Traditional and Shadow Banks*

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Abstract

We propose a theory of the coexistence of traditional and shadow banks. In our model, bankers can choose to set up a traditional or a shadow bank: shadow banks escape the costly regulation traditional banks must comply with, but forgo deposit insurance, which traditional banks can rely upon. Thus, in a crisis, shadow banks repay their creditors by selling assets at fire-sale prices to traditional banks, which fund these purchases with insured deposits. This creates a complementarity between traditional and shadow banks. We show that in equilibrium, the two bank types coexist. The analysis implies that an increase in deposit insurance leads to a *decrease* in the relative size of the traditional banking sector, and that in equilibrium, the shadow banking sector is larger than socially optimal. Our model is consistent with several facts from the 2007 financial crisis: assets and (deposit-like) liabilities migrated in large amounts from shadow banks to traditional banks, and shadow bank assets were sold to traditional banks at fire sale prices.

Keywords: Traditional banking, Shadow banking, Financial crisis, Deposit insurance

JEL Codes: E32, E44, E61, G01, G21, G23, G38.

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1 Introduction

Shadow banks now account for about a quarter of total financial intermediation worldwide (IMF, 2014).[1] This includes market-based institutions ranging from money market funds to asset-backed securities issuers, supplying credit through more or less complex intermediation chains outside of the traditional regulated banking system. By most accounts, the emergence of shadow banking has been largely motivated by regulatory arbitrage, i.e. an attempt to bypass the cost associated with the regulations traditional banks must comply with (Gorton and Metrick, 2012; Acharya et al., 2013), enabled by financial innovation allowing many of the services provided by traditional banks to be sustained by other types of banks (see e.g. Merton, 1995; Rajan, 1998a).

Following the failures of financial regulation revealed by the crisis of 2007, the need for regulatory reforms emerged as a consensus (Duffie, 2016). However, while their collapse was at the heart of the crisis, shadow banks remain difficult to regulate (FSB, 2016b). Regulatory changes and new technologies have re-shaped the modern financial landscape.[2] Policymakers and academics are concerned that tightened regulations of traditional banks might shift financial intermediation away from traditional banks and towards shadow banks (Hanson et al., 2011; Sunderam, 2015). Meanwhile, the crisis saw large amounts of assets and liabilities transferred from shadow to traditional banks, which suggests that the interactions between traditional and shadow banks are more complex than mere regulatory arbitrage. To gauge the effects of traditional banks' regulation in the presence of shadow banks, one needs to understand these interactions, and in particular why traditional and shadow banks coexist in the first place.

We propose a theory of the coexistence of traditional and shadow banks. In our model, bankers can choose to set up a traditional or a shadow bank: shadow banks escape the costly regulation traditional banks must comply with, but forgo deposit insurance, which traditional banks can rely upon in a crisis. Thus, in a crisis, shadow banks repay their creditors by selling assets at fire-sale prices to traditional banks, which fund these purchases with insured deposits. This creates a complementarity between traditional and shadow banks: the larger the relative size of the traditional banking sector, the higher these asset prices, and thus the higher a banker's incentive to set up a shadow bank in the first place. We show that in equilibrium traditional and shadow banks coexist. The analysis implies that an increase in deposit insurance leads to a *decrease* in the relative size of the traditional banking sector, and that in equilibrium, the shadow banking sector is larger than socially optimal. Our model is consistent with several facts from the 2007 financial crisis: assets and (deposit-like) liabilities migrated in large amounts from shadow banks to traditional banks, and shadow bank assets were sold to traditional banks at fire sale prices.

Specifically, we consider a model with three dates 0, 1, 2, and two groups of agents: bankers and households. At date 0, each banker can set up a traditional bank or a shadow bank. The banker invests her endowment, which constitutes the banks' equity. Banks can also issue claims to households, which

[1]This estimate is in terms of credit intermediation (see IMF, 2014). For descriptions of shadow banking, see Pozsar et al. (2013) for the United States, ESRB (2016) for the European Union, IMF (2014) and FSB (2016a) for global estimates. Globally, shadow banks' assets were worth \$80 trillion in 2014, up from \$26 trillion more than a decade earlier (FSB (2016a)).

[2]For instance, "fintech" shadow banks have emerged, see BIS (2016), Buchak et al. (2017).

we assume must be money-like claims, i.e., riskless short-term debt (henceforth "short-term debt").[3] With these funds, banks invest in risky assets which pay off at date 2. At date 1, two states are possible: Either a crisis occurs, in which case date-2 asset returns are low and uncertain, or no crisis occurs and date-2 asset returns are high and safe.

We assume two differences between traditional and shadow banks. On the one hand, traditional banks incur a cost associated with the regulation they must comply with, which shadow banks evade. This assumption captures the idea that shadow banking is largely motivated by regulatory arbitrage (Hanson et al., 2011; Acharya et al., 2013). On the other hand, traditional banks can, up to a limit, issue claims backed by deposit insurance, which shadow banks cannot. Therefore in a crisis at date 1, despite uncertain asset returns, deposit insurance enables traditional banks, but not shadow banks, to issue the riskless claims households demand. We assume that deposit insurance is actuarially fairly priced and limited, i.e. each bank can issue riskless debt only up to a fixed dollar amount. This limit may stem, for instance, from fiscal costs (see Davila and Goldstein, 2016), or ex-ante distortions in banks' behavior (Calomiris and Kahn, 1991; Diamond and Rajan, 2001).[4] There is no other built-in difference between traditional and shadow banks, which face the same choice sets.

If at date 1 there is no crisis, asset returns are high and safe. Thus all banks can issue riskless debt, which they do to refinance their assets with short-term debt. Instead, in a crisis, shadow banks are unable to roll over their short-term debt because their assets are risky and households demand riskless debt. Hence, shadow banks must liquidate assets to repay their creditors. Traditional banks can buy shadow banks' assets in a crisis, because of their unique ability to finance these purchases by issuing short-term debt backed by deposit insurance. Because of limited deposit insurance, traditional banks have limited debt capacity and therefore shadow banks' assets trade at a discount.

At date 0, when bankers choose to set up a traditional or a shadow bank, they trade off the costs and benefits associated with each type of bank, i.e. low regulation costs but need to sell assets at a discount in a crisis versus high regulation cost but ability to buy assets at a discount in a crisis. The trade-off depends on the asset discount anticipated in a crisis, itself a function of the relative size of the two banking sectors. The larger the relative size of the traditional (shadow) banking sector, the higher (lower) asset prices in a crisis, and the higher bankers' incentive to set up a shadow (traditional) bank in the first place. In that sense, traditional and shadow banks form an ecosystem. In equilibrium, expected profits in the traditional and shadow banking sectors are equal, such that bankers are indifferent between setting up a traditional or a shadow bank. This pins down asset prices in a crisis and the relative size of the traditional and shadow banking sectors in equilibrium.

Our analysis is consistent with several facts from the 2007 financial crisis.

First, in our model in a crisis, shadow banks are unable to roll over their existing debt. This is

[3]The view of banks as providers of safe money-like liabilities has its roots in Gorton and Pennacchi (1990). Recent papers show that when households have infinite risk-aversion (Gennaioli et al., 2013) or Epstein-Zin preferences with infinite relative risk aversion and infinite intertemporal elasticity of substitution (Caballero and Farhi, 2016), "money-like" short-term debt arises as the optimal financial contract between the bank and households. Likewise in our model, we show in Appendix B.1 that short-term debt arises endogenously if we assume that households are infinitely risk averse.

[4]In practice, deposit insurance only guarantees a limited level of deposits. In the U.S., this limit holds per depositor, per FDIC-insured bank. Hence, in practice, deposit insurance only guarantees a fixed level of deposits. In our model, we assume that each bank is limited in the total dollar amount of riskless debt it can issue using deposit insurance.

consistent with the evidence that in the crisis, shadow banks entirely stopped using several classes of assets as collateral and their creditors withdrew their ("repo") debt (Gorton and Metrick, 2012). In contrast to shadow banks, in our model, traditional banks are able to issue short-term debt in a crisis. This is consistent with the evidence that during the crisis, almost $600 billion of deposits went into the largest traditional banks in less than a month, following the bankruptcy of Lehman Brothers in 2008q3 (see Acharya and Mora, 2015, for a discussion).

Second, in our model, if a crisis occurs, shadow banks must liquidate assets to repay their creditors. In turn, traditional banks purchase shadow banks' assets. This is consistent with the evidence that in the crisis, about $800 billion assets flew out of shadow banks, out of which $550 billion flew into traditional banks from 2007q4 to 2009q1.[5] In our model, traditional banks finance these purchases by issuing riskless debt backed by deposit insurance. Using Call Report data, we regress traditional banks' asset purchases on deposit changes. We find evidence that traditional banks purchased assets sold by shadow banks by issuing *insured* deposits. Finally, consistent with our assumption that deposit insurance is limited, there is evidence that in the crisis, mortgage-backed government-agency securities traded at spreads well above historical norms (Merrill et al., 2012; Gagnon et al., 2011). Such high spreads for a security with no credit risk points to the scarcity of asset buyers' arbitrage capital.

Third, in our model, assets trade at a discount in a crisis. Gorton and Metrick (2012) provide evidence that in the crisis, certain higher-rated bonds traded at a higher spread than lower-rated bonds of the same category and maturity. Massive sales of higher-rated bonds pushed their price down to attract buyers. This negative spread is suggestive of fire sale prices. Other evidence of asset fire sales is documented in Krishnamurthy (2008) and Chernenko et al. (2014).

Arbitrage of regulatory costs has been an important feature of the banking industry since the first Basel accords of 1988. Some debates about the effectiveness of banking regulation thus center on the ability of shadow banks to escape regulation (Hanson et al., 2011; Buchak et al., 2017). Yet, this view does not account for the reason of the coexistence between the two sectors.

As an illustration, we study how, in our model, the level of deposit insurance affects the relative size of traditional and shadow banks. We find two competing effects.

On the one hand, traditional banks' increased debt capacity allows them to operate on a larger scale. This effect increases bankers' incentives to set up a traditional bank. On the other hand, traditional banks use their increased debt capacity to bid up shadow banks' assets prices in a crisis. In turn, higher asset prices in a crisis increases shadow banks' initial debt capacity, which allows them to operate on a larger scale, which increases bankers' incentives to set up a shadow bank.

We show that the latter effect dominates the former. To gain intuition about this result, recall that asset prices are pinned down in equilibrium so that traditional banks' regulatory costs are offset by their profits from buying shadow banks' assets at a discount, and bankers are indifferent between setting up

[5] See for instance He et al. (2010). The reason why these numbers do not exactly add up is twofold. First, traditional banks have used depost inflows in the crisis for other purpose than asset purchases (for instance, to meet credit line drawdowns, as shown in Ivashina and Scharfstein, 2010). Second, the documented figures come from balance sheets data and it need not be that, for a given volume, assets switching from one bank to the other keep a constant value. Indeed, it can be that assets trade at a discount, in which case the asset seller's balance sheets contraction is greater than the buyer's balance sheets expansion. Abbassi et al. (2015) also find that banks played an important role in providing price support to the distressed securities markets by buying fire-sold securities.

either type of bank. All else being equal, when deposit insurance expands, traditional banks use their increased debt capacity to bid up shadow banks' assets prices. Higher asset prices increases bankers' incentives to set up a shadow bank. Therefore asset prices must decrease to return to their equilibrium level. This requires an increase in the relative size of the shadow banking sector.

Last, we consider the normative implications of our analysis, and show that the equilibrium relative size of the shadow banking sector exceeds it socially optimal level. When starting a shadow bank, bankers take asset prices in a crisis as given and fail to internalize the effect of their asset sales on asset prices. Since shadow banks' ability to issue riskless debt initially depends on the collateral value of their assets in a crisis, this creates a pecuniary externality (Gromb and Vayanos, 2002; Lorenzoni, 2008). Therefore, the equilibrium fraction of bankers operating a shadow bank is larger than socially optimal. The reverse reasoning holds true for traditional banks, such that in equilibrium, the fraction of bankers operating a traditional bank is smaller than socially optimal. The social planner can improve welfare with transfers from bankers starting a shadow bank, to bankers starting a traditional bank (e.g. with lump-sum taxes).

The paper proceeds as follows. In Section 2 we document stylized facts from the crisis, which our model replicates. Section 3 presents the model, and we analyze the possible coexistence between traditional and shadow banks in Section 4. In Section 5 we discuss the implications of our model , and we develop a normative approach in Section 6. Section 7 concludes.

Related Literature Merton (1995) and Rajan (1998a,b) are early discussions of the future of traditional banks in light of increased competition from other types of banks.[6] More recently, Hanson et al. (2011) show concerns that given heightened competition, tightened regulation of traditional banks will drive a larger share of intermediation into shadow banks.[7] In this paper, we find that regulatory costs explain why traditional banks forgo investment opportunities and keep slack to purchase shadow banks' assets in a crisis. Therefore despite higher regulatory costs, traditional banks are complements to shadow banks.

Some papers study banking regulation in the presence of shadow banks, motivated by the regulatory arbitrage mechanism. In Ordonez (2013), regulation provides a commitment device for traditional banks to avoid excessive risk taking. He finds that an optimal policy is to tax shadow banks and subsidize traditional banks, allowing banks to self-select into the traditional and shadow banking sectors depending on their investment opportunities. In Plantin (2015), optimal capital requirements for traditional banks depend on shadow banks' ability to issue money-like claims to households, which comes from the liquidity of shadow banks' assets. This liquidity is determined by the degree of information asymmetry in the market for shadow banks assets. In Begenau and Landvoigt (2017), random bailouts are more likely for traditional banks than for shadow banks. In their model, households price bank debt rationally, so that shadow banks respond to higher capital requirements by increasing their size but not their riskiness. Therefore higher capital requirements for traditional banks leads to more shadow banks,

[6]Other examples include Boyd and Gertler (1994) and James and Houston (1996).

[7]Several empirical studies find evidence of regulatory arbitrage (see e.g. Houston et al., 2012; Acharya et al., 2013; Karolyi and Taboada, 2015).

but this does not reduce financial stability. Finally, Farhi and Tirole (2017) study the optimal regulatory contract that satisfies traditional banks' "participation constraint" such that they do not migrate to the shadow banking sector. This contract specifies the costs (regulation) and benefits (access to lending of last resort, and deposit insurance) for each bank type. Contract conditions are optimally differentiated between traditional and shadow banks. The existing literature studies the emergence of shadow banks as a response to tightened regulation of traditional banks, and the policy implications thereof, but not the coexistence of traditional and shadow banks.

A second group of theories assume the coexistence of traditional and shadow banks, looking at various implications. Gertler et al. (2016) build a model in which shadow banks have an advantage in managing assets, while traditional banks have an advantage in overcoming agency frictions in fund borrowing. Their objective is to account for the buildup and collapse of shadow banks, and identify the transmission of crises to the economy. In Moreira and Savov (Forthcoming), shadow banks' debt requires less collateral but is more fragile. Periods of low uncertainty are associated with expansions of shadow banks and economic booms, while shadow banks collapse under uncertainty, and in turn the economy tanks. Luck and Schempp (2016) study the conditions for runs in the shadow banking sector to spread to traditional banks. When the (exogenous) fraction of uninsurable deposits in the economy is high, Voellmy (2017) shows that the risk of bank runs on traditional banks is minimized when shadow banks absorb the bulk of uninsured deposits. Hanson et al. (2015) study which assets are held by traditional versus shadow banks. Our focus is on interactions between traditional and shadow banks to derive the conditions of their coexistence.

Our model is in line with theories of banks as issuers of riskless claims. A seminal paper is Gorton and Pennacchi (1990).[8] Our model is based on Stein (2012), however we consider two types of banks, traditional and shadow banks, which interact in equilibrium. Gennaioli et al. (2013) present a model of shadow banks catering to investors' demand for riskless debt through securitization. In our model, we also assume that banks issue riskless claims to cater to their investors' demand.[9]

Finally, some papers study the coexistence of traditional and shadow banks. LeRoy and Singhania (2017) assume that deposit insurance subsidizes traditional banks, benefitting shadow banks through different channels depending on how deposit insurance is priced. The relative size of the traditional banking sector then depends on the size of the insurance subsidy. Gornicka (2016) develops a model where shadow banking stems from regulatory arbitrage by traditional banks, and traditional banks provide exogenous guarantees to shadow banks that render both bank types complements. To the best of our knowledge, our paper is the first to provide a theory of the coexistence of traditional and shadow banks based on their interaction in a crisis, consistent with stylized facts from the 2007 financial crisis.

[8]Other recent papers include DeAngelo and Stulz (2015) and Plantin (2015).

[9]We show in Appendix B.1 that short-term debt arises endogenously in our model if we assume that households are infinitely risk averse.

2 Stylized facts

We use data from the Financial Accounts of the United States (henceforth FAUS), the Federal Reserve H8 Releases and the quarterly Call Reports. A detailed description of data construction can be found in Appendix A.1.

2.1 Fact 1: Liabilities flow from shadow to traditional banks

Table 1 in Appendix A.2 shows the evolution of short-term debt for traditional and shadow banks from 2006q4 to 2011q1. First, in the crucial phase of the crisis in the fall of 2008, investors stopped rolling over shadow banks' short-term debt. Gorton and Metrick (2012) document investors' run on one of their major yet unstable source of funding: the sale and repurchase market (the "repo" market) (see also Krishnamurthy et al. (2014) and Copeland et al. (2014)). Commercial paper, another major source of funding for shadow banks, also collapsed (Kacperczyk and Schnabl, 2010).

In the meantime, traditional banks' short-term debt increased. Figure 11 in Appendix A.2 shows a sudden \$600 billion inflow in deposits and borrowings into the largest US traditional banks, in three weeks from September 10th to October 1st, 2008. This inflow is coherent with the risk management motive proposed in Kashyap et al. (2002) to explain why traditional banks combine demand deposits with loan commitments or lines of credit: In a crisis, borrowers draw down on their credit lines while investors seek a safe haven for their wealth, turning to traditional banks because these latter provide insurance due to the government guarantee on their deposits (see also Gatev and Strahan (2006)).[10] Nevertheless, as shown in Acharya and Mora (2015), it was not until the U.S. government's intervention just before the Lehman failure on September 15, 2008 that deposit flew into traditional banks. Core deposits eventually increased by close to \$800 billion by early 2009 (see also He et al. (2010)).[11]

2.2 Fact 2: Asset flow from shadow to traditional banks

Mortgage-backed securities guaranteed by government-sponsored enterprises ("GSE MBS") are part of the assets transferred from shadow banks to traditional banks in the crisis (see Figure 13 in Appendix A.2.4). Although our data does not allow us to identify whether these changes were due to changes in the value of assets or changes in ownership, we show below that this picture does not change when accounting for the repayment/maturity rate of MBS net of the new issuance rate, suggesting asset purchases by traditional banks. Empirical work by He et al. (2010) and Bigio et al. (2016) provide estimates of the amount of assets that were transferred from shadow to traditional banks during the crisis. From 2007q4 to 2009q1, He et al. (2010) find that shadow banks decreased their holdings of securitized assets by approximately \$800 billion while traditional banks increased theirs by approximately \$550 billion. Looking at the wider period from 2007q1 to 2013q1 and considering total asset holdings, Bigio et al.

[10] This explains why there is no evidence that funds flowed into the banking system when spreads widened during the 1920s, prior to the expansion of the federal safety net with the creation of federal deposit insurance.

[11] Although many shadow banks exhibit procyclical leverage (Adrian and Shin (2010)), traditional banks attracted deposits in the crisis so that their leverage was countercyclical. Despite the increase in traditional banks' book equity in the crisis documented in Appendix A.2.3, we find that increase in deposits was such that traditional banks' market leverage increased in the crisis. Baron (2016) finds evidence of banks' countercyclical equity issuance not only in the crisis, but across credit cycles after 1980.

(2016) document a net asset outlfow of $1702 billion out of shadow banks and an asset inflow of $1595 billion into traditional banks.[12]

One testable prediction of our theory is that traditional banks finance shadow banks' asset purchases in the crisis by issuing short-term debt backed by deposit insurance. We use the Call Reports and He et al. (2010)'s estimates to test this prediction in the cross-section of traditional banks. Results and details about the sample construction are in Appendix A.3. We find support for the central mechanism of our theory: Traditional banks purchased mortgage-backed securities in the crisis by issuing more insured deposits. Besides, we find that those banks that purchased assets in the crisis did so at the expense of credit. This is in line with Shleifer and Vishny (2010) and Stein (2013) who discuss how market conditions shape the allocation of scarce bank capital across lending and asset purchases. Abbassi et al. (2015) find comparable results using German data.

2.3 Fact 3: Asset fire sales

Our illustration of asset fire sales comes from Gorton and Metrick (2012). The authors provide a snapshot of fire sales of assets in the crisis that we reproduce on Figure 14 in Appendix A.3.1. We see a negative spread between higher- and lower-rate bonds with the same maturity. Aaa-rated corporate bonds would normally trade at higher prices (i.e. lower spreads) than any lower-grade bonds with the same maturity (say, Aa-rated ones). Sales of higher-rated bonds were so massive that their price was pushed down to attract buyers. This negative spread thus is suggestive of fire sale prices.

Other examples in the literature suggest that asset prices have deviated significantly from fundamental values and were sold at fire-sale prices during the crisis. Using data on insurance companies, Merrill et al. (2012) show that risk-sensitive capital requirements, together with mark- to-market accounting, can cause financial intermediaries to engage in fire sales of RBMS securities. Krishnamurthy (2008) discusses pricing relationships reflecting similar distortions on agency MBS, and notably the increasing option-adjusted spread of Ginnie Mae MBS versus the US Treasury with the same maturity. Gagnon et al. (2011) also document substantial spreads on MBS rates - well above historical norms. Because this security has no credit risk, this evidence points to the scarcity of arbitrage capital in the marketplace and the large effects that this shortage can have on asset prices. Finally, using micro-data on insurers' and mutual funds' bond holdings, Chernenko et al. (2014) finds that in order to meet their liquidity needs during the crisis, investors traded in more liquid securities such as government-guaranteed MBS. This strategy is consistent with theories of fire sales where investors follow optimal liquidation strategies: although spreads on GSE MBS were very high in the fall of 2008, those assets remained the most liquid ones in securitization markets at that time.

[12] Another important aspect of this asset transfer is the purchase of assets from the Federal Reserve, which balance sheets increased by approximately $1954 billion (Bigio et al. (2016)).

3 Model

We consider a model with three dates 0, 1, 2, one type of goods, and two groups of agents, bankers and households.

At date 0, a unit mass of identical bankers start with a net worth n of goods. They are risk neutral and indifferent between consuming at $t = 0, 1, 2$. Bankers can each set up a traditional bank or a shadow bank, henceforth T-bank or S-bank. They invest a quantity $n^i \in [0, n]$ ($i = \{S, T\}$) into either bank type, which becomes the bank's equity.

Banks can invest in risky assets which pay off at date 2. At date 1, two states $\Omega_1 \equiv \{B, G\}$ are possible. State B occurs with probability p, in which bad news on asset returns arrives. We dub this state a "crisis". State G occurs with probability $(1 - p)$, in which good news on asset returns arrives. At date 2, three state $\Omega_2 \equiv \{GG, BG, BB\}$ are possible. In state G at date 1, there is a probability 1 that state GG occurs at date 2. In state B at date 1, there is a probability q that state BG occurs at date 2, and a probability $(1 - q)$ that state BB occurs at date 2. See below what happens at each date and state.

A unit mass of identical households is endowed with a large quantity of consumption goods at $t = \{0, 1\}$. Households cannot invest directly in risky assets, and can only invest in claims issued by banks. They can consume at each date $t = \{0, 1, 2\}$, have linear preferences over consumption at all dates, and do not discount future consumption. At date $t \in$ {0,1}, and in each state $\omega \in \Omega_1$, households' utility function is:

$$U_{t,\omega}(C_t) = C_{t,\omega} + \mathbb{E}_{t,\omega}[U_{t+1}] \tag{1}$$

with $U_{2,\omega} = C_{2,\omega}$. The timeline of the model is detailed in Figure 1.

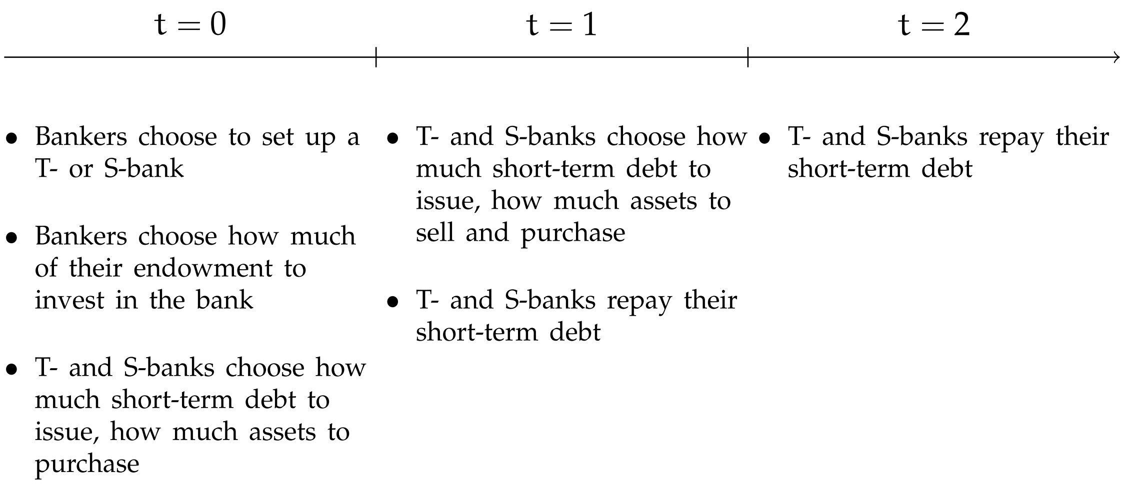


Figure 1: Timeline of the model

3.1 Risky assets

Both T- and S-banks can invest in risky assets at $t = 0$, whose payoffs are summarized on Figure 2.

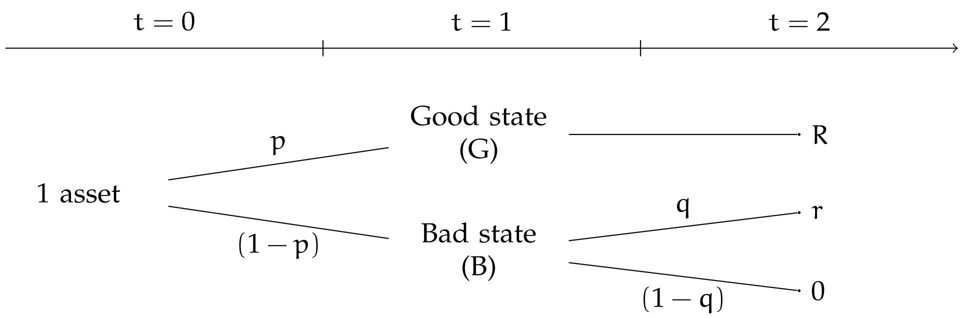


Figure 2: Asset payoff

Investing one unit of goods in the risky asset at $t = 0$ yields a risky payoff $\{R, r, 0\}$ in terms of consumption goods at $t = 2$, in each respective state of $\Omega_2 \equiv \{GG, BG, BB\}$. At $t = 2$, assets have no liquidation value. At $t = 1$, news about the occurrence of the possible states of $t = 2$ is revealed. State G occurs at $t = 1$ with probability p, and it is then known with certainty that state $\{GG\}$ occurs at $t = 2$ so that the asset payoff is $R > 0$. However, state B occurs with probability $(1 - p)$ at $t = 1$, and there is then uncertainty about asset returns at $t = 2$. At $t = 1$ in state B, there is a probability q that state BG occurs at $t = 2$, and a probability $(1 - q)$ that state BB occurs. At $t = 2$ in state BG, assets pay off $r > 0$. At $t = 2$ in state BB, assets pay off 0.

3.2 Bank's choices

At each date and state, T- and S-banks face the same choice sets, and are subject to limited liability constraints.

Time 0 At $t = 0$, i-banks ($i = \{S, T\}$) invest I_0^i units of goods in risky assets. In addition to their equity n^i, they raise an amount D_0^i of goods from households by issuing riskless short-term debt. The interest rate on short-term debt issued at $t = 0$ is r_0. The key assumption here is that households demand riskless debt.[13] This assumption captures what we see as a fundamental role of banks: their ability to act as safety and liquidity providers to households

Time 1 At $t = 1$, T- and S-banks can trade assets in a competitive secondary market, where all banks participate. In state $\omega_1 \in \{B, G\}$, i-banks purchase an amount I_{1,ω_1}^i of assets at an endogenous price p_{1,ω_1}. i-banks can also sell their assets on the market. We assume liquidation costs: a share $\varepsilon \in (0, 1]$ of assets sold is destroyed.[14] Finally, i-banks can raise an additional amount of goods D_{1,ω_1}^i from house-

[13] Alternative model specifications closer to the safe asset literature would yield similar results. For instance, in Gennaioli et al. (2013) households have an infinite risk-aversion utility function, and in Caballero and Farhi (2016) they have Epstein-Zin preferences with infinite relative risk aversion and infinite intertemporal elasticity of substitution.

[14] One can also interpret ε as the cost of breaking up a lending relationship, or the loss associated to a loosened monitoring ability induced by a change of ownership. The adjustment cost $(1 - \varepsilon)$ is a form of technological illiquidity, whose importance is emphasized in Brunnermeier and Sannikov (2014).

holds at $t = 1$, by issuing riskless short-term debt. The interest rate on short-term debt issued at $t = 1$ is r_{1,ω_1}. This debt can be used to roll over previously issued debt, or to finance asset purchases. i-banks repay their date-0 creditors an amount $r_0 D_0^i$ of goods.

Time 2 At $t = 2$, assets pay off goods, and they have no liquidation value. i-banks repay their date-1 creditors an amount $r_{1,\omega_1} D_{1,\omega_1}^i$ of goods.

3.3 Differences between T- and S-banks

We assume two differences between T- and S-banks.

Assumption 1 (Differences between traditional and shadow banks)**.** *Traditional and shadow banks differ in two ways:*

1. *T-banks can rely upon deposit insurance at $t = 1$ in state B (in a crisis), up to an amount k per bank.*
2. *T-banks must comply with costly regulation: At $t = 2$, T-banks only get a fraction $\delta \in [0, 1]$ of asset payoffs.*

On the one hand, T-banks can, up to a limit, issue claims backed by deposit insurance, which S-banks cannot. Therefore in a crisis, T-banks can use deposit insurance to back short-term debt, which shadow banks cannot because asset returns are uncertain. Specifically, the deposit insurance fund enables them to issue short-term debt at $t = 1$ in state B by insuring them against risky asset returns. We assume that the deposit insurance fund is owned by the government and actuarially fairly priced: To obtain one unit of good at $t = 2$ in state BB from the fund, T-banks have to pay $\frac{1-q}{q}$ units of goods to the fund at $t = 2$ in state BG such that the government is making zero profit in expectation. We assume that the maximum guarantee per T-bank is limited to $k > 0$. One interpretation is that the government has limited fiscal capacity at $t = 2$ in state BB and therefore cannot insure a greater amount than $k > 0$. Another interpretation is that the government's ability to enforce payments from T-banks to the fund at $t = 2$ in state BG is limited. One last interpretation for parameter k is a reduced form for informational frictions which prevents T-banks from taking too much debt at $t = 1$ in state B. In the recent crisis, investors ran on S-banks because they were not protected by deposit insurance (Gorton and Metrick, 2012). In contrast, T-banks experienced deposit inflows in the form of insured deposits (see Section 2 and He et al., 2010; Acharya and Mora, 2015), illustrating the value of deposit insurance in a crisis (Iyer et al., 2016).

On the other hand, T-banks incur a cost associated with the regulation they must comply with, which shadow banks evade.[15] We assume that T-banks face regulatory costs $\delta \in (0, 1)$, which decreases their assets payoffs to $\{\delta R, \delta r, 0\}$ at $t = 2$, in each respective state of $\Omega_2 \equiv \{GG, BG, BB\}$. This assumption captures the idea that shadow banking is largely motivated by regulatory arbitrage (Hanson et al., 2011; Acharya et al., 2013), reflecting a wide variety of costs associated to higher regulations imposed to traditional banks: regulatory compliance costs, financing of regulatory bodies, costs to generate regulatory information, etc.[16]

[15] In our model, regulation is not only a cost to T-banks, it is a social cost.

[16] From a positive perspective, it can also be interpreted more broadly as a series of costs associated to T-banks' business model:

3.4 Parametric assumptions

Assumption 2 (Assumption on asset returns). *At $t = 0$, expected asset returns satisfy:*

$$\delta \left(pR + (1-p)qr\right) > 1 \tag{2}$$

Condition (2) ensures that as of $t = 0$, investing in assets is a positive net present value investment for both T- and S-banks. This implies that each banker invests her full endowment in the bank she sets up, be it a T-bank ($n^T = n$) or a S-bank ($n^S = n$) , and that banks invest in as many assets as they can at date 0.

We make two additional assumptions on the size of the guarantee k, and the regulatory costs δ.

Assumption 3 (Assumptions on T-banks' parameters). *T-banks' regulatory costs are low enough to prevent asset trade at $t = 1$ in state G:*

$$\delta > 1 - \varepsilon \tag{3}$$

The level k of deposit insurance at $t = 1$ in state B is low enough:

$$k < k^* \equiv \frac{\delta qr}{1 - \delta qr} n \tag{4}$$

Condition (3) enables us to rule out asset transfers at $t = 1$ in state G between the two types of intermediaries: it will always be optimal for any type of bank to choose to continue their time 0 investment, at $t = 1$ in state G rather than selling assets. This assumption is simplifying but it could be relaxed at little cost. Relaxing this assumption generates asset transfers from T-banks to S-banks in the good information state: indeed, if $\delta < 1 - \varepsilon$, it is more valuable for a T-bank to sell assets to S-banks which value it more[17], and incur the illiquidity cost ε, rather than keeping assets and incurring the regulatory cost $1 - \delta$ on date-2 asset returns.

Condition (4) implies that the maximum amount of riskless debt T-banks can issue at $t = 0$ and roll over at $t = 1$ in state B is constrained by the size of the guarantee fund at $t = 1$ in state B, i.e. k. This ensures that T-banks are able to issue a maximum amount k of riskless debt both at $t = 0$ and at $t = 1$ in all states. Otherwise, the limited liability constraint at $t = 2$ in state BG might be binding, in which case the deposit insurance limit is inoperative. Because we are interested in the interactions between T- and S-banks in a crisis when the deposit insurance limit is binding, we rule out this case in the paper exposition. However, Appendix B.3.2 provides a complete characterization of equilibria when condition (4) is not satisfied.

Additional restrictions We introduce additional restrictions that we refer to in the analysis.

they have higher operating costs (e.g. bricks-and-mortar expenses associated with bank branches), employ more workers, provide more services to their customers.

[17] Each unit of assets generates a return R for a S-bank in date 2, GG instead of δR for T-banks, and none of them discount future payoffs

Assumption 4 (Additional restrictions). *At $t = 1$ in state B, expected asset returns are lower than one:*

$$qr < 1 \tag{5}$$

We assume that the cost of regulation is high enough such that banks gain from trade at $t = 1$ in state B:

$$\delta < \frac{p}{\varepsilon\left(p\left(R-1\right)+\left(1-p\right)qr\right)+p} \tag{6}$$

Condition (5) reflects the fact that in a crisis at date 1, date-2 expected asset returns are low.[18] We do not impose condition (6) on regulatory costs (δ), but we will see later that this condition is necessary for asset prices at $t = 1$ in state B to be such that there is gain from asset trade between T- and S-banks at $t = 1$ in state B, given condition (4). Note that our parametric restrictions are not mutually exclusive.

4 Model analysis

4.1 Equilibrium definition

We define a competitive equilibrium as follows.

Definition 1 (Equilibrium). *A competitive equilibrium consists of asset holdings $\left\{I_t^i\right\}_{t=0,1}^{i=T,S}$, quantities of debt issued $\left\{D_t^i\right\}_{t=0,1}^{i=T,S}$, bankers' equity investments $\left\{n^i\right\}^{i=T,S}$, interest rates $\left\{r_0, r_{1,\omega_1}\right\}_{\omega_1=B,G}$, asset prices in a crisis $\left\{p_{1,\omega_1}\right\}_{\omega_1=B,G}$, and bankers' probability to set up a S-bank χ^S, such that:*

1. *$\left\{I_t^i\right\}_{t=0,1}^{i=T,S}$ and $\left\{D_t^i\right\}_{t=0,1}^{i=T,S}$ maximize i-banks' ($i = S, T$) expected payoff $V_0^i\left(p_{1,G}, p_{1,B}, r_0, r_{1,G}, r_{1,B}, n^S\right)$.*
2. *n^i maximizes the expected payoff of a banker setting up an i-bank ($i = S, T$)*
3. *short-term debt markets clear at $t = 0$, and at $t = 1$ in states B and G for respective interest rates $\{r_0, r_{1,G}, r_{1,B}\}$.*
4. *asset market clears at $t = 1$ in states B and G at respective prices $\{p_{1,B}, p_{1,G}\}$.*
5. *bankers' probability to set up a S-bank χ^S maximizes bankers' expected payoff*

$$\max_{\chi^S \in [0;1]} \chi^S V_0^{S,B}\left(.\right) + (1-\chi^S) V_0^{T,B}\left(.\right).$$

4.2 Equilibrium implications of the assumptions

We start by detailing several equilibrium conditions implied by our assumptions.

[18] We discuss in the analysis how condition (5) is sufficient – but not necessary – to determine the effect of changes in the deposit insurance limit (k). See Section 5.2, and Appendix B.9.

4.2.1 Debt market clearing conditions

Households are endowed with a large amount of consumption goods at each date $t = 0, 1$, such that interest rates on short-term debt is pinned down by households' utility function. Given the linear utility function and no time discounting, this rate is 1 in equilibrium, i.e.

$$r_0^* = r_{1,B}^* = r_{1,G}^* = 1.$$

4.2.2 Time 1, G asset market clearing condition

At $t = 1$ state G, T- and S-banks must repay their creditors an amount $r_0 D_0^i$ ($i = S, T$), which they can do either by issuing riskless short-term debt or by selling part of their assets. They can also purchase assets from other banks. Since expected asset returns are $R > 1$ at $t = 1$ in state G, both T- and S-banks can issue an amount $D_{1,G}^i$ ($i = S, T$) of riskless short-term debt to repay their date-0 creditors, with $D_{1,G}^S \leqslant RI_0^S$ and $D_{1,G}^T \leqslant \delta RI_0^T$.

It is optimal for both T and S–banks to avoid the asset liquidation cost ε: if banks were to sell their assets, asset buyers would pay a fair price R per asset but sellers would only obtain $(1-\varepsilon)R$. This is strictly lower than asset expected returns if banks keep their assets. Even T-banks, for which expected asset returns are δRI_0^T, do not have an incentive to sell assets at date 1 in state G due to assumption (3).

In equilibrium, there is no asset supply at date 1 in state G, therefore no asset trade. Without loss of generality, we simplify T- and S-banks' programs by solving without considering the asset market at $t = 1$ in state G, as if banks' only option in this state is to repay their creditors by issuing new debt. These two problems are equivalent in equilibrium.

4.3 Shadow banks' program

We expose and solve S-banks' program backwards.

Date 1, state G. At $t = 1$ in state G, S-banks can only repay their creditors by issuing new debt. If S-banks do not default, their value function at $t = 1$ in state G writes:

$$V_{1,G}^{S,ND}\left(I_0^S, D_0^S\right) = RI_0^S - D_0^S,$$

where S-banks' asset investment level I_0^S and debt level D_0^S at $t = 0$ are taken as given. S-banks do not default if and only if their $RI_0^S - D_0^S \geqslant 0$. At $t = 1$ in state G, if S-banks have a higher debt level inherited from $t = 0$, they cannot repay their creditors and therefore default.

By assumption, T- and S-banks can only borrow using short-term debt. We take the convention to set the value function to $-\infty$ if banks default on their debt. S-banks' value function at $t = 1$ in state G thus writes:

$$V_{1,G}^{S}\left(I_0^S, D_0^S\right) = \begin{cases} RI_0^S - D_0^S & \text{if } D_0^S \leqslant RI_0^S \\ -\infty & \text{otherwise} \end{cases}$$

where I_0^S is the investment level of the S-bank at $t = 0$, and D_0^S the investment level of the S-bank at $t = 0$.

Date 1**, state** B**.** At $t = 1$ in state B, S-banks choose how much they borrow from households, how much assets they sell and how much assets they buy.

At $t = 1$ in state B there is a probability $(1 - q)$ that assets pay off zero (see Figure 2). Therefore S-banks are unable to roll over their short-term debt. Indeed, their assets are risky and creditors demand riskless debt. Hence, the only way for S-banks to pay their debt is to sell assets to pay back their creditors. Shadow banks can also choose to sell more assets than what they need to repay their debt, either to consume or to buy assets from other banks.

Remark that S-banks can only buy assets at $t = 1$ in state B by selling assets purchased at $t = 0$ and incurring a liquidation cost ε. Therefore in equilibrium, S-banks do not sell their assets to buy other banks' assets.

If S-banks do not default, their value function at $t = 1$ in state B writes:

$$V_{1,B}^{S,ND}\left(I_0^S, D_0^S, p_{1,B}\right) = \max_{\alpha_{1,B}^S \in [0;1]} \alpha_{1,B}^S q r I_0^S + (1 - \alpha_{1,B}^S)(1 - \varepsilon) p_{1,B} q r I_0^S - D_0^S$$

$$\text{s.t. } (1 - \alpha_{1,B}^S)\,(1 - \varepsilon)\, p_{1,B}\, q r I_0^S \geqslant D_0^S$$

where I_0^S is the investment level of S-banks at $t = 0$, D_0^S their investment level at $t = 0$, $p_{1,B}$ the asset price at date 1 in state B, and $\alpha_{1,B}^S$ is the share of assets purchased at $t = 0$ that S-banks do not sell at $t = 1$ in state B. S-banks are also subject to a limited liability constraint which ensures that their debt is riskless. As before, we set the value function to $-\infty$ in case of default. We show in Lemma 2 that S-banks optimally repay their debt at $t = 1$ in state B, by selling their assets. For a given amount of debt to repay, shadow banks need to sell less assets when asset prices are high, i.e. when the collateral value of their assets is high.

Lemma 1 (S-banks at $t = 1$ in state B)**.** *At $t = 1$ in state B, S-banks use the proceeds from asset sales to repay their debt.*

Proof. See Appendix B.2.1. □

Date 0**.** At $t = 0$, S-banks choose their levels of debt D_0^S and investment I_0^S to avoid default at $t = 1$ so that the debt they issue to households is riskless. They also face a funding constraint: their date-0 investment is financed with bankers' net worth and debt raised from households.[19]

[19]Due to condition (2), bankers always choose to invest all their endowment into the bank they set up (see Section 4.5 below).

The date-0 value function of a S-bank writes:

$$V_0^S\left(p_{1,B}, n^S\right) = \max_{D_0^S, I_0^S \geqslant 0} [(1-p)\left((1-\varepsilon)p_{1,B}qrI_0^S - D_0^S\right)\max\left(\frac{1}{(1-\varepsilon)p_{1,B}};1\right)$$
$$+ p\left(RI_0^S - D_0^S\right) + (D_0^S + n^S - I_0^S)]$$
$$\text{s.t. } D_0^S + n^S \geqslant I_0^S$$
$$D_0^S \leqslant (1-\varepsilon)\, p_{1,B}\, qrI_0^S$$
$$D_0^S \leqslant RI_0^S$$

Denoting $\overline{p}_1^S \equiv \frac{1}{(1-\varepsilon)(qr+\frac{p(R-1)}{1-p})} < \frac{1}{1-\varepsilon}$, we obtain Proposition 1.[20]

Proposition 1 (S-banks at $t=0$). *At $t=0$, S-banks take future asset prices as given, and optimize over the quantity of asset investment (I_0^S) and amount of borrowing from households (D_0^S):*

1. *If asset prices at $t=1$ in state B are low ($0 \leqslant p_{1,B} < \overline{p}_1^S$), S-banks do not borrow from households at $t=0$ using short-term debt, and they invest in assets backed by equity.*

2. *If asset prices at $t=1$ in state B are high ($\overline{p}_1^S < p_{1,B} < \frac{1}{(1-\varepsilon)qr}$), S-banks borrow up to the limit from households at $t=0$ using short-term debt, which they repay at $t=1$ in state B by selling their assets . This limit, as well as the amount of assets they invest in at $t=0$, are determined by the collateral value of their assets $t=1$ in state B.*

Proof. See Appendix B.2.2. □

Although S-banks do not have access to deposit insurance, they can initially issue riskless debt insofar as they are backed by the liquidation value of the assets they sell at $t=1$ in state B. When liquidating at $t=1$ in state B, proceeds from the sale of a fraction $(1-\alpha_{1,B}^S)$ of their assets are $(1-\alpha_{1,B}^S)(1-\varepsilon)p_{1,B}qrI_0^S$ where $p_{1,B}qr$ is the asset price at $t=1$ in state B.

The proceeds of assets sale depend on T-banks' ability to purchase assets in a crisis, which are finance using short-term debt backed by deposit insurance. Indirectly, S-banks therefore rely on T-banks' deposit insurance via asset sales in a crisis. We now turn to T-banks' optimization program.

4.4 Traditional banks' program

We expose and solve T-banks' program backwards.

Date 1, state G. At $t=1$ in state G, T-banks can only repay their creditors by issuing debt. As before, we define T-banks' value function at $t=1$ in state G as:

$$V_{1,G}^T\left(I_0^T, D_0^T\right) = \begin{cases} \delta RI_0^T - D_0^T & \text{if } D_0^T \leqslant \delta RI_0^T \\ -\infty & \text{otherwise} \end{cases}$$

[20] See Appendix B.2.2 for the technical solution.

where T-banks' asset investment level I_0^T and debt level D_0^T at $t = 0$ are taken as given. T-banks do not default if and only if their $\delta R I_0^T - D_0^T \geqslant 0$. At $t = 1$ in state G, if T-banks have a higher debt level inherited from $t = 0$, they cannot repay their creditors and therefore default. As before, we set T-bank's value function in case of default to $-\infty$.

Date 1**, state** B T-banks can issue riskless debt to households at $t = 1$ in state B backed by deposit insurance, which makes their debt riskless despite a non-zero probability of zero asset returns at $t = 2$. T-banks are subject to three constraints: they must (i) pay the deposit insurance fund at an actuarially fair price for backing their short-term debt, (ii) repay their debt and (iii) they cannot back more than the deposit insurance limit k.

Constraint (i) puts an upper bound on the amount of riskless debt issued at $t = 1$ in state B: this amount cannot exceed expected asset returns. Note that the deposit insurance fund does not subsidize T-banks: T-banks must repay the debt $D_{1,B}^T$ issued at date 1 in state B, either by repaying this amount $D_{1,B}^T$ at $t = 2$ in state BB, or by paying $\frac{1-q}{q} D_{1,B}^T$ to the guarantee fund at $t = 2$ in state BB, such that the expected net payment made to the deposit insurance fund at date 1 in state B is:

$$q \frac{1-q}{q} D_{1,B}^T + (1-q)(-D_{1,B}^T) = 0.$$

Constraint (ii) is T-banks' date-2 limited liability constraint in state BG, which writes:

$$\frac{1-q}{q} D_{1,B}^T + D_{1,B}^T \leqslant \delta r \left(I_{1,B}^T + \alpha_{1,B}^T I_0^T \right)$$

where I_0^T is the investment in assets at date 0, $\alpha_{1,B}^T$ is the share of T-banks' unliquidated assets, $I_{1,B}^T$ is the amount of assets purchased and $D_{1,B}^T$ is the amount of riskless short-term debt issued, all at $t = 1$ in state B. This constraint rewrites:

$$D_{1,B}^T \leqslant \delta q r \left(I_{1,B}^T + \alpha_{1,B}^T I_0^T \right).$$

Constraint (iii) is the limit on deposit insurance which stems from Assumption 1, and writes:

$$D_{1,B}^T \leqslant k.$$

In addition to these constraints, T-banks must repay date-0 creditors and finance their date-1 asset purchases either by issuing debt backed by deposit insurance, or by selling part of their assets. If T-banks do not default, their value function at $t = 1$ in state B writes:

$$V_{1,B}^{T,ND}\left(I_0^T, D_0^T, p_{1,B}\right) = \max_{\left\{\alpha_{1,B}^T, D_{1,B}^T, I_{1,B}^T\right\}} \left(\delta - p_{1,B}\right) qrI_{1,B}^T + \alpha_{1,B}^T \delta qrI_0^T + (1-\alpha_{1,B}^T) p_{1,B} qrI_0^T (1-\varepsilon) - D_0^T$$

$$\text{s.t. } (1-\alpha_{1,B}^T) p_{1,B} qrI_0^T (1-\varepsilon) + D_{1,B} \geqslant D_0^T + p_{1,B} qrI_{1,B}^T$$

$$D_{1,B} \leqslant q\delta\left(r\alpha_{1,B}^T I_0^T + rI_{1,B}^T\right)$$

$$\alpha_{1,B}^T \in [0;1]$$

$$D_{1,B}^T \in [0;k]$$

$$I_{1,B}^T \in \mathbb{R}^+$$

Again, we set T-bank's value function in case of default to $-\infty$, such that in case of default their value function is equal to $-\infty$. As shown in Lemma 3 (in Appendix B.3.1), the T-bank does not default on its debt if and only if $D_0^T \leqslant \overline{D}_0^T\left(I_0^T, p_{1,B}\right)$.[21] The value function of a T-bank at $t = 1$ in state B is given in Proposition 2.

Proposition 2 (T-banks at $t = 1$ in state B)**.** *At $t = 1$ in state B, T-banks take asset prices, the investment level (I_0^T) and debt (D_0^T) raised at $t = 0$ as given, and optimize over the quantity of asset sales ($1-\alpha_{1,B}^T$), quantity of asset purchases ($I_{1,B}^T$) and amount of borrowing from households ($D_{1,B}^T$):*

1. *If asset prices are low ($p_{1,B} < \delta$), T-banks issue as much debt backed by deposit insurance as they can, i.e. $D_{1,B}^T = k$, to repay their debt and purchase assets, and they do not sell assets.*
2. *If asset prices have intermediate values ($\delta < p_{1,B} < \frac{\delta}{1-\varepsilon}$), T-banks issue debt backed by deposit insurance to repay their debt, and they do not purchase nor sell assets.*
3. *If asset prices are high ($p_{1,B} \geqslant \frac{\delta}{1-\varepsilon}$), T-banks do not issue debt backed by deposit insurance, they use the proceeds from asset sales to repay their creditors, and they do not purchase assets.*

Proof. See Appendix B.3.2. □

Depending on asset prices, T-banks choose whether they purchase or sell assets. When asset prices are attractive enough, i.e. when they are low, T-banks borrow from households by raising short-term debt backed by deposit insurance, for which they pay an actuarially fair premium.

In Appendix B.3.2 we show that these intuitions remain true in the case where at $t = 0$, T-banks borrow more debt than the deposit insurance limit, i.e. if $D_0^T > k$. In that case, T-banks cannot fully repay their creditors by issuing debt backed by deposit insurance at $t = 1$ in state B. Then if asset prices are low at $t = 1$ in state B, T-banks issue the maximum level k of debt backed by deposit insurance to repay their creditors, and sell assets to repay the remaining debt. In that case if asset prices are high, T-banks do not raise debt at $t = 1$ in state B, and they sell assets to repay their existing debt (similarly as in Proposition 2).

Finally, if the amount of debt raised by T-banks at $t = 0$ is higher than their profits at $t = 1$ in state B, i.e. if $D_0^T > \overline{D}_0^T\left(I_0^T, p_{1,B}\right)$ (where $\overline{D}_0^T\left(I_0^T, p_{1,B}\right)$ is given in Lemma 3 in Appendix B.3.1), T-banks cannot

[21] $\overline{D}_0^T\left(I_0^T, p_{1,B}\right)$ depends on asset prices at $t = 1$ in state B as shown in Appendix B.3.1.

repay their creditors. They then default on their debt, in which case our convention is that their value function is $-\infty$.

We now turn to date 0, at which T-banks raise $D_0^T \leqslant \overline{D}_0^T\left(I_0^T, p_{1,B}\right)$ in equilibrium, so that they do not to default on their debt at $t = 1$.

Date $t = 0$**.** At $t = 0$, T-banks choose how they borrow from households in the form of short-term debt, and how much assets to invest in, which they finance with a mix of equity and short-term debt issued to households. As discussed before, T-banks choose a debt level such that they do not default at $t = 1$ (see Appendix B.3.1), in both states B and G. T-banks' value function at $t = 0$ thus writes:

$$
\begin{aligned}
V_0^T\left(p_{1,B}, n^T\right) = \max_{\{D_0^T, I_0^T \in \mathbb{R}_+^2\}} & \; p\left(\delta R I_0^T - D_0^T\right) + (1-p)V_{1,B}^{T,ND}(D_0^T, I_0^T, p_{1,B}) + (D_0^T + n^T - I_0^T) \\
\text{s.t.} & \; I_0^T \leqslant D_0^T + n^T \\
& \; D_0^T \leqslant \overline{D}_{0,B}\left(I_0^T, p_{1,B}\right) \\
& \; D_0^T \leqslant \delta R I_0^T
\end{aligned}
$$

We denote $\overline{p}_{1,L}^T < \overline{p}_{1,H}^T$ two asset price thresholds defined in Appendix B.3.2. Under condition (4), we obtain Proposition 3.[22]

Proposition 3 (T-banks at $t = 0$)**.** *At $t = 0$, T-banks take future asset prices as given, and optimize over the quantity of asset investment (I_0^T) and amount of borrowing from households (D_0^T):*

1. *If asset prices at $t = 1$ in state B are low ($0 < p_{1,B} < p_{1,L}^T$), T-banks do not borrow from households at $t = 0$ using short-term debt, and they invest in assets backed by equity.*

2. *If asset prices at $t = 1$ in state B are high ($p_{1,L}^T < p_{1,B} < p_{1,H}^T$), T-banks finance asset investment with the maximum amount of short-term debt k at $t = 0$, which they repay at $t = 1$ in state B by issuing debt backed by deposit insurance.*

Depending on the asset price at $t = 1$ in state B, T-banks choose how much short-term debt to issue at $t = 0$ to invest assets, versus how much buffer to keep to purchase assets from S-banks at $t = 1$ in state B. Although the guarantee fund enables T-banks to issue short-term debt at $t = 1$ in state B, they have to trade-off between those two investment opportunities because they can only issue a limited amount of debt backed by deposit insurance in a crisis.

If asset prices are low, T-banks' return from purchasing assets in a crisis is greater than that from investing in assets at date 0. T-banks then prefer not to issue debt at $t = 0$, to keep slack in order to purchase assets at $t = 1$ in state B. If asset prices are high, T-banks invest in assets at $t = 0$, and do not purchase assets in a crisis. For very high asset prices in a crisis, T-banks might even sell some assets. We will show that this does not occur in equilibrium.

[22] See Appendix B.3.2 for the complete technical solution both when condition (4) is met and when it is not.

One important implication is that the deposit insurance limit has an impact on T-banks' date-0 debt level. Indeed, T-banks can issue riskless debt at $t = 0$ backed by deposit insurance, but only up to an amount k, satisfying condition (4). T-banks can therefore issue up to an amount k of riskless short-term debt no matter the asset price at $t = 1$ in state B, generating a positive spillover from deposit insurance at $t = 1$ in state B onto $t = 0$. If they issue more debt at $t = 0$, they need to sell assets at $t = 1$ in state B to repay their debt.

4.5 Banker's equity investment choices

Given the value functions $V_0^i(p_{1,B}, n^i)$, bankers initially choose to set up one bank type i ($i \in \{S; T\}$), and allocate their initial net worth n between a part n^i they invest in their bank, which becomes the bank's equity, and a remaining part $(n - n^i)$ that they consume.

When investing $n^i \in [0; n]$ units of their endowment into a i-bank, bankers obtain the bank's residual payoff. At $t = 0$ the value function of setting up an i-bank is $V_0^i(p_{1,B}, n^i)$. Bankers' problem writes as follows

$$V_0^{i,B}(p_{1,B}) = \max_{n^i \in [0;n]} (n - n^i) + V_0^i\left(p_{1,B}, n^i\right)$$

Then,

1. If $p_{1,B} < \frac{1}{(1-\varepsilon)qr}$, bankers choose $n^i = n$. Indeed, each unit of banker's investment in the i-bank can at least be transformed into purchases of assets the bank can invest in. Expected asset returns are then at least $\delta(pR + (1-p)qr)$, which provides more utility to the banker than immediate consumption (assumption 2).

2. If $p_{1,B} \geqslant \frac{1}{(1-\varepsilon)qr}$ the banker is indifferent between all possible allocations of her initial net worth.

In any case, we have

$$V_0^{i,B}(p_{1,B}) = V_0^i(p_{1,B}, n).$$

4.6 Asset market clearing in a crisis

We derive the market-clearing conditions for the asset market at $t = 1$ in state B, taking the shares χ^S ($(1 - \chi^S)$) of S-banks (T-banks) as given. We first define an equilibrium on the asset market at $t = 1$ in state B.

Definition 2 (Asset market equilibrium definition)**.** *A market equilibrium at $t = 1$ in state B is defined by*

1. *A quantity $S(p_{1,B})$ of assets supplied,*

2. *A quantity $D(p_{1,B})$ of assets demanded,*

3. *A price $p_{1,B}$ such that $D(p_{1,B}) = S(p_{1,B})$.*

We obtain the following Proposition. Definitions of the thresholds $\overline{p}_1^S$, $\overline{p}_{1,L}^T$, $\overline{p}_{1,H}^T$, together with the off-equilibria solutions for $p_{1,B} > \overline{p}_{1,H}^T$, are given in Appendix B.5.

Proposition 4 (Asset supply and demand in a crisis)**.** *At date 1 in state B (in a "crisis"), with $\chi^S \in [0;1]$ the share of S-banks and $(1-\chi^S)$ that of T-banks, the aggregate demand for assets writes:*

$$D(p_{1,B}) = \begin{cases} +\infty & \text{if } p_{1,B} = 0 \\ \frac{k}{p_{1,B}\,qr}(1-\chi^S) & \text{if } 0 < p_{1,B} < \overline{p}_{1,L}^T \\ \in \left[0; \frac{k}{p_{1,B}\,qr}(1-\chi^S)\right] & \text{if } p_{1,B} = \overline{p}_{1,L}^T \\ 0 & \text{if } p_{1,B} > \overline{p}_{1,L}^T \end{cases}$$

and the aggregate supply of assets writes:

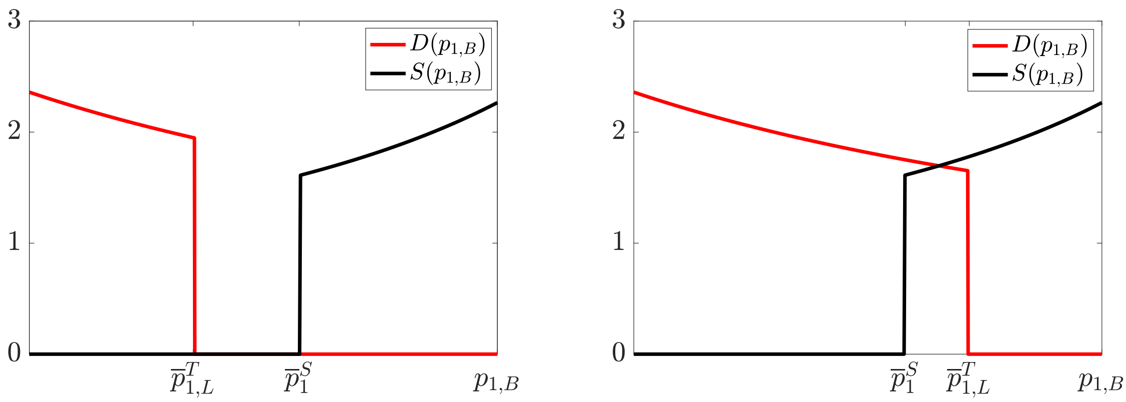
$$S(p_{1,B}) = \begin{cases} 0 & \text{if } 0 \leqslant p_{1,B} < \overline{p}_1^S \\ \in \left[0; \frac{n(1-\varepsilon)}{1-(1-\varepsilon)p_{1,B}\,qr}\chi^S\right] & \text{if } p_{1,B} = \overline{p}_1^S \\ \frac{n(1-\varepsilon)}{1-(1-\varepsilon)p_{1,B}\,qr}\chi^S & \text{if } \overline{p}_1^S < p_{1,B} < \overline{p}_{1,H}^T \\ \in \left[\frac{n(1-\varepsilon)}{1-(1-\varepsilon)p_{1,B}\,qr}\chi^S; \frac{n(1-\varepsilon)}{1-(1-\varepsilon)p_{1,B}\,qr}\chi^S + \frac{(1-\varepsilon)}{qr}\frac{(-\frac{k}{\delta})+qr(n+k)}{1-p_{1,B}(1-\varepsilon)qr}(1-\chi^S)\right] & \text{if } p_{1,B} = \overline{p}_{1,H}^T \end{cases}$$

Proof. See Appendix B.5. □

If asset prices are low, there is a high demand for assets by T-banks, such that T-banks do not invest in assets at $t = 0$ to keep slack to purchase assets at $t = 1$ in state B. If asset prices are high, T-banks prefer investing in assets at $t = 0$, which they finance by issuing debt to households.

If asset prices are low (lower than $\overline{p}_1^S$), S-banks do not issue debt at $t = 0$, because the price they would obtain by selling assets in a crisis is low, making the cost of date-0 debt too high. If asset prices are high (higher than $\overline{p}_{1,H}^T$), both T and S-banks are willing to sell assets, which induces a positive supply whatever the allocation of intermediaries between the two types of banks. This is impossible and thus restricts the set of prices that can prevail in equilibrium to $[0; \overline{p}_{1,H}^T]$.

The asset supply and demand schedules derived in Proposition 4 are illustrated in Figure 5.



(a) Case $\delta = 1$
No asset trade

(b) Case $\delta < 1$ ($\delta = 0.8$)
Asset trade

Figure 3: Numerical illustration of the asset market at $t = 1$ in state B
(T-banks' demand in red, S-banks' supply in black)

As illustrated in Figure 5, Proposition 4 implies that T- and S-banks would not trade assets if regulatory costs on T-banks are not high enough. The reason is that for T-banks to keep a buffer at $t = 0$ to purchase asset from S-banks at $t = 1$ in state B, regulatory costs need to be high enough. Otherwise T-banks would rather invest in assets at $t = 0$ than keep a buffer to purchase assets from S-banks at a price such that S-banks are willing to sell their assets. Condition (6) ensures that $\overline{p}_1^S < \overline{p}_{1,L}^T$ such that there exist parametric spaces where the asset market clears. Henceforth we assume that condition (6) is met such that asset trade is feasible. Taking the fraction of bankers initially setting up a S-bank (T-bank) as given, i.e. χ^S (respectively $(1 - \chi^S)$), we characterize in Proposition 5 the different asset market equilibria that can prevail as a function of the relative size of T- and S-banks. Definitions of the thresholds $\underline{\chi}^S$ and $\overline{\chi}^S$, together with a technical solution, are given in Appendix B.5..

Proposition 5 (Asset market equilibrium)**.** *In equilibrium, asset demand is equal to asset supply* ($D(p_{1,B}) = S(p_{1,B})$)*, and:*

1. *With T-banks only* ($\chi^S = 0$)*, no assets are traded and* $p_{1,B} \in [\overline{p}_{1,L}^T; \overline{p}_{1,H}^T]$.
2. *When T- and S-banks coexist* ($\chi^S \in (0; 1)$)*,*
   (a) *either S-banks' relative size is small* ($\chi^S \leqslant \underline{\chi}^S$)*, assets are traded at a price* $p_{1,B} = \overline{p}_{1,L}^T$.
   (b) *or S-banks' relative size is intermediate* ($\chi^S \in [\,\underline{\chi}^S; \overline{\chi}^S]$)*, assets are traded at a price* $p_{1,B} \in [\overline{p}_{1}^S; \overline{p}_{1,L}^T]$.
   (c) *or S-banks' relative size is large* ($\chi^S \geqslant \overline{\chi}^S$)*, assets are traded at a price* $p_{1,B} = \overline{p}_1^S$.
3. *With S-banks only* ($\chi^S = 1$)*, no assets are traded* ($D(p_{1,B}) = S(p_{1,B}) = 0$) *and* $p_{1,B} \in [0; \overline{p}_1^S]$. *No assets are traded.*

Proof. See Appendix B.5. □

4.7 The allocation program: Coexistence of T- and S-banks

We study bankers' choice to set up a T- or a S-bank at $t = 0$. Bankers compare expected profits in each i-bank ($i = \{T, S\}$), i.e. value functions at $t = 0$, and choose a probability χ^S ($(1 - \chi^S)$) to set up a S-bank (T-bank) such as to solve

$$\max_{\chi^S \in [0;1]} \chi^S V_0^S(p_{1,B}) + (1 - \chi^S) V_0^T(p_{1,B})$$

where $p_{1,B}$ is the asset price at $t = 1$ in state B. Figure 4 illustrates the different bankers' allocation equilibria as a function of T-banks' regulatory costs δ (see Assumption 1).[23]

Figure 4 illustrates that there exists a non-empty parameter set for which asset prices in a crisis are pinned down in equilibrium such that T- and S-banks' expected profits are equal (as described in Section 4.6). Bankers are then indifferent between setting up a T- or S-bank initially. We refer to Section 5.2 for a discussion of changes in regulatory costs on the equilibrium relative size of T- and S-banks.

[23]We set the following parameter values: $n = 1, p = 0.5, r = 1, q = 0.99, R = 1.15, k = 8.1, \varepsilon = 0.1$.

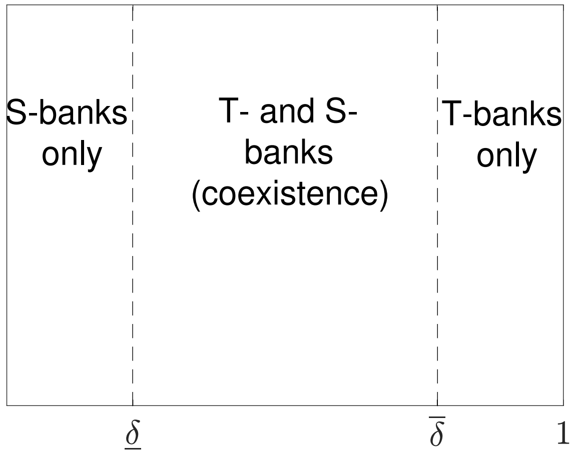


Figure 4: Conditions for the coexistence of T- and S-banks

We focus in this Section on the unique equilibrium in which T- and S-banks coexist. We refer to Appendix B.6 for a complete charaterization of bankers' equilibrium allocations between T- and S-banks. We show in Appendix B.6 that the coexistence equilibrium is uniquely pinned down by asset prices in a crisis, and Appendix B.8 gives the parametric conditions for T- and S-banks to coexist.

Proposition 6 (Coexistence of traditional and shadow banks)**.** *There exists a unique equilibrium such that (i) at $t = 0$ bankers invest all their endowment in a bank, (ii) at $t = 1$ in state G no assets are traded and banks repay their creditors by issuing debt, and (iii) at $t = 1$ in state B, S-banks sell assets to T-banks at a price such that bankers are indifferent between setting up a traditional or a shadow bank initially.*

The conclusion we draw from Proposition 6 is that both types of intermediaries can coexist and interact on the asset market (type-3 equilibrium). In that case, S-banks issue debt initially thanks to T-banks' ability to back debt in a crisis to purchase assets from shadow banks. In such a situation, asset fire-sales always occur for the following reason.

T-banks can buy S-banks' assets in a crisis, because of their unique ability to finance these purchases by issuing short-term debt backed by deposit insurance.[24] In a crisis, traditional banks can purchase assets from shadow banks by issuing short-term debt backed by deposit insurance. This is consistent with evidence from the crisis, during which traditional banks on-boarded \$550 billion worth of assets while experiencing a sudden deposit inflow.

Because of limited deposit insurance, the total quantity of assets they can finance at $t = 0$ and $t = 1$ in state B is limited. T-banks face a trade-off between issuing debt at $t = 0$ and keeping some buffer in order to issue debt at $t = 1$ in state B so as to purchase S-banks' assets. To purchase assets from S-banks in a crisis, T-banks thus need to be compensated for foregoing asset investment at $t = 0$, and S-banks asset trade at a discount. This is consistent with evidence of fire sales in the 2007 crisis (see e.g. Gorton and Metrick, 2012).

[24] Recall that we assumed households cannot purchase assets directly. This assumption could be relaxed in the presence of a premium for money-like claims issued by T-banks, as in Stein (2012), in which case T-banks would buy S-banks' assets because they could pay a higher price than households thanks to the premium they would obtain by issuing money-like claims.

Asset prices are lower than the price at which the traditional banks value the asset (i.e. δ), i.e. the fire sale is not entirely driven by the need for T-banks to be compensated for higher regulatory costs. However, asset prices cannot be too low: S-banks have to be willing to pay a cost at $t = 1$ in state B, to issue debt initially. These trade-offs are key to understand the occurrence of fire-sales in a crisis and the interaction between T- and S-banks.

5 Implications

5.1 Gains from trade and the complementarity between T- and S-banks

When bankers choose to set up a T- or a S-bank, they trade off the costs and benefits associated with each bank type. To highlight the role of asset trade in a crisis, as a thought experiment, we assume that banks cannot trade assets in a crisis. In this case, S-banks cannot rely on T-banks to purchase their assets in a crisis. As a result, S-banks cannot issue short-term debt at $t = 0$. Meanwhile, T-banks use deposit insurance to repay their creditors in a crisis. However, access to deposit insurance for T-banks comes at the cost of costly regulation. In this thought experiment, bankers trade-off expected profits for S-banks without debt versus that of T-banks without the benefits of purchasing fire-sold assets in a crisis.

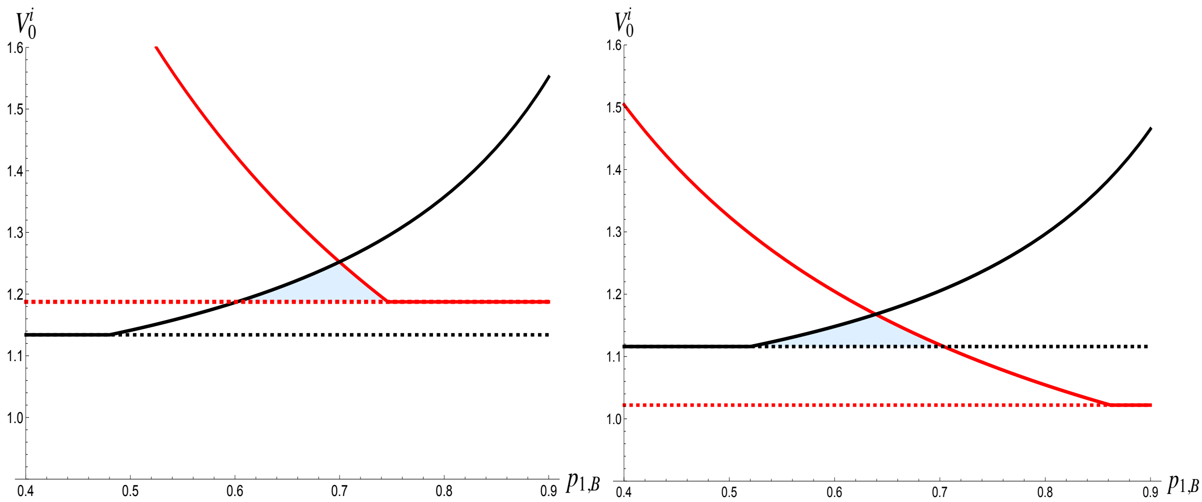
Keeping similar notations as in the model, the value function at $t = 0$ of setting up a S-bank in the absence of an asset market at $t = 1$ in state B writes:

$$V_0^{S,NM} = [pR + (1-p)qr]n \tag{7}$$

where we denote "NM" for "no market". Meanwhile, the value function at $t = 0$ of setting up a T-bank in the absence of an asset market at $t = 1$ in state B writes:

$$V_0^{T,NM} = p\,[\delta R(n+k) - k] + (1-p)\,[\delta qr(n+k) - k] \tag{8}$$

This has two implications. First, theses value functions are a lower bound for the value functions of bankers setting up a T- or S-bank in the model with an asset market. This implies that opening up the asset market generates gains from trade, thereby increasing total expected profits in the banking sector. The effect of asset trade on banks' value function is illustrated in Figure 5.



(a) Case 1: $V_0^T > V_0^S$ absent asset trade

(b) Case 2: $V_0^T < V_0^S$ absent asset trade

Figure 5: Value functions against asset price in a crisis
(V_0^T in red, V_0^S in black, dashed without an asset market)

Second, banks' value functions are modified in the presence of an asset market. Figure 5 illustrates that the introduction of an asset market, in which T-banks purchase assets from S-banks at a discount, benefits both bank types: both T- and S-banks' expected profits increase when assets are traded (plain curves in Figure 5 are above dashed lines for most asset prices). The shaded area represents banks' gains from trade on the asset market at $t = 1$ in state B.

Figure 5 illustrates the finding in Section 4.7 that depending on parameter values, either T- or S-banks' expected profits can be relatively higher absent a market for S-banks' assets in a crisis (see also Figure 4). In Figure 5a (Figure 5b), parameter values are such that expected profits without asset trade are relatively higher for T-banks (S-banks), so that absent an asset market, only T-banks (S-banks) exist in equilibrium (the red dashed line in Figure 5a is above the black dashed line, and conversely in Figure 5b).[25] In both cases, we find that banks gain from trade, so that asset trade allows for the coexistence of T- and S-banks.

We interpret this as a rationale for the coexistence of T- and S-banks. In our model, T- and S-banks coexist because they trade assets in a crisis. At date 0, when bankers choose to set up a T- or a S-bank, they trade off the costs and benefits associated with each type of bank, i.e. low regulation costs but need to sell assets at a discount in a crisis versus high regulation cost but ability to buy assets at a discount in a crisis. The trade-off depends on the asset discount anticipated in a crisis, itself a function of the relative size of the two banking sectors. The larger the relative size of the traditional (shadow) banking sector, the higher (lower) asset prices in a crisis, and the higher bankers' incentive to set up a shadow (traditional) bank in the first place. In that sense, traditional and shadow banks form an ecosystem. In equilibrium, bankers must be indifferent between setting up a traditional or a shadow bank. This pins down asset prices and thus the relative size of the T- and S-banking sectors in equilibrium.

[25] In both cases, we set $n = 1, p = 0.9, r = 1, q = 0.99, \delta = 0.9$. In Figure 5a we set $k = 8.1$ and $R = 1.15$ while in Figure 5b, $k = 4$ and $R = 1.13$.

5.2 Deposit insurance and regulatory costs: Effects on the relative size of T- and S-banks

Arbitrage of regulatory costs has been an important feature of the banking industry since the first Basel accords of 1988. Some debates about the effectiveness of banking regulation thus center on the ability of shadow banks to escape regulation (Hanson et al., 2011; Buchak et al., 2017). Yet, the regulatory arbitrage view does not account for the reason of the coexistence between the two sectors, which in our model is based on their interaction in a crisis. We study how, in our model, deposit insurance and T-banks' regulatory costs affect the relative size of the two banks types.

Changes in the deposit insurance limit k First, we study how, in our model, the level of deposit insurance affects the relative size of T- and S-banks. We focus on the unique equilibrium such that they coexist, as characterized in Proposition 6.[26]

In the coexistence equilibrium, T- and S-banks' value functions are equal at $t = 0$, so that bankers are indifferent between setting up a T- or S-bank. We consider small increases in k, but the reverse reasoning holds for small decreases in k. We then discuss the effect of greater changes in k.

On the one hand, traditional banks' increased debt capacity allows them to operate on a larger scale. This effect increases T-banks' expected profits, therefore bankers' incentives to set up a T-bank initially. On the other hand, T-banks use their increased debt capacity to bid up shadow banks' assets prices in a crisis. In turn, higher asset prices in a crisis increases S-banks' initial debt capacity, which allows them to operate on a larger scale. This effect increases S-banks' expected profits, therefore bankers' incentives to set up a S-bank initially. We show in Proposition 7 that the latter effect dominates the former.

Proposition 7 (Changes in the deposit insurance limit)**.** *An increase in deposit insurance* (k) *leads to a decrease in the relative size of the T-banking sector* ($1 - \chi^S$)*.*

Proof. See Appendix B.9. □

To gain intuition about Proposition 7, recall that asset prices are pinned down in the equilibrium in which T- and S-banks coexist, so that T-banks' regulatory costs are offset by their profits on S-banks' assets purchases, and bankers are indifferent between setting up either type of bank. All else being equal, when deposit insurance expands, because T-banks use their increased debt capacity to bid up shadow banks' assets prices, asset prices must decrease to return to their equilibrium level for T- and S-banks to coexist. This requires an increase in the relative size of the S-banking sector.

Note that this reasoning only holds for small changes in k. Holding δ fixed, when k is low enough, only one bank type exist in equilibrium. Indeed, for low levels of k, T-banks require too high a compensation for purchasing assets at $t = 1$ in state B, so that S-banks are better off not issuing debt. There is no asset trade in equilibrium, and depending on parameter values, only one bank exist in equilibrium (see Section 5.1). Finally, bankers only set up S-banks, and no debt is issued. T-banks are then wiped out, which is an effect already emphasized in existing models of shadow banking as regulatory arbitrage (see e.g. Plantin (2015), Ordonez (2013), Harris et al. (2015)).

[26]See Appendix B.8 for the parametric restrictions under which this equilibrium occurs.

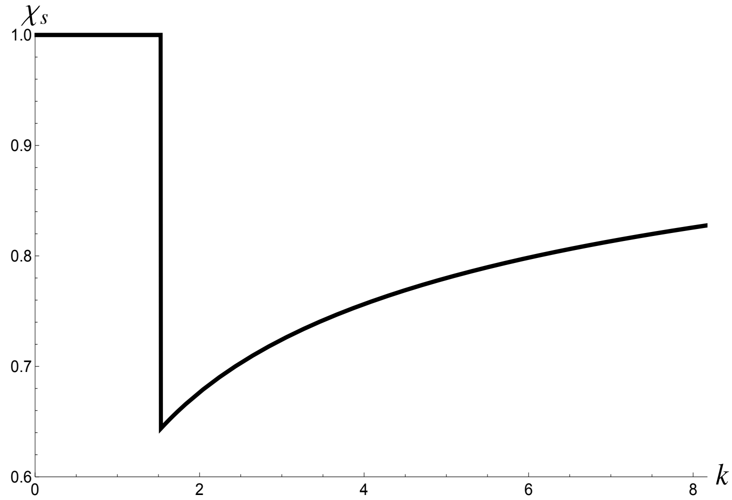


Figure 6: Share of S-banks according to the deposit insurance limit

Figure 6 displays the fraction of bankers setting up a S-bank, for different values of the deposit insurance limit (k). We use the same parameter values as in Figure 5a (condition 4 is satisfied: $0 < k < \overline{k}$). The black line represents the fraction of bankers setting up a S-bank in equilibrium. When k is low, T-banks can only issue a small amount of debt backed by deposit insurance in a crisis. As a result, S-banks cannot expect a large support from T-banks in a crisis, and S-banks cannot raise a lot of debt initially. In equilibrium, there is no asset trade and all bankers set up a T-bank ($\chi^S = 0$).[27] As k increases, T-banks bid up S-banks' asset prices in a crisis, so that S-banks can raise more debt initially. S-banks' expected profits increase, and in equilibrium T- and S-banks coexist. As shown in Proposition 7, in the coexistence equilibrium, small increases in the deposit insurance limit k provide incentives for bankers to set up more S-banks initially, so that χ^S is increasing in k for intermediate values of k (boundaries are given in Appendix B.8).

Changes in regulatory costs Second, we study how, in our model, changes in T-banks' regulatory costs (δ) affects the relative size of T- and S-banks. The message we draw from Figure 4 is that the coexistence of T- and S-banks requires regulatory costs to satisfy two conditions. First, regulatory costs must be high enough for T-banks to keep slack at $t = 0$ to purchase assets from S-banks in a crisis (at $t = 1$ in state B), i.e. $\delta < \overline{\delta}$.[28] T- and S-banks then trade assets in a crisis, so that they become complements. Second, regulatory costs must be low enough for bankers to have incentives to set up T-banks initially, i.e. $\delta > \underline{\delta}$.[29] In the coexistence equilibrium, the higher T-banks' regulatory costs, i.e. the lower δ, the higher the relative size of S-banks.

[27]Under the parameter values used here, as in Figure 5a, only T-banks exist for low values of k.
[28]Where $\overline{\delta} \equiv 1/(pR + (1-p)qr)$ is derived from condition 2. Note that condition 3 is satisfied under the parameter values chosen in Figure 4 ($\delta > 1 - \varepsilon = 0.9$).
[29]Where $\underline{\delta} \equiv p/(\varepsilon(p(R-1) + (1-p)qr) + p)$ is derived from condition 6.

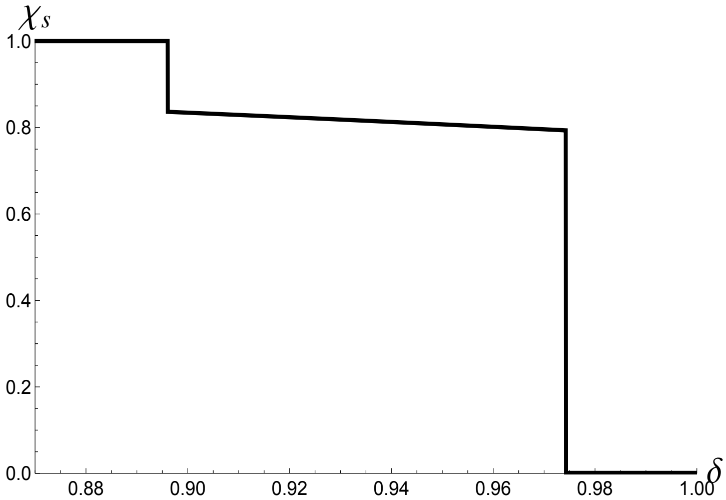


Figure 7: Share of S-banks according to T-banks' regulatory costs

5.3 Discussion

Analogy with capital requirements We consider an extension of our model in which instead of a limit on deposit insurance, T-banks have to satisfy capital requirements. Specifically, we assume that at $t = 1$ in state B, T-banks cannot issue more debt than a fraction $(1 - c)$ of their assets' market value, i.e. they are subject to the following capital requirement at $t = 1$ in state B:

$$D^T_{1,B} \leqslant (1 - c)\left[\alpha^T_{1,B} I^T_0 + p_{1,B}\, q r I^T_1\right] \tag{9}$$

where $D^T_{1,B}$ denotes T-banks' debt raised from households, $\alpha^T_{1,B}$ is the fraction of T-banks' assets that are not sold, $p_{1,B}$ the asset price, all at $t = 1$ in state B, and I^T_t the quantity of assets purchased by T-banks at $t = 0, 1$.[30]

We consider a decrease in T-banks' capital requirements (c), but the symmetric reasoning is true for an increase in capital requirements. We obtain Proposition 8.

Proposition 8 (Changes in capital requirements)**.** *An increase in T-banks' capital requirements* (c) *leads to an increase in the relative size of the T-banking sector* ($1 - \chi^S$)*.*

Proof. See Appendix B.10. □

We show in Proposition 8 that changes in capital requirements (c) have similar effects on the relative size of T- and S-banks as changes in the deposit insurance limit (k, see Section 5.2). An increase in T-banks' capital requirements reduces T-bank's debt capacity in a crisis. This reduces S-banks' asset prices in a crisis, and in turn, increases bankers' incentives to set up a T-bank *ex-ante*. Conversely, lower capital requirements in a crisis increases T-bank's debt capacity in a crisis.

We view this result as a caution for policies aimed at increasing T-banks' countercyclical capital buffers. In bad times, when T-banks use their increase debt capacity to bid up S-banks' assets prices,

[30] At $t = 1$ in state B, we assume that capital requirements are stringent enough for (9) to bind in equilibrium, and that T-banks do not sell assets to repay their creditors. See Appendix B.10.

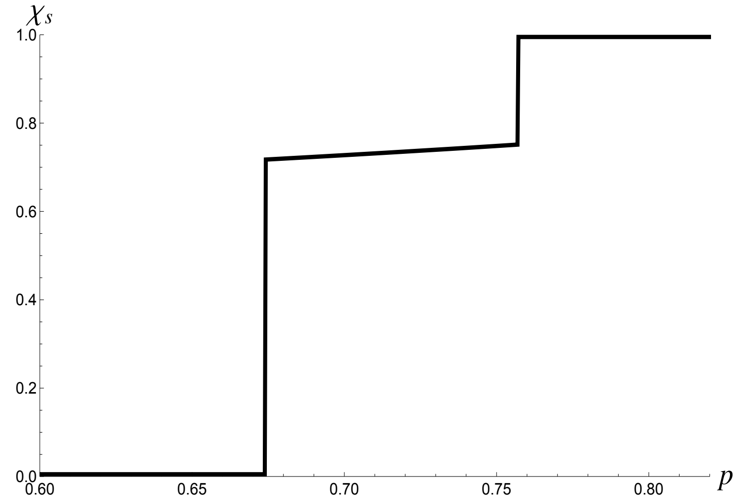


Figure 8: Share of S-banks according to the probability that no crisis occurs at $t = 1$

this might increase the relative size of S-banks. We discuss in Section 6 how the decentralized and socially optimal relative sizes of T- and S-banks might differ.

Crisis expectations and relative size of S-banks Bankers do not make expectational errors in our model. Asset fire sales in a crisis are preplanned and bankers take them into account when deciding to set up T- or S-bank at $t = 0$. However, there is accumulating research suggesting that the scope and severity of the crisis were vastly underestimated. Markets did not seem to be aware of the possibility of a large negative shock (Coval et al., 2009): investment banks and rating agencies used incorrect models, neglecting the systematic component in the risks of individual mortgages. Even securitization specialists were not aware of a large-scale housing bubble and a looming crisis in 2004–2006 (Cheng et al., 2014).

It is not entirely clear that fire sales in the crisis reflected careful deliberations that supported S-banks from an ex ante perspective. Gennaioli et al. (2013, 2015) propose models of S-banking where neglect risk of large and unlikely bank losses sow the seeds of a financial crisis. Expectational errors seem to be key to understand the crisis, but also financial instability in general.[31]

To understand the impact of neglected risks in our model, we study how the (perceived) probability of a crisis affects bankers' choice to set up a T-bank or a S-bank. An implicit assumption here is that changes in the probability (p) of a good news at $t = 1$ versus that of a bad news ($1 - p$), i.e. a crisis, reflects *all* bankers' expectations at $t = 0$. Whether these probabilities are correct or not does not matter as long as bankers share the same expectation.

Figure 8 shows that T- and S-banks only exist for intermediate values of bankers' expectations of a crisis ($1 - p$). Recall that T- and S-banks coexist because T-banks regulatory costs are offset by the purchase of S-banks' assets in a crisis, at a discount. Therefore when bankers do not anticipate a crisis, i.e. when p is high, T-banks' expected profits from S-banks' fire-sold assets are too unlikely.[32] All bankers then set up a S-bank initially; and only S-banks exist in equilibrium. Conversely when bankers

[31] See for instance Greenwood and Hanson (2013), Baron and Xiong (2016), and Bordalo et al. (2017).

[32] This corresponds to parametric values such that condition (6) is violated, i.e. such that $p > \frac{\delta \varepsilon q r}{1 - \delta \varepsilon [R - 1 - q r] - \delta}$.

anticipate a crisis with a higher probability, i.e. when p is low, S-banks' expected losses from S-banks' discounted assets in a crisis are too likely. In equilibrium, S-banks then choose not issue debt at $t = 0$ to avoid those losses in a crisis, T-banks do not keep slack at $t = 0$ and instead issue debt which they refinance at $t = 1$ in state B backed by deposit insurance (see Proposition 3). As a result, T- and S-banks do not anticipate asset trade in a crisis when p is low, and S-banks cannot issue riskless debt at $t = 0$. Under the parametric values used in Figure 8 (as in Figure 4), only T-banks exist because S-banks choose not to trade assets if a crisis is too likely to occur.

Post-crisis banking regulations There have been several policy initiatives to impose restrictions on banks' trading activities since the crisis. Prohibiting regulatory arbitrage is the paradigm in Section 619 of the Dodd-Frank Act in the U.S. (known as the "Volcker Rule"), in the Financial Services Act of 2013 in the U.K. (based on the Report of the Vickers Commission), as well as the 2012 Report of the European Commission's High-level Expert Group on Bank Structural Reform in the E.U. (known as the Liikanen Report). Those regulation proposals include a prohibition of proprietary trading by T-banks ("Volcker Rule"), a separation between different risky activities (Liikanen Report), and ring-fencing of depository institutions and systemic activities (Report of the Vickers Commission, enacted in 2013 in the Financial Services Act).

These reforms illustrate regulators' concerns about the permeability between T- and S-banks. Regulatory arbitrage was indeed an important motive for T-banks to set up off balance-sheets conduits - which Pozsar et al. (2013) refer to as "internal shadow banking" (see also Acharya et al. (2013)). However, our key finding is that, even absent contractual relationships or explicit guarantees between T- and S-banks, the two bank types coexist because they are complements. The complementarity between the two bank types comes from S-banks' asset sales to T-banks in a crisis: both bank types gain from asset trade (see Section 5). One implication of this complementarity is that T-banks channel the support from the deposit insurance to S-banks, even absent contractual relationships between the two.

This paper suggests that T-banks' profits from S-banks' fire-sold assets in a crisis outweigh the (regulatory) costs that they have to comply with. This offers an explanation for the coexistence of T- and S-banks. When discussing banking reforms one needs to consider the implications on both bank types, in light of the reasons why they coexist in the first place. This paper provides a framework to do so.

6 Normative approach

We now conduct a normative analysis. We study the aggregate surplus in the economy, hence the aggregate profit of T- and S-banks. A way to think about our exercise is to consider that bankers don't consume the profits they make, but that they pay dividends to households at date 2. This measure is comparable to that in Stein (2012).

We focus on the analysis of the efficiency of the banker's allocation between the two bank types, by considering the allocation problem faced by the social planner. The planner internalizes the impact of allocation choices on the equilibrium price $p_{1,B}$ which clears the asset market at date 1 in state B, choosing an allocation $\chi^{S*} \in [0;1]$ such as to solve the following program:

$$\chi^{S*} = \underset{\chi^S \in [0;1]}{\operatorname{argmax}} \chi^S V_0^{S,B}\left(., p_{1,B}^*\right) + (1-\chi^S) V_0^{T,B}\left(., p_{1,B}^*\right)$$

where $p_{1,B}$ is a market price for assets at $t = 1$ in state B. χ^S being fixed, T- and S-banks make the same choices as in the decentralized equilibrium and the equilibrium price $p_{1,B}^*$ is expressed as a function of χ^S as described in Proposition 6.

We focus on parametric conditions in which the decentralized equilibrium is such that T- and S-banks coexist and the asset price at $t = 1$ in state B is such that $p_{1,B} \in (\overline{p}_1^S; \overline{p}_{1,L}^T)$ (see Appendix B.6). In the decentralized equilibrium, bankers then set up a S-bank with a uniquely defined probability $\chi^{S*} \in (\underline{\chi}^S; \overline{\chi}^S)$ (see Proposition 6).

Proposition 9. *When T- and S-banks coexist in equilibrium, the fraction of bankers operating a S-bank (χ^S) is larger than socially optimal. Conversely, the fraction of bankers operating a T-bank ($1-\chi^S$) is smaller than socially optimal.*

Proof. See Appendix B.11. □

When setting up a S-bank, bankers take asset prices in a crisis as given and fail to internalize the effect of their asset sales on asset prices. Since S-banks' ability to issue riskless debt initially depends on the collateral value of their assets in a crisis, this creates a pecuniary externality (Gromb and Vayanos, 2002; Lorenzoni, 2008). Conversely, when choosing to set up a T-bank, bankers take asset prices in a crisis as given and fail to internalize the effect of their asset purchases on asset prices, hence S-banks' ability to issue riskless debt initially. Figure 9 illustrates the deviation between decentralized and socially optimal fractions of bankers setting up T- and S-banks.

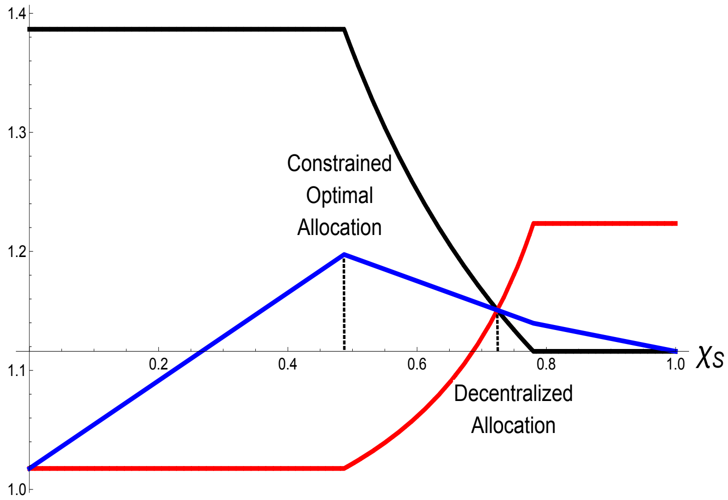


Figure 9: Decentralized and socially optimal equilibrium allocations differ
(V_0^T in red, V_0^S in black, $W(.)$ in blue)

When the constrained optimal allocation differs from the decentralized one, as in Figure 9, transfers between the two forms of intermediation technology can increase the central planner's objective function. The aim of such a transfer is to provide incentives for bankers to set up a T-bank, instead of a S-bank. For instance, lump sum taxes on S-banks that are used to subsidy T-banks can implement the socially optimal constrained allocation.

Note that the decentralized market allocation needs not be inefficient. For instance, if the decentralized allocation is one in with S-banks only (see Appendix B.6 for the parametric restrictions under which it is the case), decentralized and socially optimal allocations coincide, as illustrated in Figure 10. When T- and S-banks do not coexist in equilibrium, lump-sum taxes and subsidies between T- and S-banks need not be implemented.

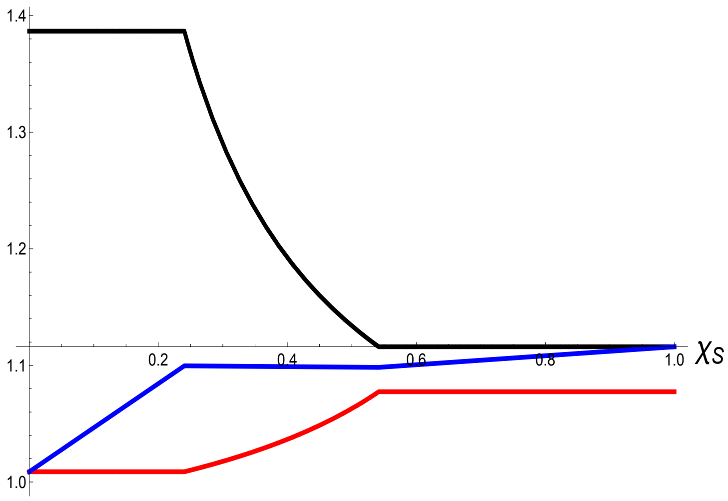


Figure 10: Decentralized and socially optimal equilibrium allocations coincide
(V_0^T in red, V_0^S in black, $W(.)$ in blue)

7 Conclusion

We propose a theory of the coexistence of traditional and shadow banks, which is consistent with several facts from the 2007 financial crisis that we document. In our model, bankers must choose to set up a traditional or a shadow bank. We assume two differences between traditional and shadow banks. On the one hand, traditional banks incur a cost associated with the regulation they must comply with, which shadow banks evade. On the other hand, traditional banks can issue claims backed by deposit insurance, which shadow banks cannot. Traditional and shadow banks otherwise face the same choice sets.

When bankers initially choose to set up a traditional or a shadow bank, they trade off the costs and benefits associated with each type of bank, i.e. low regulation costs but need to sell assets at a discount in a crisis versus high regulation cost but ability to buy assets at a discount in a crisis. The trade-off depends on the asset discount anticipated in a crisis, itself a function of the relative size of the two banking sectors. In the coexistence equilibrium, bankers are indifferent between setting up a traditional or a shadow bank. This pins down asset prices and thus the relative size of the traditional and shadow banking sectors.

We analyze our model's implications for the effect of changes in the level of deposit insurance for traditional banks, and find that expanding support to traditional banks in a crisis increases asset prices, so that more bankers set up a shadow bank initially.

Finally, we consider the normative implications of our analysis. We find that asset sales generate a pecuniary externality, and the relative size of S-banks is larger than socially optimal. Bankers indeed fail to internalize that setting up a shadow bank reduces asset prices in a crisis, hence reducing all shadow banks' ability to raise debt initially. Conversely, the relative size of traditional banks is smaller than socially optimal, because bankers fail to internalize that setting up a traditional bank increases asset prices in a crisis, hence increasing shadow banks' ability to raise debt initially.

References

Puriya Abbassi, Rajkamal Iyer, José-Luis Peydró, and Francesc R Tous. Securities trading by banks and credit supply: Micro-evidence. *Journal of Financial Economics*, 2015.

Viral V. Acharya and Nada Mora. A Crisis of Banks as Liquidity Providers. *Journal of Finance*, 70(1):1–43, 02 2015.

Viral V. Acharya, Philipp Schnabl, and Gustavo Suarez. Securitization without risk transfer. *Journal of Financial Economics*, 107(3): 515–536, 2013.

Tobias Adrian and Hyun Song Shin. Liquidity and leverage. *Journal of Financial Intermediation*, 19(3):418–437, July 2010.

Matthew Baron. Countercyclical bank equity issuance. 2016.

Matthew Baron and Wei Xiong. Credit Expansion and Neglected Crash Risk. NBER Working Papers 22695, National Bureau of Economic Research, Inc, September 2016. URL https://ideas.repec.org/p/nbr/nberwo/22695.html.

Juliane Begenau and Tim Landvoigt. Financial regulation in a quantitative model of the modern banking. *working paper*, 2017.

Saki Bigio, Juliane Begenau, and Jeremy Majerovitz. What can we learn from the financial flows of the 2008-2009 crisis? *mimeo, Harvard University*, 2016.

Bank of International Settlements BIS. Fintech credit. market structure, business models and financial stability implications. Technical report, 2016.

Pedro Bordalo, Nicola Gennaioli, Rafael La Porta, and Andrei Shleifer. Diagnostic Expectations and Stock Returns. NBER Working Papers 23863, National Bureau of Economic Research, Inc, September 2017. URL https://ideas.repec.org/p/nbr/nberwo/23863.html.

John H. Boyd and Mark Gertler. Are banks dead? Or are the reports greatly exaggerated? *Quarterly Review*, (Sum):2–23, 1994. URL https://ideas.repec.org/a/fip/fedmqr/y1994isump2-23nv.18no.3.html.

Markus K. Brunnermeier and Yuliy Sannikov. A Macroeconomic Model with a Financial Sector. *American Economic Review*, 104 (2):379–421, February 2014.

Greg Buchak, Gregor Matvos, Tomasz Piskorski, and Amit Seru. Fintech, Regulatory Arbitrage, and the Rise of Shadow Banks. NBER Working Papers 23288, National Bureau of Economic Research, Inc, March 2017. URL https://ideas.repec.org/p/nbr/nberwo/23288.html.

Ricardo J Caballero and Emmanuel Farhi. The Safety Trap. Working Paper 233766, Harvard University OpenScholar, January 2016. URL https://ideas.repec.org/p/qsh/wpaper/233766.html.

Charles W Calomiris and Charles M Kahn. The Role of Demandable Debt in Structuring Optimal Banking Arrangements. *American Economic Review*, 81(3):497–513, June 1991. URL https://ideas.repec.org/a/aea/aecrev/v81y1991i3p497-513.html.

Ing-Haw Cheng, Sahil Raina, and Wei Xiong. Wall Street and the Housing Bubble. *American Economic Review*, 104(9):2797–2829, September 2014. URL https://ideas.repec.org/a/aea/aecrev/v104y2014i9p2797-2829.html.

Sergey Chernenko, Samuel G. Hanson, and Adi Sunderam. The Rise and Fall of Demand for Securitizations. NBER Working Papers 20777, National Bureau of Economic Research, Inc, December 2014.

Adam Copeland, Antoine Martin, and Michael Walker. Repo Runs: Evidence from the Tri-Party Repo Market. *Journal of Finance*, 69(6):2343–2380, December 2014. URL https://ideas.repec.org/a/bla/jfinan/v69y2014i6p2343-2380.html.

Joshua Coval, Jakub Jurek, and Erik Stafford. The Economics of Structured Finance. *Journal of Economic Perspectives*, 23(1):3–25, Winter 2009. URL https://ideas.repec.org/a/aea/jecper/v23y2009i1p3-25.html.

Tri Vi Dang, Gary Gorton, Bengt Holmström, and Guillermo Ordoñez. Banks as secret keepers. *American Economic Review*, 107(4): 1005–29, April 2017. doi: 10.1257/aer.20140782. URL http://www.aeaweb.org/articles?id=10.1257/aer.20140782.

Eduardo Davila and Itay Goldstein. Optimal deposit insurance. *Working paper*, 2016.

Harry DeAngelo and René M. Stulz. Liquid-claim production, risk management, and bank capital structure: Why high leverage is optimal for banks. *Journal of Financial Economics*, 116(2):219–236, 2015.

Douglas W. Diamond and Raghuram G. Rajan. Liquidity Risk, Liquidity Creation, and Financial Fragility: A Theory of Banking. *Journal of Political Economy*, 109(2):287–327, April 2001. URL https://ideas.repec.org/a/ucp/jpolec/v109y2001i2p287-327.html.

Darrell Duffie. Financial Regulatory Reform after the Crisis: An Assessment. Research Papers 3440, Stanford University, Graduate School of Business, June 2016. URL https://ideas.repec.org/p/ecl/stabus/3440.html.

European Systemic Risk Board ESRB. Eu shadow banking eu shadow banking monitor. *ESRB Reports*, 1, July 2016.

Emmanuel Farhi and Jean Tirole. Shadow Banking and the Four Pillars of Traditional Financial Intermediation. NBER Working Papers 23930, National Bureau of Economic Research, Inc, October 2017. URL https://ideas.repec.org/p/nbr/nberwo/23930.html.

Financial Stability Board FSB. Global shadow banking monitoring report 2016. *Technical Report*, November 2016a.

Financial Stability Board FSB. Thematic review on the implementation of the fsb policy framework for shadow banking entities. Technical report, May 2016b.

Joseph E. Gagnon, Matthew Raskin, Julie Remache, and Brian P. Sack. Large-scale asset purchases by the Federal Reserve: did they work? *Economic Policy Review*, (May):41–59, 2011.

Evan Gatev and Philip E. Strahan. Banks' Advantage in Hedging Liquidity Risk: Theory and Evidence from the Commercial Paper Market. *Journal of Finance*, 61(2):867–892, 04 2006.

Nicola Gennaioli, Andrei Shleifer, and Robert W. Vishny. A Model of Shadow Banking. *Journal of Finance*, 68(4):1331–1363, 08 2013.

Nicola Gennaioli, Andrei Shleifer, and Robert Vishny. Neglected Risks: The Psychology of Financial Crises. *American Economic Review*, 105(5):310–314, May 2015. URL https://ideas.repec.org/a/aea/aecrev/v105y2015i5p310-14.html.

Mark Gertler, Nobuhiro Kiyotaki, and Andrea Prestipino. Wholesale Banking and Bank Runs in Macroeconomic Modelling of Financial Crises. NBER Working Papers 21892, National Bureau of Economic Research, Inc, January 2016. URL https://ideas.repec.org/p/nbr/nberwo/21892.html.

Lucyna Gornicka. Banks and shadow banks: Competitors or complements? *Journal of Financial Intermediation*, forthcoming, 2016.

Gary Gorton and Andrew Metrick. Securitized banking and the run on repo. *Journal of Financial Economics*, 104(3):425–451, 2012. doi: 10.1016/j.jfineco.2011.03. URL https://ideas.repec.org/a/eee/jfinec/v104y2012i3p425-451.html.

Gary Gorton and George Pennacchi. Financial Intermediaries and Liquidity Creation. *Journal of Finance*, 45(1):49–71, March 1990.

Robin Greenwood and Samuel G. Hanson. Issuer Quality and Corporate Bond Returns. *Review of Financial Studies*, 26(6):1483–1525, 2013. URL https://ideas.repec.org/a/oup/rfinst/v26y2013i6p1483-1525.html.

Denis Gromb and Dimitri Vayanos. Equilibrium and welfare in markets with financially constrained arbitrageurs. *Journal of Financial Economics*, 66(2-3):361–407, 2002.

Samuel G. Hanson, Anil K. Kashyap, and Jeremy C. Stein. A Macroprudential Approach to Financial Regulation. *Journal of Economic Perspectives*, 25(1):3–28, Winter 2011. URL https://ideas.repec.org/a/aea/jecper/v25y2011i1p3-28.html.

Samuel G. Hanson, Andrei Shleifer, Jeremy C. Stein, and Robert W. Vishny. Banks as patient fixed-income investors. *Journal of Financial Economics*, 117(3):449–469, 2015.

Milton Harris, Christian C. Opp, and Marcus M. Opp. The system-wide effects of bank capital regulation on credit supply and risk-taking. *mimeo*, 2015.

Zhiguo He, In Gu Khang, and Arvind Krishnamurthy. Balance sheet adjustments during the 2008 crisis. *IMF Economic Review*, 58 (1):118–156, August 2010.

Joel F. Houston, Chen Lin, and Yue Ma. Regulatory Arbitrage and International Bank Flows. *Journal of Finance*, 67(5):1845–1895, October 2012. doi: j.1540-6261.2012.01774.x. URL https://ideas.repec.org/a/bla/jfinan/v67y2012i5p1845-1895.html.

IMF. Shadow banking around the globe: How large, and how risky? Global financial stability report, International Monetary Fund, Oct 2014.

Victoria Ivashina and David Scharfstein. Bank lending during the financial crisis of 2008. *Journal of Financial Economics*, 97(3): 319–338, September 2010.

Rajkamal Iyer, Thais Jensen, Niels Johannesen, and Adam Sheridan. The Run for Safety: Financial Fragility and Deposit Insurance. EPRU Working Paper Series 1602, Economic Policy Research Unit (EPRU), University of Copenhagen. Department of Economics, May 2016. URL https://ideas.repec.org/p/kud/epruwp/1602.html.

Christopher James and Joel Houston. Evolution Or Extinction: Where Are Banks Headed? *Journal of Applied Corporate Finance*, 9 (2):8–23, 1996. URL https://ideas.repec.org/a/bla/jacrfn/v9y1996i2p8-23.html.

Marcin Kacperczyk and Philipp Schnabl. When Safe Proved Risky: Commercial Paper during the Financial Crisis of 2007-2009. *Journal of Economic Perspectives*, 24(1):29–50, Winter 2010.

G. Andrew Karolyi and Alvaro G. Taboada. Regulatory Arbitrage and Cross-Border Bank Acquisitions. *Journal of Finance*, 70(6): 2395–2450, December 2015. URL https://ideas.repec.org/a/bla/jfinan/v70y2015i6p2395-2450.html.

Anil K. Kashyap, Raghuram Rajan, and Jeremy C. Stein. Banks as Liquidity Providers: An Explanation for the Coexistence of Lending and Deposit-Taking. *Journal of Finance*, 57(1):33–73, 02 2002.

Arvind Krishnamurthy. Fundamental value and limits to arbitrage. *mimeo*, December 4 2008.

Arvind Krishnamurthy and Annette Vissing-Jorgensen. The impact of Treasury supply on financial sector lending and stability. *Journal of Financial Economics*, 118(3):571–600, 2015.

Arvind Krishnamurthy, Stefan Nagel, and Dmitry Orlov. Sizing up repo. *Journal of Finance*, 69(6):2381–2417, 2014. URL https://EconPapers.repec.org/RePEc:bla:jfinan:v:69:y:2014:i:6:p:2381-2417.

Stephen LeRoy and Rish Singhania. Deposit insurance and the coexistence of commercial and shadow banks. *mimeo*, 2017.

Guido Lorenzoni. Inefficient credit booms. *Review of Economic Studies*, 75(3):809–833, 2008.

Stephan Luck and Paul Schempp. Banks, Shadow Banking, and Fragility. Working paper, 2016.

Patrick E. McCabe. The cross section of money market fund risks and financial crises. *FEDS Working Paper No. 2010-51.*, September 2010.

Craig B. Merrill, Taylor Nadauld, Rene M. Stulz, and Shane M. Sherlund. Did Capital Requirements and Fair Value Accounting Spark Fire Sales in Distressed Mortgage-Backed Securities? Working Paper Series 2012-12, Ohio State University, Charles A. Dice Center for Research in Financial Economics, July 2012.

Robert C. Merton. Financial innovation and the management and regulation of financial institutions. *Journal of Banking & Finance*, 19(3-4):461–481, June 1995. URL https://ideas.repec.org/a/eee/jbfina/v19y1995i3-4p461-481.html.

Alan Moreira and Alexi Savov. The macroeconomics of shadow banking. *The Journal of Finance*, Forthcoming.

Guillermo Ordonez. Sustainable Shadow Banking. NBER Working Papers 19022, National Bureau of Economic Research, Inc, May 2013.

Guillaume Plantin. Shadow Banking and Bank Capital Regulation. *Review of Financial Studies*, 28(1):146–175, 2015.

Zoltan Pozsar, Tobias Adrian, Adam B. Ashcraft, and Hayley Boesky. Shadow banking. *Economic Policy Review*, (Dec):1–16, 2013.

Raghuram G. Rajan. Do we still need commercial banks? In *NBER*, 1998a.

Raghuram G Rajan. The Past and Future of Commercial Banking Viewed through an Incomplete Contract Lens. *Journal of Money, Credit and Banking*, 30(3):524–50, August 1998b. URL https://ideas.repec.org/a/mcb/jmoncb/v30y1998i3p524-50.html.

Andrei Shleifer and Robert W. Vishny. Unstable banking. *Journal of Financial Economics*, 97(3):306–318, September 2010.

Jeremy C. Stein. Monetary Policy as Financial Stability Regulation. *The Quarterly Journal of Economics*, 127(1):57–95, 2012.

Jeremy C. Stein. The fire-sales problem and securities financing the fire-sales problem and securities financing transactions. *Speech by Governor Jeremy C. Stein at the Federal Reserve Bank of New York during the Workshop on Fire Sales as a Driver of Systemic Risk in Triparty Repo and other Secured Funding Markets, New York.*, October 2013.

Adi Sunderam. Money Creation and the Shadow Banking System. *Review of Financial Studies*, 28(4):939–977, 2015.

Lukas Voellmy. Shadow banking and financial stability under limited deposit insurance. 2017.

Appendix

A The data

A.1 Stylized balance sheets of US financial intermediaries

In the FAUS data, we identify traditional banks as private depository institutions (L.110). Those institutions are composed of U.S.-chartered depository institutions (L.111), foreign banking offices (L.112), banks in U.S.-affiliated areas (L.113) and credit unions (L.114). The stylized facts that we document do not hinge on a particular definition of shadow banks, and our goal is not to provide an accurate measure of shadow banking (for global estimates of shadow banking, see IMF (2014) and FSB (2016a)). We define shadow banks as chains of market-based transactions among legal institutions which, taken together, perform maturity transformation activities comparable to that of traditional banks. In this section, shadow banks are the sum of money market mutual funds (L.121), mutual funds (L.122), issuers of asset-backed securities (L.127) and security brokers and dealers (L.130).[33]

We aggregate those financial intermediaries that we include in our definition of the shadow banking sector, and define short-term debt using the FAUS by using Krishnamurthy and Vissing-Jorgensen (2015)'s definition of short-term debt in the FAUS data, 60% of which is composed of small time and savings deposits in the 2007-09 period. The list of FAUS items included in shadow banks' short-term debt is: Security repurchase agreements (net), Depository institution loans n.e.c., Trade payables, Security credit (Customer credit balances), Security credit (U.S.-chartered institutions), Security credit (foreign banking offices in U.S.), Taxes payable, Commercial paper, Open market paper.

We obtain stylized balance-sheets of traditional and shadow banks by consolidation of the financial balance sheets of the legal institutions for which we have data in the Financial Accounts of the United States (FAUS).

A.2 Fact 1: Liabilities flow from shadow to traditional banks

A.2.1 Table 1

We take the definition of the largest US bank-holding companies on Figure 11 from the Federal Reserve's website (https://www.ffiec.gov/nicpubweb/nicweb/HCSGreaterThan10B.aspx/).

[33] Earlier descriptive studies adopt similar approaches to shadow banks, see e.g. Pozsar et al. (2013), or Adrian and Shin (2010). Acharya et al. (2013), McCabe (2010) and Krishnamurthy et al. (2014) use more micro data to measure shadow banks' short-term debt and its collapse.

	Cumulative flows since 2006q4	
	Shadow banks ($Bill)	Traditional banks ($Bill)
2007q2	+437	+106
2007q3	+606	+230
2007q4	+514	+454
2008q1	+378	+591
2008q2	+623	+732
2008q3	+284	+751
2008q4	+505	+1104
2009q1	-277	+1733
2009q2	-670	+1659
2009q3	-966	+ 1428
2009q4	-1132	+1436
2010q1	-1353	+1409
2010q2	-1354	+1431
2010q3	-1412	+1317
2010q4	-1440	+1420
2011q1	-1471	+1596
2011q2	-1398	+2011

Table 1: Traditional and shadow banks: short-term debt inflows (negative values denote outflows) source: Financial Accounts of the United States. We define traditional, shadow banks, and short-term debt in Appendix A.1.

A.2.2 Figure 11

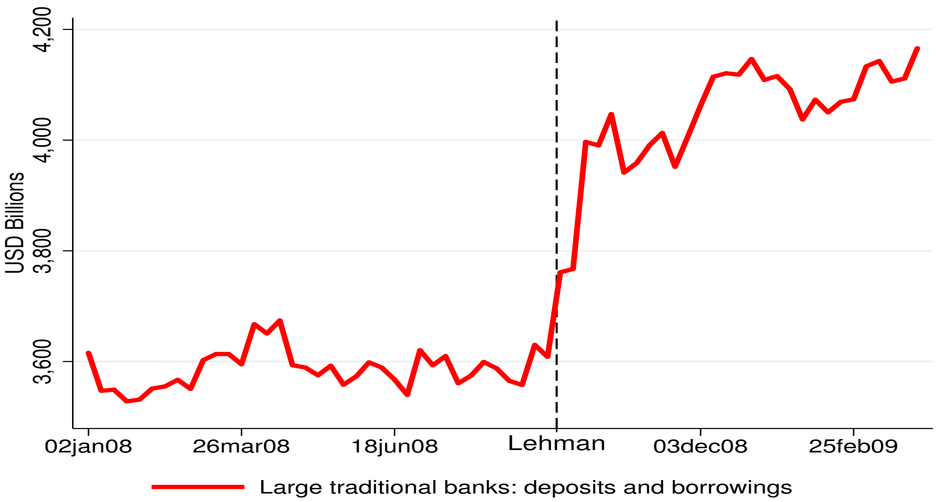


Figure 11: Large traditional banks: deposits and borrowings (stocks in $ bn)

source: Fed H8 Releases

A.2.3 Book versus market value of equity

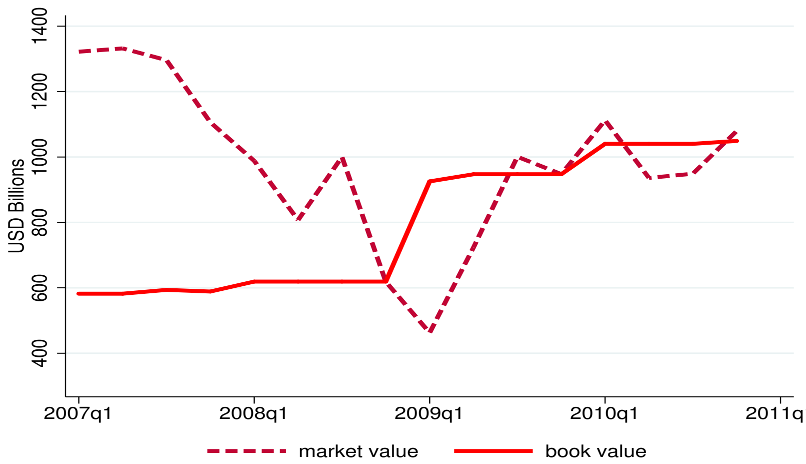


Figure 12: Traditional banks: Book versus market value of equity
source: CRSP, Call Reports

He et al. (2010) and Bigio et al. (2016) also find that traditional banks' book equity increased by around US $250 billion during the crisis. Figure 12 provides evidence of this increase in the stock of book equity of the US traditional banking sector through the crisis. This Figure is based on reported book value of equity, which is the leverage measure most used for regulatory purposes. However, there are reasons to believe that the true level of capital for the traditional banking sector was lower. We use data from CRSP to measure the market value of traditional banks' equity and we see that most of the increase in book value of equity disappears when one looks at market value of equity.

A.2.4 Fact 2: Asset flow from shadow to traditional banks

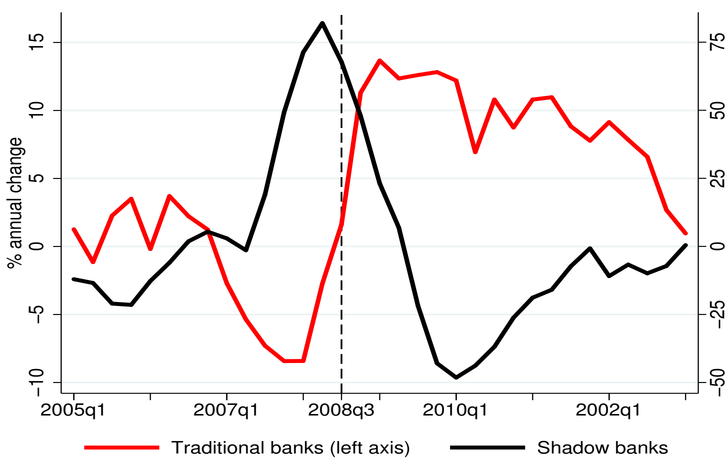


Figure 13: Traditional and shadow banks: agency- and GSE-backed securities holdings (% annual growth)
source: Financial Accounts of the United States

A.3 Regression: Traditional banks' MBS purchases in the crisis

One main testable prediction of our theory is that traditional banks are able to purchase assets from shadow banks in a crisis, insofar as they benefit from a guarantee on their deposits. This guarantee indeed enables them to attract deposits precisely when shadow banks have to repay their creditors. Publicly available data on purchases/sales of assets by traditional and shadow banks during the crisis is not available. Therefore we try to estimate purchases/sales of mortgage-backed securities (henceforth MBS) applying He et al. (2010)'s methodology on traditional banks' regulatory data from the Call Reports. We observe the total value of MBS holdings by each traditional bank before the crisis (denote it $P\_2007q4 * MBS\_2007q4_i$ where P_2007q4 is the fair price of MBS securities in 2007q4 and MBS_2007q4 is the quantity of MBS held by bank i in 2007q4) and after the crisis ($P\_2009q1 * MBS\_2009q1_i$). Besides, denoting f the repayment/maturity rate of MBS net of the new issuance rate during the period from 2007q4 to 2009q1, the International Financial Reporting Standards (IFRS) give us the following accounting identity:

$$P\_2009q1 * MBS\_2009q1_i - P\_2007q4 * MBS\_2007q4_i * (1-f) = MBSPurchases_i - MBSLosses_i$$

As in He et al. (2010), we test three different scenarii based on (i) the total losses that traditional banks incurred on MBS assets during the 2008 crisis, and (ii) Bloomberg WDCI estimates for the net repayment rate f. Under scenario 1 the repayment rate used to construct the MBS_Purchases variable is 7% and total losses imputed to the financial sector are $500 billion.[34] Under scenario 2, the repayment rate is 12% and total losses are $176 billion. Under the "naive" scenario, we do not correct for the net repayment rate nor total losses.

We analyze the data formally by running the following OLS regression on changes in various items of traditional banks' balance sheets from 2007q4 to 2009q1:

$$\begin{aligned} MBSPurchases_i = {} & \beta_1.Change\_insured\_deposits_i \\ & + \beta_2.Change\_uninsured\_deposits_i + \beta_3.Change.\_Credit_i \\ & + \beta_4.Unused\_commitments\_ratio\_2007q4_i + \beta_5.Capital\_ratio\_2007q4\_i \\ & + \beta_6.Log\_assets\_2007q4\_i + \beta_7.Controls\_i + \epsilon_i \end{aligned}$$

where $MBSPurchases_i$ is our estimated purchases/sales of mortgage-backed securities by traditional bank i normalized by total assets (banks are aggregated to the top holder level in the Call Reports). The data come from the quarterly Call Reports and He et al. (2010)'s estimates. We use the procedure described in Acharya and Mora (2015) to construct our sample. All missing observations are consid-

[34]Note that the only available estimate on MBS losses in the crisis is an aggregate over the traditional banking sector from the IMF's Global Financial Stability Report of October 2008 and Bloomberg WDCI (which explains why we test two scenarii thereafter). Denote those estimates for the entire traditional banking sector losses on MBS assets $MBSLosses$. Although we try to estimate MBS purchases/losses by taking into account potential losses on those assets when using the change in MBS holdings from 2007q4 to 2009q1 adjusted for the net repayment/maturity rate, we cannot account for differences in losses across traditional banks. We therefore assume that losses incurred by traditional banks are proportional to the amount of MBS they hold, so that $MBSLosses_i = \frac{MBS\_2007q4_i}{\sum_k MBS\_2007q4_k} * MBSLosses$ and $\sum_k MBSLosses_k = MBSLosses$.

ered equal to zero, and banks are aggregated to top holder level (RSSD9348). Table 2 above details the construction of variables.

Variable	Variable Name	Call Report Items
Insured deposits	insured_deposits	rconf049 +rconf045
Uninsured deposits	uninsured_deposits	rcon2365 (brokered deposits)
Interest rate on large deposits	ir_large_deposits	rconf049 + rconf045
Unused commitment		rcfd3814+rcfd3816+rcfd3817+rcfd3818+rcfd6550+rcfd3411
Credit	Credit	rcfd1400 + Unused_commitments
Unused commitments ratio	Unused_commitments_ratio	unused commitments/(unused commitments+rcfd1400)
Cash		rcfd0010
Federal Funds Sold		rcfd1350+rconb987
		(rconb987+rcfdb989 if after 2002/03/30)
MBS		rcfd1699+rcfd1705+rcfd1710+rcfd1715+rcfd1719+rcfd1734
		+rcfd1702+rcfd1707+rcfd1713+rcfd1717+rcfd1732+rcfd1736
Securities (MBS excluded)		rcfd1754+rcfd1773-(rcfd8500+rcfd8504+rcfdc026+rcfd8503+rcfd8507+rcfdc027)
Liquid assets		Securities (MBS excluded)+ Federal Funds Sold+Cash
Liquidity ratio	Liquidity_ratio	Liquid Assets/rcfd2170
Wholesale funding		rcon2604+rcfn2200+rcfd3200+rconb993+rcfdb995+rcfd3190
Wholesale funding ratio		Wholesale funding/rcfd2170
Net Wholesale fund ratio	Net_Wholesale_fund	Wholesale funding -(Securities (MBS excluded)+Federal Funds Sold+Cash)
Non performing loan		rcfd1407+rcfd1403
Non performing loan ratio	NPL_ratio	Non performing loan/rcfd1400
Capital ratio	Capital_ratio	(rcfd3210+rcfd3838)/rcfd2170
Real Estate Loan Share	Real_Estate	rcfd1410/rcfd1400
Residential Mortgages		(rcfdf070+rcfdf071)/rcfd2170
Financial Assets		rcfd0081+rcfd0071+rcfda570+rcfda571+rcona564+rcona565
		+rcfd1350+rcfda549+rcfda550+rcfda556+rcfda248
Short Term Liabilities		rcon2210+rcona579+rcona580+rcona584+rcona585+rcfd2800+rcfd2651+rcfdb571
Maturity Gap	Mat_Gap	(Financial Assets- Short Term Liabilities) / rcfd2170
Tag deposits	Tag_deposits	rcong167

Table 2: Variables definitions

Results are reported in Table 3. Variables ending in 2007q4 represent variable levels as of 2007q4. Variables starting with "Change" are growth rates from 2007q4 to 2009q1, normalized by total assets as of 2007q4. The dependent variable MBS_Purchases represents purchases of mortgage-backed securities by traditional banks between 2007q4 and 2009q1, normalized by total assets as of 2007q4. As in He et al. (2010) we test different scenarios in terms of MBS repayment rate and total losses on assets, to make sure that what our dependent variables capture are actual purchases of MBS by traditional banks. We report three of these scenarios, including the "naive" one. Under scenario 1 the repayment rate (net of new issuances) used to construct the MBS_Purchases variable is 7% and total losses imputed to the financial sector are $500 billion. Under scenario 2, the (net) repayment rate is 12% and total losses are $176 billion. Under the "naive" scenario, we do not correct for the net repayment rate nor total losses. Standard errors are clustered by insurer. Control variables are of three types. The first type are change variables: interest rates on large deposits, capital ratio, net wholesale funds, real estate, non-performing loans, liquidity. The second type are stock variables as of 2007q4: liquidity ratio, non-performing loans ratio, maturity gap, interest rate on large desposits, net wholesale fund ratio, unused commitments ratio, real estate ratio, capital ratio. The third type is a dummy variable for traditional banks' use of the "tag deposit" facility (equal to one if the bank used the facility).

	Scenario 1	Scenario 2	"Naive" Scenario
Change_uninsured_deposits	0.02	0.03	0.05
	(0.05)	(0.04)	(0.04)
Change_insured_deposits	0.23***	0.23***	0.22***
	(0.05)	(0.04)	(0.04)
Change_Liquidity	-0.36***	-0.33***	-0.30***
	(0.04)	(0.03)	(0.03)
Change_Credit	-0.11***	-0.10***	-0.09***
	(0.03)	(0.03)	(0.02)
Capital_ratio_2007q4	-0.06	-0.05	-0.03
	(0.06)	(0.05)	(0.05)
Unused_commitments_ratio_2007q4	-0.00	-0.02	-0.04*
	(0.03)	(0.03)	(0.02)
Controls	yes	yes	yes
Adjusted R^2	0.20	0.22	0.24
Observations	3954	3954	3954

Table 3: Traditional banks: determinants of MBS purchases in the crisis

Source: Call Reports and He et al. (2010)'s estimates. ***, **, and * mean statistically significant at the 1%, 5%, and 10% levels, respectively. We use White robust standard errors.

A.3.1 Fact 3: Asset fire sales

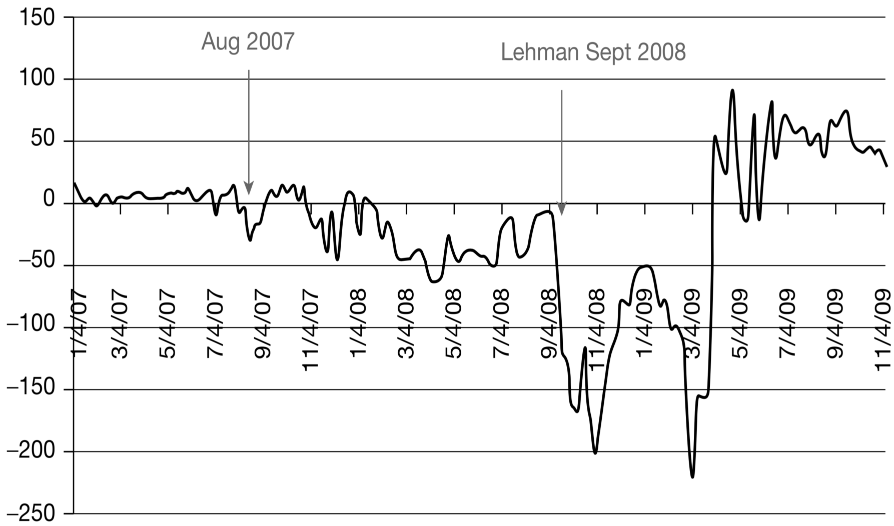


Figure 14: Interest rate spread: 5year AA-AAA Industrials
source: Gorton and Metrick (2012)

B The model

B.1 Optimal financial contracts when households are infinitely risk averse

We show in this Section that when households are infinitely risk averse, short-term debt contract arise endogenously in our model. We assume as in Gennaioli et al. (2013) that households have the following utility function:

$$U = C_0 + (\beta + \gamma)\,(\min C_1 + \min C_2) \tag{10}$$

where C_t is consumption at $t = 0, 1, 2$. Households value future stochastic consumption streams at their worst-case scenario. This assumption that households value safe (money-like) claims *only* is aimed at capturing the information-insensitivity properties of such claims (see e.g. Gorton and Pennacchi (1990), Stein (2012) or DeAngelo and Stulz (2015), Dang et al. (2017)).

We assume that each i-bank contracts with a household at $t = 0, 1$, and perfect information between the two parties. We consider all feasible contracts in which a bank borrows at t from a household, and promises positive repayments in the following dates (i.e. we assume the household cannot credibly promise to refinance the bank). We obtain the following Proposition.

Proposition 10. *For both T- and S-banks, at each date $t = 0, 1$ and in the two state $\{B, G\}$ the optimal financial contract is a short-term debt contract.*

Proof. The proof is made of three parts. First, we show that financial contracts are necessarily debt contrats. Second, we show that S-banks optimally borrow short-term. Third, we show that T-banks optimally borrow short-term. We can finally conclude that both T- and S-banks borrow using short-term debt.

We denote $D^i_{0\to1,G}, D^i_{0\to1,B}, D^i_{0\to2,BG}, D^i_{0\to2,BB}, D^i_{0\to2,GG}) \in \mathbb{R}^5_+$ the repayment schedule of i-banks when borrowing an amount $D^S_0 > 0$ from a household at $t = 0$.

First, from household's utility function (10), we know that households only value the contracts' lowest possible payoff. Using $D^S_{0\to2,BB} = D^T_{0\to2,BB} = 0$, we obtain $\min(D^i_{0\to2,BG}, D^i_{0\to2,BB}, D^i_{0\to2,GG}) \leqslant 0$ where $i = S, T$. Therefore any i-bank sets

$$D^i_{0\to2,BG} = D^i_{0\to2,BB} = D^i_{0\to2,GG} = 0,$$

such that all contracts are debt contracts.

Second, the financial contract between a S-bank and a household must satisfy the household's participation constraint, i.e. make the household at least indifferent between accepting and refusing the terms of the contract:

$$D^S_0 \leqslant \min(D^S_{0\to1,G}, D^S_{0\to1,B}) + \min(D^S_{0\to2,BG}, D^S_{0\to2,BB}, D^S_{0\to2,GG})$$

Now, remark that S-banks invest in risky assets whose payoff is 0 at $t = 2$ in state BB (see Figure 2). Therefore S-bank cannot credibly commit to reimburse a positive amount of funds at $t = 2$ in state BB, implying $D^S_{0\to2,BB} = 0$ such that S-banks borrow short-term.

Third, the only way for a T-bank to transfer funds to $t = 2, BB$ is to use deposit insurance. Because the deposit insurance fund only guarantees short-term claims, no long-term contract can be credibly set up at $t = 0$ between a T-bank and a household, such that $D^T_{0\to 2,BB} = 0$ and T-banks borrow short-term.

To conclude, we find that any financial contract between an infinitely risk averse household and an i-bank must be a short-term debt contract. QED. □

B.2 Shadow banks' program

B.2.1 S-banks: optimization program at $t = 1$ in state B (proof of lemma 2)

We rewrite lemma 2 more technically as follows.

Lemma 2 (S-banks at $t = 1$ in state B). *At $t = 1$, in state B, S-banks do not default on their debt if and only if $D^S_0 \leqslant (1-\varepsilon)\, p_{1,B}\, q r I^S_0$.*

If $p_{1,B} > 0$, their value function writes

$$V^S_{1,B}\left(I^S_0, D^S_0, p_{1,B}\right) = \begin{cases} \left((1-\varepsilon)p_{1,B}\, qrI^S_0 - D^S_0\right)\max\left(\frac{1}{(1-\varepsilon)p_{1,B}}; 1\right) & \textit{if } D^S_0 \leqslant (1-\varepsilon)\, p_{1,B}\, qrI^S_0 \\ -\infty & \textit{otherwise} \end{cases}$$

If $p_{1,B} = 0$, their value function writes

$$V^S_{1,B}\left(I^S_0, D^S_0, p_{1,B}\right) = \begin{cases} qrI^S_0 & \textit{if } D^S_0 = 0 \\ -\infty & \textit{otherwise} \end{cases}$$

Proof. No default occurs at $t = 1$ in state B if and only if the S-bank is able to obtain enough funds when selling assets, to finance its debt level D^S_0 .

For any $t = 0$ investment level $I^S_0 > 0$, $t = 1$-state B asset purchases $p_{1,B}\, qr > 0$, there is an upper level $\overline{D}^S_0\left(I^S_0, p_{1,B}\right)$ of debt that can be reimbursed at $t = 1$ in state B:

$$\begin{aligned} \overline{D}^S_0\left(I^S_0\right) &= \max_{\alpha^1_S \in [0;1]} \left(\left(1-\alpha^S_1\right)(1-\varepsilon)\, p_{1,B}\, qrI^S_0\right) \\ &= (1-\varepsilon)\, p_{1,B}\, qrI^S_0 \end{aligned}$$

where $(1-\alpha^S_1)$ is the share of S-bank's assets that is liquidated. at $t = 1$ in state B. If $D^S_0 > (1-\varepsilon)\, p_{1,B}\, qrI_{0,S}$, the S-bank must default on its debt issued at $t = 0$. In case of default, we set $V^{S,D}_{1,B}\left(I^S_0, D^S_0, p_{1,B}\right) = -\infty$. This ensures that the S-bank is not willing to default on its debt at $t = 1$ in state B.

In case of no-default, the program writes:

$$\begin{aligned} V^{S,ND}_{1,B}\left(I^S_0, D^S_0, p_{1,B}\right) = \max_{\alpha^S_{1,B}\in[0;1]} \alpha^S_{1,B}\, qrI^S_0 + (1-\alpha^S_{1,B})(1-\varepsilon)p_{1,B}\, qrI^S_0 - D^S_0 \\ \text{s.t. } (1-\alpha^S_{1,B})\,(1-\varepsilon)\, p_{1,B}\, qrI^S_0 \geqslant D^S_0 \end{aligned}$$

Denoting $\nu_{\alpha^S_{1,B}\geqslant 0}$ the Lagrange multiplier associated to the constraint $\alpha^S_{1,B} \geqslant 0$, $\nu_{\alpha^S_{1,B}\leqslant 1}$ the Lagrange multiplier associated to the constraint $\alpha^S_{1,B} \leqslant 1$ and $\mu^S_{1,B}$ the Lagrange multiplier associated to the funding constraint, the Lagrangian of the problem rewrites:

$$\begin{aligned} L = & \alpha^S_{1,B} qrI^S_0 + (1-\alpha^S_{1,B})(1-\varepsilon)p_{1,B} qrI^S_0 - D^S_0 \\ & + \nu_{\alpha^S_{1,B}\geqslant 0}\alpha^S_{1,B} + \nu_{\alpha^S_{1,B}\leqslant 1}\left(1-\alpha^S_{1,B}\right) \\ & + \mu^S_{1,B}\left((1-\alpha^S_{1,B})\,(1-\varepsilon)\,p_{1,B} qrI^S_0 - D^S_0\right) \end{aligned}$$

and the first order condition writes as follows

$$\frac{dL}{d\alpha^S_{1,B}} = qrI^S_0\left(1-(1-\varepsilon)p_{1,B}\right) + \nu_{\alpha^S_{1,B}\geqslant 0} - \nu_{\alpha^S_{1,B}\leqslant 1} - \mu^S_{1,B}\,(1-\varepsilon)\,p_{1,B} qrI^S_0 = 0$$

And solves as follows:

1. If $1-(1-\varepsilon)p_{1,B} < 0$, $\nu_{\alpha^S_{1,B}\geqslant 0} > 0$ and $\alpha^S_{1,B} = 0$, $V^S_{1,B}\left(I^S_0, D^S_0, p_{1,B}\right) = (1-\varepsilon)p_{1,B} qrI^S_0 - D^S_0$
2. If $1-(1-\varepsilon)p_{1,B} = 0$, $\nu_{\alpha^S_{1,B}\geqslant 0} - \nu_{\alpha^S_{1,B}\leqslant 1} - \mu^S_{1,B}\,(1-\varepsilon)\,p_{1,B} qrI^S_0 = 0$ and, either $\mu^S_{1,B} > 0$ and $\nu_{\alpha^S_{1,B}\geqslant 0} > 0$ and $\alpha^S_{1,B} = 0$, $(1-\varepsilon)\,p_{1,B} qrI^S_0 = D^S_0$ or any $\alpha^S_{1,B} \in [0;1]$ such that $(1-\alpha^S_{1,B})\,(1-\varepsilon)\,p_{1,B} qrI^S_0 \geqslant D^S_0$ is an equilibrium solution, and $V^S_{1,B}\left(I^S_0, D^S_0, p_{1,B}\right) = qrI^S_0 - D^S_0$
3. If $1-(1-\varepsilon)p_{1,B} > 0$, $\nu_{\alpha^S_{1,B}\leqslant 1} + \mu^S_{1,B}\,(1-\varepsilon)\,p_{1,B} qrI^S_0 > 0$. Hence, either $\alpha^S_{1,B} = 1$, which can hold if and only if $D^S_0 = 0$, or $\mu^S_{1,B} > 0$. In this case, $\alpha^S_{1,B} = 1 - \frac{D^S_0}{(1-\varepsilon)p_{1,B} qrI^S_0}$ and $V^S_{1,B}\left(I^S_0, D^S_0, p_{1,B}\right) = \alpha^S_{1,B} qrI^S_0 = \frac{(1-\varepsilon)p_{1,B} qrI^S_0 - D^S_0}{(1-\varepsilon)p_{1,B}}$

To summarize, either $p_{1,B} > 0$ and in any case, $V^{S,ND}_{1,B}\left(I^S_0, D^S_0, p_{1,B}\right) = \left((1-\varepsilon)\,p_{1,B} qrI^S_0 - D^S_0\right)\max\left(\frac{1}{(1-\varepsilon)p_{1,B}};1\right)$, which holds true if $D^S_0 = I^S_0 = 0$. Besides, if $I^S_0 > 0$ and $p_{1,B} qr > 0$, we have

$$\alpha^S_{1,B} = \begin{cases} 0 & \text{if } p_{1,B} < \frac{1}{1-\varepsilon} \\ \in \left[0; 1 - \frac{D^S_0}{(1-\varepsilon)p_{1,B} qrI^S_0}\right] & \text{if } p_{1,B} = \frac{1}{1-\varepsilon} \\ 1 - \frac{D^S_0}{(1-\varepsilon)p_{1,B} qrI^S_0} & \text{if } p_{1,B} > \frac{1}{1-\varepsilon} \end{cases}.$$

Or $p_{1,B} = 0$, and $\overline{D}^S_0\left(I^S_0, p_{1,B}\right) = 0$. In this case, the value function in case of no-default, writes $V^{S,ND}_{1,B}\left(I^S_0, D^S_0, p_{1,B}\right) = qrI^S_0$. This proves lemma 2. □

B.2.2 S-banks: optimization progam at $t = 0$

For $p_{1,B} > 0$, S-banks' value function at $t = 0$ writes:

$$\begin{aligned}
V_0^S\left(p_{1,B}, n_S\right) = \max_{D_0^S, I_0^S \geqslant 0} & [(1-p)\left((1-\varepsilon)p_{1,B}\,qrI_0^S - D_0^S\right)\max\left(\frac{1}{(1-\varepsilon)p_{1,B}};1\right) \\
& + p\left(RI_0^S - D_0^S\right) + (D_0^S + n_S - I_0^S)] \\
\text{s.t. } & D_0^S, I_0^S \geqslant 0 \\
& D_0^S \leqslant (1-\varepsilon)\,p_{1,B}\,qrI_0^S \\
& D_0^S \leqslant RI_0^S
\end{aligned}$$

For $p_{1,B} = 0$, $V_0^S(0, n) = (1-p)qrn + pRn$ and $I_0^S = n$, $D_0^S = 0$.

Case $p_{1,B} < \frac{1}{1-\varepsilon}$. If $\frac{1}{(1-\varepsilon)} > p_{1,B}$, $(1-\varepsilon)\,p_{1,B}\,qrI_0^S < RI_0^S$. Therefore we ignore the last constraint and the program rewrites:

$$\begin{aligned}
V_0^S\left(p_{1,B}, n_S\right) = \max_{D_0^S, I_0^S \geqslant 0} & (1-p)\left(qrI_0^S - \frac{D_0^S}{(1-\varepsilon)p_{1,B}}\right) + p\left(RI_0^S - D_0^S\right) + \left(D_0^S + n - I_0^S\right) \\
\text{s.t. } & D_0^S + n_S \geqslant I_0^S \\
& D_0^S \leqslant (1-\varepsilon)\,p_{1,B}\,qrI_0^S
\end{aligned}$$

Denoting $\nu_{D_0^S \geqslant 0}$ the Lagrange multiplier associated to the constraint $D_0^S \geqslant 0$, $\nu_{I_0^S \geqslant 0}$ the Lagrange multiplier associated to the constraint $I_0^S \geqslant 0$, μ_0^S the Lagrange multiplier associated to the funding constraint and $\lambda_{1,B}^S$ the Lagrange multiplier associated to the debt constraint, the Lagrangian of the problem rewrites:

$$\begin{aligned}
L = & (1-p)\left(qrI_0^S - \frac{D_0^S}{(1-\varepsilon)p_{1,B}}\right) + p\left(RI_0^S - D_0^S\right) + \left(D_0^S + n - I_0^S\right) \\
& + \nu_{I_0^S \geqslant 0} I_0^S + \nu_{D_0^S \geqslant 0} D_0^S + \mu_0^S\left(D_0^S + n - I_0^S\right) + \lambda_{1,B}^S\left((1-\varepsilon)\,p_{1,B}\,qrI_0^S - D_0^S\right)
\end{aligned}$$

and the first order condition on I_0^S yields:

$$\frac{dL}{dI_0^S} = (1-p)qr + pR - 1 + \nu_{I_0^S \geqslant 0} + \lambda_{1,B}^S\,(1-\varepsilon)\,p_{1,B}\,qr - \mu_0^S = 0$$

This implies $\mu_0^S > 0$ and $D_0^S + n = I_0^S$.

One can replace I_0^S and rewrite the problem as

$$\begin{aligned}
V_0^S\left(p_{1,B}, n\right) = \max_{D_0^S \geqslant 0} & (1-p)\left(qrI_0^S - \frac{D_0^S}{(1-\varepsilon)p_{1,B}}\right) + p\left(RI_0^S - D_0^S\right) \\
\text{s.t. } & D_0^S + n = I_0^S \\
& D_0^S \leqslant (1-\varepsilon)\,p_{1,B}\,qrI_0^S
\end{aligned}$$

Or

$$V_0^S(p_{1,B}, n) = \max_{D_0^S \geqslant 0} \left[(1-p)\left(qr - \frac{1}{(1-\varepsilon)p_{1,B}}\right) + p(R-1)\right] D_0^S + (pR + (1-p)qr)\, n$$
$$\text{s.t. } D_0^S + n = I_0^S$$
$$D_0^S \leqslant (1-\varepsilon)\, p_{1,B}\, qr \left(D_0^S + n\right).$$

We denote $\overline{p}_1^S \equiv \frac{1}{(1-\varepsilon)\left(qr+\frac{p(R-1)}{1-p}\right)}$. From 2 we have $qr + \frac{p(R-1)}{1-p} > 1$ and therefore $\overline{p}_1^S < \frac{1}{(1-\varepsilon)}$. In this first case, the first order condition solves as follows.

1. If $0 < p_{1,B} < \overline{p}_1^S$, $\left[(1-p)\left(qr - \frac{1}{(1-\varepsilon)p_{1,B}}\right) + p(R-1)\right] < 0$ and $D_0^S = 0$, $I_0^S = n$, $V_0^S(p_{1,B}, n) = (pR + (1-p)qr)\, n$.
2. If $p_{1,B} = \overline{p}_1^S$, $\left[(1-p)\left(qr - \frac{1}{(1-\varepsilon)p_{1,B}}\right) + p(R-1)\right] = 0$, and any $D_0^S \in \left[0; \frac{(1-\varepsilon)\overline{p}_1^S qr}{1-(1-\varepsilon)\overline{p}_1^S qr} n\right]$ is an equilibrium value of D_0^S, $I_0^S = n + D_0^S$ and $V_0^S(p_{1,B}) = (pR + (1-p)qr)\, n$.
3. If $\overline{p}_1^S < p_{1,B} < \frac{1}{1-\varepsilon}$, $D_0^S = \frac{(1-\varepsilon)p_{1,B}qr}{1-(1-\varepsilon)p_{1,B}qr} n$, $I_0^S = n + D_0^S = \frac{n}{1-(1-\varepsilon)p_{1,B}qr}$, and $V_0^S(p_{1,B}) = p\left(RI_0^S - D_0^S\right) = p\left(\frac{R-(1-\varepsilon)p_{1,B}qr}{1-(1-\varepsilon)p_{1,B}qr}\right) n$

Case $p_{1,B} \geqslant \frac{1}{1-\varepsilon}$. If $p_{1,B} \geqslant \frac{1}{1-\varepsilon}$, the program rewrites

$$V_0^S(p_{1,B}, n_S) = \max_{D_0^S, I_0^S \geqslant 0} (1-p)\left((1-\varepsilon)p_{1,B}qrI_0^S - D_0^S\right) + p\left(RI_0^S - D_0^S\right) + \left(D_0^S + n - I_0^S\right)$$
$$\text{s.t. } D_0^S + n_S \geqslant I_0^S$$
$$D_0^S \leqslant (1-\varepsilon)\, p_{1,B}\, qrI_0^S$$
$$D_0^S \leqslant RI_0^S$$

Denoting $\nu_{D_0^S \geqslant 0}$ the Lagrange multiplier associated to the constraint $D_0^S \geqslant 0$, $\nu_{I_0^S \geqslant 0}$ the Lagrange multiplier associated to the constraint $I_0^S \geqslant 0$, μ_0^S the Lagrange multiplier associated to the funding constraint, $\lambda_{1,B}^S$ the Lagrange multiplier associated to the debt constraint $D_0^S \leqslant (1-\varepsilon)\, p_{1,B}\, qrI_0^S$, and $\lambda_{1,G}^S$ the Lagrange multiplier associated to the debt constraint $D_0^S \leqslant RI_0^S$, the Lagrangian of the problem writes:

$$L = (1-p)\left((1-\varepsilon)p_{1,B}qrI_0^S - D_0^S\right) + p\left(RI_0^S - D_0^S\right) + \left(D_0^S + n - I_0^S\right)$$
$$+ \nu_{I_0^S \geqslant 0} I_0^S + \nu_{D_0^S \geqslant 0} D_0^S + \mu_0^S\left(D_0^S + n - I_0^S\right) + \lambda_{1,B}^S\left((1-\varepsilon)\, p_{1,B}\, qrI_0^S - D_0^S\right)$$
$$+ \lambda_{1,G}^S\left(RI_0^S - D_0^S\right)$$

and first order condition on I_0^S yields:

$$\frac{dL}{dI_0^S} = (1-p)(1-\varepsilon)p_{1,B}qr + pR - 1 + \nu_{I_0^S \geqslant 0} + \lambda_{1,B}^S(1-\varepsilon)\, p_{1,B}\, qr + \lambda_{1,G}^S R - \mu_0^S = 0$$

This implies $\mu_0^S > 0$ and $D_0^S + n = I_0^S$. Therefore the debt constraint $D_0^S \leqslant RI_0^S$ is always satisfied, so that we can rewrite the problem as:

$$V_0^S\left(p_{1,B}, n_S\right) = \max_{D_0^S \geqslant 0} \left[(1-p)\left((1-\varepsilon)p_{1,B}qr - 1\right) + p\left(R-1\right)\right] D_0^S + (1-p)(1-\varepsilon)p_{1,B}qrn + pRn$$
$$\text{s.t. } D_0^S + n_S = I_0^S$$
$$D_0^S \leqslant (1-\varepsilon)\, p_{1,B}qrI_0^S$$

which implies :

1. If $\frac{1}{1-\varepsilon} \leqslant p_{1,B} < \frac{1}{(1-\varepsilon)qr}$, $D_0^S = \frac{(1-\varepsilon)p_{1,B}qr}{1-(1-\varepsilon)p_{1,B}qr}n$, $I_0^S = \frac{n}{1-(1-\varepsilon)p_{1,B}qr}$, and $V_0^S\left(p_{1,B}\right) = p\left(RI_0^S - D_0^S\right) = p\left(\frac{R-(1-\varepsilon)p_{1,B}qr}{1-(1-\varepsilon)p_{1,B}qr}\right)n$
2. If $p_{1,B} \geqslant \frac{1}{(1-\varepsilon)qr}$, $D_0^S = +\infty$, $I_0^S = +\infty$,and $V_0^S\left(p_{1,B}\right) = +\infty$

Putting together all the above cases, we summarize S-banks' optimal choices in Proposition 1.

Proposition 11 (S-banks' at $t = 0$)**.** *At $t = 0$, S-banks take the following decisions.*

1. *If $0 \leqslant p_{1,B} < \overline{p}_1^S$, $D_0^S = 0$, $I_0^S = n^S$, and $V_0^S\left(p_{1,B}, n^S\right) = (1-p)qrn^S + pRn^S$. S-banks do not issue short-term debt at $t = 0$*
2. *If $p_{1,B} = \overline{p}_1^S$, any $D_0^S \in \left[0; \frac{(1-\varepsilon)\overline{p}_1^S qr}{1-(1-\varepsilon)\overline{p}_1^S qr}n^S\right]$ is an equilibrium, $I_0^S = n^S + D_0^S$ and $V_0^S\left(p_{1,B}, n^S\right) = (1-p)qrn^S + pRn^S$. S-banks sell a fraction $\frac{D_0^S}{(1-\varepsilon)p_{1,B}qr\left(D_0^S + n^S\right)}$ of their assets at $t = 1$ in state B, to repay their debt.*
3. *If $\overline{p}_1^S < p_{1,B} < \frac{1}{(1-\varepsilon)qr}$, $D_0^S = \frac{(1-\varepsilon)p_{1,B}qr}{1-(1-\varepsilon)p_{1,B}qr}n^S$, $I_0^S = n^S + D_0^S$ and $V_0^S\left(p_{1,B}, n^S\right) = p\left(\frac{R-(1-\varepsilon)p_{1,B}qr}{1-(1-\varepsilon)p_{1,B}qr}\right)n^S$. S-banks sell all their assets at $t = 1$ in state B, to repay their debt.*
4. *If $p_{1,B} \geqslant \frac{1}{(1-\varepsilon)qr}$, $D_0^S = +\infty$, $I_0^S = +\infty$ and $V_0^S\left(p_{1,B}, n^S\right) = +\infty$.*

B.3 Traditional bank's program

B.3.1 T-banks' debt constraint

Lemma 3. *For a given level of investment $I_0^T \geqslant 0$ at $t = 0$, the maximum amount of short term debt that can be reimbursed at $t = 1$ in state B is:*

$$\overline{D}_{0,B}^T\left(I_0^T, p_{1,B}\right) = \begin{cases} k + \frac{p_1}{\delta}\left(\delta qrI_0^T - k\right)_- + \frac{p_{1,B}(1-\varepsilon)}{\delta}\left(\delta qrI_0^T - k\right)_+ & \textit{if } 0 \leqslant p_{1,B} \leqslant \delta \\ k + \left(\delta qrI_0^T - k\right)_- + \frac{p_{1,B}(1-\varepsilon)}{\delta}\left(\delta qrI_0^T - k\right)_+ & \textit{if } \delta \leqslant p_{1,B} \leqslant \frac{\delta}{1-\varepsilon} \\ p_{1,B}qrI_0^T(1-\varepsilon) & \textit{if } p_{1,B} \geqslant \frac{\delta}{1-\varepsilon} \end{cases}$$

Similarly, the maximum amount of short term debt that can be reimbursed at $t = 1$ in state G is:

$$\overline{D}_{0,G}\left(I_0\right) = \delta RI_0^T.$$

Proof. At $t = 1$ in state B, T-banks can generate funds either by selling a share $1 - \alpha^T_{1,B}$ of their assets, or by newly raising funds $D^T_{1,B}$. They are subject to a (i) limited liability constraint at $t = 2$, and (ii) a limit k on the amount of debt that can be guaranteed at $t = 1$ in state B. For a given level of investment $I^T_0 > 0$ at $t = 0$ and asset purchases $p_{1,B}qr > 0$, the maximum amount of debt issued at $t = 0$ that can be reimbursed at $t = 1$ in state B is:

$$
\begin{aligned}
\overline{D}^T_{0,B}\left(I^T_0, p_{1,B}\right) = \max_{\alpha^T_{1,B}, I^T_{1,B}, D^T_{1,B}} & (1-\alpha^T_{1,B})p_{1,B}qrI^T_0(1-\varepsilon) + D^T_{1,B} - p_{1,B}qrI^T_{1,B} \\
\text{s.t. } & D^T_{1,B}, I^T_{1,B}, \alpha^T_{1,B} \geqslant 0, \alpha^T_{1,B} \leqslant 1 \\
& D^T_{1,B} \leqslant q\delta\left(\alpha^T_{1,B} rI^T_0 + rI^T_{1,B}\right) \\
& D^T_{1,B} \leqslant k
\end{aligned}
$$

Denoting $\lambda^T_{1,B} \geqslant 0$ the Lagrange multiplier associated to the funding constraint $D^T_{1,B} \leqslant q\delta\left(\alpha^T_{1,B} rI^T_0 + rI^T_{1,B}\right)$, $\nu_{D^T_{1,B}\leqslant k}$ the Lagrange multiplier associated to the constraint $D^T_{1,B} \leqslant k$, $\nu_{D^T_{1,B}\geqslant 0}$ the Lagrange multiplier associated to the constraint $D^T_{1,B} \geqslant 0$, $\nu_{I^T_{1,B}\geqslant 0}$ the Lagrange multiplier associated to the constraint $I^T_{1,B} \geqslant 0$, $\nu_{\alpha^T_{1,B}\geqslant 0}$ the Lagrange multiplier associated to the debt constraint $\alpha^T_{1,B} \geqslant 0$, and $\nu_{\alpha^T_{1,B}\leqslant 1}$ the Lagrange multiplier associated to the constraint $1 - \alpha^T_{1,B} \geqslant 0$, the Lagrangian of the problem writes:

$$
\begin{aligned}
L = & (1-\alpha^T_{1,B})p_{1,B}qrI^T_0(1-\varepsilon) + D^T_{1,B} - p_{1,B}qrI^T_{1,B} \\
& + \lambda^T_{1,B}\left[q\delta\left(\alpha^T_{1,B} rI^T_0 + rI^T_{1,B}\right) - D^T_{1,B}\right] \\
& + \nu_{D^T_{1,B}\leqslant k}\left(k - D^T_{1,B}\right) \\
& + \nu_{D^T_{1,B}\geqslant 0} D^T_{1,B} + \nu_{I^T_{1,B}\geqslant 0} I^T_{1,B} \\
& + \nu_{\alpha^T_{1,B}\geqslant 0}\alpha^T_{1,B} + \nu_{\alpha^T_{1,B}\leqslant 1}\left(1 - \alpha^T_{1,B}\right)
\end{aligned}
$$

The first order conditions yield

$$
\begin{aligned}
\frac{dL}{d\alpha^T_{1,B}} &= -p_{1,B}qrI^T_0(1-\varepsilon) + \lambda^T_{1,B}q\delta rI^T_0 + \nu_{\alpha^T_{1,B}\geqslant 0} - \nu_{\alpha^T_{1,B}\leqslant 1} = 0 \\
\frac{dL}{dD^T_{1,B}} &= 1 - \lambda^T_{1,B} - \nu_{D^T_{1,B}\leqslant k} + \nu_{D^T_{1,B}\geqslant 0} = 0 \\
\frac{dL}{dI^T_{1,B}} &= -p_{1,B}qr + \lambda^T_{1,B}q\delta r + \nu_{I^T_{1,B}\geqslant 0} = 0
\end{aligned}
$$

Hence

$$\left(\lambda^T_{1,B}\delta - p_{1,B}(1-\varepsilon)\right)qrI^T_0 + \nu_{\alpha^T_{1,B}\geqslant 0} - \nu_{\alpha^T_{1,B}\leqslant 1} = 0 \tag{11}$$

$$1 + \nu_{D^T_{1,B}\geqslant 0} = \lambda^T_{1,B} + \nu_{D^T_{1,B}\leqslant k} \tag{12}$$

$$\lambda^T_{1,B}q\delta r + \nu_{I^T_{1,B}\geqslant 0} = p_{1,B}qr \tag{13}$$

First, we solve the problem for $I^T_0 > 0$.

1. If $p_{1,B} = 0, \lambda^T_{1,B} = \nu_{I^T_{1,B} \geqslant 0} = 0, \nu_{D^T_{1,B} \leqslant k} > 0, \nu_{\alpha^T_{1,B} \leqslant 1} = \nu_{\alpha^T_{1,B} \geqslant 0} = 0$. We then have $\overline{D}^T_{0,B}\left(I^T_0, p_{1,B}\right) = k$.

2. If $0 < p_{1,B} < \delta$, using (13) we have $\lambda^T_{1,B} < 1$, and using (12) we have $\nu_{D^T_{1,B} \leqslant k} > 0$ and $D^T_{1,B} = k$. Then, either $\lambda^T_{1,B} = 0$ and $q\delta\left(\alpha^T_{1,B} r I^T_0 + r I^T_{1,B}\right) > D^T_{1,B}$, in which case $\nu_{I^T_{1,B} \geqslant 0} > 0$, $\nu_{\alpha^T_{1,B} \geqslant 0} > 0$ which implies $\alpha^T_{1,B} = 0$, $I^T_{1,B} = 0$. This is impossible because $D^T_{1,B} = k > 0$. We must then have $\lambda^T_{1,B} > 0$ and $q\delta\left(\alpha^T_{1,B} r I^T_0 + r I^T_{1,B}\right) = D^T_{1,B}$. In that case, (11) and (13) yield $\left(\lambda^T_{1,B}\delta - p_{1,B}(1-\varepsilon)\right) qrI^T_0 = p_{1,B} qr\varepsilon I^T_0 - \nu_{I^T_{1,B} \geqslant 0} I^T_0 = \nu_{\alpha^T_{1,B} \leqslant 1} - \nu_{\alpha^T_{1,B} \geqslant 0}$. Putting (11) and (13) together we obtain $\nu_{\alpha^T_{1,B} \leqslant 1} + \nu_{I^T_{1,B} \geqslant 0} I^T_0 = \nu_{\alpha^T_{1,B} \geqslant 0} + p_{1,B} qr\varepsilon I^T_0$. Two subcases arise.

   (a) Either $\nu_{\alpha^T_{1,B} \leqslant 1} > 0$, and $I^T_{1,B} = \frac{k - q\delta r I^T_0}{q\delta r}$, $k = q\delta\left(rI^T_0 + rI^T_{1,B}\right)$. We then have $\overline{D}^T_{0,B}\left(I^T_0, p_{1,B}\right) = D^T_{1,B} - p_{1,B} qrI^T_{1,B} = k + \frac{p_{1,B}}{\delta}\left(q\delta r I^T_0 - k\right)$. This solution is an optimum if and only if $\delta qrI^T_0 \leqslant k$.

   (b) Or $\nu_{I^T_{1,B} \geqslant 0} > 0$, $D^T_{1,B} = q\delta\alpha^T_{1,B} r I^T_0 = k$. We then have $\overline{D}^T_{0,B}\left(I^T_0, p_{1,B}\right) = \left(q\delta r I^T_0 - k\right)\frac{p_{1,B}(1-\varepsilon)}{\delta} + k$. This solution is an optimum if and only if $\delta qrI^T_0 \geqslant k$.

   In a nutshell $\overline{D}^T_{0,B}\left(I^T_0, p_{1,B}\right) = k + \frac{p_{1,B}}{\delta}\left(\delta qrI^T_0 - k\right)_- + \frac{p_{1,B}(1-\varepsilon)}{\delta}\left(\delta qrI^T_0 - k\right)_+$.

3. If $p_{1,B} = \delta$, two subcases arise:

   (a) Either $\lambda^T_{1,B} = 1$, and $\nu_{I^T_{1,B} \geqslant 0} = \nu_{D^T_{1,B} \geqslant 0} = \nu_{D^T_{1,B} \leqslant k} = 0, \nu_{\alpha^T_{1,B} \leqslant 1} > 0$. We then have $\overline{D}^T_{0,B}\left(I^T_0, p_{1,B}\right) = q\delta r I^T_0$. This solution is an optimum if and only if $\delta qrI^T_0 \leqslant k$.

   (b) Or $0 < \lambda^T_{1,B} < 1$, $\nu_{I^T_{1,B} \geqslant 0} > 0, \nu_{D^T_{1,B} \leqslant k} > 0$ and $D^T_{1,B} = k$, $I^T_{1,B} = 0$, $q\delta\alpha^T_{1,B} r I^T_0 = D^T_{1,B} = k$. We then have $\overline{D}^T_{0,B}\left(I^T_0, p_{1,B}\right) = (1-\varepsilon)\left(\delta qrI^T_0 - k\right) + k$. This solution is an optimum if and only if $\delta qrI^T_0 \geqslant k$.

   In a nutshell $\overline{D}^T_{0,B}\left(I^T_0, p_{1,B}\right) = k + \left(\delta qrI^T_0 - k\right)_- + (1-\varepsilon)\left(\delta qrI^T_0 - k\right)_+$.

4. If $\delta < p_{1,B} < \frac{\delta}{1-\varepsilon}$, by (13) we have either $\lambda^T_{1,B} > 1$ or $\nu_{I^T_{1,B} \geqslant 0} > 0$. If $\lambda^T_{1,B} > 1$, $\nu_{D^T_{1,B} \geqslant 0} > 0$ by (12). This implies $\alpha^T_{1,B} = I^T_{1,B} = 0$. However $\lambda^T_{1,B} > 1$ also implies $\left(\lambda^T_{1,B}\delta - p_{1,B}(1-\varepsilon)\right) qrI^T_0 > 0$, which imposes $\nu_{\alpha^T_{1,B} \leqslant 1} > 0$ by (11) and contradicts $\alpha^T_{1,B} = 0$. Hence, in equilibrium $\nu_{I^T_{1,B} \geqslant 0} > 0$ and $0 < \lambda^T_{1,B} \leqslant 1$.

   (a) If $\lambda^T_{1,B} = 1$, $\nu_{D^T_{1,B} \geqslant 0} = \nu_{D^T_{1,B} \leqslant k} = 0$, $\nu_{\alpha^T_{1,B} \leqslant 1} > 0$, $D^T_{1,B} = q\delta r I^T_0 \leqslant k$. We then have $\overline{D}^T_{0,B}\left(I^T_0, p_{1,B}\right) = q\delta r I^T_0$. This solution is an optimum if and only if $\delta qrI^T_0 \leqslant k$.

   (b) If $\lambda^T_{1,B} < 1, \nu_{D^T_{1,B} \leqslant k} > 0, D^T_{1,B} = \alpha^T_{1,B} q\delta r I^T_0 = k$. We then have $\overline{D}^T_{0,B}\left(I^T_0, p_{1,B}\right) = \frac{(1-\varepsilon)p_{1,B}}{\delta}\left(\delta qrI^T_0 - k\right) + k$. This solution is an optimum if and only if $\delta qrI^T_0 \geqslant k$.

   In a nutshell $\overline{D}^T_{0,B}\left(I^T_0, p_{1,B}\right) = k + \left(\delta qrI^T_0 - k\right)_- + \frac{(1-\varepsilon)p_{1,B}}{\delta}\left(\delta qrI^T_0 - k\right)_+$.

5. If $p_{1,B} = \frac{\delta}{1-\varepsilon}$, by (13) we have either $\lambda^T_{1,B} > 1$ or $\nu_{I^T_{1,B} \geqslant 0} > 0$. If $\lambda^T_{1,B} > 1$, $\nu_{D^T_{1,B} \geqslant 0} > 0$ by (12). This implies $\alpha^T_{1,B} = I^T_{1,B} = 0$. However $\lambda^T_{1,B} > 1$ also implies $\left(\lambda^T_{1,B}\delta - p_{1,B}(1-\varepsilon)\right) qrI > 0$, which

imposes $\nu_{\alpha^T_{1,B}\leqslant 1} > 0$ by (11) and contradicts $\alpha^T_{1,B} = 0$. Hence, in equilibrium $\nu_{I^T_{1,B}\geqslant 0} > 0$ and $0 < \lambda^T_{1,B} \leqslant 1$.

(a) If $\lambda^T_{1,B} = 1, \nu_{D_{1,B}T\geqslant 0} = \nu_{D^T_{1,B}\leqslant k} = 0, \nu_{\alpha^T_{1,B}\leqslant 1} = 0, D^T_{1,B} = \alpha^T_{1,B} q\delta r I^T_0 \leqslant k$. We then have $\overline{D}_0 = q\delta r I^T_0$. This solution is an optimum if and only if $\delta q r I^T_0 \leqslant k$.

(b) If $\lambda^T_{1,B} < 1, \nu_{D^T_{1,B}\leqslant k} > 0, D^T_{1,B} = \alpha^T_{1,B} q\delta r I = k$. We then have $\overline{D}^T_{0,B}\left(I^T_0, p_{1,B}\right) = \left(\delta q r I^T_0 - k\right) + k$. This solution is an optimum if and only if $\delta q r I^T_0 \geqslant k$.

In a nutshell $\overline{D}^T_{0,B}\left(I^T_0, p_{1,B}\right) = \delta q r I^T_0$.

6. If $p_{1,B} > \frac{\delta}{1-\varepsilon}$, by (11) either $\nu_{\alpha^T_{1,B}\leqslant 1} > 0$, which implies $\left(\lambda^T_{1,B}\delta - p_{1,B}(1-\varepsilon)\right) > 0$ and $\lambda^T_{1,B} > 1$, therefore $\nu_{D^T_{1,B}\geqslant 0} > 0$ by (12) which contradicts $\left(\lambda^T_{1,B}\delta - p_{1,B}(1-\varepsilon)\right) > 0$ therefore this not possible. We must then have $\nu_{\alpha^T_{1,B}\leqslant 1} = 0$, and $\nu_{I^T_{1,B}\geqslant 0} > 0$ by (13). If $\lambda^T_{1,B}\delta - p_{1,B}(1-\varepsilon) = 0$, then $\lambda^T_{1,B} > 1$, $\nu_{D^T_{1,B}\geqslant 0} > 0$ by (12) and $D^T_{1,B} = I^T_{1,B} = \alpha^T_{1,B} = 0$. Otherwise $\lambda^T_{1,B}\delta - p_{1,B}(1-\varepsilon) < 0$ and $\nu_{\alpha^T_{1,B}\geqslant 0} > 0$. We then have $\overline{D}^T_{0,B}\left(I^T_0, p_{1,B}\right) = p_{1,B} q r I^T_0 (1-\varepsilon)$.

Second, we solve the problem for $I^T_0 = 0$. In this case the program rewrites

$$\overline{D}^T_{0,B}\left(I^T_0, p_{1,B}\right) = \max_{I^T_{1,B}, D^T_{1,B}} D^T_{1,B} - p_{1,B} q r I^T_{1,B}$$
$$D^T_{1,B} \leqslant q\delta\left(r I^T_{1,B}\right)$$
$$D^T_{1,B} \leqslant k$$
$$D^T_{1,B}, I^T_{1,B} \geqslant 0$$

It is easily shown that the previous optima also hold true when $I^T_0 = 0$. Lemma 3 obtains.

□

B.3.2 T-Banks: optimization program at $t = 1$ in state B

We focus on the case where T-banks do not default (hereafter "ND" for "no default"), i.e. when $D^T_{0,B} \in [0; \overline{D}^T_{0,B}\left(I^T_0, p_{1,B}\right)]$ where $\overline{D}^T_{0,B}$ is defined in Lemma 3.

Proposition 12. *For a given level of investment $I^T_0 \geqslant 0$ at $t = 0$, $p_{1,B} > 0$ and $D^T_{0,B} \in [0; \overline{D}^T_{0,B}\left(I^T_0, p_{1,B}\right)]$, value functions at $t = 1$ in state B solve as follows:*

1. *If $D^T_0 \leqslant k$ then*

   (a) *If $0 < p_{1,B} < \delta$, then the equilibrium is $\alpha^T_{1,B} = 1, D^T_{1,B} = k, I^T_{1,B} = \frac{k - D^T_0}{p_{1,B} q r}, V^{T,ND}_{1,B}\left(I^T_0, D^T_0, p_{1,B}\right) = \frac{(\delta - p_{1,B})}{p_{1,B}}\left(k - D^R_0\right) + \delta q r I^T_0 - D^T_0$.*

   (b) *If $p_{1,B} = \delta$, then equilibria are such that $0 \leqslant I^T_{1,B} \leqslant \frac{k - D^T_0}{\delta q r}$, $D^T_{1,B} \in [D^T_0 + \delta q r I^T_{1,B}; \delta q r I^T_0 + \delta q r I^T_1]$, $\alpha^T_{1,B} = 1$, $V^{T,ND}_{1,B}\left(I^T_0, D^T_0, p_{1,B}\right) = \delta q r I^T_0 - D^T_0$.*

   (c) *If $\delta < p_{1,B} < \frac{\delta}{1-\varepsilon}$, then equilibria are such that $\alpha^T_{1,B} = 1$, $I^T_1 = 0$, $D^T_{1,B} \in [D^T_0; \min(\delta q r I^T_0; k)]$ and $V^{T,ND}_{1,B}\left(I^T_0, D^T_0, p_{1,B}\right) = \delta q r I^T_0 - D^T_0$.*

(d) *If* $p_{1,B} = \frac{\delta}{1-\varepsilon}$*, then equilibria are such that* $I^T_{1,B} = 0$*,* $\alpha^T_{1,B} \in [0;1]$*, and* $D^T_{1,B} \in [0; \min(k; \delta q \alpha^T_{1,B} r I^T_0)]$*,* $D^T_0 \leqslant D^T_{1,B} + (1-\alpha^T_{1,B}) p_{1,B} q r I^T_0 (1-\varepsilon)$ *and* $V^{T,ND}_{1,B}\left(I^T_0, D^T_0, p_{1,B}\right) = \delta q r I^T_0 - D^T_0$.

(e) *If* $p_{1,B} > \frac{\delta}{1-\varepsilon}$*, then the equilibrium is such that* $D^T_{1,B} = \alpha = I^T_{1,B} = 0$ *and* $V^{T,ND}_{1,B}\left(I^T_0, D^T_0, p_{1,B}\right) = (1-\varepsilon)\, p_{1,B} q r I^T_0 - D^T_0$.

2. *If* $D^T_0 \geqslant k$*, then*

(a) *If* $0 < p^T_{1,B} < \frac{\delta}{1-\varepsilon}$*, then the equilibrium is such that* $I^T_{1,B} = 0$*,* $D^T_{1,B} = k$*,* $\alpha = 1 - \frac{D^T_0 - k}{p_{1,B} q r I^T_0 (1-\varepsilon)}$*,* $V^{T,ND}_{1,B}\left(I^T_0, D^T_0, p_{1,B}\right) = \left(1 - \frac{D^T_0 - k}{p_{1,B} q r I^T_0 (1-\varepsilon)}\right) \delta q r I^T_0 - k$.

(b) *If* $p_{1,B} = \frac{\delta}{1-\varepsilon}$*, then equilibria are such that* $I^T_{1,B} = 0$*,* $\alpha^T_{1,B} \in [0;1]$*, and* $D^T_{1,B} \in [0; \min(k; \delta q \alpha^T_{1,B} r I^T_0)]$*, with* $D^T_0 \leqslant D^T_{1,B} + (1-\alpha^T_{1,B}) p_{1,B} q r I^T_0 (1-\varepsilon)$ *and* $V^{T,ND}_{1,B}\left(I^T_0, D^T_0, p_{1,B}\right) = \delta q r I^T_0 - D^T_0$.

(c) *If* $p_{1,B} > \frac{\delta}{1-\varepsilon}$*, then the equilibrium is such that* $D^T_{1,B} = \alpha^T_{1,B} = I^T_{1,B} = 0$ *and* $V^{T,ND}_{1,B}\left(I^T_0, D^T_0, p_{1,B}\right) = p_{1,B} (1-\varepsilon)\, q r I^T_0 - D^T_0$.

Morevover, if $p_{1,B} = 0$*, then* $V^{T,ND}_{1,B}\left(I^T_0, D^T_0, p_{1,B}\right) = +\infty$.

Proof. We first solve for T-banks' program at $t = 1$ in state B, taking D^T_0 and I^T_0 as given, in the set which ensures no-default at $t = 1$ in state B. In this no-default case, T-banks' value function writes:

$$
\begin{aligned}
V^{T,ND}_{1,B}\left(I^T_0, D^T_0, p_{1,B}\right) = \max_{\alpha^T_{1,B}, D^T_{1,B}, I^T_{1,B}} \; & \left(\delta - p_{1,B}\right) q r I^T_{1,B} + \alpha^T_{1,B} \delta r I^T_0 + (1-\alpha^T_{1,B}) p_{1,B} q r I^T_0 \,(1-\varepsilon) - D^T_0 \\
\text{s.t. } & D^T_{1,B}, I^T_{1,B}, \alpha^T_{1,B} \geqslant 0, \alpha^T_{1,B} \leqslant 1 \\
& D^T_{1,B} \leqslant q\delta \left(\alpha^T_{1,B} r I^T_0 + r I^T_{1,B}\right) \\
& D^T_{1,B} \leqslant k \\
& (1-\alpha^T_{1,B}) p_{1,B} q r I^T_0 \,(1-\varepsilon) + D_{1,B} \geqslant D^T_0 + p_{1,B} q r I^T_{1,B} \\
& D_{1,B} \leqslant q\delta \left(r \alpha^T_{1,B} I^T_0 + r I^T_{1,B}\right)
\end{aligned}
$$

The Lagrangian writes as follows:

$$
\begin{aligned}
L^T_{1,B} = & \left(\delta - p_{1,B}\right) q r I^T_{1,B} + \alpha^T_{1,B} \delta q r I^T_0 + (1-\alpha^T_{1,B}) p_{1,B} q r I^T_0 \,(1-\varepsilon) - D^T_0 \\
& + \lambda_1 \left((1-\alpha^T_{1,B}) p_{1,B} q r I^T_0 \,(1-\varepsilon) + D^T_{1,B} - D^T_0 - p_{1,B} q r I^T_{1,B}\right) \\
& + \lambda_2 \left(\delta q \left(\alpha^T_{1,B} r I^T_0 + r I^T_1\right) - D^T_{1,B}\right) \\
& + \nu_{D^T_{1,B} \leqslant k} \left(k - D^T_{1,B}\right) \\
& + \nu_{D^T_{1,B} \geqslant 0} D^T_{1,B} + \nu_{\alpha^T_{1,B} \geqslant 0} \alpha^T_{1,B} + \nu_{\alpha^T_{1,B} \leqslant 1} \left(1 - \alpha^T_{1,B}\right) + \nu_{I^T_{1,B} \geqslant 0} I^T_{1,B}
\end{aligned}
$$

The first-order conditions yield:

$$\frac{dL_{1,B}^T}{dD_{1,B}^T} = \lambda_1 - \lambda_2 - \nu_{D_{1,B}^T \leqslant k} + \nu_{D_{1,B}^T \geqslant 0} = 0 \tag{14}$$

$$\frac{dL_{1,B}^T}{d\alpha_{1,B}^T} = qrI_0^T \left(\delta - p_{1,B}\left(1-\varepsilon\right)\right) - \lambda_1 p_{1,B} qrI_0^T \left(1-\varepsilon\right) + \lambda_2 \delta qrI_0^T + \nu_{\alpha_{1,B}^T \geqslant 0} - \nu_{\alpha_{1,B}^T \leqslant 1} = 0 \tag{15}$$

$$\frac{dL_{1,B}^T}{dI_{1,B}^T} = \left(\delta - p_{1,B}\right) qr - \lambda_1 p_{1,B} qr + \lambda_2 \delta qr + \nu_{I_{1,B}^T \geqslant 0} = 0 \tag{16}$$

We focus on cases where $k > 0$. First, we solve for $I_0^T > 0$.

We use (14) to replace λ_1 in (16) and obtain:

$$\left(\delta - p_{1,B}\right) qr \left(1 + \lambda_2\right) + \nu_{I_{1,B}^T \geqslant 0} + \left(\nu_{D_{1,B}^T \geqslant 0} - \nu_{D_{1,B}^T \leqslant k}\right) p_{1,B} qr = 0 \tag{17}$$

We use (14) to replace λ_1 in (15) and obtain:

$$qrI_0^T \left(\delta - p_{1,B}\left(1-\varepsilon\right)\right)\left(1+\lambda_2\right) + \nu_{\alpha_{1,B}^T \geqslant 0} - \nu_{\alpha_{1,B}^T \leqslant 1} + \left(\nu_{D_{1,B}^T \geqslant 0} - \nu_{D_{1,B}^T \leqslant k}\right) p_{1,B} qrI_0^T \left(1-\varepsilon\right) = 0 \tag{18}$$

Finally, we multiply (17) by $(1-\varepsilon)I_0^T$ and subtract (18) to obtain:

$$\varepsilon qrI_0^T \delta \left(1+\lambda_2\right) + \nu_{\alpha_{1,B}^T \geqslant 0} = \nu_{\alpha_{1,B}^T \leqslant 1} + \nu_{I_{1,B}^T \geqslant 0} I_0^T \left(1-\varepsilon\right) \tag{19}$$

We treat different cases sequentially.

Case $p_{1,B} = 0$ In this case, equations (14), (15) and (16) rewrite:

$$\begin{aligned}
\lambda_1 - \lambda_2 - \nu_{D_{1,B} \leqslant k} + \nu_{D_{1,B} \geqslant 0} &= 0 \\
qrI_0^T \delta + \lambda_2 \delta qrI_0^T + \nu_{\alpha_{1,B}^T \geqslant 0} - \nu_{\alpha_{1,B}^T \leqslant 1} &= 0 \\
\delta qr + \lambda_2 \delta qr + \nu_{I_{1,B}^T \geqslant 0} &= 0
\end{aligned}$$

Hence, $I_1^T = +\infty$, $\alpha_{1,B}^T = 1$ and $V_{1,B}^T = +\infty$.

Case $0 < p_{1,B} < \delta$ In this case, (16) obtains $\lambda_1 > 0$ and (17) obtains $\nu_{D_{1,B} \leqslant k} > 0$.

Hence the constraint associated to the Lagrange multipliers λ_1 and $\nu_{D_{1,B} \leqslant k}$ bind, and respectively $(1-\alpha_{1,B}^T)p_{1,B} qrI_0^T \left(1-\varepsilon\right) + D_{1,B}^T = D_0^T + p_{1,B} qrI_{1,B}^T$ and $D_{1,B} = k$. It follows that $\nu_{D_{1,B} \geqslant 0} = 0$.

From (19), we have two possible cases:

1. Either $\nu_{\alpha_{1,B}^T \leqslant 1} > 0$ and $\alpha_{1,B}^T = 1$. It follows that $D_{1,B} = k$, $I_{1,B}^T = \frac{k - D_0^T}{p_{1,B} qr}$. In this case $V_{1,B}^T = \frac{(\delta - p_{1,B})}{p_{1,B}}\left(k - D_0^T\right) + \delta qrI_0^T - D_0^T$. This solution is an equilibrium if and only if $D_0^T \leqslant k$.
2. Or $\nu_{I_{1,B}^T \geqslant 0} > 0$, and $I_{1,B}^T = 0$. It follows that $(1-\alpha)p_{1,B} qrI_{0,B}^T \left(1-\varepsilon\right) + k = D_0^T$, hence $\alpha_{1,B}^T = 1 - \frac{D_0^T - k}{p_{1,B} qrI_0^T (1-\varepsilon)}$. In this case, $V_{1,B}^T = \left(1 - \frac{D_0^T - k}{p_{1,B}^T qrI_0^T (1-\varepsilon)}\right) \delta qrI_0^T - k$. This solution is an equilibrium

if and only if $D_0^T \geqslant k$.

Case $p_{1,B} = \delta$ In this case, again from (19) we have two possible cases:

1. Either $\nu_{\alpha_{1,B}^T \leqslant 1} > 0$ and $\alpha_{1,B}^T = 1$.

   (a) In which case either $\lambda_1 = \lambda_2$, $\nu_{D_{1,B}^T \leqslant k} = \nu_{D_{1,B}^T \geqslant 0} = 0$, $\nu_{I_{1,B}^T \geqslant 0} = 0$. In this case any $0 \leqslant I_{1,B}^T \leqslant \frac{k - D_0^T}{\delta q r}$ is an equilibrium value, any $D_{1,B}^T$ such that $\delta q r I_0^T + \delta q r I_{1,B}^T \geqslant D_{1,B}^T \geqslant D_0^T + \delta q r I_{1,B}^T$, $V_{1,B}^T = \delta q r I_{0,B}^T - D_0^T$, and these solutions are equilibria if and only if $D_0^T \leqslant k$.

   (b) Or $\lambda_1 > \lambda_2$, $\nu_{I_{1,B}^T \geqslant 0} > 0$, $\nu_{D_{1,B}^T \leqslant k} > 0$ and $I_{1,B}^T = 0$, $D_{1,B}^T = k$, $\alpha_{1,B}^T = 1 - \frac{D_0^T - k}{\delta q r I_0^T (1-\varepsilon)} = 1$. In this case, $V_{1,B}^T = \delta q r I_0^T - k$. This solution is an equilibrium if and only if $D_0^T = k$.

2. Or $\nu_{I_{1,B}^T \geqslant 0} > 0$ and $I_{1,B}^T = 0$.

   (a) In this case, $\lambda_2 < \lambda_1$, $\lambda_1 > 0$, $\nu_{D_{1,B}^T \leqslant k} > 0$, $\alpha_{1,B}^T = 1 - \frac{D_0^T - k}{\delta q r I_0^T (1-\varepsilon)}$, and $I_{1,B}^T = 0$, $D_{1,B}^T = k$. In this case, $V_{1,B}^T = \left(1 - \frac{D_0^T - k}{\delta q r I_0^T (1-\varepsilon)}\right) \delta q r I_0^T - k$. This solution is an equilibrium if and only if $k \leqslant D_0^T$.

Case $\delta < p_{1,B} < \frac{\delta}{1-\varepsilon}$ We rewrite (17) and (18) as follows:

$$\nu_{D_{1,B}^T \geqslant 0} p_{1,B} q r + \nu_{I_{1,B}^T \geqslant 0} = -\left(\delta - p_{1,B}\right) q r \left(1 + \lambda_2\right) + \nu_{D_{1,B}^T \leqslant k} p_{1,B} q r$$

$$\left(\nu_{D_{1,B}^T \leqslant k} - \nu_{D_{1,B}^T \geqslant 0}\right) p_{1,B} q r I_0^T \left(1 - \varepsilon\right) + \nu_{\alpha_{1,B}^T \leqslant 1} = q r I_0^T \left(\delta - p_{1,B}\left(1 - \varepsilon\right)\right)\left(1 + \lambda_2\right) + \nu_{\alpha_{1,B}^T \geqslant 0}$$

We have $q r I_0^T \left(\delta - p_{1,B}\left(1-\varepsilon\right)\right)\left(1 + \lambda_2\right) > 0$, therefore $-\left(\delta - p_{1,B}\right) q r \left(1+\lambda_2\right) > 0$.

Hence, either $\nu_{I_{1,B}^T \geqslant 0} > 0$ or $\nu_{I_{1,B}^T \geqslant 0} = 0$. In the latter case, $\nu_{D_{1,B}^T \geqslant 0} > \nu_{D_{1,B}^T \leqslant k} = 0$, $\nu_{\alpha_{1,B}^T \leqslant 1} > 0$ and $\lambda_2 > 0$. This is impossible because $I_0^T > 0$. We therefore have $\nu_{I_{1,B}^T \geqslant 0} > 0$.

1. Then, either $\nu_{D_{1,B}^T \geqslant 0} > \nu_{D_{1,B}^T \leqslant k} = 0$, $\nu_{\alpha_{1,B}^T \leqslant 1} > 0$ and $\lambda_2 > 0$ which is impossible because $I_0^T > 0$.

2. Finally $\nu_{D_{1,B}^T \geqslant 0} = 0$.

   (a) Then, either $\nu_{D_{1,B}^T \leqslant k} = 0$, $\nu_{\alpha_{1,B}^T \leqslant 1} > 0$, in which case $\alpha_{1,B}^T = 1$, $I_{1,B}^T = 0$, $q \delta r I_0^T \geqslant D_{1,B}^T \geqslant D_0^T$, and $D_{1,B}^T \leqslant k$. In this case $V_{1,B}^T = \delta q r I_0^T - D_0^T$. This solution is an equilibrium if and only if $D_0^T \leqslant k$.

   (b) Or $\nu_{D_{1,B}^T \leqslant k} > 0$, $\lambda_1 > 0$, in which case $\alpha_{1,B}^T = 1 - \frac{D_0^T - k}{\delta q r I_0^T (1-\varepsilon)}$, $I_{1,B}^T = 0$, $D_{1,B}^T = k$ and this solution is an equilibrium if and only if $k \leqslant D_0^T$. In this case, $V_{1,B}^T = \left(1 - \frac{D_0^T - k}{\delta q r I_0^T (1-\varepsilon)}\right) \delta q r I_0^T - k$.

Case $p_{1,B} = \frac{\delta}{1-\varepsilon}$ Again, $\nu_{D_{1,B}^T \geqslant 0} > 0 = \nu_{D_{1,B}^T \leqslant k}$, $\lambda_2 > 0$, and $\nu_{\alpha_{1,B}^T \leqslant 1} > 0$ is impossible because $I_0^T > 0$. We then have $\nu_{I_{1,B}^T \geqslant 0} > 0$ and $\nu_{D_{1,B}^T \geqslant 0} = 0$. In this case,

$$\nu_{\alpha_{1,B}^T \geqslant 0} = \nu_{\alpha_{1,B}^T \leqslant 1} + \nu_{D_{1,B}^T \leqslant k} \delta q r I_0^T$$

and $\nu_{\alpha_{1,B}^T\leqslant 1}=0$.

Finally, $I_{1,B}^T=0$, any $\alpha_{1,B}^T\in[0;1]$, and any $D_{1,B}^T\in[0;\min(k;\delta q\alpha_{1,B}^T rI_0^T)]$ such that $D_0^T\leqslant D_{1,B}^T+(1-\alpha_{1,B}^T)p_{1,B}qrI_0^T(1-\varepsilon)$ is an equilibrium, and $V_{1,B}^T=\delta qrI_0^T-D_0^T$.

Case $p_{1,B}>\frac{\delta}{1-\varepsilon}$ In this case, (17) yields

$$\nu_{I_{1,B}^T\geqslant 0}+\nu_{D_{1,B}^T\geqslant 0}p_{1,B}qr=\nu_{D_{1,B}^T\leqslant k}p_{1,B}qr+\left(p_{1,B}-\delta\right)qr\left(1+\lambda_2\right)>0$$

and (18) yields

$$\nu_{\alpha_{1,B}^T\geqslant 0}+\nu_{D_{1,B}^T\geqslant 0}p_{1,B}qrI_0^T\left(1-\varepsilon\right)=\nu_{\alpha_{1,B}^T\leqslant 1}+\nu_{D_{1,B}^T\leqslant k}p_{1,B}qrI_0^T\left(1-\varepsilon\right)+qrI_0^T\left(p_{1,B}\left(1-\varepsilon\right)-\delta\right)\left(1+\lambda_2\right)>0$$

Two possible cases arise:

1. Either $\nu_{\alpha_{1,B}^T\geqslant 0}>0,\nu_{\alpha_{1,B}^T\leqslant 1}=0$, in which case we have from (19) $\nu_{I_{1,B}^T\geqslant 0}>0$. Hence, $I_{1,B}^T=0,\alpha_{1,B}^T=0,D_{1,B}^T=0,V_{1,B}^T=p_{1,B}\left(1-\varepsilon\right)qrI_0^T-D_0^T$.
2. Otherwise $\nu_{\alpha_{1,B}^T\geqslant 0}=0$, $\nu_{D_{1,B}^T\geqslant 0}>0$, $\lambda_2>0$ and $\alpha_{1,B}^T=I_{1,B}^T=0$.

In both cases, $D_{1,B}^T=\alpha_{1,B}^T=I_{1,B}^T=0$ and $V_{1,B}^T=p_{1,B}\left(1-\varepsilon\right)qrI_0^T-D_0^T$.

Second, if $I_0^T=0$ the program rewrites

$$\begin{aligned}V_{1,B}^{T,ND}\left(0,D_0^T,p_{1,B}\right)=\max_{(D_{1,B}^T,I_{1,B}^T)\in[0;k]\times\mathbb{R}_+}&\left(\delta-p_{1,B}\right)qrI_{1,B}^T-D_0^T\\ \text{s.t. }&D_{1,B}^T\geqslant D_0^T+p_{1,B}qrI_{1,B}^T\\ &D_{1,B}^T\leqslant q\delta\left(rI_{1,B}^T\right)\end{aligned}$$

and the same results about the value function hold true.

□

B.3.3 T-banks: optimization program at $t=0$

We define $k^*\equiv\frac{\delta qrn}{1-\delta qr}$, $p_{1,L}^T\equiv\frac{\delta}{\delta qr+\frac{p(\delta R-1)}{1-p}}$, $p_{1,H}^{TB}\equiv\frac{p_{1,L}^T}{1-\varepsilon}$, and solve solve for T-bank's optimization program at $t=0$, taking n and $p_{1,B}$ as given.

Proposition 13. *The solution to T-bank's optimization program is as follows:*

1. *If* $0<k\leqslant k^*$
   (a) *If* $0<p_{1,B}<p_{1,L}^T$, $D_0^T=0, I_0^T=n$, $V_0^T=p\left(\delta Rn\right)+\left(1-p\right)\left(\delta qrn-k+\frac{\delta}{p_{1,B}}k\right)$
   (b) *If* $p_{1,B}=p_{1,L}^T$, *any* $D_0^T\in[0;k]$, $I_0^T=n+D_0^T$ *is an equilibrium solution and* $V_0^T=p\left(\delta R(n+k)-k\right)+\left(1-p\right)\left(\delta qr(n+k)-k\right)$
   (c) *If* $p_{1,L}^T<p_{1,B}<p_{1,H}^T$, $D_0^T=k, I_0^T=k+n$ *and* $V_0^T=p\left(\delta R(n+k)-k\right)+\left(1-p\right)\left(\delta qr(n+k)-k\right)$

(d) *If* $p_{1,B} = p^T_{1,H}$*, any* $D^T_0 \in [k; \frac{k\left(1-\frac{p_{1,B}(1-\varepsilon)}{\delta}\right)+(1-\varepsilon)p_{1,B}qrn}{1-p_{1,B}(1-\varepsilon)qr}]$ *is an equilibrium solution,* $I^T_0 = n + D^T_0$ *and* $V^T_0 = p\left(\delta R(n+k)-k\right) + (1-p)\left(\delta qr(n+k)-k\right)$

(e) *If* $p^T_{1,H} < p_{1,B} \leqslant \frac{\delta}{1-\varepsilon}$, $D^T_0 = \frac{k\left(1-\frac{p_{1,B}(1-\varepsilon)}{\delta}\right)+(1-\varepsilon)p_{1,B}qrn}{1-p_{1,B}(1-\varepsilon)qr}$, $I^T_0 = n + D^T_0$ *and* $V^T_0 = p\left(\delta R\left(D_0+n\right)-D_0\right)$

(f) *If* $\frac{\delta}{1-\varepsilon} \leqslant p_{1,B} < \frac{1}{(1-\varepsilon)qr}$, $D^T_0 = \frac{p_{1,B}(1-\varepsilon)qrn}{(1-p_{1,B}(1-\varepsilon)qr)}$, $I^T_0 = D^T_0 + n$*, and* $V^T_0 = p\left(\delta R\left(D_0+n\right)-D_0\right)$

(g) *If* $p_{1,B} \geqslant \frac{1}{(1-\varepsilon)qr}$, $D^T_0 = +\infty$, $I^T_0 = +\infty$ *and* $V^T_0 = +\infty$

2. *If* $k > k^*$

(a) *If* $0 < p_{1,B} \leqslant p^T_{1,L}$, $D^T_0 = 0, I^T_0 = n$, $V^T_0 = p\left(\delta Rn\right) + (1-p)\left(\delta qrn - k + \frac{\delta}{p_{1,B}}k\right)$

(b) *If* $p_{1,B} = p^T_{1,L}$*, any* $D^T_0 \in [0; \frac{k\left(1-\frac{p_{1,B}}{\delta}\right)+p_{1,B}qrn}{1-p_{1,B}qr}]$ *is an equilibrium solution,* $I^T_0 = D^T_0 + n$ *and* $V^T_0 = p\left(\delta R(n+k)-k\right) + (1-p)\left(\delta qr(n+k)-k\right)$

(c) *If* $p^T_{1,L} < p_{1,B} < \delta$, $D_0 = \frac{k\left(1-\frac{p_{1,B}}{\delta}\right)+p_{1,B}qrn}{1-p_{1,B}qr}$, $I^T_0 = D^T_0 + n$ *and* $V^T_0 = p\left(\delta R\left(D^T_0+n\right)-D^T_0\right)$

(d) *If* $\delta \leqslant p_{1,B} \leqslant \frac{\delta}{1-\varepsilon}$, $D^T_0 = \frac{\delta qrn}{1-\delta qr}$, $I^T_0 = D^T_0 + n$ *and* $V^T_0 = p\left(\delta RI - D_0\right) + (1-p)\left(\delta qrI - D^R_0\right)$.

(e) *If* $\frac{\delta}{1-\varepsilon} \leqslant p_{1,B} < \frac{1}{(1-\varepsilon)qr}$, $D^T_0 = \frac{p_{1,B}(1-\varepsilon)qrn}{(1-p_{1,B}(1-\varepsilon)qr)}$, $I^T_0 = D^T_0 + n$*, and* $V^T_0 = p\left(\delta R\left(D_0+n\right)-D_0\right)$

(f) *If* $p_{1,B} \geqslant \frac{1}{(1-\varepsilon)qr}$, $D^T_0 = +\infty$, $I^T_0 = +\infty$ *and* $V^T_0 = +\infty$

Moreover, if $p_{1,B} = 0$*, then* $V^T_0 = +\infty$.

Proof. As S-banks, T-banks can only raise funds in the form of riskless short term debt. They will choose a debt level which ensures that all debt is reimbursed at $t = 1$, with certainty. T-banks' program for a given level n of own funds and a given price $p_{1,B}$ at $t = 1$ in state B market writes

$$V^T_0(p_{1,B}, n) = \max_{I^T_0, D^T_0} (D^T_0 + n - I^T_0) + pV^{T,ND}_{1,G}\left(I^T_0, D^T_0, p_{1,B}\right) + (1-p)V^{T,ND}_{1,B}\left(I^T_0, D^T_0, p_{1,B}\right) \tag{20}$$

$$D^T_0 \leqslant \delta RI^T_0 \tag{21}$$

$$D^T_0 \leqslant \overline{D}^T_0\left(p_{1,B}, I^T_0\right) \tag{22}$$

$$D^T_0 + n \geqslant I^T_0 \tag{23}$$

$$I^T_0, D^T_0 \geqslant 0 \tag{24}$$

with

$$\overline{D}^T_0\left(I^T_0, p_{1,B}\right) = \begin{cases} k + \frac{p_{1,B}}{\delta}\left(\delta qrI^T_0 - k\right)_- + \frac{p_{1,B}(1-\varepsilon)}{\delta}\left(\delta qrI^T_0 - k\right)_+ & \text{if } 0 \leqslant p_{1,B} \leqslant \delta \\ k + \left(\delta qrI^T_0 - k\right)_- + \frac{p_{1,B}(1-\varepsilon)}{\delta}\left(\delta qrI^T_0 - k\right)_+. & \text{if } \delta \leqslant p_{1,B} \leqslant \frac{\delta}{1-\varepsilon} \\ p_{1,B}qrI^T_0(1-\varepsilon) & \text{if } p_{1,B} \geqslant \frac{\delta}{1-\varepsilon} \end{cases}$$

$$V^{T,ND}_{1,G}\left(I^T_0, D^T_0, p_{1,B}\right) = \delta RI^T_0 - D^T_0$$

and, as shown in Proposition 12,

$$V_{1,B}^{T,ND}\left(I_0^T, D_0^T, p_{1,B}\right) = \left(\frac{(\delta - p_{1,B})_+}{p_{1,B}}\left(k - D_0^T\right)_+ + \left(k - D_0^T\right)_- \frac{(\delta - p_{1,B}(1-\varepsilon))_+}{p_{1,B}(1-\varepsilon)}\right)$$
$$+ (p_{1,B}(1-\varepsilon) - \delta)_+ qrI_0^T + \delta qrI_0^T - D_0^T$$

The first thing to notice is that, as for S-banks, T-banks are always binding their date 0 funding constraint (equation (23)): indeed, they always prefer investing one unit of funds in the assets available at date 0, which yields at least an expected $\delta(pR + (1-p)qr) > 1$ than consume it at date 0 and obtain 1.

The program of the bank at date 0 can then be rewritten:

$$\begin{aligned}
V_0^T(p_{1,B}, n) = \max_{D_0^T \geqslant 0}\ & pV_{1,G}^{T,ND}\left(I_0^T, D_0^T, p_{1,B}\right) + (1-p)V_{1,B}^{T,ND}\left(I_0^T, D_0^T, p_{1,B}\right) \\
& D_0^T \leqslant \overline{D}_0^T\left(p_{1,B}, I_0^T\right) \\
& D_0^T + n = I_0^T
\end{aligned}$$

We solve this program according to the values of $p_{1,B}$, and split it into three subprograms to ease the resolution. The solution to our program is then the maximum of the solution of the three subprograms.

Case $p_{1,B} = 0$**.** If $p_{1,B} = 0$, as $V_{1,B}^{T,ND}(I_0^T, D_0^T, 0) = +\infty$ whatever D_0^T and I_0^T, we have $V_0^T(0, n) = +\infty$

Case $0 < p_{1,B} < \delta$**.** If $0 < p_{1,B} < \delta$, we get $\overline{D}_0^T(I_0^T, p_{1,B}) = k + \frac{p_{1,B}}{\delta}(\delta qrI_0^T - k)_- + \frac{p_{1,B}(1-\varepsilon)}{\delta}(\delta qrI_0^T - k)_+$ and

$$V_{1,B}^{T,ND}\left(I_0^T, D_0^T, p_{1,B}\right) = \left(\frac{(\delta - p_{1,B})_+}{p_{1,B}}\left(k - D_0^T\right)_+ + \left(k - D_0^T\right)_- \frac{(\delta - p_{1,B}(1-\varepsilon))_+}{p_{1,B}(1-\varepsilon)}\right) + \delta qrI_0^T - D_0^T$$

First subprogram We first focus on the subprogram where we look for solutions I_0^T such that $\delta qrI_0^T \leqslant k$. In this case, $\overline{D}_0^T(p_{1,B}, I_0^T) \leqslant k + p_{1,B}(qrI_0^T - \frac{k}{\delta}) \leqslant k$ and the program rewrites as:

$$\begin{aligned}
V_0^T(p_{1,B}, n) = \max_{I_0^T \geqslant 0, D_0^T \geqslant 0}\ & p\left(\delta RI_0^T - D_0^T\right) + (1-p)\left(\delta qrI_0^T - k + \frac{\delta}{p_{1,B}}\left(k - D_0^T\right)\right) \\
& D_0^T \leqslant k + p_{1,B}\left(qrI_0^T - \frac{k}{\delta}\right) \\
& \delta qrI_0^T \leqslant k \\
& D_0^T + n = I_0^T
\end{aligned}$$

and replacing for I_0^T, we obtain:

$$V_0^T(p_{1,B},n) = \max_{D_0^T \geqslant 0} \left(p\delta R + (1-p)\delta qr - 1 - (1-p)\frac{\delta - p_{1,B}}{p_{1,B}}\right) D_0^T + (p\delta R + (1-p)\delta qr)\, n + (1-p)\frac{\delta - p_{1,B}}{p_{1,B}} k$$

$$D_0^T \leqslant k + p_{1,B}\left(qrI_0^T - \frac{k}{\delta}\right)$$

$$\delta qrI_0^T \leqslant k$$

$$D_0^T + n = I_0^T$$

This sub-program has a non-empty set of solutions if and only if $\delta qrn \leqslant k$, in which case:

1. $p\delta R + (1-p)\delta qr - 1 - (1-p)\frac{\delta - p_{1,B}}{p_{1,B}} < 0$, $D_0^T = 0$ and $V_0^T(p_{1,B},n) = (p\delta R + (1-p)\delta qr)\,n + (1-p)\frac{\delta - p_{1,B}}{p_{1,B}} k$
2. $p\delta R + (1-p)\delta qr - 1 - (1-p)\frac{\delta - p_{1,B}}{p_{1,B}} = 0$ and any D_0^T such that $0 \leqslant D_0^T \leqslant \min\left(\frac{k\left(1-\frac{p_{1,B}}{\delta}\right)+p_{1,B}qrn}{1-p_{1,B}qr}; \frac{k-\delta qrn}{\delta qr}\right)$ is an optimum. In this case, $V_0^T(p_{1,B},n) = (p\delta R + (1-p)\delta qr)\,n + (1-p)\frac{\delta - p_{1,B}}{p_{1,B}} k$
3. $p\delta R + (1-p)\delta qr - 1 - (1-p)\frac{\delta - p_{1,B}}{p_{1,B}} > 0$ and $D_0^T = \min\left(\frac{k\left(1-\frac{p_{1,B}}{\delta}\right)+p_{1,B}qrn}{1-p_{1,B}qr}; \frac{k-\delta qrn}{\delta qr}\right)$. In this case, $V_0^T(p_{1,B},n) = p\left(\delta R(D_0^T + n) - D_0^T\right)$

Notice that $\frac{k\left(1-\frac{p_{1,B}}{\delta}\right)+p_{1,B}qrn}{1-p_{1,B}qr} \leqslant \frac{k-\delta qrn}{\delta qr}$ if and only if $k \geqslant \overline{k}$

Second subprogram Let's now turn to the subprogram where we look for solutions D_0^T and I_0^T such that $\delta qrI_0^T \geqslant k$ and $D_0^T \leqslant k$. In this case, the program writes

$$V_0^T(p_{1,B},n) = \max_{D_0^T \geqslant 0} p\left(\delta RI_0^T - D_0^T\right) + (1-p)\left(\delta qrI_0^T - k + \frac{\delta}{p_{1,B}}\left(k - D_0^T\right)\right)$$

$$D_0^T \leqslant k$$

$$\delta qrI_0^T \geqslant k$$

$$D_0^T + n = I_0^T$$

This program has a non-empty set of solutions if and only if $\frac{k-\delta qrn}{\delta qr} \leqslant k$ (which rewrites $k \leqslant k^\star$).

1. if $p\delta R + (1-p)\delta qr - 1 - (1-p)\frac{\delta - p_{1,B}}{p_{1,B}} < 0$ then $D_0^T = \max(0; \frac{k-\delta qrn}{\delta qr})$ is the solution to this program. We also have $V_0^T(p_{1,B},n) = (p\delta R + (1-p)\delta qr)\,n + (1-p)\frac{\delta - p_{1,B}}{p_{1,B}} k$ if $k \leqslant \delta qrn$ and $V_0^T(p_{1,B},n) = p\left(\delta R(\frac{k-\delta qrn}{\delta qr} + n) - \frac{k-\delta qrn}{\delta qr}\right) + (1-p)\left(\delta qr(\frac{k-\delta qrn}{\delta qr} + n) - k + \frac{\delta}{p_{1,B}}\left(k - \frac{k-\delta qrn}{\delta qr}\right)\right)$ otherwise.
2. if $p\delta R + (1-p)\delta qr - 1 - (1-p)\frac{\delta - p_{1,B}}{p_{1,B}} = 0$ then any $D_0^T \in [\max(0; \frac{k-\delta qrn}{\delta qr}); k]$ is a solution to this program. We also have $V_0^T(p_{1,B},n) = (p\delta R + (1-p)\delta qr)\,n + (1-p)\frac{\delta - p_{1,B}}{p_{1,B}} k$ if $k \leqslant \delta qrn$ and $V_0^T(p_{1,B},n) = p\left(\delta R(\frac{k-\delta qrn}{\delta qr} + n) - \frac{k-\delta qrn}{\delta qr}\right) + (1-p)\left(\delta qr(\frac{k-\delta qrn}{\delta qr} + n) - k + \frac{\delta}{p_{1,B}}\left(k - \frac{k-\delta qrn}{\delta qr}\right)\right)$ otherwise.
3. if $p\delta R + (1-p)\delta qr - 1 - (1-p)\frac{\delta - p_{1,B}}{p_{1,B}} > 0$ then $D_0^T = k$ is the solution to this program. We also have $V_0^T(p_{1,B},n) = \delta(pR + (1-p)qr)(n+k) - k$.

Third subprogram Finally, in the last subprogram we look for solutions D_0^T and I_0^T such that $\delta qrI_0^T \geqslant k$ and $D_0^T \geqslant k$. In this case, the program writes

$$V_0^T(p_{1,B}, n) = \max_{D_0^T \geqslant 0} p\left(\delta RI_0^T - D_0^T\right) + (1-p)\left(\delta qrI_0^T - k + \frac{\delta}{p_{1,B}(1-\varepsilon)}\left(k - D_0^T\right)\right)$$

$$k \leqslant D_0^T \leqslant p_{1,B}(1-\varepsilon)\left(qrI_0^T - \frac{k}{\delta}\right) + k$$

$$\delta qrI_0^T \geqslant k$$

$$D_0^T + n = I_0^T$$

It has a non-empty set of solutions if and only if $\frac{k-\delta qrn}{\delta qr} \leqslant \frac{k\left(1-\frac{p_{1,B}(1-\varepsilon)}{\delta}\right)+(1-\varepsilon)p_{1,B}qrn}{1-p_{1,B}(1-\varepsilon)qr}$ (which rewrites $k \leqslant k^\star$). Then in this case,

1. if $p\delta R + (1-p)\delta qr - 1 - (1-p)\frac{\delta-(1-\varepsilon)p_{1,B}}{p_{1,B(1-\varepsilon)}} < 0$, then $D_0^T = \max(k; \frac{k-\delta qrn}{\delta qr}) = k$, and $V_0^T(p_{1,B}, n) = \delta(pR + (1-p)qr)(n+k) - k$.
2. if $p\delta R + (1-p)\delta qr - 1 - (1-p)\frac{\delta-(1-\varepsilon)p_{1,B}}{(1-\varepsilon)p_{1,B}} = 0$ then any $D_0 \in \left[\max(k; \frac{k-\delta qrn}{\delta qr}); \frac{k\left(1-\frac{p_{1,B}(1-\varepsilon)}{\delta}\right)+(1-\varepsilon)p_{1,B}qrn}{1-p_{1,B}(1-\varepsilon)qr}\right] = \left[k; \frac{k\left(1-\frac{p_{1,B}(1-\varepsilon)}{\delta}\right)+(1-\varepsilon)p_{1,B}qrn}{1-p_{1,B}(1-\varepsilon)qr}\right]$ is an equilibrium, and $V_0^T(p_{1,B}, n) = \delta(pR + (1-p)qr)(n+k) - k$.
3. if $p\delta R + (1-p)\delta qr - 1 - (1-p)\frac{\delta-(1-\varepsilon)p_{1,B}}{(1-\varepsilon)p_{1,B}} > 0$ then $D_0^T = \frac{k\left(1-\frac{p_{1,B}(1-\varepsilon)}{\delta}\right)+(1-\varepsilon)p_{1,B}qrn}{1-p_{1,B}(1-\varepsilon)qr}$. In this case, $V_0^T(p_{1,B}, n) = p\left(\delta R(D_0^T + n) - D_0^T\right)$.

We now distinguish between three cases according to the value of δ.

Case $\delta qrn \geqslant k$ In this case, the first program delivers an empty set of solutions. The overall maximum is therefore the maximum of the two other subprograms.

1. If $p\delta R + (1-p)\delta qr - 1 - (1-p)\frac{\delta-p_{1,B}}{p_{1,B}} < 0, D_0^T = 0,$ and $V_0^T(p_{1,B}, n) = p(\delta Rn) + (1-p)\left(\delta qrn - k + \frac{\delta}{p_{1,B}}k\right)$
2. If $p\delta R + (1-p)\delta qr - 1 - (1-p)\frac{\delta-p_{1,B}}{p_{1,B}} = 0$, any $D_0^T \in [0; k]$ is an optimum debt level and $V_0^T(p_{1,B}, n) = p(\delta R(n+k) - k) + (1-p)(\delta qr(n+k) - k)$
3. If $p\delta R + (1-p)\delta qr - 1 - (1-p)\frac{\delta-p_{1,B}}{p_{1,B}} > 0$, and $p_{1,B} < \delta$, and $p\delta R + (1-p)\delta qr - 1 - (1-p)\frac{\delta-(1-\varepsilon)p_{1,B}}{p_{1,B(1-\varepsilon)}} < 0$, $D_0^T = k$,and $V_0^T(p_{1,B}, n) = p(\delta R(n+k) - k) + (1-p)(\delta qr(n+k) - k)$
4. If $p\delta R + (1-p)\delta qr - 1 - (1-p)\frac{\delta-p_{1,B}}{p_{1,B}} > 0$, and $p_{1,B} < \delta$, and $p\delta R + (1-p)\delta qr - 1 - (1-p)\frac{\delta-(1-\varepsilon)p_{1,B}}{(1-\varepsilon)p_{1,B}} = 0$, any $D_0^T \in [k; \frac{k\left(1-\frac{p_{1,B}(1-\varepsilon)}{\delta}\right)+(1-\varepsilon)p_{1,B}qrn}{1-p_{1,B}(1-\varepsilon)qr}]$ is an optimum debt level and $V_0^T(p_{1,B}, n) = p(\delta R(n+k) - k) + (1-p)(\delta qr(n+k) - k)$
5. If $p\delta R + (1-p)\delta qr - 1 - (1-p)\frac{\delta-(1-\varepsilon)p_{1,B}}{(1-\varepsilon)p_{1,B}} > 0$, $p_{1,B} < \delta$, and $D_0^T = \frac{k\left(1-\frac{p_{1,B}(1-\varepsilon)}{\delta}\right)+(1-\varepsilon)p_{1,B}qrn}{1-p_{1,B}(1-\varepsilon)qr}$, $V_0^T(p_{1,B}, n) = p\left(\delta R\left(D_0^T + n\right) - D_0^T\right)$

Case $\delta q r n < k \leqslant \frac{\delta q r n}{1-\delta q r}$ In this case the three sub-programs admit a non-empty set of solutions. The solution to the program is identical to the case above.

Case $k > \frac{\delta q r n}{1-\delta q r}$ In this case, the last two programs always admit empty set of solutions. Indeed $(k > \delta q r\,(n+k))$. We end up in the first program, and:

1. If $p\delta R+(1-p)\delta q r-1-(1-p)\frac{\delta-p_{1,B}}{p_{1,B}} < 0, D_0^T = 0$, and $V_0^T(p_{1,B}, n) = p\,(\delta R n)+(1-p)\left(\delta q r n - k + \frac{\delta}{p_{1,B}} k\right)$
2. If $p\delta R + (1-p)\delta q r - 1 - (1-p)\frac{\delta-p_{1,B}}{p_{1,B}} = 0$, any $D_0^T \in [0; \frac{k\left(1-\frac{p_{1,B}}{\delta}\right)+p_{1,B} q r n}{1-p_{1,B} q r}]$ is optimal and $V_0^T(p_{1,B}, n) = p\,(\delta R(n+k)-k)+(1-p)\,(\delta q r(n+k)-k)$
3. If $p\delta R + (1-p)\delta q r - 1 - (1-p)\frac{\delta-p_{1,B}}{p_{1,B}} > 0$, and $p_{1,B} < \delta$, $D_0^T = \frac{k\left(1-\frac{p_{1,B}}{\delta}\right)+p_{1,B} q r n}{1-p_{1,B} q r}$,and $V_0^T(p_{1,B}, n) = p\left(\delta R\left(D_0^T+n\right)-D_0^T\right)$

Case $\delta \leqslant p_{1,B} \leqslant \frac{\delta}{(1-\varepsilon)}$. Let's now focus on the case where $\delta \leqslant p_{1,B} \leqslant \frac{\delta}{(1-\varepsilon)}$.

In this case,$\overline{D}_0^T\left(I_0^T, p_{1,B}\right) = k + \left(\delta q r I_0^T - k\right)_- + \frac{p_{1,B}(1-\varepsilon)}{\delta}\left(\delta q r I_0^T - k\right)_+$

$$V_{1,B}^{T,ND}\left(I_0^T, D_0^T, p_{1,B}\right) = \left(\left(k-D_0^T\right)_- \frac{\left(\delta-p_{1,B}\,(1-\varepsilon)\right)_+}{p_{1,B}\,(1-\varepsilon)}\right)+\delta q r I_0^T - D_0^T$$

The program rewrites again in three subprograms.

First subprogram We first focus on the subprogram where we additionally constrain I_0^T to evolve in the set where $\delta q r I_0^T \leqslant k$. In this case

$$\begin{aligned} V_0^T(p_{1,B}, n) = \max_{D_0^T \geqslant 0} p\left(\delta R I_0^T - D_0^T\right)+(1-p)\left(\delta q r I_0^T - D_0^T\right) \\ D_0^T \leqslant \delta q r I_0^T \\ \delta q r I_0^T \leqslant k \\ D_0^T + n = I_0^T \end{aligned}$$

This program has a non-empty set of solution when $q\delta r n \leqslant k$, in which case $D_0^T = \min\left(\frac{k-q\delta r n}{q\delta r}; \frac{\delta q r n}{1-\delta q r}\right)$.

Second subprogram Let's now turn to the subprogram where we additionally constrain D_0^T and I_0^T to evolve in the set where $\delta q r I_0^T \geqslant k$ and $D_0^T \leqslant k$.

In this case, the program writes

$$\begin{aligned} V_0^T(p_{1,B}, n) = \max_{D_0^T \geqslant 0} p\left(\delta R I_0^T - D_0^T\right)+(1-p)\left(\delta q r I_0^T - D_0^T\right) \\ D_0^T \leqslant k \\ \delta q r I_0^T \geqslant k \\ D_0^T + n = I_0^T \end{aligned}$$

This program has a non-empty set of solutions if and only if $\frac{k-q\delta rn}{q\delta r} \leqslant k$, in which case $D_0^T = k$.

Third subprogram Finally, in the last subprogram we additionally constrain D_0^T and I_0^T to evolve in the set where $\delta qrI_0^T \geqslant k$ and $D_0^T \geqslant k$.

In this case,

$$\begin{aligned}
V_0^T(p_{1,B}, n) = &\max_{D_0^T \geqslant 0} p\left(\delta RI_0^T - D_0^T\right) + (1-p)\left(\delta qrI_0^T - k + \frac{\delta}{p_{1,B}(1-\varepsilon)}\left(k - D_0^T\right)\right) \\
&k \leqslant D_0^T \leqslant p_{1,B}(1-\varepsilon)\left(qrI_0^T - \frac{k}{\delta}\right) + k \\
&\delta qrI_0^T \geqslant k \\
&D_0^T + n = I_0^T
\end{aligned}$$

Again, this program has a non-empty set of solutions if and only if $k \leqslant \bar{k}$. In this range,

1. Either $p\delta R + (1-p)\delta qr - 1 - (1-p)\frac{\delta-(1-\varepsilon)p_{1,B}}{p_{1,B(1-\varepsilon)}} < 0$, $D_0^T = \max(k; \frac{k-\delta qrn}{\delta qr}) = k$, and $V_0^T(p_{1,B}, n) = \delta(pR + (1-p)qr)(n+k) - k$.
2. Or $p\delta R + (1-p)\delta qr - 1 - (1-p)\frac{\delta-(1-\varepsilon)p_{1,B}}{(1-\varepsilon)p_{1,B}} = 0$ and any $D_0 \in [\max(k; \frac{k-\delta qrn}{\delta qr}); \frac{k\left(1-\frac{p_{1,B}(1-\varepsilon)}{\delta}\right)+(1-\varepsilon)p_{1,B}qrn}{1-p_{1,B}(1-\varepsilon)qr}] = [k; \frac{k\left(1-\frac{p_{1,B}(1-\varepsilon)}{\delta}\right)+(1-\varepsilon)p_{1,B}qrn}{1-p_{1,B}(1-\varepsilon)qr}]$ is an equilibrium, and $V_0^T(p_{1,B}, n) = \delta(pR + (1-p)qr)(n+k) - k$.
3. Or $p\delta R + (1-p)\delta qr - 1 - (1-p)\frac{\delta-(1-\varepsilon)p_{1,B}}{(1-\varepsilon)p_{1,B}} > 0$ and $D_0^T = \frac{k\left(1-\frac{p_{1,B}(1-\varepsilon)}{\delta}\right)+(1-\varepsilon)p_{1,B}qrn}{1-p_{1,B}(1-\varepsilon)qr}$. In this case, $V_0^T(p_{1,B}, n) = p\left(\delta R(D_0^T + n) - D_0^T\right)$.

We can now solve for the time 0 bank program.

Case $\delta qrn \geqslant k$. In this case, as before, we necessarily end up in the last two programs. Hence,

1. If $p\delta R + (1-p)\delta qr - 1 - (1-p)\frac{\delta-(1-\varepsilon)p_{1,B}}{p_{1,B(1-\varepsilon)}} < 0$, $D_0^T = k$,and $V_0^T(p_{1,B}, n) = p\left(\delta R(n+k) - k\right) + (1-p)\left(\delta qr(n+k) - k\right)$
2. If $p\delta R + (1-p)\delta qr - 1 - (1-p)\frac{\delta-(1-\varepsilon)p_{1,B}}{p_{1,B(1-\varepsilon)}} = 0$, any $D_0^T \in [k; \frac{k\left(1-\frac{p_{1,B}(1-\varepsilon)}{\delta}\right)+(1-\varepsilon)p_{1,B}qrn}{1-p_{1,B}(1-\varepsilon)qr}]$ is an optimum and $V_0^T(p_{1,B}, n) = p\left(\delta R(n+k) - k\right) + (1-p)\left(\delta qr(n+k) - k\right)$
3. If $p\delta R + (1-p)\delta qr - 1 - (1-p)\frac{\delta-(1-\varepsilon)p_{1,B}}{(1-\varepsilon)p_{1,B}} > 0$, $D_0^T = \frac{k\left(1-\frac{p_{1,B}(1-\varepsilon)}{\delta}\right)+(1-\varepsilon)p_{1,B}qrn}{1-p_{1,B}(1-\varepsilon)qr}$, and $V_0^T(p_{1,B}, n) = p\left(\delta R\left(D_0 + n\right) - D_0\right)$

Case $\delta qrn < k \leqslant \frac{\delta qrn}{1-\delta qr}$ The three subprograms have a non-empty set of solutions, and the equilibria of the T-bank at time 0 are the same as in the case above. Indeed, on this range, the value function of the first subprogram is weakly dominated by the one of the second subprogram (strictly if $k < \bar{k}$), and we can disregard it and perform the same analysis as in the case above.

Case $k > \frac{\delta qrn}{1-\delta qr}$ Here, the only subprogram with non-empty solution set is the first one. In this case the optimum is such that $D_0^T = \frac{\delta qrn}{1-\delta qr}$, $V_0^T(p_{1,B}, n) = p(\delta RI - D_0) + (1-p)\left(\delta qrI - D_0^R\right)$.

Case $\frac{\delta}{(1-\varepsilon)} \leqslant p_{1,B} < \frac{1}{(1-\varepsilon)qr}$. Here, the program rewrites

$$V_0^T(p_{1,B}, n) = \max_{D_0^T \geqslant 0} p\left(\delta RI_0^T - D_0^T\right) + (1-p)\left(p_{1,B}(1-\varepsilon)qrI_0^T - D_0^T\right)$$

$$D_0^T \leqslant p_{1,B}(1-\varepsilon)qrI_0^T$$

$$D_0^T + n = I_0^T$$

And $D_0^T = \frac{p_{1,B}(1-\varepsilon)qrn}{1-p_{1,B}((1-\varepsilon)qr)}$ and $V_0^T(p_{1,B}, n) = p\left(\delta RI_0^T - D_0^T\right)$

Case $p_{1,B} \geqslant \frac{1}{(1-\varepsilon)qr}$. Finally, if $p_{1,B} \geqslant \frac{1}{(1-\varepsilon)qr}$, $D_0^T = +\infty$ and $V_0^T(p_{1,B}, n) = +\infty$.

□

B.4 Asset demand and supply schedules: Proof of Proposition 4

Proof. Using lemmas B.2.1 and 3, Propositions 12 and 13, and Condition 4, we summarize the optimal choices of T- and S-banks.

1. If $p_{1,B} = 0$, $V_0^T(p_{1,B}, n) = +\infty$,

2. If $0 < p_{1,B} < p_{1,L}^T$, $D_0^T = 0$, $V_0^T(p_{1,B}, n) = p(\delta Rn) + (1-p)\left(\delta qrn - k + \frac{\delta}{p_{1,B}}k\right)$, $I_{1,B}^T = \frac{k}{p_{1,B}qr}$, $\alpha_{1,B}^T = 1$

3. If $p_{1,B} = p_{1,L}^T$, any $D_0^T \in [0; k]$ is an equilibrium debt level, and $V_0^T(p_{1,B}, n) = p(\delta R(n+k) - k) + (1-p)(\delta qr(n+k) - k)$, $I_{1,B}^T = \frac{k - D_0^T}{p_{1,B}qr} \in [0; \frac{k}{p_{1,B}qr}]$, $\alpha_{1,B}^T = 1$

4. If $p_{1,L}^T < p_{1,B} < p_{1,H}^T$, $D_0^T = k$,and $V_0^T(p_{1,B}, n) = p(\delta R(n+k) - k) + (1-p)(\delta qr(n+k) - k)$, $I_{1,B}^T = 0$, $\alpha_{1,B}^T = 1$

5. If $p_{1,B} = p_{1,H}^T$, any $D_0^T \in [k; \frac{k\left(1 - \frac{p_{1,B}(1-\varepsilon)}{\delta}\right) + (1-\varepsilon)p_{1,B}qrn}{1 - p_{1,B}(1-\varepsilon)qr}]$ is an optimal debt level, $V_0^T(p_{1,B}, n) = p(\delta R(n+k) - k) + (1-p)(\delta qr(n+k) - k)$, $I_{1,B}^T = 0$, $(1 - \alpha_{1,B}^T) = \frac{D_0^T - k}{p_{1,B}qrI_0^T(1-\varepsilon)}$ hence $\alpha_{1,B}^T \in [1 - \frac{qr(n+k) - \frac{k}{\delta}}{qr\left(k\left(1 - \frac{p_{1,B}(1-\varepsilon)}{\delta}\right) + n\right)}; 1]$,

6. If $p_{1,H}^T < p_{1,B} < \frac{\delta}{1-\varepsilon}$, $D_0^T = \frac{k\left(1 - \frac{p_{1,B}(1-\varepsilon)}{\delta}\right) + (1-\varepsilon)p_{1,B}qrn}{1 - p_{1,B}(1-\varepsilon)qr}$, $V_0^T(p_{1,B}, n) = p(\delta R(D_0 + n) - D_0)$, $I_{1,B}^T = 0$, $\alpha_{1,B}^T = 1 - \frac{qr(n+k) - \frac{k}{\delta}}{qr\left(k\left(1 - \frac{p_{1,B}(1-\varepsilon)}{\delta}\right) + n\right)}$

7. If $p_{1,B} = \frac{\delta}{1-\varepsilon}$, $D_0^T = \frac{(1-\varepsilon)p_{1,B}qrn}{1 - p_{1,B}(1-\varepsilon)qr}$, $V_0^T(p_{1,B}, n) = p(\delta R(D_0 + n) - D_0)$, $I_{1,B}^T = 0$, $\alpha_{1,B}^T \in \left[0; \frac{k(1-\delta qr)}{qrn}\right]$

8. If $p_{1,B} > \frac{\delta}{1-\varepsilon}$, $D_0^T = \frac{p_{1,B}(1-\varepsilon)qrn}{(1-p_{1,B}(1-\varepsilon)qr)_+}$, $V_0^T(p_{1,B}, n) = p\left(\delta R\left(D_0^T + n\right) - D_0^T\right)$, $I_{1,B}^T = 0$, $\alpha_{1,B}^T = 0$

The net aggregate demand of assets by T-banks at $t = 1$ in state G asset market is

$$D^{TB}(p_{1,B}) = \frac{k - D_0^T(p_{1,B}, n)}{p_{1,B}qr} \times (1 - \chi^S). \tag{25}$$

This is strictly negative when $p_{1,B} > p_{1,H}^T$. As S-banks net supply is always positive, no market clearing can occur for such price levels.

For $0 \leqslant p_{1,B} \leqslant p_{1,H}^T$, the aggregate demand for assets is

$$D(p_{1,B}) = \begin{cases} \frac{k}{p_{1,B}qr}(1-\chi^S) & \text{if } 0 \leqslant p_{1,B} < \overline{p}_{1,L}^T \\ \in \left[0; \frac{k}{p_{1,B}qr}(1-\chi^S)\right] & \text{if } p_{1,B} = \overline{p}_{1,L}^T \\ 0 & \text{if } p_{1,B} > \overline{p}_{1,L}^T \end{cases}$$

while the aggregate supply of assets is:

$$S(p_{1,B}) = \begin{cases} 0 & \text{if } 0 \leqslant p_{1,B} < \overline{p}_1^S \\ \in \left[0; \frac{n(1-\varepsilon)}{1-(1-\varepsilon)p_{1,B}qr}\chi^S\right] & \text{if } p_{1,B} = \overline{p}_1^S \\ \frac{n(1-\varepsilon)}{1-(1-\varepsilon)p_{1,B}qr}\chi^S & \text{if } \overline{p}_1^S < p_{1,B} < \overline{p}_{1,H}^T \\ \in \left[\frac{n(1-\varepsilon)}{1-(1-\varepsilon)p_{1,B}qr}\chi^S; \frac{n(1-\varepsilon)}{1-(1-\varepsilon)p_{1,B}qr}\chi^S + \frac{(1-\varepsilon)}{qr}\frac{\left(-\frac{k}{\delta}\right)+qr(n+k)}{1-p_{1,B}(1-\varepsilon)qr}\left(1-\chi^S\right)\right] & \text{if } p_{1,B} = \overline{p}_{1,H}^T \end{cases}$$

Proposition 4 obtains. □

Depending on the value of χ^S, the asset market clears at prices, as detailed in Proposition 5 and Appendix B.5.

B.5 Asset market equilibria: Proof of Proposition 5

Proof. Using Proposition 4, we obtain that in equilibrium:

1. If $\chi^S = 0$, then $D(p_{1,B}) = S(p_{1,B}) = 0$, and $\overline{p}^T_{1,L} \leqslant p_{1,B} \leqslant \overline{p}^T_{1,H}$. No assets are traded.
2. If $\chi^S \in (0;1)$, $D(p_{1,B}) = S(p_{1,B})$, and
   (a) either $\chi^S \geqslant \frac{\frac{k}{\overline{p}^S_1 qr}}{\frac{n(1-\varepsilon)}{1-(1-\varepsilon)\overline{p}^S_1 qr} + \frac{k}{\overline{p}^S_1 qr}}$ and $D(p_{1,B}) = S(p_{1,B}) = \frac{k}{\overline{p}^S_1 qr}(1-\chi^S)$, and $p_{1,B} = \overline{p}^S_1$.
   (b) Or $\chi^S \in \left[\frac{\frac{k}{\overline{p}^T_{1,L} qr}}{\frac{k}{\overline{p}^T_{1,L} qr} + \frac{n(1-\varepsilon)}{1-(1-\varepsilon)\overline{p}^T_{1,L} qr}}; \frac{\frac{k}{\overline{p}^S_1 qr}}{\frac{n(1-\varepsilon)}{1-(1-\varepsilon)\overline{p}^S_1 qr} + \frac{k}{\overline{p}^S_1 qr}}\right]$, $D(p_{1,B}) = S(p_{1,B}) = \frac{k}{p_{1,B} qr}(1-\chi^S) = \frac{n(1-\varepsilon)}{1-(1-\varepsilon)p_{1,B} qr}\chi^S$, and $p_{1,B} = \frac{1}{\frac{\chi^S n}{k(1-\chi^S)}+1}\frac{1}{qr(1-\varepsilon)} \in [\overline{p}^S_{1,}; \overline{p}^T_{1,L}]$.
   (c) Or $\chi^S \leqslant \frac{\frac{k}{\overline{p}^T_{1,L} qr}}{\frac{k}{\overline{p}^T_{1,L} qr} + \frac{n(1-\varepsilon)}{1-(1-\varepsilon)\overline{p}^T_{1,L} qr}}$ and $D(p_{1,B}) = S(p_{1,B}) = \frac{n(1-\varepsilon)}{1-(1-\varepsilon)\overline{p}^T_{1,L} qr}\chi^S$, $p_{1,B} = \overline{p}^T_{1,L}$.
3. If $\chi^S = 1$, $D(p_{1,B}) = S(p_{1,B}) = 0$, and $0 \leqslant p_{1,B} \leqslant \overline{p}^S_1$. No assets are traded.

Defining $\underline{\chi}^S = \frac{\frac{k}{\overline{p}^S_1 qr}}{\frac{n(1-\varepsilon)}{1-(1-\varepsilon)\overline{p}^S_1 qr} + \frac{k}{\overline{p}^S_1 qr}}$ and $\overline{\chi}^S = \frac{\frac{k}{\overline{p}^T_{1,L} qr}}{\frac{k}{\overline{p}^T_{1,L} qr} + \frac{n(1-\varepsilon)}{1-(1-\varepsilon)\overline{p}^T_{1,L} qr}}$, Proposition 5 obtains. □

B.6 The complete allocation program

We now endogenize bankers' choice to initially set up a T- or a S-bank. Bankers compare expected profits for each i-bank ($i = \{T, S\}$), i.e. value functions at $t = 0$, and choose a probability χ^S ($(1-\chi^S)$) to set up a S-bank (T-bank) such as to solve

$$\max_{\chi^S \in [0;1]} \chi^S V^S_0 (p_{1,B}) + (1-\chi^S) V^T_0 (p_{1,B})$$

where $p_{1,B}$ is the asset price at $t = 1$ in state B. We define:

$$\Delta (p_{1,B}) \equiv V^T_0 (p_{1,B}) - V^S_0 (p_{1,B})$$

Recall that equilibrium asset prices at $t = 1$ in state B belong to the interval $[0; \overline{p}^T_{1,H}]$. It is therefore sufficient to study $\Delta (p_{1,B})$ on this interval. $\Delta (p_{1,B})$ is a continuous, strictly decreasing function on $[0; \overline{p}^T_{1,H}]$. Since $\Delta(0) = +\infty$, it can cancel at most once on this interval. We obtain the following Proposition:

Proposition 14 (Allocation program)**.** *Defining* $S = \Delta^{-1}(0) \cap [0; \overline{p}^T_{1,H}]$, *the allocation program solves as follows*

1. *If* $S = \emptyset$, $\chi^S = 0$.
2. *Otherwise, denoting* $p^*_{1,B} \equiv \Delta^{-1}(0) \cap [0; \overline{p}^T_{1,H}]$, *we have:*

$$\chi^S = \begin{cases} 0 & \text{if } p_{1,B} \in [0; p^*_{1,B}) \\ \in [0;1] & \text{if } p_{1,B} = p^*_{1,B} \\ 1 & \text{if } p_{1,B} \in (p^*_{1,B}; \overline{p}^T_{1,H}]. \end{cases}$$

B.7 Bankers' equilibrium choices between T- and S-banks: Proof of Proposition 6

Proof. We have detailed the different parts of our equilibrium, which we can now characterize. In all equilibria, bankers invest all their initial endowment in their bank at $t = 0$, which becomes the bank's equity and is entirely invested in assets. At $t = 1$ in state G no assets are traded and banks repay their creditors by issuing debt ($p_{1,G} \in [1; \frac{\delta}{1-\varepsilon}]$). At each date in all states, all riskless short-term debt (if any) is sold at par to households. Finally, bankers' equilibrium allocation between the T- and S-banking sectors depends on $p_{1,B}$. It can be of 5 types as follows:

1. Either $\Delta(\overline{p}_1^S) < 0$. In this case, there is a unique $p_{1,B}^* \in (0; \overline{p}_1^S)$ such that $\Delta\left(p_{1,B}^*\right) = 0$. Then $\chi^S = 1$ is the unique equilibrium allocation, and any $p_{1,B} \in [p_{1,B}^*; \overline{p}_1^S]$ is an equilibrium asset price at $t = 1$ in state B. No assets are traded in these equilibria, and only S banks exist. They don't issue any form of risk-free debt.

2. Or $\Delta(\overline{p}_1^S) = 0$. In this case, any χ^S such that $\frac{\frac{k}{\overline{p}_1^S qr}}{\frac{n(1-\varepsilon)}{1-(1-\varepsilon)\overline{p}_1^S qr} + \frac{k}{\overline{p}_1^S qr}} \leqslant \chi^S$ is an equilibrium allocation, and $p_{1,B} = \overline{p}_1^S$. Either only S-banks exist, which don't issue debt and invest all their endowment in assets, or T- and S-banks coexist, and T-banks issue debt both at $t = 0$ to invest in assets, and at $t = 1$ in state B to purchase assets from S-banks.

3. Or $\Delta(\overline{p}_1^S) > 0$ and $\Delta(\overline{p}_1^T) < 0$. Then, there is a unique $p_{1,B}^* \in (\overline{p}_1^S; \overline{p}_1^T)$ such that $\Delta\left(p_{1,B}^*\right) = 0$. In this case $\chi^S = \frac{1}{1+\frac{p_{1,B}^* qr}{1-(1-\varepsilon)p_{1,B}^* qr}\frac{n(1-\varepsilon)}{k}}$ is the unique equilibrium allocation, and $p_{1,B} = p_{1,B}^*$ is the unique equilibrium market price at $t = 1$ in state B. T- and S-banks coexist, and T-banks do not issue debt at $t = 0$, but issue debt at $t = 1$ in state B to purchase assets from S-banks. S-banks issue debt at $t = 0$, and sell assets at $t = 1$ in state B to repay their creditors.

4. Or $\Delta(\overline{p}_1^T) = 0$. In this case, any χ^S such that $\frac{\frac{k}{\overline{p}_1^T qr}}{\frac{n(1-\varepsilon)}{1-(1-\varepsilon)\overline{p}_1^T qr} + \frac{k}{\overline{p}_1^T qr}} \geqslant \chi^S$ is an equilibrium allocation, and $p_{1,B} = \overline{p}_1^T$. Either T-banks only exist, in which case they issue an amount k of debt at $t = 0$. Or T- and S-banks coexist, in which case T-banks issue less debt at $t = 0$, to keep slack and issue debt at $t = 1$ in state B to purchase assets from S-banks.

5. Or $\Delta(\overline{p}_1^T) > 0$. In this case $\chi^S = 0$ and any $p_{1,B} \in \left[\overline{p}_{1,L}^T; \overline{p}_{1,H}^T\right]$ such that $\Delta(p_{1,B}) \geqslant 0$ is an equilibrium market price. No assets are traded in equilibrium. Only T-banks exist, and they issue k units of debt at $t = 0$, which they repay by issuing debt at $t = 1$, both in state B and in state G.

Proposition 6 describes the above equilibrium of type 3, i.e. the coexistence equilibrium. □

B.8 Parametric conditions for T- and S-banks to coexist

For T- and S-banks to coexist in equilibrium, the parameter set needs to satisfy two conditions to reach an equilibrium of type 3 (see Appendix B.7 above): $\Delta(\overline{p}_1^S) > 0$ and $\Delta(\overline{p}_1^T) < 0$. Recall first that

$$\Delta : p_{1,B} \to V_0^{T,B}\left(p_{1,B}\right) - V_0^{S,B}\left(p_{1,B}\right).$$

Now, recall that:

$$\begin{aligned}
V_0^S\left(\overline{p}_1^S\right) &= (1-p)qrn + pRn && \text{from Proposition 11}\\
V_0^S\left(\overline{p}_{1,L}^T\right) &= pn\left(\frac{R-(1-\varepsilon)\,\overline{p}_{1,L}^T qr}{1-(1-\varepsilon)\,\overline{p}_{1,L}^T qr}\right) && \text{from Proposition 11}\\
V_0^T\left(\overline{p}_1^S\right) &= p\left(\delta Rn\right) + (1-p)\left(\delta qrn - k + \frac{\delta}{\overline{p}_1^S}k\right) && \text{from Proposition 13}\\
V_0^T\left(\overline{p}_{1,L}^T\right) &= p\left(\delta R(n+k) - k\right) + (1-p)\left(\delta qr(n+k) - k\right) && \text{from Proposition 13}
\end{aligned}$$

where

$$\begin{aligned}
\overline{p}_1^S &\equiv \frac{1}{(1-\varepsilon)(qr + \frac{p(R-1)}{1-p})}\\
\overline{p}_{1,L}^T &\equiv \frac{\delta}{\delta qr + \frac{p(\delta R-1)}{1-p}}
\end{aligned}$$

The first parametric restriction $\Delta(\overline{p}_1^S) > 0$ yields

$$k > \frac{n(1-\delta)\left(pR + (1-p)qr\right)}{\left(p(R-1) + (1-p)qr\right)\delta(1-\epsilon) - (1-p)},$$

and the second parametric restriction $\Delta(\overline{p}_{1,L}^T) < 0$ yields

$$k < \left[p\frac{R\left[\delta\left(pR + (1-p)qr\right) - p\right] - \delta(1-p)qr(1-\epsilon)}{\delta\left(pR + (1-p)qr\right) - p - \delta(1-p)qr(1-\epsilon)} - \delta\left(pR + (1-p)qr\right)\right]\frac{n}{\delta\left(pR + (1-p)qr\right) - 1}.$$

B.9 Changes in the deposit insurance limit: Proof of Proposition 7

Proof. In the coexistence equilibrium, $V_0^S = V_0^T$ with $V_0^S = \left[\frac{p(R-(1-\varepsilon)p_{1,B}^* qr)}{1-(1-\varepsilon)p_{1,B}^* qr}\right]n$, $V_0^T = \left[p\left(\delta R\right) + (1-p)\left(\delta qr + \frac{\delta - p_{1,B}^*}{p_{1,B}^*}\frac{k}{n}\right)\right]n$, and $p_{1,B}^* == \frac{\frac{k}{n}}{(1-\varepsilon)qr\left(\frac{\chi^{S*}}{(1-\chi^{S*})} + \frac{k}{n}\right)}$ is the market equilibrium price for assets at $t = 1$ in state B. Recall that for parameter values such that this equilibrium exists, it is uniquely defined (see Appendix B.8). We obtain:

$$\begin{aligned}
V_0^S &= npR + pk(R-1)\frac{1-\chi^{S*}}{\chi^{S*}}\\
V_0^T &= np\delta R + (1-p)\left(\delta qrn + \left(\delta\left(1-\varepsilon\right)qr - 1\right)k + \delta\left(1-\varepsilon\right)qr\frac{\chi^{S*}}{(1-\chi^{S*})}n\right)
\end{aligned}$$

Increases in k changes banks' value function: $\frac{\partial V_0^T}{\partial k} = (1-p)\left(\delta(1-\varepsilon)qr - 1\right)$, and $\frac{\partial V_0^S}{\partial k} = p\,(R-1)\,\frac{\chi^{S*}}{(1-\chi^{S*})}$. Condition 5 is a sufficient, though not necessary, condition for $\frac{\partial V_0^S}{\partial k} > \frac{\partial V_0^S}{\partial k}$, i.e. $qr < \frac{1}{\delta(1-\varepsilon)}\left[1 + \frac{p}{1-p}\frac{1-\chi^S}{\chi^S}(R-1)\right]$. QED. □

B.10 Changes in capital requirements: Proof of Proposition 8

Proof. We replace T-banks' deposit insurance limit (Assumption 1) by capital requirements as given in (9). We assume that the capital constraint is stringent enough (rhs) for this constraint to bind in equilibrium, and that T-banks do not sell assets to repay their creditors (lhs): $\delta(1-\varepsilon)qr \leqslant (1-c) \leqslant \delta qr$.

We obtain that in equilibrium, T-banks do not default at $t = 1$ in state B. As in Appendix B.3.1, this implies that

$$D_0^T \leqslant (1-c)I_0^T,$$

and T-banks' value function at $t = 1$ in state B obtains (as in Appendix B.3.2):

$$V_{1,B}^T = \delta qr I_0^T - D_0^T + \left(\frac{\delta - p_{1,B}}{c p_{1,B}}\right)\left((1-c)I_0^T - D_0^T\right)_+ .$$

Now, at $t = 0$, T-banks' budget constraint is binding, i.e. $I_0^T = D_0^T + n$, such that capital requirements at $t = 1$ imply $D_0 \leqslant \frac{1-c}{c}n$, and for asset prices such that T- and S-banks coexist ($p_{1,B} < p_{1,L}^T$), T-banks' value function at $t = 0$ writes:

$$V_0^T = p\delta Rn + (1-p)\left(\delta qrn + (1-c)\frac{\delta - p_{1,B}}{c p_{1,B}}\right).$$

For asset prices such that T- and S-banks trade assets at $t = 1$ in state B (as in Appendix B.5), we have that the equilibrium asset price is:

$$p_{1,B}^* = \frac{1}{(1-\varepsilon) + \frac{c}{1-c}\frac{\chi^S}{1-\chi^S}},$$

and in the coexistence equilibrium (see Appendix B.8), $\Delta(\overline{p}_1^S) > 0$ and $\Delta(\overline{p}_1^T) < 0$. We then have $V_0^T = V_0^S$, which rewrites:

$$\left[p\alpha R + (1-p)\delta + (1-p)\frac{(1-c)}{c}\left(\delta\,(1-\varepsilon) - 1\right) + (1-p)\alpha\frac{\chi^S}{1-\chi^S}\right] = pR + p\,(R-1)\,(1-\varepsilon)\,\frac{1-\chi^S}{\chi^S}\frac{1-c}{c}.$$

We make the following change of variables: $u = \frac{(1-c)}{c}$ and $v = \frac{1-\chi^S}{\chi^S}$. The implicit function theorem yields

$$\frac{dv}{du} = \frac{(1-p)\left(\alpha\,(1-\varepsilon) - 1\right) - p\,(R-1)\,(1-\varepsilon)\,v}{p\,(R-1)\,(1-\varepsilon) + \frac{(1-p)\delta}{v^2}} < 0,$$

Therefore the relative size of T-banks $(1-\chi^S)$ is increasing in capital requirements (c). QED. □

B.11 Normative Approach: Proof of Proposition 9

Proof. Using Propositions 1 and 3 together with the conclusions from Appendix B.6, we rewrite the objective function of the central planner $W(\chi^S) = \chi^S V_0^{S,B}(p_{1,B}) + (1-\chi^S)V_0^{T,B}(p_{1,B})$ as a piecewise linear function of χ^S:

$$W(\chi^S) = \begin{cases} \chi^S V_0^{S,B}(\overline{p}_{1,L}^T) + (1-\chi^S)V_0^{T,B}(\overline{p}_{1,L}^T) & \text{if } \chi^S \in [0; \underline{\chi}^S] \\ \beta + \gamma\chi^S & \text{if } \chi^S \in [\underline{\chi}^S; \overline{\chi}^S] \\ \chi^S V_0^{S,B}(\overline{p}_1^S) + (1-\chi^S)V_0^{T,B}(\overline{p}_1^S) & \text{if } \chi^S \in [\overline{\chi}^S; 1] \end{cases}$$

where $\beta = p\,(R-1)\,k + np\delta R + (1-p)\,[\delta qrn + (\delta(1-\varepsilon)qr-1)k]$, $\gamma = n[pR - \delta[pR + (1-p)\varepsilon qr]] + k[1 - [pR + (1-p)\delta(1-\varepsilon)qr]]$, $\underline{\chi}^S = \frac{\frac{k}{\overline{p}_1^S qr}}{\frac{n(1-\varepsilon)}{1-(1-\varepsilon)\overline{p}_1^S qr} + \frac{k}{\overline{p}_1^S qr}}$, and $\overline{\chi}^S = \frac{\frac{k}{\overline{p}_{1,L}^T qr}}{\frac{k}{\overline{p}_{1,L}^T qr} + \frac{n(1-\varepsilon)}{1-(1-\varepsilon)\overline{p}_{1,L}^T qr}}$.

We focus on parametric conditions in which the decentralized equilibrium is such that T- and S-banks coexist (see Appendix B.6), i.e. $\chi^{S*} \in [0,1]$. Under these parametric conditions, $W(.)$ is strictly increasing on $[0; \underline{\chi}^S]$ and strictly decreasing on $[\overline{\chi}^S; 1]$. Moreover, $V_0^{T,B}(\overline{p}_1^S) > V_0^{S,B}(\overline{p}_1^S)$ which rewrites $[pR + (1-p)qr][1-\delta]n < k[(1-\varepsilon)\delta[p(R-1) + (1-p)qr] - (1-p)]$, implying that $\gamma < 0$. Therefore $W(.)$ is strictly decreasing on $[\underline{\chi}^S; \overline{\chi}^S]$ and the constrained optimum allocation (χ^{S**}) is uniquely obtained for $\chi^{S**} = \underline{\chi}^S < \chi^{S*}$. QED. $\square$