump in the real-state price density when the economy shifts from recession to boom (see Bhamra, Kuehn, and Strebulaev, 2010b, Proposition 1).

To link nominal to real values, I specify a stochastic price index as

$$\frac{\mathrm{d}P_t}{P_t} = \pi \mathrm{d}t + \sigma^{P,C}\mathrm{d}W_t^C + \sigma^{P,id}\mathrm{d}W_t^P, \tag{A-10}$$

with W_t^P being a Brownian motion describing the idiosyncratic price index shock, independent of the consumption shock Brownian W_t^C and the Markov chain. π denotes the expected inflation rate, and $\sigma^{P,C} < 0, \sigma^{P,id} > 0$ are the volatilities of the stochastic price index associated with the consumption shock and the idiosyncratic price index shock, respectively. The nominal interest rate r_i^n is then given by

$$r_i^n = r_i + \pi - \sigma_P^2 - \sigma^{P,C}\eta_i, \tag{A-11}$$

with $\sigma_P := \sqrt{\left(\sigma^{P,C}\right)^2 + \left(\sigma^{P,id}\right)^2}$ being the total volatility of the stochastic price index.

At any time, the real cash flow process X of a firm follows

$$\frac{\mathrm{d}X_{t,real}}{X_{t,real}} = \mu_{i,real}\mathrm{d}t + \sigma_{i,real}^{X,C}\mathrm{d}W_t^C + \sigma^{X,id}\mathrm{d}W_t^f, \quad i = B, R, \tag{A-12}$$

in which W_t^f is a standard Brownian motion describing an idiosyncratic shock, independent of the aggregate output shock W_t^C, the consumption price index shock W_t^P, and the Markov chain. $\mu_{i,real}$ are the real regime-dependent drifts, $\sigma_{i,real}^{X,C} > 0$ the real firm-specific regime-dependent volatilities associated with the aggregate output process, and $\sigma^{X,id} > 0$ the firm-specific volatility associated with the idiosyncratic Brownian shock. The idiosyncratic shocks W_t^f are independent across firms.

The nominal cash flow process can now be written as

$$\frac{\mathrm{d}X_t}{X_t} = \mu_i\mathrm{d}t + \sigma_i^{X,C}\mathrm{d}W_t^C + \sigma^{P,id}\mathrm{d}W_t^P + \sigma^{X,id}\mathrm{d}W_t^f, \quad i = B, R, \tag{A-13}$$

in which $\mu_i = \mu_{i,real} + \pi + \sigma^{P,C}\sigma_{i,real}^{X,C}$ are the nominal regime-dependent drifts, and $\sigma_i^{X,C} = \sigma_{i,real}^{X,C} + \sigma^{P,C} > 0$ the nominal firm-specific regime-dependent volatilities associated with the aggregate output process. As suggested by the literature, I posit that $\sigma_B^{X,C} < \sigma_R^{X,C}$ (Ang and Bekaert, 2004). Defining

$$\sigma_i = \sqrt{\left(\sigma_i^{X,C}\right)^2 + \left(\sigma^{P,id}\right)^2 + \left(\sigma^{X,id}\right)^2}, \tag{A-14}$$

and a $\mathbb{P}$-Brownian Z_t yields the cash flow dynamics as stated in (1).

The expected growth rates of the firm's nominal cash flow under the risk-neutral measure $\mathbb{Q}$, $\tilde{\mu}_i$, are given by

$$\tilde{\mu}_i := \mu_i - \sigma_i^{X,C}\left(\eta_i + \sigma^{P,C}\right) - \left(\sigma^{P,id}\right)^2, \tag{A-15}$$

and let $\tilde{\lambda}_i$ denote the risk-neutral transition intensities, determined as

$$\tilde{\lambda}_i = e^{\kappa_i}\lambda_i. \tag{A-16}$$

Following Chen (2010) and Bhamra, Kuehn, and Strebulaev (2010b), the unlevered asset value can be written as

$$V_t = (1-\tau)\, X_t y_i \quad \text{for } I_t = i, \tag{A-17}$$

with y_i being the price-cash flow ratio in state i determined by

$$y_i^{-1} = r_i^n - \tilde{\mu}_i + \frac{\left(r_j^n - \tilde{\mu}_j\right) - \left(r_i^n - \tilde{\mu}_i\right)}{r_j^n - \tilde{\mu}_j + \tilde{p}}\tilde{p}\tilde{f}_j. \tag{A-18}$$

$\tilde{p} := \tilde{\lambda}_i + \tilde{\lambda}_j$ is the risk-neutral rate of news arrival, and $\left(\tilde{f}_B, \tilde{f}_R\right) = \left(\frac{\lambda_R}{\tilde{p}}, \frac{\lambda_B}{\tilde{p}}\right)$ is the long-run risk-neutral distribution. y^{-1} can be interpreted as a discount rate, in which the first two terms constitute the standard expression if the economy stayed in regime i forever, and the last term accounts for future time spent in regime j. As in Bhamra, Kuehn, and Strebulaev (2010b), the price-cash flow ratio in the main analysis is higher in boom than in recession, i.e., $y_B > y_R$.

Finally, the volatility of the cash flow process in regime i is

$$\tilde{\sigma}_i = \sqrt{\left(\sigma_i^{X,C}\right)^2 + \left(\sigma^{P,id}\right)^2 + \left(\sigma^{X,id}\right)^2}. \tag{A-19}$$

A.2. The value of corporate securities before investment

The solutions for the values of corporate debt, tax shield, and bankruptcy costs is based on Hackbarth, Miao, and Morellec (2006). Without loss of generality, I assume that the default boundary in boom is lower than the one in recession, i.e., $\hat{D}_B < \hat{D}_R$.

The valuation of corporate debt. An investor holding corporate debt requires an instantaneous return equal to the risk-free rate r_i^n. Once the firm defaults, debt-holders receive a fraction α_i of the asset value $(1-\tau)\, X y_i$. The required rate of return on debt must be equal to the realized rate of return plus the coupon proceeds from debt. The coupon proceeds from debt are determined by the coupon after investment, c_n. Therefore, an application of Ito's lemma yields that the value of debt satisfies the following system of ODEs. For $0 \leq X \leq \hat{D}_B$:

$$\begin{cases} \hat{d}_B(X) &= \alpha_B\,(1-\tau)\, X y_B \\ \hat{d}_R(X) &= \alpha_R\,(1-\tau)\, X y_R. \end{cases} \tag{A-20}$$

For $\hat{D}_B < X \leq \hat{D}_R$:

$$\begin{cases} r_B^n \hat{d}_B(X) &= c_n + \tilde{\mu}_B X \hat{d}_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 \hat{d}_B''(X) + \tilde{\lambda}_B\left(\alpha_R\,(1-\tau)\, X y_R - \hat{d}_B(X)\right) \\ \hat{d}_R(X) &= \alpha_R\,(1-\tau)\, X y_R. \end{cases} \tag{A-21}$$

For $X > \hat{D}_R$:

$$\begin{cases} r^n_B \hat{d}_B(X) &= c_n + \tilde{\mu}_B X \hat{d}'_B(X) + \frac{1}{2}\tilde{\sigma}^2_B X^2 \hat{d}''_B(X) + \tilde{\lambda}_B \left(\hat{d}_R(X) - \hat{d}_B(X)\right) \\ r^n_R \hat{d}_R(X) &= c_n + \tilde{\mu}_R X \hat{d}'_R(X) + \frac{1}{2}\tilde{\sigma}^2_R X^2 \hat{d}''_R(X) + \tilde{\lambda}_R \left(\hat{d}_B(X) - \hat{d}_R(X)\right). \end{cases} \tag{A-22}$$

The boundary conditions are given by

$$\lim_{X\to\infty} \frac{\hat{d}_i\,(X)}{X} < \infty, \quad i = B, R, \tag{A-23}$$

$$\lim_{X\searrow \hat{D}_R} \hat{d}_B(X) = \lim_{X\nearrow \hat{D}_R} \hat{d}_B(X), \tag{A-24}$$

$$\lim_{X\searrow \hat{D}_R} \hat{d}'_B(X) = \lim_{X\nearrow \hat{D}_R} \hat{d}'_B(X), \tag{A-25}$$

$$\lim_{X\searrow \hat{D}_B} \hat{d}_B\,(X) = \alpha_B\,(1-\tau)\,D_B y_B, \tag{A-26}$$

and

$$\lim_{X\searrow \hat{D}_B} \hat{d}_R\,(X) = \alpha_R\,(1-\tau)\,D_R y_R. \tag{A-27}$$

Condition (A-23) is the no-bubbles condition. The remaining boundary conditions are the value-matching conditions (A-24), (A-26), and (A-27), and the smooth-pasting condition at the higher default threshold $\hat{D}_R$ for the debt function in boom $\hat{d}_B(\cdot)$, Eq. (A-25). The functional form of the solution is

$$\hat{d}_i(X) = \begin{cases} \alpha_i\,(1-\tau)\,X y_i & X \le \hat{D}_i & i = B, R \\ \hat{C}_1 X^{\beta^B_1} + \hat{C}_2 X^{\beta^B_2} + C_3 X + \hat{C}_4 & \hat{D}_B < X \le \hat{D}_R, & i = B \\ \hat{A}_{i1} X^{\gamma_1} + \hat{A}_{i2} X^{\gamma_2} + \hat{A}_{i6} & X > \hat{D}_R, & i = B, R, \end{cases} \tag{A-28}$$

in which $\hat{A}_{B1}, \hat{A}_{B2}, \hat{A}_{R1}, \hat{A}_{R2}, A_{B5}, A_{R5}, \hat{C}_1, \hat{C}_2, C_3, \hat{C}_4, \gamma_1, \gamma_2, \beta^B_1$, and β^B_2 are real-valued parameters to be determined.

Solving the system of ODEs (A-20)-(A-22) subject to its boundary conditions (A-23)-(A-27), I find that

$$C_3 = \frac{\tilde{\lambda}_B \alpha_R\,(1-\tau)\,y_R}{r^n_B + \tilde{\lambda}_B - \tilde{\mu}_B}, \tag{A-29}$$

$$\hat{C}_4 = \frac{c_n}{r^n_B + \tilde{\lambda}_B}, \tag{A-30}$$

and

$$\hat{A}_{i6} = \frac{c_n \left(r^n_j + \tilde{\lambda}_i + \tilde{\lambda}_j\right)}{r^n_i r^n_j + r^n_j \tilde{\lambda}_i + r^n_i \tilde{\lambda}_j} = \frac{c_n}{r^p_i}. \tag{A-31}$$

Next, $\hat{A}_{Bk}$ is a multiple of $\hat{A}_{Rk}$, $k = 1, 2$, with the factor $l_k := \frac{1}{\tilde{\lambda}_B}(r_B^n + \tilde{\lambda}_B - \tilde{\mu}_B\gamma_k - \frac{1}{2}\tilde{\sigma}_B^2\gamma_k(\gamma_k - 1))$, i.e., $\hat{A}_{Rk} = l_k\hat{A}_{Bk}$, and γ_1 and γ_2 are the negative roots of the quartic equation[16]

$$\left(\tilde{\mu}_R\gamma + \frac{1}{2}\tilde{\sigma}_R^2\gamma(\gamma-1) - \tilde{\lambda}_R - r_R^n\right)\left(\tilde{\mu}_B\gamma + \frac{1}{2}\tilde{\sigma}_B^2\gamma(\gamma-1) - \tilde{\lambda}_B - r_B^n\right) = \tilde{\lambda}_R\tilde{\lambda}_B. \tag{A-32}$$

The reason for taking the negative roots is the no-bubbles condition for debt stated in Eq. (A-23).

The remaining unknown parameters $\hat{A}_{B1}, \hat{A}_{B2}, \hat{C}_1$, and $\hat{C}_2$ solve

$$\begin{bmatrix} \hat{A}_{B1} & \hat{A}_{B2} & \hat{C}_1 & \hat{C}_2 \end{bmatrix}^T = \hat{M}^{-1}\hat{b}, \tag{A-33}$$

in which

$$\hat{M} := \begin{bmatrix} \hat{D}_R^{\gamma_1} & \hat{D}_R^{\gamma_2} & -\hat{D}_R^{\beta_1^B} & -\hat{D}_R^{\beta_2^B} \\ \gamma_1\hat{D}_R^{\gamma_1} & \gamma_2\hat{D}_R^{\gamma_2} & -\beta_1^B\hat{D}_R^{\beta_1^B} & -\beta_2^B\hat{D}_R^{\beta_2^B} \\ 0 & 0 & \hat{D}_B^{\beta_1^B} & \hat{D}_B^{\beta_2^B} \\ l_1\hat{D}_R^{\gamma_1} & l_2\hat{D}_R^{\gamma_2} & 0 & 0 \end{bmatrix} \tag{A-34}$$

and

$$\hat{b} := \begin{bmatrix} C_3\hat{D}_R + \hat{C}_4 - A_{B5} \\ C_3\hat{D}_R \\ \alpha_B(1-\tau)\hat{D}_B y_B - C_3\hat{D}_B - \hat{C}_4 \\ \alpha_R(1-\tau)\hat{D}_R y_R - A_{R5} \end{bmatrix}. \tag{A-35}$$

Bankruptcy costs. Once the firm defaults, the bankruptcy costs correspond to one minus the recovery rate times the value of the unlevered assets. Hence, the system of ODEs for bankruptcy costs $\hat{b}_i(X)$ is given by For $0 \le X \le \hat{D}_B$:

$$\begin{cases} \hat{b}_B(X) &= (1-\alpha_B)(1-\tau)Xy_B \\ \hat{b}_R(X) &= (1-\alpha_R)(1-\tau)Xy_R. \end{cases} \tag{A-36}$$

For $\hat{D}_B < X \le \hat{D}_R$:

$$\begin{cases} r_B^n\hat{b}_B(X) &= \tilde{\mu}_B X\hat{b}'_B(X) + \frac{1}{2}\tilde{\sigma}_B^2X^2\hat{b}''_B(X) + \tilde{\lambda}_B\left((1-\alpha_R)(1-\tau)Xy_R - \hat{b}_B(X)\right) \\ \hat{d}_R(X) &= (1-\alpha_R)(1-\tau)Xy_R. \end{cases} \tag{A-37}$$

For $X > \hat{D}_R$:

$$\begin{cases} r_B^n\hat{b}_B(X) &= \tilde{\mu}_B X\hat{b}'_B(X) + \frac{1}{2}\tilde{\sigma}_B^2X^2\hat{b}''_B(X) + \tilde{\lambda}_B\left(\hat{b}_R(X) - \hat{b}_B(X)\right) \\ r_R^n\hat{b}_R(X) &= \tilde{\mu}_R X\hat{b}'_R(X) + \frac{1}{2}\tilde{\sigma}_R^2X^2\hat{b}''_R(X) + \tilde{\lambda}_R\left(\hat{b}_B(X) - \hat{b}_R(X)\right). \end{cases} \tag{A-38}$$

[16] By arguments of Guo (2001), this quartic equation always has four distinct real roots, two of them negative, and two of them positive.

The system is subject to the boundary conditions

$$\lim_{X\to\infty}\frac{\hat{b}_i(X)}{X}<\infty,\quad i=B,R, \tag{A-39}$$

$$\lim_{X\searrow\hat{D}_R}\hat{b}_B(X)=\lim_{X\nearrow\hat{D}_R}\hat{b}_B(X), \tag{A-40}$$

$$\lim_{X\searrow\hat{D}_R}\hat{b}_B'(X)=\lim_{X\nearrow\hat{D}_R}\hat{b}_B'(X), \tag{A-41}$$

$$\lim_{X\searrow\hat{D}_B}\hat{b}_B(X)=(1-\alpha_B)(1-\tau)D_B y_B, \tag{A-42}$$

and

$$\lim_{X\searrow\hat{D}_B}\hat{b}_R(X)=(1-\alpha_R)(1-\tau)D_R y_R. \tag{A-43}$$

This system (A-36)-(A-38) with its boundary conditions (A-39)-(A-43) corresponds to the system for debt, (A-20)-(A-22), with its boundary conditions (A-23)-(A-27), in which the recovery rate is replaced by $1-\alpha_i$ and the coupon by zero. Therefore, the solution for bankruptcy costs is analogous to the solution for the value of corporate debt, (A-29)-(A-35), with a recovery rate of $1-\alpha_i$ and a coupon of zero.

The value of the tax shield. Coupon payments are shielded from taxation, resulting in an instantaneous tax shield of τc_n. Once the firm defaults, the tax shield is zero. Thus, the system of ODEs for the value of the tax shield corresponds to For $0\le X\le\hat{D}_B$:

$$\begin{cases}\hat{t}_B(X) &= 0\\ \hat{t}_R(X) &= 0.\end{cases} \tag{A-44}$$

For $\hat{D}_B<X\le\hat{D}_R$:

$$\begin{cases} r_B^n\hat{t}_B(X) &= \tau c_n+\tilde{\mu}_B X\hat{t}_B'(X)+\frac{1}{2}\tilde{\sigma}_B^2X^2\hat{t}_B''(X)+\tilde{\lambda}_B\left(0-\hat{t}_B(X)\right)\\ \hat{t}_R(X) &= 0.\end{cases} \tag{A-45}$$

For $X>\hat{D}_R$:

$$\begin{cases} r_B^n\hat{t}_B(X) &= \tau c_n+\tilde{\mu}_B X\hat{t}_B'(X)+\frac{1}{2}\tilde{\sigma}_B^2X^2\hat{t}_B''(X)+\tilde{\lambda}_B\left(\hat{t}_R(X)-\hat{t}_B(X)\right)\\ r_R^n\hat{t}_R(X) &= \tau c_n+\tilde{\mu}_R X\hat{t}_R'(X)+\frac{1}{2}\tilde{\sigma}_R^2X^2\hat{t}_R''(X)+\tilde{\lambda}_R\left(\hat{t}_B(X)-\hat{t}_R(X)\right).\end{cases} \tag{A-46}$$

The boundary conditions read

$$\lim_{X\to\infty}\frac{\hat{t}_i(X)}{X}<\infty,\quad i=B,R, \tag{A-47}$$

$$\lim_{X\searrow\hat{D}_R}\hat{t}_B(X)=\lim_{X\nearrow\hat{D}_R}\hat{t}_B(X), \tag{A-48}$$

$$\lim_{X\searrow\hat{D}_R}\hat{t}_B'(X)=\lim_{X\nearrow\hat{D}_R}\hat{t}_B'(X), \tag{A-49}$$

$$\lim_{X\searrow\hat{D}_B}\hat{t}_B(X)=0, \tag{A-50}$$

and

$$\lim_{X \searrow \hat{D}_B} \hat{b}_R(X) = 0. \tag{A-51}$$

This system (A-44)-(A-46) with its boundary conditions (A-47)-(A-51) corresponds to the system for debt, (A-20)-(A-22), with its boundary conditions (A-23)-(A-27), in which the recovery rate is replaced by zero and the coupon by τc_n. Therefore, the solution for the tax shield is analogous to the solution for the value of corporate debt, (A-29)-(A-35), with a recovery rate of zero and a coupon of τc_n.

The expected value of net future cash flows. As long as the firm is solvent, the manager diverts a fraction of the firm's net cash flow as private benefits. Once the firm defaults, the expected value of future cash flows is zero. Thus, an application of Ito's lemma shows that the system of ODEs for the value future cash flows is

For $0 \leq X \leq \hat{D}_B$:

$$\begin{cases} \hat{n}_B(X) &= 0 \\ \hat{n}_R(X) &= 0. \end{cases} \tag{A-52}$$

For $\hat{D}_B < X \leq \hat{D}_R$:

$$\begin{cases} r_B^n \hat{n}_B(X) &= (1-\tau)(X - c_n) + \tilde{\mu}_B X \hat{n}_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 \hat{n}_B''(X) + \tilde{\lambda}_B (0 - \hat{n}_B(X)) \\ \hat{n}_R(X) &= 0. \end{cases} \tag{A-53}$$

For $X > \hat{D}_R$:

$$\begin{cases} r_B^n \hat{n}_B(X) &= (1-\tau)(X - c_n) + \tilde{\mu}_B X \hat{n}_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 \hat{n}_B''(X) + \tilde{\lambda}_B (\hat{n}_R(X) - \hat{n}_B(X)) \\ r_R^n \hat{n}_R(X) &= (1-\tau)(X - c_n) + \tilde{\mu}_R X \hat{n}_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 \hat{n}_R''(X) + \tilde{\lambda}_R (\hat{n}_B(X) - \hat{n}_R(X)). \end{cases} \tag{A-54}$$

The boundary conditions read

$$\lim_{X \to \infty} \frac{\hat{n}_i(X)}{X} < \infty, \quad i = B, R, \tag{A-55}$$

$$\lim_{X \searrow \hat{D}_R} \hat{n}_B(X) = \lim_{X \nearrow \hat{D}_R} \hat{t}_B(X), \tag{A-56}$$

$$\lim_{X \searrow \hat{D}_R} \hat{n}_B'(X) = \lim_{X \nearrow \hat{D}_R} \hat{t}_B'(X), \tag{A-57}$$

$$\lim_{X \searrow \hat{D}_B} \hat{n}_B(X) = 0, \tag{A-58}$$

and

$$\lim_{X \searrow \hat{D}_B} \hat{n}_R(X) = 0. \tag{A-59}$$

Technically, the difference between this system for the expected value of future cash flow and the systems for debt is the linear term in X, as long as the firm is solvent. This linear term X corresponds to the firm's current cash flow. Consequently, the solution is slightly different. The functional form for the solution of system (A-52)-(A-54) with boundary conditions (A-55)-(A-139)is now given by

$$\hat{n}_i(X) = \begin{cases} 0 & X \leq \hat{D}_i & i = B, R \\ \hat{C}_1 X^{\beta_1^B} + \hat{C}_2 X^{\beta_2^B} + C_3 X + C_4 & \hat{D}_B < X \leq \hat{D}_R, & i = B \\ \hat{A}_{i1} X^{\gamma_1} + \hat{A}_{i2} X^{\gamma_2} + \hat{A}_{i5} X + \hat{A}_{i6} & X > \hat{D}_R, & i = B, R, \end{cases} \tag{A-60}$$

in which $\hat{A}_{B1}, \hat{A}_{B2}, \hat{A}_{R1}, \hat{A}_{R2}, \hat{A}_{B4}, \hat{A}_{R4}, A_{B5}, A_{R5}, \hat{C}_1, \hat{C}_2, C_3, \hat{C}_4, \gamma_1, \gamma_2, \beta_1^B$, and β_2^B are real-valued parameters to be determined.

Solving, I find that

$$C_3 = \frac{1-\tau}{r_B^n + \tilde{\lambda}_B - \tilde{\mu}_B}, \tag{A-61}$$

$$\hat{C}_4 = \frac{c_n (1-\tau)}{r_B^n + \tilde{\lambda}_B}, \tag{A-62}$$

$$\hat{A}_{i5} = \frac{\left(r_j^n - \tilde{\mu}_j + \tilde{\lambda}_i + \tilde{\lambda}_j\right)}{\left(r_i^n - \tilde{\mu}_i\right)\left(r_j^n - \tilde{\mu}_j\right) + \left(r_j^n - \tilde{\mu}_j\right)\tilde{\lambda}_i + \left(r_i^n - \tilde{\mu}_i\right)\tilde{\lambda}_j}, \tag{A-63}$$

and

$$\hat{A}_{i6} = \frac{(1-\tau)\, c_n \left(r_j^n + \tilde{\lambda}_i + \tilde{\lambda}_j\right)}{r_i^n r_j^n + r_j^n \tilde{\lambda}_i + r_i^n \tilde{\lambda}_j} = \frac{(1-\tau)\, c_n}{r_i^p}. \tag{A-64}$$

Next, $\hat{A}_{Bk}$ is a multiple of $\hat{A}_{Rk}$, $k = 1, 2$, with the factor $l_k := \frac{1}{\tilde{\lambda}_B}(r_B^n + \tilde{\lambda}_B - \tilde{\mu}_B \gamma_k - \frac{1}{2}\tilde{\sigma}_B^2 \gamma_k(\gamma_k - 1))$, i.e., $\hat{A}_{Rk} = l_k \hat{A}_{Bk}$, and γ_1 and γ_2 are the negative roots of the quartic equation

$$\left(\tilde{\mu}_R \gamma + \frac{1}{2}\tilde{\sigma}_R^2 \gamma(\gamma - 1) - \tilde{\lambda}_R - r_R^n\right)\left(\tilde{\mu}_B \gamma + \frac{1}{2}\tilde{\sigma}_B^2 \gamma(\gamma - 1) - \tilde{\lambda}_B - r_B^n\right) = \tilde{\lambda}_R \tilde{\lambda}_B. \tag{A-65}$$

The remaining unknown parameters $\hat{A}_{B1}, \hat{A}_{B2}, \hat{C}_1$, and $\hat{C}_2$ solve

$$\begin{bmatrix} \hat{A}_{B1} & \hat{A}_{B2} & \hat{C}_1 & \hat{C}_2 \end{bmatrix}^T = \hat{M}^{-1}\hat{b}, \tag{A-66}$$

in which

$$\hat{M} := \begin{bmatrix} \hat{D}_R^{\gamma_1} & \hat{D}_R^{\gamma_2} & -\hat{D}_R^{\beta_1^B} & -\hat{D}_R^{\beta_2^B} \\ \gamma_1 \hat{D}_R^{\gamma_1} & \gamma_2 \hat{D}_R^{\gamma_2} & -\beta_1^B \hat{D}_R^{\beta_1^B} & -\beta_2^B \hat{D}_R^{\beta_2^B} \\ 0 & 0 & \hat{D}_B^{\beta_1^B} & \hat{D}_B^{\beta_2^B} \\ l_1 \hat{D}_R^{\gamma_1} & l_2 \hat{D}_R^{\gamma_2} & 0 & 0 \end{bmatrix} \tag{A-67}$$

and

$$\hat{b} := \begin{bmatrix} C_3\hat{D}_R + \hat{C}_4 - \hat{A}_{B4}\hat{D}_R - \hat{A}_{B5} \\ C_3\hat{D}_R - \hat{A}_{B4}\hat{D}_R \\ -C_3\hat{D}_B - \hat{C}_4 \\ -\hat{A}_{R4}\hat{D}_R - \hat{A}_{R5} \end{bmatrix}. \tag{A-68}$$

A.3. The value functions before investment

The value of the growth option. The following Proposition 1 states the value of the growth option for any given pair of exercise boundaries. The proposition is as in Arnold, Wagner, and Westermann (forthcoming).

Proposition 1. *(i) For any given pair of exercise boundaries $[X_B, X_R]$, the value of the growth option in regime i is given by*

$$G_i(X) = \begin{cases} \bar{A}_{i3}X^{\gamma_3} + \bar{A}_{i4}X^{\gamma_4} & X < X_B, \qquad i = B, R \\ \bar{C}_1 X^{\beta_1^R} + \bar{C}_2 X^{\beta_2^R} + \tilde{\lambda}_R \frac{(s-1)y_B X}{r_R^n - \tilde{\mu}_R + \tilde{\lambda}_R} - \tilde{\lambda}_R \frac{K}{r_R^n + \tilde{\lambda}_R} & X_B \le X < X_R, \quad i = R \\ (s-1)(1-\tau) X y_i - K & X \ge X_i, \qquad i = B, R, \end{cases} \tag{A-69}$$

in which $\gamma_k, k = 3, 4,$ are the positive roots of the quartic equation

$$\left(\tilde{\mu}_R\gamma + \frac{1}{2}\tilde{\sigma}_R^2\gamma(\gamma-1) - \tilde{\lambda}_R - r_R^n\right)\left(\tilde{\mu}_B\gamma + \frac{1}{2}\tilde{\sigma}_B^2\gamma(\gamma-1) - \tilde{\lambda}_B - r_B^n\right) = \tilde{\lambda}_R\tilde{\lambda}_B, \tag{A-70}$$

and β_k^R, $k = 1, 2,$ are given by

$$\beta_{1,2}^R = \frac{1}{2} - \frac{\tilde{\mu}_R}{\tilde{\sigma}_R^2} \pm \sqrt{\left(\frac{1}{2} - \frac{\tilde{\mu}_R}{\tilde{\sigma}_R^2}\right)^2 + \frac{2\left(r_R^n + \tilde{\lambda}_R\right)}{\tilde{\sigma}_R^2}}. \tag{A-71}$$

$\bar{A}_{Rk}$, $k = 3, 4,$ is a multiple of $\bar{A}_{Bk}$ with the factor

$$\bar{l}_k := \frac{1}{\tilde{\lambda}_B}(r_B^n + \tilde{\lambda}_B - \tilde{\mu}_B\gamma_k - \frac{1}{2}\tilde{\sigma}_B^2\gamma_k(\gamma_k - 1)). \tag{A-72}$$

$\left[\bar{A}_{B3}, \bar{A}_{B4}, \bar{C}_1, \bar{C}_2\right]$ solve the linear system

$$\begin{bmatrix} \bar{A}_{B3} & \bar{A}_{B4} & \bar{C}_1 & \bar{C}_2 \end{bmatrix}^T = \bar{M}^{-1}\bar{b}, \tag{A-73}$$

in which

$$\bar{M} = \begin{bmatrix} \bar{l}_3 X_B^{\gamma_3} & \bar{l}_4 X_B^{\gamma_4} & -X_B^{\beta_1^R} & -X_B^{\beta_2^R} \\ \bar{l}_3\gamma_3 X_B^{\gamma_3} & \bar{l}_4\gamma_4 X_B^{\gamma_4} & -\beta_1^R X_B^{\beta_1^R} & -\beta_2^R X_B^{\beta_2^R} \\ 0 & 0 & X_R^{\beta_1^R} & X_R^{\beta_2^R} \\ X_B^{\gamma_3} & X_B^{\gamma_4} & 0 & 0 \end{bmatrix}, \tag{A-74}$$

and

$$\bar{b} := \begin{bmatrix} \bar{C}_3 X_B + \bar{C}_4 \\ \bar{C}_3 X_B \\ -\bar{C}_3 X_R - \bar{C}_4 + (s-1)(1-\tau) y_R X_R - K \\ (s-1)(1-\tau) y_B X_B - K \end{bmatrix}. \tag{A-75}$$

(ii) The unlevered value of the growth option, i.e., using value-maximizing exercise boundaries can be calculated using the formulas (A-69)-(A-72). $\left[\bar{A}_{B3}, \bar{A}_{B4}, \bar{C}_1, \bar{C}_2, X_B^{unlev}, X_R^{unlev}\right]$ *are determined by the non-linear six-dimensional equation*

$$\bar{M} \begin{bmatrix} \bar{A}_{B3} & \bar{A}_{B4} & \bar{C}_1 & \bar{C}_2 \end{bmatrix}^T = \bar{b}, \tag{A-76}$$

in which

$$\bar{M} = \begin{bmatrix} \bar{l}_3 X_B^{\gamma_3} & \bar{l}_4 X_B^{\gamma_4} & -X_B^{\beta_1^R} & -X_B^{\beta_2^R} \\ \bar{l}_3 \gamma_3 X_B^{\gamma_3} & \bar{l}_4 \gamma_4 X_B^{\gamma_4} & -\beta_1^R X_B^{\beta_1^R} & -\beta_2^R X_B^{\beta_2^R} \\ 0 & 0 & X_R^{\beta_1^R} & X_R^{\beta_2^R} \\ X_B^{\gamma_3} & X_B^{\gamma_4} & 0 & 0 \\ 0 & 0 & \beta_1^R X_R^{\beta_1^R} & \beta_2^R X_R^{\beta_2^R} \\ \gamma_3 X_B^{\gamma_3} & \gamma_4 X_B^{\gamma_4} & 0 & 0 \end{bmatrix}, \tag{A-77}$$

and

$$\bar{b} := \begin{bmatrix} \bar{C}_3 X_B + \bar{C}_4 \\ \bar{C}_3 X_B \\ -\bar{C}_3 X_R - \bar{C}_4 + (s-1)(1-\tau) y_R X_R - K \\ (s-1)(1-\tau) y_B X_B - K \\ -\bar{C}_3 X_R + (s-1)(1-\tau) y_R X_R \\ (s-1)(1-\tau) y_B X_B \end{bmatrix}. \tag{A-78}$$

Proof. For the proof, see Arnold, Wagner, and Westermann (forthcoming). □

The value of corporate securities. Define r_i^p as the perpetual risk-free rate given by

$$r_i^p = r_i + \frac{r_j - r_i}{\tilde{p} + r_j} \tilde{p} \tilde{f}_j, \tag{A-79}$$

in which $\tilde{p} = \tilde{\lambda}_1 + \tilde{\lambda}_2$ is the risk-neutral rate of news arrival, and $\left(\tilde{f}_B, \tilde{f}_R\right) = \left(\frac{\lambda_R}{\tilde{p}}, \frac{\lambda_B}{\tilde{p}}\right)$ the long-run risk-neutral distribution.

The following proposition is states the values of corporate securities. I first state the general functional form of the value functions of interest (debt, bankruptcy costs, tax shield, and expected value of future cash flows), and then present the parameters of the general functional form for each value function.

Proposition 2. *For any given set of default and exercise boundaries* $[D_B, D_R, X_B, X_R]$*, the general functional form for the value functions of interest in regime* i *is given by*

$$f_i(X) = \begin{cases} E_{i1}X + E_{i2}G_i^{unlev}(X) + E_{i3}G_i(X) & X \leq D_i, \quad i = B, R, \\ C_1 X^{\beta_1^B} + C_2 X^{\beta_2^B} + C_3 X + C_4 + C_5 X^{\gamma_3} + C_6 X^{\gamma_4} & D_B < X \leq D_R, \quad i = B \\ A_{i1}X^{\gamma_1} + A_{i2}X^{\gamma_2} + A_{i3}X^{\gamma_3} + A_{i4}X^{\gamma_4} + A_{i5}X + A_{i6} & D_R < X \leq X_B, \quad i = B, R \\ B_1 X^{\beta_1^R} + B_2 X^{\beta_2^R} + B_3 X + B_4 & X_B < X \leq X_R, \quad i = R \\ F_{i1}X + F_{i2} & X > X_i, \quad i = B, R. \end{cases} \tag{A-80}$$

G_i^{unlev} *denotes the unlevered option value in regime* i *(see Proposition 1), and*

$$\beta_{1,2}^i = \frac{1}{2} - \frac{\tilde{\mu}_i}{\tilde{\sigma}_i^2} \pm \sqrt{\left(\frac{1}{2} - \frac{\tilde{\mu}_i}{\tilde{\sigma}_i^2}\right)^2 + \frac{2\left(r_i^n + \tilde{\lambda}_i\right)}{\tilde{\sigma}_i^2}} \tag{A-81}$$

$\gamma_k, k = 1, 2, 3, 4$ *are the roots of the quartic equation*

$$\left(\tilde{\mu}_R\gamma + \frac{1}{2}\tilde{\sigma}_R^2\gamma(\gamma-1) - \tilde{\lambda}_R - r_R^n\right)\left(\tilde{\mu}_B\gamma + \frac{1}{2}\tilde{\sigma}_B^2\gamma(\gamma-1) - \tilde{\lambda}_B - r_B^n\right) = \tilde{\lambda}_R\tilde{\lambda}_B. \tag{A-82}$$

A_{Rk}*,* $k = 1, 2, 3, 4$*, is a multiple of* A_{Bk} *with the factor*

$$l_k := \frac{1}{\tilde{\lambda}_B}(r_B^n + \tilde{\lambda}_B - \tilde{\mu}_B\gamma_k - \frac{1}{2}\tilde{\sigma}_B^2\gamma_k(\gamma_k - 1)). \tag{A-83}$$

$[A_{B1}\ A_{B2}\ A_{B3}\ A_{B4}\ C_1\ C_2\ B_1\ B_2]^T$ *solve a linear system*

$$M\begin{bmatrix} A_{B1} & A_{B2} & A_{B3} & A_{B4} & C_1 & C_2 & B_1 & B_2 \end{bmatrix}^T = b, \tag{A-84}$$

in which

$$M = \begin{bmatrix} D_R^{\gamma_1} & D_R^{\gamma_2} & D_R^{\gamma_3} & D_R^{\gamma_4} & -D_R^{\beta_1^B} & -D_R^{\beta_2^B} & 0 & 0 \\ \gamma_1 D_R^{\gamma_1} & \gamma_2 D_R^{\gamma_2} & \gamma_3 D_R^{\gamma_3} & \gamma_4 D_R^{\gamma_4} & -\beta_1^B D_R^{\beta_1^B} & -\beta_2^B D_R^{\beta_2^B} & 0 & 0 \\ 0 & 0 & 0 & 0 & D_B^{\beta_1^B} & D_B^{\beta_2^B} & 0 & 0 \\ l_1 D_R^{\gamma_1} & l_2 D_R^{\gamma_2} & l_3 D_R^{\gamma_3} & l_4 D_R^{\gamma_4} & 0 & 0 & 0 & 0 \\ l_1 X_B^{\gamma_1} & l_2 X_B^{\gamma_2} & l_3 X_B^{\gamma_3} & l_4 X_B^{\gamma_4} & 0 & 0 & -X_B^{\beta_1^R} & -X_B^{\beta_2^R} \\ l_1\gamma_1 X_B^{\gamma_1} & l_2\gamma_2 X_B^{\gamma_2} & l_3\gamma_3 X_B^{\gamma_3} & l_4\gamma_4 X_B^{\gamma_4} & 0 & 0 & -\beta_1^R X_B^{\beta_1^R} & -\beta_2^R X_B^{\beta_2^R} \\ X_B^{\gamma_1} & X_B^{\gamma_2} & X_B^{\gamma_3} & X_B^{\gamma_4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & X_R^{\beta_1^R} & X_R^{\beta_2^R} \end{bmatrix} \tag{A-85}$$

and

$$b = \begin{bmatrix} -A_{B5}D_R - A_{B6} + C_3D_R + C_4 + C_5D_R^{\gamma_1} + C_6D_R^{\gamma_2} \\ -A_{B5}D_R + C_3D_R + \gamma_1 C_5D_R^{\gamma_1} + \gamma_2 C_6D_R^{\gamma_2} \\ -C_3D_B - C_4 - C_5D_B^{\gamma_3} - C_6D_B^{\gamma_4} + E_{B1}D_B + E_{B2}G_B^{unlev}(D_B) + E_{B3}G_B(D_B) \\ -A_{R5}D_R - A_{R6} + E_{R1}D_R + E_{R2}G_R^{unlev}(D_R) + E_{R3}G_R(D_R) \\ -A_{R5}X_B - A_{R6} + B_3X_B + B_4 \\ -A_{R5}X_B + B_3X_B \\ -A_{B5}X_B - A_{B6} + F_{B1}X_B + F_{B2} \\ -B_3X_R - B_4 + F_{R1}X_R + F_{R2} \end{bmatrix}. \tag{A-86}$$

(i) The value of corporate debt of debtholders before investment is determined by the following parameters.

$$E_{i1} = \alpha_i y_i (1-\tau) \tag{A-87}$$

$$E_{i2} = \alpha_i (1-\tau) \tag{A-88}$$

$$E_{i3} = 0 \tag{A-89}$$

$$C_3 = \tilde{\lambda}_B \frac{\alpha_R (1-\tau) y_R}{r_B^n - \tilde{\mu}_B + \tilde{\lambda}_B} \tag{A-90}$$

$$C_4 = \frac{c_o}{r_B^n + \tilde{\lambda}_B} \tag{A-91}$$

$$C_5 = \alpha_R (1-\tau) \frac{\bar{l}_3}{l_3} \bar{A}_{B3}^{unlev} \tag{A-92}$$

$$C_6 = \alpha_R (1-\tau) \frac{\bar{l}_4}{l_4} \bar{A}_{B4}^{unlev} \tag{A-93}$$

$$A_{i5} = 0 \tag{A-94}$$

$$A_{i6} = \frac{c_o}{r_i^p} \tag{A-95}$$

$$B_3 = 0 \tag{A-96}$$

$$B_4 = \frac{\tilde{\lambda}_R f_B^d + c_o}{r_R^n + \tilde{\lambda}_R} \tag{A-97}$$

$$F_{i1} = 0 \tag{A-98}$$

$$F_{i2} = f_i^d \tag{A-99}$$

f_i^d *is the value of debt owned by the debtholders before investment at the time of exercise.*

(ii) Bankruptcy costs are calculated using the following parameters.

$$E_{i1} = (1 - \alpha_i)(1 - \tau) y_i \tag{A-100}$$

$$E_{i2} = -\alpha_i (1 - \tau) \tag{A-101}$$

$$E_{i3} = 1 \tag{A-102}$$

$$C_3 = \tilde{\lambda}_B \frac{\alpha_R (1 - \tau) y_R}{r_B^n - \tilde{\mu}_B + \tilde{\lambda}_B} \tag{A-103}$$

$$C_4 = \frac{c_o}{r_B^n + \tilde{\lambda}_B} \tag{A-104}$$

$$C_5 = \frac{\bar{l}_3}{l_3} \left(\bar{A}_{B3}^{lev} - \alpha_R (1 - \tau) \bar{A}_{B3}^{unlev} \right) \tag{A-105}$$

$$C_6 = \frac{\bar{l}_4}{l_4} \left(\bar{A}_{B4}^{lev} - \alpha_R (1 - \tau) \bar{A}_{B4}^{unlev} \right) \tag{A-106}$$

$$A_{i5} = 0 \tag{A-107}$$

$$A_{i6} = 0 \tag{A-108}$$

$$B_3 = \tilde{\lambda}_R \frac{s f_B^b}{r_R^n - \tilde{\mu}_R + \tilde{\lambda}_R} \tag{A-109}$$

$$B_4 = 0 \tag{A-110}$$

$$F_{i1} = s f_i^b \tag{A-111}$$

$$F_{i2} = 0 \tag{A-112}$$

f_i^b *is the factor to calculate bankruptcy costs of a firm with only invested assets given the manager-selected coupon* c_n.

(iii) The value of the tax shield uses the following parameters.

$$E_{i1} = 0 \tag{A-113}$$

$$E_{i2} = 0 \tag{A-114}$$

$$E_{i3} = 0 \tag{A-115}$$

$$C_3 = 0 \tag{A-116}$$

$$C_4 = \frac{c_o \tau}{r_B^n + \tilde{\lambda}_B} \tag{A-117}$$

$$C_5 = 0 \tag{A-118}$$

$$C_6 = 0 \tag{A-119}$$

$$A_{i5} = \frac{\left(r_j^n - \tilde{\mu}_j + \tilde{\lambda}_i + \tilde{\lambda}_j\right)}{\left(r_i^n - \tilde{\mu}_i\right)\left(r_j^n - \tilde{\mu}_j\right) + \left(r_j^n - \tilde{\mu}_j\right)\tilde{\lambda}_i + \left(r_i^n - \tilde{\mu}_i\right)\tilde{\lambda}_j} \tau\psi \tag{A-120}$$

$$A_{i6} = \frac{\tau c_o}{r_i^p} \tag{A-121}$$

$$B_3 = \frac{\tilde{\lambda}_R s f_B^t}{r_R^n - \tilde{\mu}_R + \tilde{\lambda}_R} \tag{A-122}$$

$$B_4 = \frac{\tau c_o}{r_R^n + \tilde{\lambda}_R} \tag{A-123}$$

$$F_{i1} = s f_i^t \tag{A-124}$$

$$F_{i2} = 0 \tag{A-125}$$

f_i^t is the factor to calculate tax shield of a firm with only invested assets given the manager-selected coupon c_n.

(iv) The value of the net future cash flows is determined by the following parameters.

$$E_{i1} = 0 \tag{A-126}$$

$$E_{i2} = 0 \tag{A-127}$$

$$E_{i3} = 0 \tag{A-128}$$

$$C_3 = \frac{1-\tau}{r_B^n - \tilde{\mu}_B + \tilde{\lambda}_B} \tag{A-129}$$

$$C_4 = -\frac{c_o\left(1-\tau\right)}{r_B^n + \tilde{\lambda}_B} \tag{A-130}$$

$$C_5 = 0 \tag{A-131}$$

$$C_6 = 0 \tag{A-132}$$

$$A_{i5} = \frac{\left(r_j^n - \tilde{\mu}_j + \tilde{\lambda}_i + \tilde{\lambda}_j\right)}{\left(r_i^n - \tilde{\mu}_i\right)\left(r_j^n - \tilde{\mu}_j\right) + \left(r_j^n - \tilde{\mu}_j\right)\tilde{\lambda}_i + \left(r_i^n - \tilde{\mu}_i\right)\tilde{\lambda}_j} \tag{A-133}$$

$$A_{i6} = -\frac{c_o\left(1-\tau\right)}{r_i^p} \tag{A-134}$$

$$B_3 = \frac{\tilde{\lambda}_R s f_i^n + 1 - \tau}{r_R^n - \tilde{\mu}_R + \tilde{\lambda}_R} \tag{A-135}$$

$$B_4 = -\frac{c_o\left(1-\tau\right)}{r_R^n + \tilde{\lambda}_R} \tag{A-136}$$

$$F_{i1} = f_i^n \tag{A-137}$$

$$F_{i2} = 0 \tag{A-138}$$

f_i^n is the factor to calculate the value of future cash flows of a firm with only invested assets given the manager-selected coupon c_n.

Proof. (i) Using a no-arbitrage argument, corporate debt requires an instantaneous return equal to the nominal risk-free rate r_i^n. An application of Ito's lemma with regime switches shows that debt satisfies the following system of ODEs. For $0 \leq X \leq D_B$:

$$\begin{cases} d_B\left(X\right) &= \alpha_B\left(1-\tau\right)\left(Xy_B + G_B^{unlev}\left(X\right)\right) \\ d_R\left(X\right) &= \alpha_R\left(1-\tau\right)\left(Xy_R + G_R^{unlev}\left(X\right)\right). \end{cases} \tag{A-139}$$

For $D_B < X \leq D_R$:

$$\begin{cases} r_B^n d_B\left(X\right) &= c_o + \tilde{\mu}_B X d_B'\left(X\right) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 d_B''\left(X\right) \\ & \quad + \tilde{\lambda}_B\left(\alpha_R\left(1-\tau\right)\left(Xy_R + G_R^{unlev}\left(X\right)\right) - d_B\left(X\right)\right) \\ d_R\left(X\right) &= \alpha_R\left(1-\tau\right)\left(Xy_R + G_R^{unlev}\left(X\right)\right). \end{cases} \tag{A-140}$$

For $D_R < X < X_B$:

$$\begin{cases} r_B^n d_B\left(X\right) &= c_o + \tilde{\mu}_B X d_B'\left(X\right) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 d_B''\left(X\right) + \tilde{\lambda}_B\left(d_R\left(X\right) - d_B\left(X\right)\right) \\ r_R^n d_R\left(X\right) &= c_o + \tilde{\mu}_R X d_R'\left(X\right) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 d_R''\left(X\right) + \tilde{\lambda}_R\left(d_B\left(X\right) - d_R\left(X\right)\right). \end{cases} \tag{A-141}$$

For $X_B \leq X < X_R$:

$$\begin{cases} d_B(X) &=& f_B^d \\ r_R^n d_R(X) &=& c_o + \tilde{\mu}_R X d_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 d_R''(X) + \tilde{\lambda}_R \left(f_B^d - d_R(X) \right). \end{cases} \quad \text{(A-142)}$$

For $X \geq X_R$:

$$\begin{cases} d_B(X) &=& f_B^d \\ d_R(X) &=& f_R^d. \end{cases} \quad \text{(A-143)}$$

System (A-139) corresponds to the firm being in the default region in both boom and recession. At default, debt-holders receive $\alpha_i \left(X(1-\tau) y_i + G_i^{unlev}(X) \right)$. System (A-140) represents the case in which the firm is in the continuation region in boom and in the default region in recession. For the continuation region, the left-hand side of the equation is the rate of return required by investors for holding corporate debt. The right-hand side corresponds to the realized rate of return, which consists of the expected change in the value of debt as given by Ito's lemma plus the coupon payment c_o. The last term captures the change in the value of debt in case of a regime switch, and, hence, immediate default. Eqs. (A-141) present the case in which the firm is in the continuation region in both boom and recession. The next equations, Eqs. (A-142), corresponds to the case in which the firm is in the exercise region in boom and in the continuation region in recession. At exercise, the manager issues additional debt with coupon $c_n - c_o$, such that the firm now faces total debt obligations with total coupon payments c_n. New debt is issued with the same maturity and seniority as already existing debt. Due to the scaling property, the value of old debt is homogenous of degree zero in X given the regime, and denoted by f_i^d. Finally, (A-143) represents the case in which the firm is in the exercise region in both boom and recession. For consistency, it is assumed that the firm exercised the option at X_i with the corresponding manager-selected coupon. The boundary conditions are as follows.

$$\lim_{X \searrow D_R} d_B(X) = \lim_{X \nearrow D_R} d_B(X), \quad \text{(A-144)}$$

$$\lim_{X \searrow D_R} d_B'(X) = \lim_{X \nearrow D_R} d_B'(X), \quad \text{(A-145)}$$

$$\lim_{X \searrow D_B} d_B(X) = \alpha_B \left((1-\tau) D_B y_B + G_B^{unlev}(D_B) \right), \quad \text{(A-146)}$$

$$\lim_{X \searrow D_R} d_R(X) = \alpha_R \left(D_R y_R (1-\tau) + G_R^{unlev}(D_R) \right), \quad \text{(A-147)}$$

$$\lim_{X \searrow X_B} d_R(X) = \lim_{X \nearrow X_B} d_R(X), \quad \text{(A-148)}$$

$$\lim_{X \searrow X_B} d_R'(X) = \lim_{X \nearrow X_B} d_R'(X), \quad \text{(A-149)}$$

$$\lim_{X \nearrow X_B} d_B(X) = f_B^d, \quad \text{(A-150)}$$

and

$$\lim_{X \nearrow X_R} d_R(X_R) = f_R^d. \quad \text{(A-151)}$$

Eqs. (A-144) and (A-145) correspond to the value-matching and smooth-pasting conditions for the debt value in boom at the default boundary in recession. Similarly, Eqs. (A-148) and (A-149) are the value-matching and smooth-pasting conditions for the debt value in recession at the option exercise boundary in boom. Eqs. (A-146) and (A-147) represent the value-matching conditions at the default thresholds, and Eqs. (A-150) and (A-151) are the value-matching conditions at the option exercise boundaries. The functional form of the system of ODE (A-139)-(A-143) and its boundary conditions (A-144)-(A-151) is given in (A-80). Solving with standard techniques, I find that the parameters correspond to the ones in (A-87)-(A-99). The linear system determining the remaining unknowns $A_{B1}, A_{B2}, A_{B3}, A_{B4}, C_1, C_2, B_1, B_2$, (A-84)-(A-86), is given by the boundary conditions (A-144)-(A-151).

(ii) Similarly, the system of ODEs for bankruptcy costs is given by the following.

For $0 \leq X \leq D_B$:

$$\begin{cases} b_B(X) &= (1-\alpha_B)(1-\tau)Xy_B + G_B(X) - \alpha_B G_B^{unlev}(X) \\ b_R(X) &= (1-\alpha_R)(1-\tau)Xy_R + G_R(X) - \alpha_R G_R^{unlev}(X) \end{cases} \tag{A-152}$$

For $D_B < X \leq D_R$:

$$\begin{cases} r_B^n b_B(X) &= \tilde{\mu}_B X b_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 b_B''(X) + \tilde{\lambda}_B\left((1-\alpha_R)(1-\tau)Xy_R + G_R^{lev}(X) - \alpha_R G_R^{unlev}(X) - b_B(X)\right) \\ b_R(X) &= (1-\alpha_R)Xy_R(1-\tau) + G_R(X) - \alpha_R G_R^{unlev}(X) \end{cases} \tag{A-153}$$

For $D_R < X < X_B$:

$$\begin{cases} r_B^n b_B(X) &= \tilde{\mu}_B X b_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 b_B''(X) + \tilde{\lambda}_B(b_R(X) - b_B(X)) \\ r_R^n b_R(X) &= \tilde{\mu}_R X b_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 b_R''(X) + \tilde{\lambda}_R(b_B(X) - b_R(X)). \end{cases} \tag{A-154}$$

For $X_B \leq X < X_R$:

$$\begin{cases} b_B(X) &= f_B^b X \\ r_R^n b_R(X) &= \tilde{\mu}_R X b_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 b_R''(X) + \tilde{\lambda}_R\left(f_B^b X - b_R(X)\right). \end{cases} \tag{A-155}$$

For $X \geq X_R$:

$$\begin{cases} b_B(X) &= f_B^b X \\ b_R(X) &= f_R^b X. \end{cases} \tag{A-156}$$

The system is subject to the following boundary conditions.

$$\lim_{X \searrow D_R} b_B(X) = \lim_{X \nearrow D_R} b_B(X), \tag{A-157}$$

$$\lim_{X \searrow D_R} b_B'(X) = \lim_{X \nearrow D_R} b_B'(X), \tag{A-158}$$

$$\lim_{X \searrow D_B} b_B(X) = (1-\alpha_B)(1-\tau)D_B y_B + G_B(X) - \alpha_B G_B^{unlev}(X), \tag{A-159}$$

$$\lim_{X \searrow D_R} b_R(X) = (1-\alpha_R)(1-\tau)D_R y_R + G_R(X) - \alpha_R G_R^{unlev}(X), \tag{A-160}$$

$$\lim_{X \searrow X_B} b_R(X) = \lim_{X \nearrow X_B} b_R(X), \tag{A-161}$$

$$\lim_{X \searrow X_B} b_R'(X) = \lim_{X \nearrow X_B} b_R'(X), \tag{A-162}$$

$$\lim_{X \nearrow X_B} b_B(X) = f_B^b X_B, \tag{A-163}$$

and

$$\lim_{X \nearrow X_R} b_R(X_R) = f_R^b X_R. \tag{A-164}$$

The functional form of the solution is given in (A-80). For bankruptcy costs, the parameters are as in (A-100)-(A-112).

(iii) Next, the system of ODEs for the tax shield is presented.

For $0 \leq X \leq D_B$:

$$\begin{cases} t_B(X) &= 0 \\ t_R(X) &= 0. \end{cases} \tag{A-165}$$

For $D_B < X \leq D_R$:

$$\begin{cases} r_B^n t_B(X) &= \tau c_o + \tilde{\mu}_B X t_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 t_B''(X) + \tilde{\lambda}_B(0 - t_B(X)) \\ t_R(X) &= 0 \end{cases} \tag{A-166}$$

For $D_R < X < X_B$:

$$\begin{cases} r_B^n t_B(X) &= \tau c_o + \tilde{\mu}_B X t_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 t_B''(X) + \tilde{\lambda}_B(t_R(X) - t_B(X)) \\ r_R^n t_R(X) &= \tau c_o + \tilde{\mu}_R X t_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 t_R''(X) + \tilde{\lambda}_R(t_B(X) - t_R(X)). \end{cases} \tag{A-167}$$

For $X_B \leq X < X_R$:

$$\begin{cases} t_B(X) &= f_B^t \\ r^n t_R(X) &= \tau c_o + \tilde{\mu}_R X t_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 t_R''(X) + \tilde{\lambda}_R(f_B^t X - t_R(X)). \end{cases} \tag{A-168}$$

For $X \geq X_R$:

$$\begin{cases} t_B(X) &= f_B^t X \\ t_R(X) &= f_R^t X. \end{cases} \tag{A-169}$$

The system is subject to the following boundary conditions.

$$\lim_{X \searrow D_R} t_B(X) = \lim_{X \nearrow D_R} t_B(X), \tag{A-170}$$

$$\lim_{X \searrow D_R} t_B'(X) = \lim_{X \nearrow D_R} t_B'(X), \tag{A-171}$$

$$\lim_{X \searrow D_B} t_B(X) = 0, \tag{A-172}$$

$$\lim_{X \searrow D_R} t_R(X) = 0, \tag{A-173}$$

$$\lim_{X \searrow X_B} t_R(X) = \lim_{X \nearrow X_B} t_R(X), \tag{A-174}$$

$$\lim_{X \searrow X_B} t_R'(X) = \lim_{X \nearrow X_B} t_R'(X), \tag{A-175}$$

$$\lim_{X \nearrow X_B} t_B(X) = f_B^t X_B, \tag{A-176}$$

and

$$\lim_{X \nearrow X_R} t_R(X_R) = f_R^t X_R. \tag{A-177}$$

The fact that managerial compensation is shielded from taxation results in an additional tax shield of $\tau\psi X$ as long as the firm is solvent (cf. Eqs. (A-166)-(A-168)). The functional form of the solution is given in (A-80). The parameters in the solution are given in (A-113)-(A-125).

(iv) The system of ODEs for the expected value of future cash flow is given as follows.

For $0 \leq X \leq D_B$:

$$\begin{cases} n_B(X) &= 0 \\ n_R(X) &= 0. \end{cases} \tag{A-178}$$

For $D_B < X \leq D_R$:

$$\begin{cases} r_B^n n_B(X) &= (1-\tau)(X-c_o) + \tilde{\mu}_B X n_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 n_B''(X) + \tilde{\lambda}_B(0 - n_B(X)) \\ t_R(X) &= 0 \end{cases} \tag{A-179}$$

For $D_R < X < X_B$:

$$\begin{cases} r_B^n n_B(X) &= (1-\tau)(X-c_o) + \tilde{\mu}_B X n_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 n_B''(X) + \tilde{\lambda}_B(n_R(X) - n_B(X)) \\ r_R^n n_R(X) &= (1-\tau)(X-c_o) + \tilde{\mu}_R X n_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 n_R''(X) + \tilde{\lambda}_R(n_B(X) - n_R(X)). \end{cases} \tag{A-180}$$

For $X_B \leq X < X_R$:

$$\begin{cases} n_B(X) &= f_B^n X \\ r_R^n n_R(X) &= (1-\tau)(X-c_o) + \tilde{\mu}_R X n_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 n_R''(X) + \tilde{\lambda}_R(f_B^n X - n_R(X)). \end{cases} \tag{A-181}$$

For $X \geq X_R$:

$$\begin{cases} n_B(X) &= f_B^n X \\ n_R(X) &= f_R^n X. \end{cases} \tag{A-182}$$

The system is subject to the following boundary conditions.

$$\lim_{X \searrow D_R} n_B(X) = \lim_{X \nearrow D_R} n_B(X), \tag{A-183}$$

$$\lim_{X \searrow D_R} n_B'(X) = \lim_{X \nearrow D_R} n_B'(X), \tag{A-184}$$

$$\lim_{X \searrow D_B} n_B(X) = 0, \tag{A-185}$$

$$\lim_{X \searrow D_R} n_R(X) = 0, \tag{A-186}$$

$$\lim_{X \searrow X_B} n_R(X) = \lim_{X \nearrow X_B} n_R(X), \tag{A-187}$$

$$\lim_{X \searrow X_B} n_R'(X) = \lim_{X \nearrow X_B} n_R'(X), \tag{A-188}$$

$$\lim_{X \nearrow X_B} n_B(X) = f_B^n X_B, \tag{A-189}$$

and

$$\lim_{X \nearrow X_R} n_R(X_R) = f_R^n X_R. \tag{A-190}$$

Eq. (A-80) corresponds to the functional form of the solution, and the parameters are as in (A-126)-(A-138).

□