Means of Payment and Timing of Mergers and Acquisitions in a Dynamic Economy*

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Abstract

We develop a theory of acquisition timing and means of payment when potential acquirers are financially constrained. Bidders with private valuations choose when to approach the target and whether to bid in cash or stock. A bidder's ability to pay cash is limited by a cash constraint. We solve for the equilibrium initiation strategies and study the interrelation between bidders' cash constraints, acquisition timing, and properties of the deal. Because of ability to bid in stock, a cash constraint has no effect on the bidder's maximum willingness to pay. Yet it affects a bidder's incentive to initiate a bid. While a cash constraint usually makes a bidder reluctant to initiate a bid for the target, the effect can be opposite if the target is a high-growth high-synergy firm. Cash constraints of other bidders typically make a bidder more reluctant to initiate a bid. The model delivers many implications, both novel and consistent with existing evidence. For example, high-synergy targets tend to be approached when they are young and small, and acquired for cash. In contrast, low-synergy targets are acquired after they have grown, and using stock. Acquisitions are driven not only by fundamentals but also by bidders' ability to pay cash. Finally, some targets are never acquired despite positive synergies.

Keywords: Auctions, financial constraints, mergers and acquisitions, real options, security design.

The decision to acquire a target is one of the most important choices that the firm's management and board of directors face, with the potential to gain or lose millions and billions in profit.[1] It is therefore important to understand how these multifaceted decisions are made and what factors affect them. While according to the neoclassical theory of mergers, the only driver of a merger should be the net total gains created from the deal, it appears that the ability of bidders to pay cash and, more broadly, access to finance is also important.[2]

While appealing, the link between bidders' cash constraints on their propensity to make acquisitions is not obvious. After all, if a bidder and the target find the deal worthwhile, they can agree to make the payment in stock, in case the bidder is unable to pay cash. The goal of this paper is provide a theoretical analysis of a bidder's decision to bid for the target in the presence of a cash constraint. We build a real-options model of acquisitions based on two simple assumptions: (i) a bidder can choose when to approach the target with an offer; (ii) its ability to pay cash is limited by a cash constraint. Our analysis has three main insights. First, a bidder's cash constraint as well as cash constraints of other potential acquirers matter for the decision of a bidder to initiate a bid for the target. This is so despite its ability to bid in stock. Second, a bidder's cash constraint matters not in an obvious way. In particular, while a cash constraint usually makes a bidder reluctant to initiate a bid, it is not always so. For some targets (high-growth and high-synergy), inability of a bidder to pay cash increases incentives to initiate a bid. Finally, the model delivers many implications on the relation of means of payment in acquisitions, synergies, cash constraints, and the distribution of gains among the contest participants. In addition, we provide several novel predictions. Many of these implications are consistent with existing empirical evidence, and some have not been looked at yet.

More specifically, we consider a dynamic model in which there are three agents: a target and two potential bidders. The target is a growth firm: its assets and cash flows grow over time with some uncertainty. Both bidders are mature companies: the bidder's assets and cash flows do not grow unless it acquires the target.[3] The bidders have privately-known synergies with the target: an acquisition improves productivity of the target in a combined company by a bidder-specific multiple. At any time each bidder can approach the target with an offer. Once a bidder makes a bid, the auction between

[1] In 2007 alone, the value of world-wide deal volume exceeded $4.8 trillion.

[2] For neoclassical arguments, see, e.g., Mitchell and Mulherin (1996), Jovanovic and Rousseau (2002), and Lambrecht (2004). For evidence on relation of M&A to the costs of borrowing see Harford (2005).

[3] An alternative interpretation of the framework is that the target's assets and cash flows change relative to those of the bidders.

the first bidder and the competitor is initiated, and the bidder who submits the highest bid wins the auction. A bidder's decision when to approach the target reflects the following trade-off. On one hand, approaching the target early leads to an earlier increase in its productivity. On the other hand, a deal involves a cost: If the bidder loses the auction, its post-merger value will diminish, because it will face a stronger competitor. In addition, initiating the bid for the target today destroys the option to acquire the target in the future. If the bidder's valuation of the target is low, it is optimal to wait until the target grows in size so that the increase in its productivity outweighs the cost of the acquisition.

The second building block of the model is information asymmetry between the target and the bidders. Similarly to the literature on auctions, but unlike the prior literature that considers takeovers in the real-options framework, we assume that potential synergies from acquiring the target are the private information of the bidder. As shown in the literature on securities auctions, this feature makes bids in stock and in cash not equivalent, in contrast to the case when bidders do not have any private information. Specifically, because the value of a bid in stock (but not in cash) depends on the bidder's private information, it is costlier for a bidder to separate itself from a marginally lower type in a stock auction than in a cash auction. Even if both stock bidders offer the same proportion of the combined company to the target's shareholders, the bidder with the higher valuation will end up paying more in cash equivalent. Because of this effect, each bidder wants to bid in cash whenever possible. The ability to do this is, however, limited by the financing constraint of the bidder. We model it by assuming that the bidder cannot pay in cash above a certain limit.

We initially solve for the equilibrium initiation strategies and terms of takeovers in three special cases of the model: both bidders are unconstrained and thus bid in cash; both bidders are extremely constrained and thus bid in stock; and one bidder is unconstrained, while the other is extremely constrained. This model is convenient for the analysis of the effects of cash constraints on the timing of acquisitions but is limited, because it does not have endogenous means of payment.

Our first result concerns the link between a bidder's cash constraint and its decision to initiate a bid. We show that there are two opposite effects. The first, static, effect is that bidding in stock transfers surplus from the winning bidder to the seller. As a result, all else equal, the bidder's expected payoff from the auction is lower if the bidder is more cash constrained. This higher payoff from option exercise leads to an earlier exercise, i.e., an earlier bidding. The second, dynamic, effect is that the fraction of the total surplus that the winning bidder obtains, all else equal, decreases as the target grows over time. Intuitively, if the target is very small, there is little difference between bids in cash and in stock. However, the difference is substantial if the target is large. This dynamic effect has the opposite impact on the bidder's decision to bid for the target: because of it, a more constrained bidder benefits from not postponing the bid. If the target does not grow very quickly or the bidder's synergy is not too high, the first effect always dominates, and cash constraints always make the bidder reluctant to bid for the target. However, if the target operates in a very high-growth industry and the bidder has a high enough valuation of the target, the second effect may dominate, and constraints can speed-up the acquisition.

Second, we show that a bidder's decision to initiate a bid is affected not only by its own cash constraint, but also by the cash constraint of its rival. In the "normal" case when cash constraints delay acquisitions, cash constraints of the rival bidder have the same directional effect as the bidder's own cash constraints. In other words, an unconstrained bidder is more reluctant to initiate a bid if the rival bidder is constrained than if it is unconstrained. This occurs because of learning of a bidder about the valuation of its rival from observing that the rival has not initiated the bid for the target yet. When the rival is constrained, it is more reluctant to bid for the target for the same valuation. Hence, observing that the constrained rival has not approached the target yet, the other bidder does not update its estimate of the rival bidder's valuation as much. In its eyes, the bidder faces a stronger competitor at each date. As a result, this bidder obtains a lower payoff from the auction in expectation, which make it reluctant to bid for the target too.

To endogenize means of payment, we provide solution of the general model with arbitrary cash constraints. Here, we show that high-synergy targets are typically acquired young and for cash, while low-synergy targets are typically acquired old (if at all, despite positive synergies) and for stock. Intuitively, if the bidder expects high synergies, it does not pay off to wait, so the target is acquired when small. As a result, for an acquirer, the required payment is likely to be below the financing constraint, leading to deals done in cash. Because of high synergies, such deals are also likely to result in high takeover premiums (relative to the current value of the target under its current management). Thus, the model predicts that in a sample of deals, cash deals can be associated with higher takeover premiums, despite that stock deals are perceived as more expensive by bidders. This finding is broadly consistent with empirical evidence (e.g., Betton, Eckbo, and Thorburn, 2008). While this evidence can seem inconsistent with predictions of security-bid auctions literature, it becomes consistent once dynamic selection of targets by bidders into cash and stock deals is taken into account.

The model delivers interesting comparative statics as to which deals are likely to be done in cash versus in stock and when. For example, all else equal, the option to delay approaching the target is more valuable if the value of the target's assets is more volatile. Thus, such targets are acquired later, when the financial constraint of the acquirer is less likely to be satisfied, and hence are more likely to be done in stock. All else equal, stock deals for these targets are also, on average, better than stock deals for lower-risk targets: they have higher average synergies and higher average takeover premiums.

Our paper is related to three strands of research. First, it is related to literature that studies mergers and acquisitions as real options. Lambrecht (2004) studies a setting in which mergers are driven by economies of scale and shows that the merger takes place once the price of the industry output rises to a sufficiently high threshold, thereby providing a rationale for the procyclicality of mergers. Hackbarth and Morellec (2008) apply a similar framework to a setting with incomplete information between the market and the merging firms to study the dynamics of stock returns and risk in M&A. Other papers that study mergers and acquisitions as real-options problems include Morellec and Zhdanov (2005), Alvarez and Stenbacka (2006), Lambrecht and Myers (2007), Margsiri, Mello, and Ruckes (2008), Morellec and Zhdanov (2008), and Hackbarth and Miao (2012). To our knowledge, all prior literature assumes that the target and the acquire have the same information about the value of the combined company. This assumption has a crucial effect, because it makes cash and stock bids equivalent, and thus bidders' ability to pay cash irrelevant. To make it relevant, we follow the traditional literature on auctions in assuming that bidders have private information about their valuations of the target.

Second, our paper is related to information theories of means of payment in mergers and acquisitions and, more generally, in auctions in which bidders can make bids in securities.[4] These models are static, and do not explore strategic timing in the presence of financing constraints. An exception is Cong (2012) who studies the interplay between post-auction moral hazard and the seller's strategic timing of auctioning the asset in a security-bid auction framework. Perhaps, the most relevant paper in this literature is Fishman (1989), as it delivers many of our empirical implications for means of payment using a different mechanism, in a static model with a two-sided information asymmetry between bidders and the target.[5] The advantage of a stock bid is that it reduces the adverse selection problem, inducing a more efficient accept/reject decision of the target. A cash bid is, however, used when a bidder has a high enough valuation to preempt competition by signaling a high valuation. In contrast to Fishman (1989), our paper shows that a one-sided information asymmetry in which only bidders have private information is sufficient to capture empirical evidence on means of payment, once dynamic aspects are taken into account. It also explains why stock bids are often perceived as more expensive by bidders,

[4]Security-bid auctions are studied by Hansen (1985), Rhodes-Kropf and Viswanathan (2000), DeMarzo, Kremer, and Skrzypacz (2005), Gorbenko and Malenko (2011), and Liu (2013). Skrzypacz (2013) provides a review of the literature.

[5]Other models of means of payment based on two-sided information asymmetry are provided by Hansen (1987), Eckbo, Giammarino, and Heinkel (1990), and Berkovitch and Narayanan (1990). Shleifer and Vishny (2003) and Rhodes-Kropf and Viswanathan (2004) develop theories relating means of payment in mergers to merger waves.

yet look smaller in the data. The way to test the relative importance of the two explanations for the observed means of payment would be to account for the timing of acquisitions, such as size of the target and its age, and financing constraints of bidders.

Finally, our paper is related to literature on auctions with cash-constrained bidders. Che and Gale (1998, 2000) and Che, Gale, and Kim (2013) consider buyers with exogenous budget constraints, as we do here. Zheng (2001), Rhodes-Kropf and Viswanathan (2005), Board (2007), and Vladimirov (2012) have bidders that can raise capital in the financial market to finance their cash bids. All these papers restrict bids to be made in cash. Our contributions to this literature are that we allow bidders to time the decision to bid strategically and to make bids in securities.

The remainder of the paper is organized in the following way. Section I outlines the setup of the model. Section II solves for the equilibrium in the auction taking its timing exogenous. The next two sections endogenize its timing. Specifically, Section III solves for the full equilibrium of the model in three special cases: when both bidders are unconstrained, when both bidders are extremely constrained, and when one bidder is unconstrained, and the other is extremely constrained. Section IV considers the general case of the model, thereby endogenizing the means of payment. Section V provides the comparative statics analysis. Section VI studies the properties of the equilibrium and the predictions of the model, and discusses testable hypotheses. Section VII concludes. All proofs appear in Appendix A. Appendix B contains the details of numerical solutions.

I Model Setup

We consider a setting in which the risk-neutral target attracts two potential risk-neutral acquirers, or bidders. The roles of the target and the bidders are exogenous. The value of the target as a separate entity at time t is given by X_t, where X_t evolves as a geometric Brownian motion:

$$dX_t = \mu X_t dt + \sigma X_t dB_t, \ X_0 = x. \tag{1}$$

Here, μ and $\sigma > 0$ are constant growth rate and volatility, and dB_t is the increment of a standard Brownian motion. The discount rate is constant at r. To guarantee finite values, we assume that $r > \mu$. Process $(X_t)_{t>0}$ is a reduced-form specification of the present value of the target's assets. For example, this value can be obtained by assuming that the target produces cash flow $(r - \mu) X_t$ per unit of time. We interpret X_t as the current size of the target. It accounts for all exogenous shocks to their value, such as changes in the price of the final product and inputs, as well as for the endogenous response of the target firm to them.[6] The initial value of each bidder as a separate entity is constant at Π_b.[7] If bidder i acquires the target at time t, the value of the combined firm is

$$\Pi_b + v_i X_t, \tag{2}$$

where $v_i \in [\underline{v}, \bar{v}]$, $\bar{v} > \underline{v} > 1$ is the multiple that characterizes an improvement in operations of the target due to a change in ownership.[8] We refer to v_i as bidder i's valuation of the target. Importantly, each bidder's valuation is its private information that is known to it before the start of the acquisition process.[9] Each valuation is an i.i.d. draw from distribution with p.d.f. $f(v) > 0$ on $[\underline{v}, \bar{v}]$. Each bidder knows its valuation, but not the valuation of its competitor, except for the distribution. We assume that the distribution of valuations satisfies the restriction that the payoff of the winning bidder monotonically increases in its valuation v in all specifications.[10] This assumption intuitively means that the direct effect on the winner's payoff of having a higher valuation is stronger than the indirect effect of a higher expected payment.

To have a non-trivial timing of the acquisition, the deal has to entail a cost. We capture this cost by assuming that the losing bidder is also affected by the acquisition: its value changes from Π_b to $\Pi_o < \Pi_b$. Intuitively, the acquisition makes the winning bidder a stronger competitor for the losing bidder, resulting in the lower post-acquisition value of the latter.[11] For example, the recent acquisition of Instagram by Facebook made Facebook a stronger competitor for other social network firms. This loss in the losing bidder's value is a source of delay of the acquisition in the model. Of course, other potential sources of delay such as direct costs of initiating the takeover contest are possible too. We

[6]In this paper, we focus on fundamental rather than market prices of the target (that is, prices clear of market expectations about the potential acquisition). This is consistent with related empirical studies, in which target prices are typically cleared of pre-acquisition runups.

[7]Bidders' values are equal for simplicity of exposition; this assumption does not affect the main trade-offs of the model. This setup captures a situation in which a relatively mature company aims to acquire a growing company. An additional assumption could be that the growth rate of the target decreases as it grows, so that it becomes a more mature company. Although more realistic, this assumption results in less tractability and does not alter the economics behind our results. Similarly, it is possible to extend our setup by allowing bidders to grow over time. Our results hold in this setup as long as the cash balances of each bidder, defined below, do not grow at a faster rate than the target.

[8]Allowing $\underline{v}$ below 1 does not add to the model intuition in any way.

[9]Introducing the additional private information that the bidder can learn at the beginning of the contest does not affect the results of the model qualitatively. It is only the ex-ante private information that defines bidders' strategies to initiate the takeover contest.

[10]For example, in the model of Section II.B this restriction is equivalent to a restriction that $v - \mathbb{E}[w|w \leq v]$ is a strictly increasing function of v. An example of distribution that satisfies these restrictions is uniform distribution.

[11]Spiegel and Tookes (2013) quantify this effect at 1.86% of the rival firm value on average. Horizontal mergers also feature an opposite effect, because the losing bidder faces fewer competitors. This effect is not present in our setup, because the target is not a direct competitor of the bidder.

denote the value loss of the losing bidder as $\Delta \equiv \Pi_b - \Pi_o$.

In practice, acquisitions by strategic buyers are usually initiated by a potential bidder, rather than the target (Fidrmuc et al., 2012). To reflect this practice, we assume that each bidder has a real option to approach the target at any time. If a bidder approaches the target at time t, the takeover contest is initiated and both bidders compete for the target in an open ascending-bid auction, formally defined below. Payments can be in cash, stock of the combined company, or their combination. The ability to submit bids in cash is potentially limited by a bidder's cash constraint. For simplicity, we assume that bidder i can pay up to C_i units of cash, and the cash constraint is infinitely rigid after that.[12]

I.A The Auction

We extend the formalization of the English auction for bids from different security sets. The following definition puts a formal structure on the English auction:

Definition (English auction for bids in combinations of stock and cash). The auctioneer sets the starting price to zero and gradually raises it. A price p corresponds to either a payment of p dollars in cash or a payment of any $b \in [0, p]$ dollars in cash and a fraction $\alpha(b, p)$ in the stock of the combined company defined below. As p gradually rises, a bidder confirms its participation until it decides to withdraw from the auction. As soon as only one bidder remains, it is declared the winner and pays any element of its choice from set $\{(b, \alpha(b, p), b \in [0, p])\}$, corresponding to price p at which its competitor dropped. $\alpha(b, p)$ is such that a bidder who withdraws at price p is indifferent between all elements of set $\{(b, \alpha(b, p), b \in [0, p])\}$:

$$\alpha(b, p) = \frac{p - b}{\Pi_o + p}. \tag{3}$$

This formalization extends the standard "button" model of an English auction for all-cash bids (Milgrom and Weber, 1982), as well as the analogous model for all-stock bids (Hansen, 1985). If bidders always bid in cash, the definition is equivalent to an auction in which the seller gradually raises the cash price, which the winner pays once its rival withdraws. Similarly, if bidders always bid in stock, the definition is equivalent to an auction in which the seller gradually raises the proportion of the combined company, which the winner pays once its rival withdraws. The indifference condition

[12] Modeling the cash constraint with a rigid limit is common in models of auctions with budget-constrained bidders (e.g., see a model of Section 3 in Che and Gale, 1998). A more general formulation of the cash constraint can be an interesting extension.

for $\alpha(b,p)$ means that the decision of a bidder to drop from the auction is only driven by its valuation and not the security it is bidding with. To obtain (3), note that the bidder with valuation v withdraws at price p if and only if

$$(1-\alpha)(\Pi_b + vX_t) = b + \Pi_o \;\Rightarrow\; vX_t + \Pi_b = \frac{b+\Pi_o}{1-\alpha}. \tag{4}$$

The indifference condition requires $(b+\Pi_o)/(1-\alpha)$ to be the same for all $b \in [0,p]$ and yields (3). In the case considered here, the stock bidder does not have cash, so for it, $b = 0$, $\alpha(b,p) = \frac{p}{\Pi_o + p}$.

As we show in the next section, similarly to all-cash English auction and an all-stock English auction, this auction model has a simple equilibrium in weakly dominant strategies, in which each bidder drops out when price p reaches its value of the combined company less the post-auction value of the bidder as a stand-alone company.

I.B Equilibrium Concept

At the auction, we focus on the equilibrium in weakly dominant strategies, specified in the next section. Prior to the auction, a strategy of bidder i at time t is a mapping from the history of the game H_t to a binary action $a_{i,t} \in 0, 1$, where $a_{i,t} = 1$ stands for "initiating a bid" and $a_{i,t} = 0$ stands for "waiting." If the rival initiates a bid at time t, it is a weakly dominant strategy for bidder i to join the auction. Because the game ends once the auction takes place, the history of the game H_t can be summarized by a sample path of $X(s)$, $s \leq t$ and the fact that the auction has not been initiated yet. The equilibrium concept is Markov Perfect Bayesian equilibrium. In the class of MPBE, we look for separating equilibria in continuous threshold strategies: (i) the strategy of a bidder with type v is to initiate a bid at the first instant when $X(t)$ reaches some upper threshold $\bar{X}(v)$ for the first time; (2) $\bar{X}(v_1) = \bar{X}(v_2) < \infty$ if and only if $v_1 = v_2$; (3) $\bar{X}(v)$ is continuous. Continuity and separation imply that $\bar{X}(v)$ is strictly monotone in v. Because at any finite threshold $\bar{X}$ types with valuations close enough to $\underline{v}$ obtain a negative payoff in the auction, $\bar{X}(v)$ must be strictly decreasing in v.

II Equilibrium in the Auction

In this section, we show that there exists an equilibrium in the auction in weakly dominant strategies. This result generalizes the equilibrium in weakly dominant strategies in the standard cash English auction.

Proposition 1. *It is a weakly dominant strategy for bidder $i \in \{1, 2\}$ to drop out once the price reaches its valuation of the combined company less its post-auction value as a stand-alone firm:*

$$p(v_i) = v_i X_t + \Delta. \tag{5}$$

Conditional on winning, it is optimal for the bidder to pay the winning bid y using as much cash as possible, if $v_i > p^{-1}(y)$, and as much stock as possible, if $v_i < p^{-1}(y)$. In equilibrium, the winning bidder pays the winning bid using as much cash as possible: $b = \min\{y, C_i\}$.

The reason why bidding up to (5) is a weakly dominant strategy generalizes from that in the standard cash English auction. At (5), the bidder with valuation v_i is exactly indifferent between winning the auction at price $p(v_i)$ and losing the auction and getting Π_o. Conditional on this valuation, the value of this break-even bid does not depend on the mix of cash and stock. Dropping out below (5) is suboptimal, because it leads to potentially not winning the auction when the payoff from winning is higher than that from losing. Dropping out above (5) leads to potentially winning the auction at a price y above the break-even level. In Appendix A, we show that in this case, even though the bidder can pay less than y by making an all-stock bid, it is still better off losing the auction. Thus, the dominant strategy result for a standard cash auction extends to our setting.

An interesting property is that the break-even bid strategy is independent of the cash position of the bidder. Intuitively, the bidder type that marginally wins the auction is indifferent between paying in stock, cash or combinations. The cash position of a bidder, however, does affect the equilibrium division of the surplus between the target and the winner. To see this, consider a bidder with valuation v and cash position C. It wins the auction if and only if the valuation of its competitor w is below v. If $C \geq wX_t + \Delta$, the winner acquires the target by paying $wX_t + \Delta$ in cash. Otherwise, it pays C in cash and fraction $\alpha(C, wX_t + \Delta)$ in stock. In the former case, the change in the value of the winner relative to its pre-auction value is

$$(v - w) X_t - \Delta. \tag{6}$$

In the latter case, it is

$$\begin{aligned} & \frac{\Pi_o + C}{\Pi_b + wX_t} (v - w) X_t - C - \Pi_b \\ = & (\Pi_o + C) \frac{(v - w) X_t}{\Pi_b + wX_t} - \Delta. \end{aligned} \tag{7}$$

Value (6) is strictly higher that (7) for any $v > w$, because a stock bid, but not a cash bid, is worth more if the bidder's type is higher, and the type of the winning bidder is higher than the type of the rival (that determines the winning bid).

In the following sections, we will solve for bidders' decision of when to bid for a target. The results there will be driven by two key effects that are evident from the comparison of (6) and (7). The first, *static*, effect is that (6) exceeds (7), and, more generally, (7) is strictly increasing in the amount $C \leq wX_t + \Delta$ of cash portion in the bid. It implies that all else equal, a less cash-constrained bidder obtains a higher payoff, conditional on winning. The second, *dynamic*, effect is that (6) and (7) change differently, as the target grows over time. Specifically, when a bidder pays the bid in cash, its payoff from winning is increasing linearly in the size of the target X_t. However, when the marginal dollar of the bid is paid in stock, the bidder's payoff is increasing in X_t at a decreasing rate. Specifically, as the target grows relative to the bidder, a lower fraction of the total surplus from the auction remains with the bidder and a higher fraction is transferred to the target. If the target is very small, there is little difference between bids in cash and in stock. However, this difference can be significant if the target is large. Because of the first, static, effect, a cash-constrained bidder benefits from letting the target grow more internally compared to an unconstrained bidder. At the same time, the impact of the second, dynamic, effect is opposite: a cash-constrained bidder benefits from acquiring the target early, because it would retain a smaller share of the combined company if the target were allowed to grow further. Because of the dynamic effect, using a static security-bid auction model to analyze strategic initiation can lead to misleading results.

III Model with Absent or Extreme Cash Constraints

Before analyzing the general version of the model, we consider special cases of it when each bidder is either unconstrained ($C_i = \infty$) or extremely constrained ($C_i = 0$). In the first case, both bidders are unconstrained, and as a result, always compete in cash bids. In the second case, both bidders are extremely constrained, and as a result, always compete in stock bids. Finally, in the third case, one bidder is constrained and thus competes in cash bids, while the other is extremely constrained. These special cases are useful for developing intuition about how cash constraints, both a bidder's and its rival, affect incentives to initiate a bid. Their limitation is that the means of payment are exogenous and given by whether the bidder is unconstrained or extremely constrained. The model with partial cash constraints studied in the next section endogenizes the means of payment.

III.A Two Unconstrained Bidders

Consider the case in which both bidders are unconstrained. By Proposition 1, the payment of the winning bidder is always in cash. Suppose that the auction is initiated at time t and both bidders compete for the target in an English auction. If the bidder with valuation v wins the auction against the bidder with valuation w, the change in its value relative to the stand-alone level is given by (6). If, on the other hand, the bidder loses, the corresponding difference is $-\Delta$. If τ is the first passage time by $X(t)$ of an upper threshold $\bar{X}$, then the present value of a security that pays a unit at time τ equals $\mathbb{E}\left[e^{-r\tau}\right] = \left(\frac{X_0}{\bar{X}}\right)^{\beta}$, where β is the positive root of the fundamental quadratic equation $\frac{1}{2}\sigma^2\beta\left(\beta-1\right)+\mu\beta-r=0$(e.g., Dixit and Pindyck, 1994):

$$\beta = \frac{1}{\sigma^2}\left[-\left(\mu-\frac{\sigma^2}{2}\right)+\sqrt{\left(\mu-\frac{\sigma^2}{2}\right)^2+2r\sigma^2}\right] > 1. \tag{8}$$

If the bidder with valuation v follows the strategy of approaching the target at threshold $\bar{X}$, while its rival follows the strategy of approaching the target at threshold $\bar{X}_c\left(w\right)$, where w is its type (c stands for "cash"), then the expected payoff of the bidder at the initial date is

$$\begin{aligned}&\left(\frac{X_0}{\bar{X}}\right)^{\beta}\int_{\underline{v}}^{\bar{X}_c^{-1}(\bar{X})}\left(\bar{X}\max\left\{v-w,0\right\}-\Delta\right)dF\left(w\right) \\ &+\int_{\bar{X}_c^{-1}(\bar{X})}^{\overline{v}}\left(\frac{X_0}{\bar{X}_c\left(w\right)}\right)^{\beta}\left(\bar{X}_c\left(w\right)\max\left\{v-w,0\right\}-\Delta\right)dF\left(w\right).\end{aligned} \tag{9}$$

Intuitively, the auction is initiated either by the bidder (if $\bar{X}<\bar{X}_c\left(w\right)$) or by its rival (if $\bar{X}>\bar{X}_c\left(w\right)$). In the former case, the auction takes place at threshold $\bar{X}$ and conditional on rival initiating the auction before, its type must be below $\bar{X}_c^{-1}\left(\bar{X}\right)$. The payoff corresponding to this case is given by the first term in (9). The latter case occurs if the valuation of the rival bidder is above $\bar{X}_c^{-1}\left(\bar{X}\right)$. Conditional on this valuation being w, the auction occurs when $X\left(t\right)$ reaches threshold $\bar{X}_c\left(w\right)$. Integrating over all realizations of w above $\bar{X}_c^{-1}(\bar{X})$ yields the second term of (9).

Maximizing (9) with respect to $\bar{X}$ and applying the equilibrium condition that the maximum is reached at $\bar{X}_c(v)$, we obtain

$$\bar{X}_c(v) = \frac{\beta}{\beta - 1} \frac{\Delta}{v - \mathbb{E}[w|w \leq v]}. \tag{10}$$

This equation is intuitive. Because of the option to delay approaching the target, a bidder approaches the target only at a point when its expected surplus from initiating the contest exceed the costs by a high enough margin. The increase in the target's efficiency that is captured by the acquirer in expectation is $(v - \mathbb{E}[w|w \leq v]) X_t$, and the cost of approaching the target is Δ. The term $\beta/(\beta - 1) > 1$ captures the degree to which the option to delay approaching the target is important. It is higher if the target grows faster (μ is higher), is more volatile (σ is higher), or if the discount rate r is lower.

Assume that the distribution of types is such that the expected surplus of the winning bidder, $v - \mathbb{E}[w|w \leq v]$, is strictly increasing in its type. This property holds for many distributions. For example, it holds for uniform distribution.[13] Then, there indeed exists a unique equilibrium in separating threshold strategies:

Proposition 2. *Assume that $v - \mathbb{E}[w|w \leq v]$, is strictly increasing in v. Then, there exists a unique equilibrium in separating threshold strategies. In this equilibrium, a bidder with valuation v initiates the auction at threshold $\bar{X}_c(v)$, given by (10), provided that no bidder has initiated the auction before.*

The equilibrium has three properties. First, a deal with a higher synergy occur earlier in time, before the target has grown much. Second, among the two potential bidders, the bidder that approaches the target is the bidder with the higher valuation. It follows that in equilibrium, the bidder that approaches the target always wins the auction. This property will not hold if the bidders are asymmetric in their cash constraints.[14] Finally, all bidders with valuations $v > \underline{v}$ find it optimal to approach the target at some finite $\bar{X}_c(v)$. This is because, as (6) shows, there always exists high enough X_t such that the winning bidder receives a positive surplus for any $w < v$.

[13]Intuitively, there cannot be too "few" low types.

[14]In a more general setting, in which bidders can update their valuations after the contest initiation (e.g., during due diligence), this result would not hold, but the bidder that initiates the contest would always win with a higher probability than its competitor, provided that the degree of initial information is the same for both bidders.

In the special case of the uniform distribution of v over $[\underline{v}, \bar{v}]$, $\mathbb{E}[w|w<v] = (\underline{v}+v)/2$. Therefore,

$$\bar{X}_c(v) = \frac{\beta}{\beta-1}\frac{2\Delta}{v-\underline{v}}. \tag{11}$$

It is easy to see that $\bar{X}_c(v)$ is indeed a decreasing function of v.

III.B Two Extremely Constrained Bidders

Now, consider the opposite case: Assume that both bidders are extremely constrained and always make offers in stock. Suppose that the auction is initiated at time t. If the bidder with valuation v wins the auction against the bidder with valuation w, the change in its value relative to the stand-alone level is given by (7). If the bidder loses, this difference is $-\Delta$. Thus, if the bidder with valuation v follows the strategy of approaching the target at threshold $\bar{X}$, while its rival follows the strategy of approaching the target at threshold $\bar{X}_s(w)$, where w is its type (s stands for "stock"), then the expected payoff of the bidder at the initial date is

$$\begin{aligned} &\left(\frac{X_0}{\bar{X}}\right)^{\beta}\int_{\underline{v}}^{\bar{X}_s^{-1}(\bar{X})}\left(\frac{\Pi_o}{\Pi_b+w\bar{X}}\bar{X}\max\{v-w,0\}-\Delta\right)dF(w) \\ &+\int_{\bar{X}_s^{-1}(\bar{X})}^{\bar{v}}\left(\frac{X_0}{\bar{X}_s(w)}\right)^{\beta}\left(\frac{\Pi_o}{\Pi_b+w\bar{X}_s(w)}\bar{X}_s(w)\max\{v-w,0\}-\Delta\right)dF(w), \end{aligned} \tag{12}$$

Similarly to the case of two unconstrained bidders, the first (second) term of (12) reflects the case in which the bidder with valuation v (its competitor) initiates the auction.

Maximizing (12) with respect to $\bar{X}$ and applying the equilibrium condition that the maximum is reached at $\bar{X}_s(v)$, we obtain

$$\mathbb{E}\left[\frac{\Pi_o\left(\Pi_b+\frac{\beta}{\beta-1}w\bar{X}_s(v)\right)}{\left(\Pi_b+w\bar{X}_s(v)\right)^2}(v-w)\,|w\le v\right]\bar{X}_s(v) = \frac{\beta}{\beta-1}\Delta. \tag{13}$$

The left-hand side is a strictly increasing function of $\bar{X}$, which implies that the optimal approaching policy of each bidder is given by the upper trigger $\bar{X}_s(v)$. In particular, monotonicity implies that if the trigger exists, it is unique.

However, (13) does not have a solution for some v. By monotonicity, the highest value of the left-hand side of (13) is

$$\lim_{\bar{X}\to\infty}\mathbb{E}\left[\frac{\Pi_o\left(\Pi_b+\frac{\beta}{\beta-1}w\bar{X}\right)\bar{X}}{\left(\Pi_b+w\bar{X}\right)^2}\left(v-w\right)|w\leq v\right]=\frac{\beta}{\beta-1}\Pi_o\mathbb{E}\left[\frac{v-w}{w}|w\leq v\right]. \tag{14}$$

This value decreases in v and reaches zero when $v=\underline{v}$.[15] Thus, once v decreases to a sufficiently low level v^*, given by

$$\mathbb{E}\left[\frac{v^*-w}{w}|w\leq v^*\right]=\frac{\Delta}{\Pi_o}, \tag{15}$$

no bidder finds it optimal to approach the target, even though it is socially optimal to do so when X_t is high enough. This result is driven by the dynamic effect of a stock auction and can be seen from (7). As X_t increases, for the same v, the bidder has to give away a larger portion of the combined company to the target. As a result, the expected revenue of the bidder with valuation v is also limited from above as $X_t\to\infty$. For sufficiently low valuations, the bidder never has an incentive to initiate a bid for the target.

The equilibrium is summarized in the following proposition:

Proposition 3. *The equilibrium in separating threshold strategies must have the following characterization. If the valuation of a bidder is $v>v^*$, where v^* is defined by (15), then it approaches the target at threshold $\bar{X}_s(v)$, given by (13), provided that no bidder has approached the target before. If $v\leq v^*$, then a bidder never approaches the target.*

A sufficient condition for existence is that the distribution of types is such that the left-hand side of (10) is strictly increasing in v. This condition is analogous to that in the case of unconstrained bidders. It holds for many distributions: in particular, for uniform distribution, and more generally, for any distribution with a non-increasing density on its support.

While there is no analytical solution for $\bar{X}_s(v)$, it is easy to study its properties. In particular, it is interesting to see how (13) relates to (10). For this purpose, it is convenient to decompose (13) into

[15] To see that the value decreases in v, differentiate it with respect to v. The derivative is

$$-\frac{\beta}{\beta-1}\Pi_o\int_{\underline{v}}^{v}\frac{v-w}{w}\frac{f(w)f(v)}{F(v)^2}dw<0.$$

two parts:

$$\mathbb{E}\left[\frac{\Pi_o\left(v-w\right)\bar{X}}{\Pi_b+w\bar{X}}|w\leq v\right]+\frac{1}{\beta-1}\mathbb{E}\left[\frac{\Pi_o\left(v-w\right)w\bar{X}^2}{\left(\Pi_b+w\bar{X}\right)^2}|w\leq v\right]=\frac{\beta}{\beta-1}\Delta. \tag{16}$$

The left-hand side of (16) consists of two components. The first component is the surplus that the bidder obtains in expectation. It is always below the left-hand side of (10), because separation is costlier is stock than in cash. If this were the only term on the left-hand side of (16), then each bidder would always find it optimal to approach the target later if it bids in stock. However, (16) contains an additional positive second term. It corresponds to the effect that the delay causes the surplus of the bidder to increase at a slower pace when the bidder makes bids in stock. Alternatively, one can think of this term as a part of the delay cost on the right-hand side of (16): when X_t is higher, further delay is less costly to the bidder as further increase in X_t has a negative effect of a smaller magnitude on the bidder revenue. The magnitude of this effect depends on the value of delay parameter $\beta/\left(\beta-1\right)$. The following proposition shows that if $\beta/\left(\beta-1\right)$ is not too high, then the first effect dominates, so bidders approach the target earlier if they are unconstrained:

Proposition 4. *Suppose that the measure of the option value of delay, $\beta/\left(\beta-1\right)$, is not too high:*

$$\frac{\beta}{\beta-1}<2\frac{\Pi_b}{\Pi_o}. \tag{17}$$

Then, $\bar{X}_s\left(v\right)>\bar{X}_c\left(v\right)$ for any *v.*

In most calibrations in the literature, the multiplier of the delay option, $\beta/\left(\beta-1\right)$, does not exceed 2 for the average US publicly-traded firm. As a consequence, condition (17) is likely to hold for a wide range of firms, so we refer to this case as the standard case. According to Proposition 4, if bidders are unconstrained, they are more likely to undertake an acquisition over any finite time interval $[0,t]$ than if bidders are extremely constrained.

However, if the target grows very quickly or with very high volatility or if the interest rate is very low, then Proposition 4 no longer applies. Because $\lim_{v\downarrow v^*}\bar{X}_s\left(v\right)=\infty$ and $\bar{X}_c\left(v^*\right)<\infty$, constraints delay the auction for low enough types even in this case. However, constrained bidders with high valuations may initiate the bid for the target earlier than unconstrained bidders, despite obtaining a lower fraction of the total surplus from the auction. Figure 1 presents two examples: the standard case, in which constraints delay initiation of the auction for all realizations of valuations, and the non-standard case, in which they speed up initiation for high realizations of valuations.

The results of this and the previous subsections highlight that a bidder's cash constraint has a non-trivial effect on its decision to bid for the target. First, while a constraint usually makes a bidder more reluctant to initiate a bid, this is not always so. If the target is a very high-growing or high-volatility company and a bidder has a high valuation, a constraint may make a bidder more willing to bid for the target. Second, constraints make bidders with positive but low synergies never willing to initiate the bid for the target. This leads to some positive-NPV deals never occurring in equilibrium.

III.C An Unconstrained vs. An Extremely Constrained Bidder

Finally, consider the case in which one bidder is unconstrained and thus bids in cash, and the other bidder is extremely constrained, and thus always bids in stock. Without loss of generality, we refer to the unconstrained bidder as "bidder 1" and to the constrained bidder as "bidder 2." Let $\bar{X}_i(v)$ denote the (possibly infinite) initiation threshold of bidder $i \in 1, 2$ with valuation v. We do not make any assumptions about ordering of the two strategies but later provide conditions under which such ordering can be established.

First, if bidder 1 with valuation v approaches the target at threshold $\bar{X}$, its expected payoff at the initial date equals[16]

$$\left(\frac{X_0}{\bar{X}}\right)^{\beta} \int_{\underline{v}}^{\bar{X}_2^{-1}(\bar{X})} \left(\bar{X} \max\left\{v - w, 0\right\} - \Delta\right) dF(w) + \int_{\bar{X}_2^{-1}(\bar{X})}^{\bar{v}} \left(\frac{X_0}{\bar{X}_2\left(w\right)}\right)^{\beta} \left(\bar{X}_2(w) \max\left\{v - w, 0\right\} - \Delta\right) dF(w). \tag{18}$$

Intuitively, if valuation of bidder 2 is below $\bar{X}_2^{-1}(\bar{X})$, bidder 1 initiates the auction at threshold $\bar{X}$. Otherwise, the auction is initiated by bidder 2. If the auction is initiated at some X_t and valuation of bidder 1, v, is above valuation of bidder 2, w, then bidder 1 wins the auction, makes a payment in cash and is left with the revenue equal to $X_t\left(v - w\right) - \Delta$. If $v < w$, it loses the auction and suffers the loss of Δ. Maximizing (18) with respect to $\bar{X}$ and applying the equilibrium condition that the maximum is reached at $\bar{X}_1\left(v\right)$, we obtain

$$\bar{X}_1\left(v\right) = \frac{\beta}{\beta - 1} \frac{\Delta}{v - \mathbb{E}\left[w | w \leq \Omega(v)\right]} \Psi(v), \tag{19}$$

[16]Here and hereafter, we use $\bar{X}_i^{-1}(\bar{X})$, $i = \{1, 2\}$ instead of the more precise $\min\{\bar{X}_i^{-1}(\bar{X}), \bar{v}\}$ to save on notation.

where for bidder i and its competitor $-i$, $\Omega(v) = \min\left\{v, \bar{X}_{-i}^{-1}\left(\bar{X}_i(v)\right)\right\}$ and $\Psi\left(v\right) \equiv \max\left\{1, \frac{F\left(\bar{X}_{-i}^{-1}\left(\bar{X}_i(v)\right)\right)}{F(v)}\right\}$. Note that (19) is very similar to (10). To see the intuition for the difference, consider $\bar{X}_1\left(v\right) < \bar{X}_2\left(v\right)$. Then for bidder 1, $\Omega(v) = v$, $\Phi(v) \geq 1$. Consequently, bidder 1 delays approaching the target compared to the case in which it faces another cash bidder: $\bar{X}_1(v) \leq \bar{X}_c(v)$. Intuitively, because other things equal bidder 2 with the same valuation approaches the target later than bidder 1, upon approaching bidder 1 faces a stronger competitor than if it faced a cash bidder. Because of this, bidder 1 faces a lower probability of winning the auction, which decreases its expected surplus. Consequently, it further delays approaching the target.

Second, if bidder 2 with valuation v approaches the target at threshold $\bar{X}$, its expected payoff at time 0 is equal to

$$\left(\frac{X_0}{\bar{X}}\right)^{\beta}\int_{\underline{v}}^{\bar{X}_1^{-1}(\bar{X})}\left(\frac{\Pi_0}{\Pi_b + w\bar{X}}\bar{X}\max\left\{v-w,0\right\}-\Delta\right)dF(w)$$
$$+\int_{\bar{X}_1^{-1}(\bar{X})}^{\bar{v}}\left(\frac{X_0}{\bar{X}_1\left(w\right)}\right)^{\beta}\left(\frac{\Pi_0}{\Pi_b + w\bar{X}_1(w)}\bar{X}_1(w)\max\left\{v-w,0\right\}-\Delta\right)dF(w). \tag{20}$$

This expression is similar to (18), with the only difference that bidder 2 pays stock if it wins the contest and is left with its payoff equal to $\left(\frac{\Pi_0}{\Pi_b + wX_t}X_t \max\left\{v-w,0\right\}-\Delta\right)$. Maximizing (20) with respect to $\bar{X}$ and applying the equilibrium condition that the maximum is reached at $\bar{X}_2\left(v\right)$, we obtain

$$\mathbb{E}\left[\frac{\Pi_o\left(\Pi_b + \frac{\beta}{\beta-1}w\bar{X}_2\left(v\right)\right)}{\left(\Pi_b + w\bar{X}_2\left(v\right)\right)^2}\left(v-w\right)|w \leq \Omega(v)\right]\bar{X}_2\left(v\right) = \frac{\beta}{\beta-1}\Delta\Psi(v). \tag{21}$$

Note that (21) is very similar to (13). To see the intuition for the difference, again, consider $\bar{X}_1\left(v\right) < \bar{X}_2\left(v\right)$, so that for bidder 2, $\Omega(v) < v$ and $\Psi(v) = 1$. Because w takes lower values compared to the case in which bidder 2 faces another stock bidder, bidder 2 accelerates approaching the target: $\bar{X}_2(v) \geq \bar{X}_s(v)$. Intuitively, because other things equal bidder 1 with the same valuation approaches the target earlier than bidder 2, upon approaching bidder 2 faces a weaker competitor than if it faced another stock bidder. Because of this, bidder 2 obtains a higher expected surplus from the auction, which accelerates its decision to approach the target.

The equilibrium is summarized in the following proposition:

Proposition 5. *The equilibrium in separating threshold strategies must have the following characterization. The initiation strategy of bidder 1 (the unconstrained bidder) with valuation v_1 is to* *approach the target at threshold $\bar{X}_1(v_1)$, given by (19), provided that no bidder has approached the target before. The initiation strategy of bidder 2 (the constrained bidder) with valuation $v_2 > v_2^*$ is to approach the target at threshold $\bar{X}_2(v_2)$, given by (21), provided that no bidder has approached the target before. If $v_2 \leq v_2^*$, then bidder 2 never approaches the target first. The boundary type v_2^* is given by*

$$v_2^* = \frac{\Pi_b}{\Pi_o}\underline{v} > \underline{v}. \tag{22}$$

As in the case of two constrained bidders, expecting low payoff from acquiring the target in stock, the constrained bidder does not initiate the takeover contest for low enough valuations. There is no analytical solution for the jointly determined $\bar{X}_1(v)$ and $\bar{X}_2(v)$ but two closed form equations can be obtained for $\bar{X}_1^{-1}(X)$ and $\bar{X}_2^{-1}(X)$ which make the numerical analysis of the strategies easy. Appendix B provides more detail.

In the normal case, when the option value of delay is not too high so that financial constraints delay acquisition, Proposition 6 establishes the ordering of strategies in the three cases of the model:

Proposition 6. *Suppose that $\frac{\beta}{\beta-1} < 2\frac{\Pi_b}{\Pi_o}$ and that equilibria in separating threshold strategies exist in all three cases of the model. Then, the equilibrium strategies are ordered: $\bar{X}_s(v) > \bar{X}_2(v) > \bar{X}_1(v) > \bar{X}_c(v)$ for any v.*

For the numerical example, we choose the benchmark model parametrization: $r = 0.05$, $\mu = 0.01$, $\sigma = 0.25$, $\underline{v} = 1.1$, $\bar{v} = 1.5$, $v \sim$ Uniform$[\underline{v}, \bar{v}]$, $\Pi_b = 100$, $\Pi_o = 95$. These values are also reported in Table I. Specifically, the benchmark case considers acquisition of a target whose assets grow at the risk-adjusted rate μ, typically used in dynamic models of the firm, and that has the average COMPUSTAT asset volatility σ. The losing bidder's profits are 5% below the pre-acquisition levels. The average synergies are equal to 30% of the target's core business. The interest rate is set at 5%. The benchmark parametrization satisfies $\beta/(\beta-1) < 2$. The non-standard case features identical parameters except $\mu = 0.035$.

Figure 1 shows the four thresholds as functions of bidders' valuations, v, in the standard and non-standard cases. Consider the standard case. A higher probability of losing the takeover contest makes a constrained bidder that competes against an unconstrained bidder more cautious compared to the case when it competes against another unconstrained bidder. As a result, its initiation threshold increases.

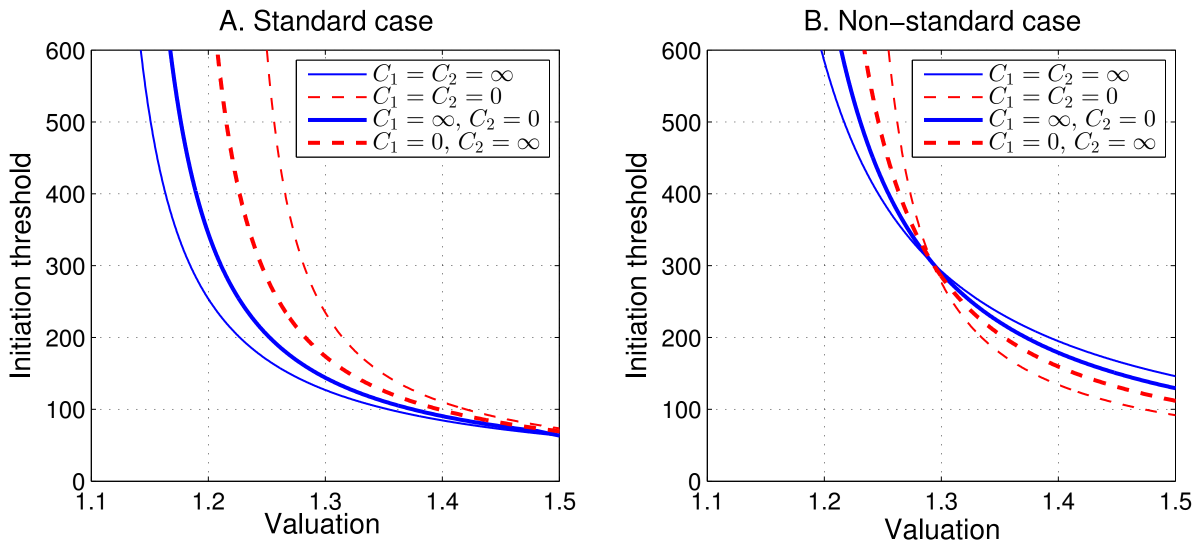

Figure 1: Initiation strategies of unconstrained and constrained bidders facing different types of competitors. The figure shows the optimal initiation strategies of bidders as a function of their valuations, v. The thin solid (thin dashed) line is the strategy of an unconstrained (extremely constrained) bidder facing another unconstrained (extremely constrained) bidder; the thick solid (thick dashed) line is the strategy of an unconstrained (extremely constrained) bidder facing an extremely constrained (unconstrained) bidder.

The opposite is also true: a lower probability of losing the takeover contest makes a constrained bidder more aggressive when it competes against an unconstrained bidder. As a result, its initiation threshold decreases. In the non-standard case, cash constraints speed up initiation of the bid for bidders with high enough valuations. In either case, constraints of the rival bidder matter. Another interesting result is that competing against an unconstrained bidder also makes constrained bidders with lower valuations willing to initiate in the first place: $v_2^* < v^*$.

The main result of this subsection is that a bidder's decision to initiate a bid depends not only on its own cash constraint but also on the cash constraints of its competitors. This is so despite the fact that the bidding strategy is "myopic" in the sense that it is independent of cash constraints of other bidders. Intuitively, when deciding whether to initiate a bid, a bidder cares about the type of its competitors. Whether the rival is constrained or not impacts its own decision to initiate a bid, and thus indirectly affects the learning of the other bidder. In the normal case, if the rival is constrained, it delays its decision to approach the target for every possible realization of its valuation. Thus, conditional on the rival not initiating a bid, the bidder believes that the rival is more pessimistic about its valuation, if the rival is unconstrained. Therefore, cash constraints of the rival reduce the expected payoff of the other bidder from the auction at any point, and consequently make it reluctant to approach the target. This result also implies that in empirical analysis changes in financial constraints in the economy should be accounted for even if they do not have an effect on a particular bidder.

IV Model with General Constraints

The special cases of the previous section highlighted the role of bidders' financial constraints in acquisitions decisions. However, means of payment were uniquely determined by the constraint of the acquirer. In this section, we develop richer implications for means of payment by introducing considering general cash constraints of bidders: specifically, bidder i can only bid up to $C_i \geq 0$ in cash. We show endogenous timing of an acquisition leads to an interconnection between bidders' financial constraints, means of payment, and synergies. High-synergy targets are acquired when they are young and small and for cash. In contrast, low-synergy targets are acquired (if at all) after they have grown and with the help of stock. Because of this selection, cash acquisitions can feature a higher average takeover premium despite the fact that bidders perceive acquisitions in stock as more expensive. We also show that in the general model the impact of constraints is non-trivial even in the standard case. Throughout the section, we assume that the separating equilibrium in threshold strategies, $X_1(v)$ and $X_2(v)$, exists. This is the case in all of our numerical examples.

Consider the decision of bidder i with valuation v to approach the target. If bidder i approaches the target at threshold $\bar{X}$, its expected payoff at the initial date equals

$$\begin{aligned}&\left(\frac{X_0}{\bar{X}}\right)^{\beta}\int_{\underline{v}}^{\bar{X}_{-i}^{-1}(\bar{X})}\left(\min\left\{\frac{\Pi_o+C_i}{\Pi_b+w\bar{X}},1\right\}\bar{X}\max\{v-w,0\}-\Delta\right)dF(w) \\ &+\int_{\bar{X}_{-i}^{-1}(\bar{X})}^{\bar{v}}\left(\frac{X_0}{\bar{X}_{-i}(w)}\right)^{\beta}\left(\min\left\{\frac{\Pi_o+C_i}{\Pi_b+w\bar{X}_{-i}(w)},1\right\}\bar{X}_{-i}(w)\max\{v-w,0\}-\Delta\right)dF(w).\end{aligned} \tag{23}$$

Intuitively, if the valuation of the competitor is below $\bar{X}_{-i}^{-1}\left(\bar{X}\right)$, bidder i approaches the target at threshold $\bar{X}$. Otherwise, the competitor approaches the target at threshold $\bar{X}_{-i}(w)$. In both cases, if $v > w$, bidder i wins the auction and makes a payment either in cash or in a combination of cash and stock. If $v < w$, it loses the auction and suffers the loss of Δ. Maximizing (23) with respect to $\bar{X}$ and using the equilibrium condition that the maximum is reached at $\bar{X}_i(v)$, we obtain

$$\begin{aligned}&\mathbb{E}\left[\min\left\{\frac{\Pi_o+C_i}{\Pi_b+w\bar{X}_i(v)},1\right\}(v-w)\,|w\leq\Omega(v)\right]\bar{X}_i(v) \\ &+\frac{1}{\beta-1}\int_{\min\left(\frac{C_i-\Delta}{\bar{X}_i(v)},\Omega(v)\right)}^{\Omega(v)}(\Pi_o+C_i)\frac{(v-w)\,w\bar{X}_i(v)^2}{\left(\Pi_b+w\bar{X}_i(v)\right)^2}\frac{f(w)}{F(\Omega(v))}dv \\ =\;&\frac{\beta}{\beta-1}\Delta\Psi(v),\end{aligned} \tag{24}$$

where $\Omega(v) \equiv \min\left\{v, \bar{X}_{-i}^{-1}\left(\bar{X}_i(v)\right)\right\}$ and $\Psi(v) \equiv \max\left\{1, \frac{F\left(\bar{X}_{-i}^{-1}\left(\bar{X}_i(v)\right)\right)}{F(v)}\right\}$. The system of equations (24) for bidders 1 and 2 jointly determines equilibrium thresholds $\bar{X}_1(v)$ and $\bar{X}_2(v)$. Note that this solution embeds solutions for three special cases, studied in Section II. The following proposition summarizes the equilibrium:

Proposition 7. *The separating threshold equilibrium in the general model must take the following form. Bidder i with valuation $v_i > v_i^*$ initiates the auction the at threshold $\bar{X}_i(v_i)$, provided that it has not been approached before, where $\bar{X}_i(v)$ satisfies (24) and v_i^* is defined in Appendix A, provided that the rival bidder has not initiated the auction yet. If $v_i \leq v_i^*$, bidder i never initiates the auction.*

As long as $C_1 < \infty$ and $C_2 < \infty$, each bidder never approaches the target for valuations equal to or below, correspondingly, v_1^* and v_2^*. Appendix B provides more detail on the numerical solution for $\bar{X}_1(v)$ and $\bar{X}_2(v)$.

Figure 2, Panel A shows the four thresholds (cash vs. cash bidders, stock vs. stock bidders, and bidders with internal cash $C_1 = 125$ and $C_2 = 0$ competing against each other) for our benchmark parametrization as a function of bidders' valuations, v. An interesting new effect compared to the case of exogenous means of payment is that for intermediate valuations, constrained bidders can choose to accelerate initiation even relative to the case of two cash bidders. This happens because they attempt to "fit into" their cash constraints. Consider Figure 2, Panels B and C that show expected bidder revenue from non-cash and cash-only deals. As the valuation of bidder 1 decreases, it initiates contests for a larger target and eventually finds itself unable to complete all deals in cash (the dashed vertical line on the right-hand side of all panels). At this stage, bidder 1 trades off costs of inefficiently early initiation against its benefits (a smaller probability that the deal is non-cash, resulting in a higher expected revenue from the auction). If the latter dominates, bidder 1 can approach a smaller target compared to the case when it is unconstrained ($C_1 = 0$) or even to the case when both bidders are unconstrained. As the valuation of bidder 1 decreases even further (beyond the dashed vertical line on the left-hand side of all panels), any successful contest requires the payment of at least C_1 that makes fitting into cash not possible. Then, bidder 1's initiation threshold increases faster, similarly to an all-stock bidder.

Consider bidder 2 who competes against bidder 1 with $C_1 < \infty$ instead of $C_1 \to \infty$. Bidder 1 attempts to fit into cash and, for intermediate valuations, accelerates its initiation compared to $C_1 \to \infty$, so bidder 2 becomes a stronger bidder with higher expected revenues. As a result, it is optimal for bidder 2 to also accelerate initiation for intermediate valuations.

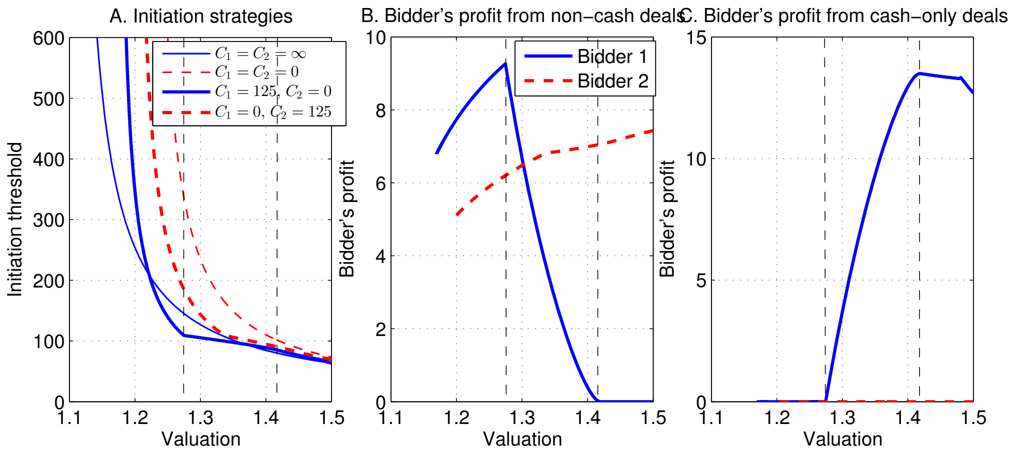

Figure 2: Initiation strategies of bidders facing different types of competitors. Panel A shows the equilibrium initiation thresholds of bidders as functions of their valuations, v. The thin solid (thin dashed) line is the strategy of an unconstrained (extremely constrained) bidder facing another unconstrained (extremely constrained) bidder; the thick solid (thick dashed) line is the strategy of a bidder with cash $C_1 = 125$ ($C_1 = 0$) facing a bidder with cash $C_2 = 0$ ($C_2 = 125$). Panel B (C) shows the part of the total expected payoff of a bidder with valuation v at the date of the auction that comes from non-cash (cash) deals for bidders with internal cash $C_1 = 125$ and $C_2 = 0$.

V Comparative Statics

In this section, we investigate the effects of target and bidder characteristics on initiation strategies. Proposition 8 establishes comparative statics results:

Proposition 8. *Assume that each bidder is, in any combination, either severely constrained ($C_i < \Delta$) or unconstrained ($C_i \to \infty$), and that (17) holds. Consider an equilibrium in decreasing initiation strategies $\bar{X}_i(v)$. For any v, $\bar{X}_i(v)$, $i \in \{1, 2\}$:*

1. *increase in μ;*

2. *increase in σ;*

3. *decrease in r;*

4. *increase in Δ (keeping Π_b fixed);*

5. *weakly decrease in Π_b (keeping Δ fixed).*

Proposition 8 provides sufficient conditions for monotone comparative statics; in addition, the numerical analysis shows that the same results hold for a wide range of constraints of both bidders that do not satisfy the conditions of Proposition 8. These results are intuitive. *(1)* When μ is higher, bidders wait longer before approaching the target: the present value of costs associated with losing the deal increases due to X_t reaching the initiation threshold of a competitor faster, and this increase dominates an increase in the present value of synergies in case of success. *(2)* For the same reason, when the discount rate r is lower, the costs of losing the deal loom larger, so the takeover contest is initiated later. *(3)* Similarly, higher σ implies a higher likelihood of the competitor reaching the initiation threshold fast, which in turn increases costs of losing the deal and leads to delay in initiation. *(4)* When costs of losing the contest, Δ, are high, the winning bidder has to pay more to separate itself from the losing bidder: the value of the winning bidder's outside option (losing) is a negative function of Δ. As a result, the bidders' expected payoffs from the contest decrease, so they initiate later. *(5)* The additional restrictions here make the motive to fit into cash weak, resulting in monotone comparative statics. The initiation strategies of two unconstrained bidders competing with each other are constant in Π_b keeping Δ fixed. For a severely constrained bidder, however, a larger Π_b results in its bidding a smaller portion of the combined company, which leads to earlier initiation, no matter the constraints of the competitor.

In case *(5)*, it is easy to notice that when an unconstrained bidder competes against a severely constrained bidder, its initiation threshold also decreases in Π_b. The reason is that a higher Π_b speeds up initiation by the constrained bidder. Thus, conditional on the constrained bidder not initiating yet, the unconstrained bidder faces, on average, a weaker competitor. As a result, at any hypothetical initiation threshold, the expected payoff of the unconstrained bidder from initiating the contest is higher, leading to a lower initiation threshold.

Figure 3 shows the comparative statics of the four equilibrium initiation strategies corresponding to the model in Sections III.A–III.C. The strategies are built for the benchmark model parametrization, for a bidder with the average valuation, $v = 1.3$. The comparative statics are with respect to the five model parameters highlighted in Proposition 8 as well as the dispersion of the bidders' valuations. As the dispersion of the valuations increases, a bidder with valuation v becomes better separated from bidders with lower valuations, and therefore on average pays less in a successful contest. As a result, the bidder initiates the auction earlier. The initiation strategies seem to be particularly sensitive to the costs of losing the deal and the dispersion of the bidders' valuations. In fact, when costs of losing the deal (the dispersion of valuations) are sufficiently high (low), the stock bidder with the average valuation never initiates the contest: its valuation is below the threshold v^* (v_2^*) obtained in Proposition 3 (4).

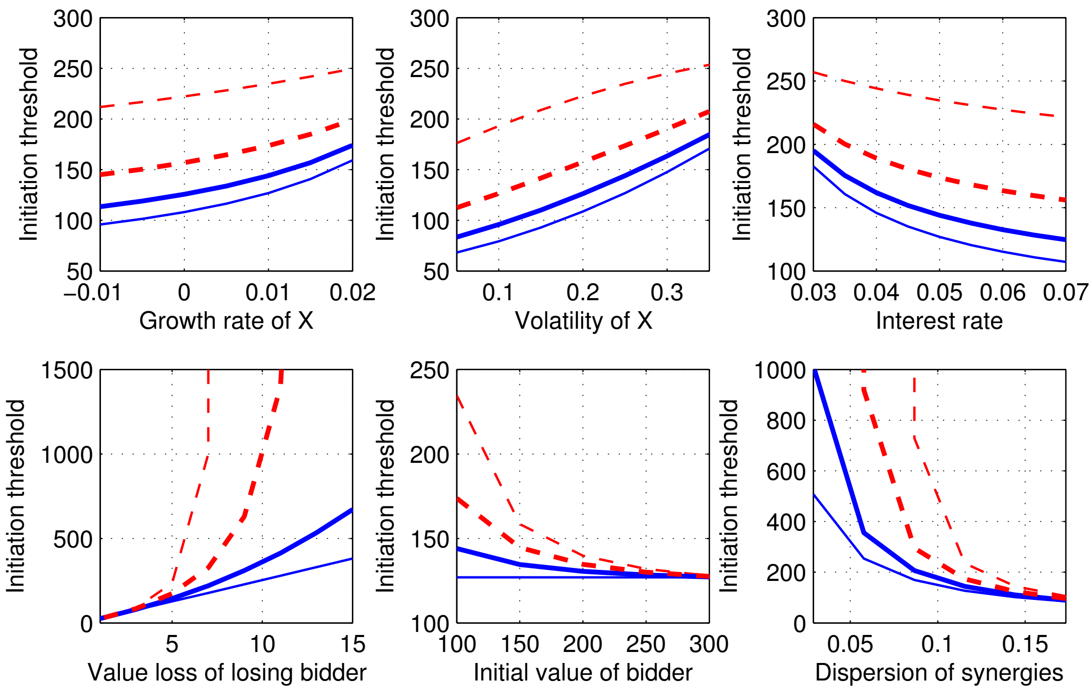

Figure 3: Initiation strategies of unconstrained and extremely constrained bidders as a function of model parameters. The figure shows the comparative statics of the four initiation strategies for the benchmark model parametrization (Table I) and a bidder with the average valuation, $v = 1.3$. The thin solid (thin dashed) line is the strategy of an unconstrained (extremely constrained) bidder facing another unconstrained (extremely constrained) bidder; the thick solid (thick dashed) line is the strategy of an unconstrained (extremely constrained) bidder facing an extremely constrained (unconstrained) bidder. The comparative statics are with respect to *(i)* the growth rate of a target's assets, μ, *(ii)* the volatility of a target's assets, σ, *(iii)* the interest rate, r, *(iv)* costs of losing the contest, Δ, *(v)* the initial value of bidders, P_b (keeping Δ fixed), and *(vi)* the dispersion of the bidders' valuations, $D(v)$ (keeping the average valuation fixed).

Figure 4 depicts the comparative statics of the four equilibrium initiation strategies (two unconstrained bidders, two extremely constrained bidders ($C_1 = C_2 = 0$), and bidders with internal cash $C_1 = 125$ and $C_2 = 0$ competing against each other) for the benchmark model parametrization as a function of the same six model parameters. The strategies are plotted for the bidder with the average valuation, $v = 1.3$. Incentives to fit into cash constraints are strong when μ, σ or P_b are higher, and when r is smaller. In all these cases, the combined company has a higher expected value. When means of payment are endogenous, the bidders are unwilling to share this highly-valued company with the target and choose to predominantly pay cash at the cost of earlier initialization.

Figure 5 shows the comparative statics of the optimal initiation strategies for the benchmark model parametrization and bidders with cash constraints C_1 and $C_2 = 0$, with respect to C_1 for the bidder with the average valuation, $v = 1.3$. For intermediate ranges of C_1, bidder 1 has incentives to fit into cash and bidder 2, recognizing that now it faces a weaker competitor, follows by decreasing its own initiation threshold. For low and high values of C_1, all deals either require all available cash to be done or are always done in cash only, weakening the motives to fit into cash. As a result, strategies of both cash-constrained bidders lie between the strategies of two unconstrained and two extremely constrained bidders competing against each other.

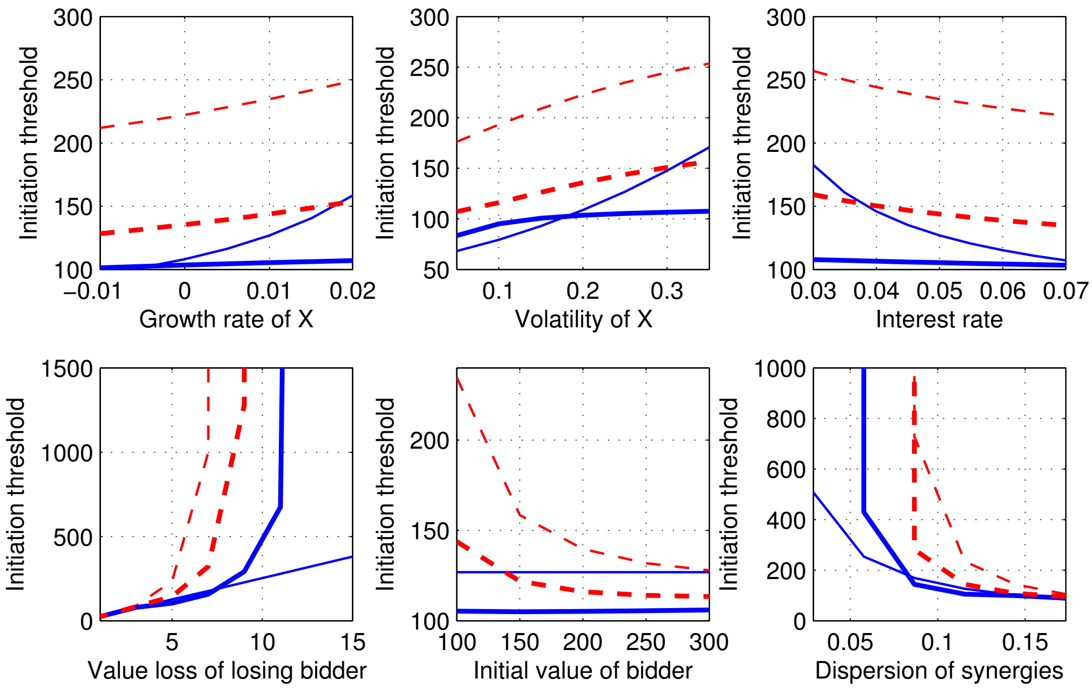

Figure 4: Initiation strategies of constrained bidders as a function of model parameters. The figure shows the comparative statics of the four initiation strategies for the benchmark model parametrization (Table I) and a bidder with the average valuation, $v = 1.3$. The thin solid (thin dashed) line is the strategy of an unconstrained (extremely constrained) bidder facing another unconstrained (extremely constrained) bidder; the thick solid (thick dashed) line is the strategy of a bidder with cash $C_1 = 125$ ($C_1 = 0$) facing a bidder with cash $C_2 = 0$ ($C_2 = 125$). The comparative statics are with respect to *(i)* the growth rate of a target's assets, μ, *(ii)* the volatility of a target's assets, σ, *(iii)* the interest rate, r, *(iv)* costs of losing the contest, Δ, *(v)* the initial value of bidders, Π_b (keeping Δ fixed), and *(vi)* the dispersion of the bidders' valuations, $D(v)$ (keeping the average valuation fixed).

VI Analysis

The results obtained in previous sections yield a number of implications. Below, we list and discuss each of them.

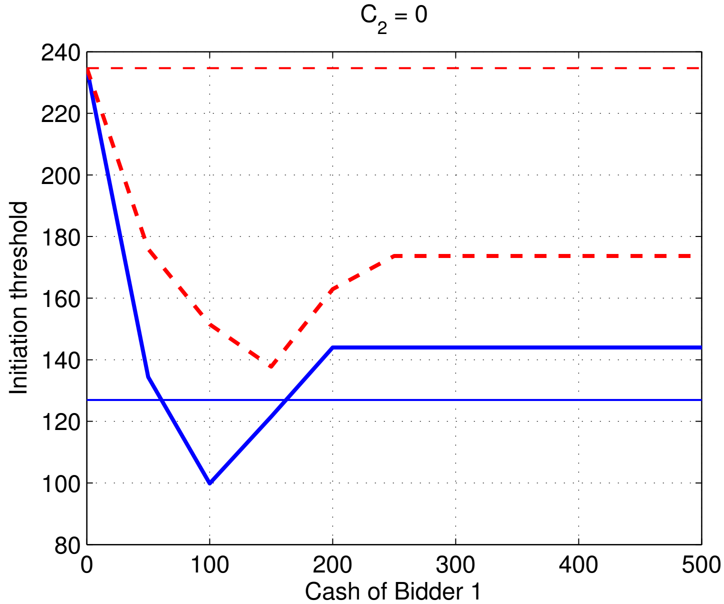

Figure 5: Initiation strategies of constrained bidders as a function of asymmetries in cash constraints. The figure shows the comparative statics of the four initiation strategies for the benchmark model parametrization (Table I) as a function of the cash position of bidder 1, C_1. The comparative statics are calculated for the bidder with the average valuation, $v = 1.3$. The thick solid (thick dashed) line is the strategy of a bidder with cash C_1 ($C_2 = 0$) competing against a bidder with cash $C_2 = 0$ (C_1). The thin solid (thin dashed) line is the strategy of an unconstrained (extremely constrained) bidder facing another unconstrained (extremely constrained) bidder.

VI.A Initiation of Takeover Contests, Means of Payment, and Premiums

A1. *Companies acquired in stock are (usually) larger and older than companies acquired in cash.*

Bidders with lower valuations have higher benefits to wait until the target grows, and when the target is larger, the cash constraint of the bidder is more likely to bind. Thus, these targets tend to be acquired with the help of stock. This is always the case for targets with low enough valuations. To see this, note that $\lim_{v \downarrow v_i^*} \bar{X}_i(v) = \infty$. Thus, targets with synergies of the winning bidder close to $\min(v_1^*, v_2^*)$ are always acquired with the help of stock. In contrast, targets with higher synergies can be acquired either in cash or in stock, depending on whether the valuation of the rival bidder is high enough so that the cash constraint of the winning bidder binds. Thus, the probability of a cash deal conditional on the valuation v of the acquirer always increases in v, when v is low. In theory, this probability may be non-monotone in v for larger v, because of the countervailing effect: an increase in the strength of the rival, as v increases, may dominate the effect of the target getting acquired smaller. However, we found it to be monotone in the numerical specifications we have tried.

This effect can be important for empirical research as it highlights an omitted variable in the link between the size of the target and means of payment. Not only are large companies acquired in non-cash deals because the acquirer does not have sufficient cash to finance a large payment; such companies were allowed to grow large because potential synergies were not high enough for bidders to acquire them small.

Figure 6 shows, for the benchmark parametrization and $C_1 = 125$, $C_2 = 0$, probabilities that cash and non-cash deals are completed in years 1, 2–5, 6–10, 11–25, and 26–100[17] as well as average acquisition size in deals completed by the end of year 1, 5, 10, 25, and 100. The starting value of the target is such that it is on the verge of being acquired by the highest bidder with the lowest cash constraints: $X_0 = \bar{X}_1(\bar{v})$. Cash deals mostly happen within the first five years of the target's life while non-cash deals reach their peak in years 2–5 and continue to be dominant types of acquisition in years 6–10. Cash deals are on average smaller and the gap in average size of cash and non-cash deals increases with the sample horizon as more and more non-cash deals are made for large targets by bidders with the lowest valuations.

While we do not directly model shocks to cash constraints, the above results make it evident that takeover activity can be spurred by two types of shocks: technology shocks that affect the gains from a takeover and shocks to bidders' cash constraints. The effect of technology shocks is clear: a contest is triggered once technology shocks shoot the cash flow variable X_t up to the upper acquisition threshold. The effect of shocks to cash constraints is more subtle. According to a naive argument, cash constraints should have no effect, because even if a bidder is cash constrained, it can always pay the target the proportion of the stock of the combined firm. In the setting with bidders' private information about valuations, this naive argument is not valid, because a severely cash constrained bidder initiates a contest at a higher threshold than an unconstrained bidder. As a result, the change in economic environment that relaxes the bidders'

[17] Formally, for each given realization of the two bidders' valuations, v_1 and v_2, the conditional probability that a contest is initiated over a finite time horizon T is

$$\mathbb{P}[\text{acquisition}|v_1, v_2, X_t, T] = \min\left\{1, N\left(\frac{-\log\frac{\min\{\bar{X}_1(v_1),\bar{X}_2(v_2)\}}{X_t} + (\mu - \sigma^2/2)T}{\sigma\sqrt{T}}\right)\right.$$
$$\left. + \exp\left\{\frac{2(\mu-\sigma^2/2)\log\frac{\min\{\bar{X}_1(v_1),\bar{X}_2(v_2)\}}{X_t}}{\sigma^2}\right\} N\left(\frac{-\log\frac{\min\{\bar{X}_1(v_1),\bar{X}_2(v_2)\}}{X_t} - (\mu - \sigma^2/2)T}{\sigma\sqrt{T}}\right)\right\}.$$

Then, the conditional probability that a contest is initiated over a finite time horizon T for any v_1 and v_2 is

$$\mathbb{P}[\text{acquisition}|X_t, T] = \mathbb{E}_{v_1,v_2}\left[\mathbb{I}[v_1 > v_1^*, v_2 > v_2^*]\mathbb{P}[\text{acquisition}|v_1, v_2, X_t, T]\right],$$

where $\mathbb{I}[\cdot]$ is the indicator function equal to one if the condition in brackets is satisfied and zero otherwise.

cash constraints decreases the threshold on the level of cash flows at which each bidder initiates an acquisition and thereby sparks merger activity.[18]

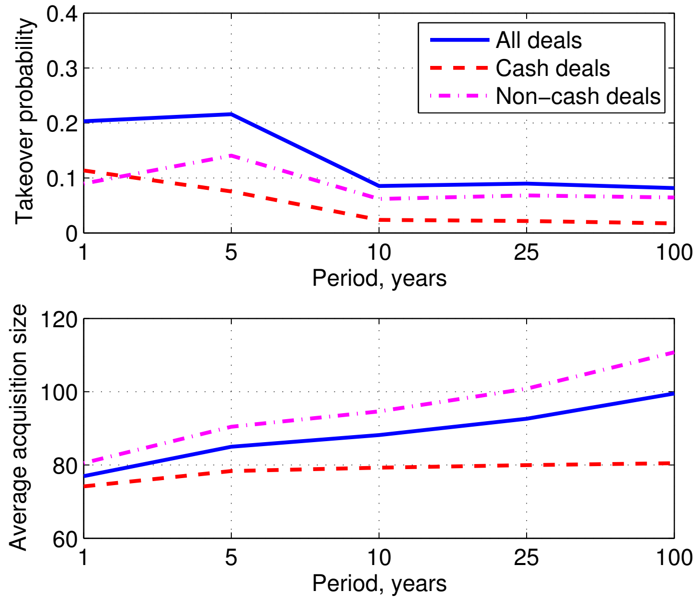

Figure 6: Takeover probability and average acquisition size in cash and non-cash deals. The figure corresponds to prediction A1. For the benchmark parametrization (Table I) and $C_1 = 125$, $C_2 = 0$, the top panel shows the frequency of takeovers initiated and completed in years 1, 2–5, 6–10, 11–25, and 26–100. The bottom panel shows the average acquisition size in deals completed by the end of years 1, 5, 10, 25, and 100. The starting value of the target is $X_0 = \bar{X}_1(\bar{v})$. The solid line corresponds to all types of deals. The dashed (dash-dotted) line corresponds to cash (non-cash) deals.

A2. *Conditional on its valuation and the size of the target, the acquirer pays a higher takeover premium if the deal is done in stock.*

Conditional on the valuation v of the winning bidder and size of the target at acquisition $\bar{X}$, whether the deal is done in cash or not is driven by the variation in the valuation w of the rival bidder. The deal is done in cash if $w < (C - \Delta)/\bar{X}$, where C is the cash position of the acquirer, and is done with the help of stock otherwise. Thus, the takeover premium in the stock deal is higher for two reasons: first, the stock bid transfers wealth from the winning bidder to the seller; and, second, the acquirer is more likely to use stock if its rival is stronger.

At the same time, without conditioning on the acquirer's valuation of the target, the average takeover premium can be higher in cash deals than in stock deals:

[18] Allowing cash constraints to evolve in time, e.g., switch from high to low level only strengthens this result. Intuitively, in states with low cash, bidders have extra incentives to delay the acquisition until their cash constraints are relaxed and acquisition in mostly cash is possible.

A3. *For some parameterizations of the model, bidders pay higher takeover premiums to acquire companies in cash.*

Despite the fact that acquirers give away a smaller portion of their valuations in cash deals, they tend to be bidders with higher valuations. They give away a smaller portion of a larger pie. As a result, there exist parameterizations for which the effect of a pie increase dominates the effect of a smaller pie share and cash bidders on average pay higher takeover premiums (as a percentage of the target's value).

Figure 7 shows the average takeover premiums in cash and non-cash deals, both conditional on observing the highest bidder valuation and sample-wide unconditional, where the sample consists of takeover contests that differ only in valuations of participating bidders. As expected, the conditional takeover premiums are higher in non-cash deals for any value of highest valuation. However, in the case when both bidders have non-zero internal cash (Panels B and D), best deals are done exclusively in stock while worst deals are done exclusively in combinations on cash and stock which leads to an inverse relationship between the sample-wide unconditional average takeover premiums. This result is obtained without assuming either adverse selection about the bidders' assets or private information of the acquirer about its own firm as in the previous literature. It is the takeover timing-determined positive correlation between cash deals and high-synergy deals that is responsible for the result.

An empirical implication of A2 - A3 is that, if a good proxy of synergies can be found, then conditional on this proxy, takeover premiums in cash deals should be lower than those in non-cash deals. Conditional on the recovered valuation of the highest bidder, takeover premiums in cash deals should be lower than those in non-cash deals, despite the empirical evidence that they are higher unconditionally.

A4. *Stock bidders receive usually lower acquirer premiums than cash bidders.*

Not only do stock bidders give away a larger portion of their valuations, but also they have lower valuations, so the two effects complement each other.

These properties are consistent with existing empirical evidence. The first property is consistent with the evidence that takeovers paid in cash are for smaller firms than those partially or fully paid in stock (e.g., Betton, Eckbo, and Thorburn, 2008). The third property is consistent with a number of studies (e.g., Franks, Harris, and Mayer, 1988; Eckbo and Langohr, 1989) that historically, offer premiums were greater in all-cash offers, even controlling for the differential tax impact. The fourth

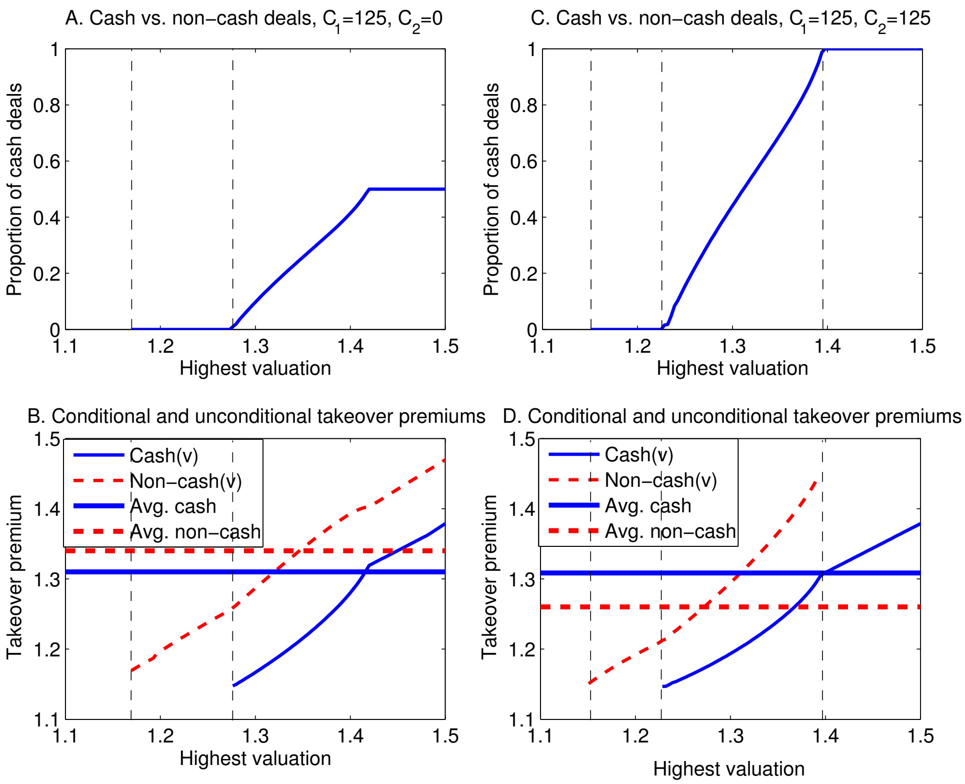

Figure 7: Conditional and unconditional takeover premiums in cash and non-cash deals. The figure corresponds to prediction A3. Panels A and C show, for the two cases: *(i)* $C_1 = 125$, $C_2 = 0$, *(ii)* $C_1 = 125, C_2 = 125$, the probability that a takeover contest is completed in cash as a function of the highest bidder valuation. Panels B and D show, for the same two cases, the average takeover premiums in cash and non-cash deals, both conditional on observing the highest bidder valuation (thick solid and dashed lines) and sample-wide unconditional (extra thick solid and dashed lines).

property is consistent with the findings by Eckbo, Giammarino, and Heinkel (1990) and Berkovitch and Narayanan (1990) that the larger the cash portion of the deal, the higher are acquirer abnormal announcement returns.

VI.B Properties of Initiating and Winning Bidders

B1. *If the impact of an acquisition on the losing bidder is negative, bidders with sufficiently low valuations and cash constraints never initiate a contest.*

As a result, there is a non-zero probability that neither cash-constrained bidder initiates a contest and a valuable target continues as a stand-alone. Figure 8, region (1) shows that for the benchmark model parametrization and cash constraints $C_1 = 125$, $C_2 = 0$, the probability that the target is never acquired is approximately 4%. In the case of two stock bidders competing against each other, this probability is almost 9%.

B2. *In initiated contests, the distribution of participating bidders' valuations is determined endogenously and can be asymmetric.*

This result holds true even if the unconditional distribution of valuations is the same for the bidders. Figure 8, left-most dashed line shows valuations of bidders 1 and 2, v and w, at which they initiate contest at the same threshold, $\bar{X}_1(v) = \bar{X}_2(w)$. In contests initiated by any bidder, the highest possible valuation of the more constrained bidder is higher than that of the less constrained bidder; the less constrained bidder also faces a stronger competitor on average. Interestingly, in the sample of takeovers that differ only in valuations of participating bidders, this result is reversed: because more constrained bidders are less likely to initiate takeover contests in the first place, their average valuation across all initiated contests is lower than that of less constrained bidders. Figure 9 shows how average valuations of the bidders with cash constraints $C_1 = 125$, $C_2 = 0$ change with respect to the parameters that have the strongest effect on the probability that a contest is never initiated: the value of the losing bidder, P_0 and the cash constraint of one of the bidders, specifically, C_1. Lower P_0 and C_1 correspond to a larger gap between v_1^* and v_2^* and result in a larger difference between average valuations in the sample of similar takeovers.

B3. *Some initial bids of a less constrained bidder will be rejected in favor of a more constrained bidder. Under some parameterizations of the model, initial bids in cash have a smaller probability to be rejected compared to initial bids that include stock.*

The second prediction is consistent with empirical evidence (Betton, Eckbo, and Thorburn, 2009) while the first prediction is novel. The two predictions might seem contradictory at first. However, a less constrained bidder and a bidder who completes the deal in cash are not equivalent. The latter bidder is more likely to have both high cash balances and high valuation so that it approaches the target while the deal can still be sealed in cash. For the benchmark parametrization and $C_1 = 125$, $C_2 = 0$, Figure 8, regions (2) and (4) show contests initiated by the less constrained bidder 1 in which the initial bidder bids in combinations of cash and stock. Region (4) shows contests in which such bidder loses to bidder 2 who bids in stock. Region (6) shows contests initiated by bidder 2 who wins in stock. The conditional probability of the initiating non-cash bidder losing the contest is the area of region (4) divided by the combined areas of regions (2), (4), and (6) and is equal to approximately 10%. In contrast, regions (3) and (5) show contests initiated by the less constrained bidder 1 in which the initial bidder bids in cash. Region (5) shows contests in which such bidder loses to bidder 2 who bids in stock. The conditional provability of the initiating cash bidder losing the contest is the area of region (5) divided by the combined area of regions (3) and (5) and is equal to approximately 2.6%. Hence, for a given parametrization, cash bids by the initiating bidder indeed have a smaller probability to be rejected compared to non-cash bids. It is easy to construct an example in which the opposite is true: take $C_1 \to \infty$, $C_2 = 0$. In this case, there is zero correlation between cash bids and cash bidder valuations and only initial cash but not stock bids can be rejected.

VI.C Target's Preference for Cash versus Stock Bids

An important result in the static security design literature (Hansen, 1985; DeMarzo, Kremer, and Skrzypacz, 2005) is that auctions in stock dominate contests in cash in terms of the expected revenues of the seller. As a result, if in a static setting the target can commit to accept only stock bids, it will do so. However, practical cases of such commitment in takeover contests are rare. An interesting question is to study whether the target would have incentives to commit to accept only stock bids in a dynamic setting, when bidders can time an acquisition. In this paper, we do not aim to provide a rigorous answer to this question. One of the complications that can arise is that the target, upon learning about bidder valuations from their initiation (and non-initiation) decisions, can change the preferred security design of the takeover contest dynamically. Instead, to provide a flavor of the more general case, we consider

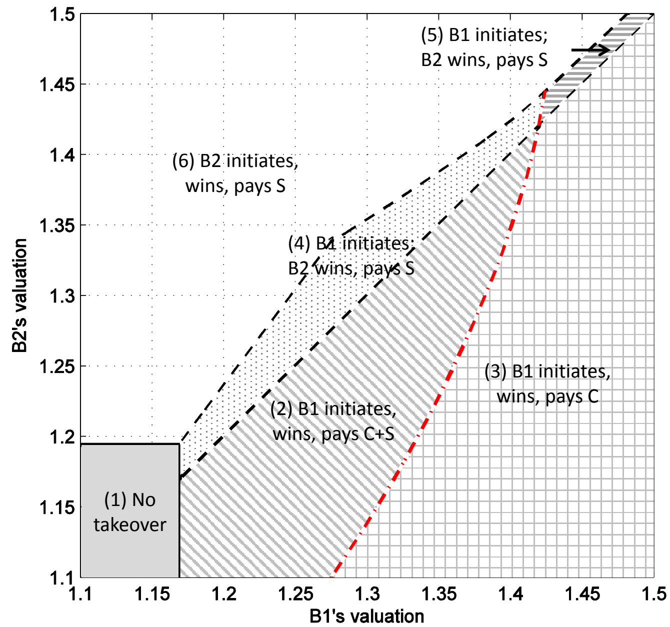

Figure 8: Initiation, acquisition and means of payment in takeover contests with cash constrained bidders. For the benchmark parametrization (Table I) and cash constraints of bidders 1 and 2 equal to $C_1 = 125$, $C_2 = 0$, the figure shows regions of valuations for which bidders initiate and win takeover contests, as well as the resulting type of the deal (cash, cash and stock, stock). The dash-dotted line separates the cases in which bidder 1 makes cash and non-cash final bids.

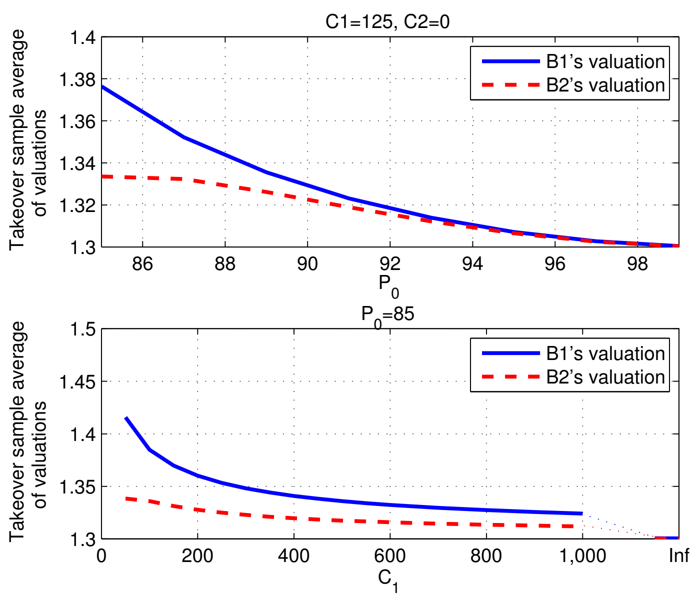

Figure 9: Average valuations of cash constrained bidders in initiated contests. The figure shows average valuations of cash constrained bidders for the benchmark parametrization (Table I) as a function of *(i)* the value of the losing bidder, P_0, assuming cash constraints $C_1 = 125, C_2 = 0$, and *(ii)* cash constraint of bidder 1, C_1, assuming $C_2 = 0$ and $P_0 = 85$. The solid (dashed) line is the average valuation of bidder 1 (2).

a simpler setting in which the target has to commit to the security design at time zero.[19] We also focus on the case in which both bidders are exogenously unconstrained: $C_1 \to \infty, C_2 \to \infty$. Our results in this section are related to Cong (2012), who shows that an auctioneer selling a real option, such as a lease to explore an oil well, can prefer the auction in cash over the auction in stock, because of the post-auction moral hazard that affects the timing of the option exercise. Our argument is different, because the timing of actions is reverse: a bidder exercises its option (approaches the target) before the auction takes place.

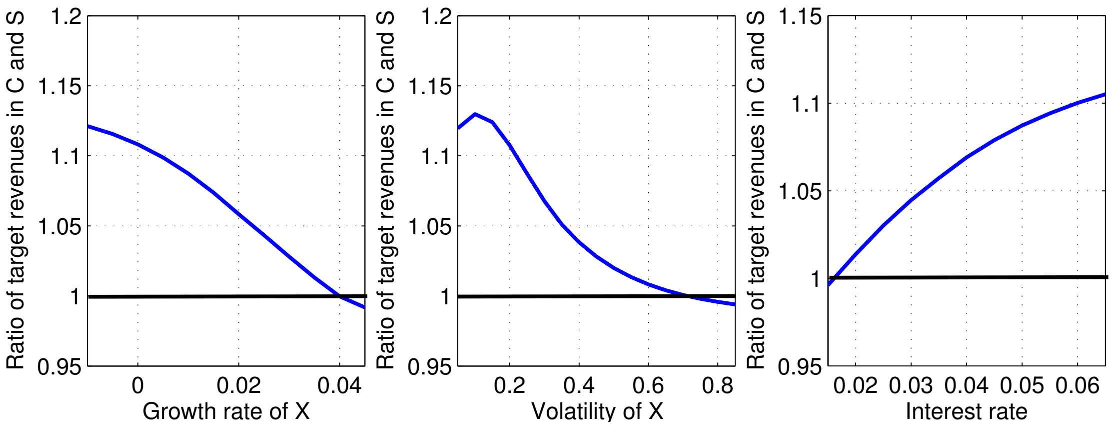

Figure 10: The ratio of the target revenue (present value) from contests in cash and in stock. For the benchmark parametrization (Table I), the figure shows the ratio of present values of target revenues in cash and stock deals as a function of *(i)* the growth rate of the target's assets, μ, *(ii)* the volatility of the target's assets, σ, *(iii)* the interest rate r.

Figure 10 shows the ratio of present values of target revenues in cash and stock contests as a function of μ, σ, and r. For realistic parameters, the target prefers not to commit to restricting bids to stock. When μ and σ are well above realistic parameters (or r is very low), contests in stock start to dominate contests in cash in terms of target revenue. Intuitively, if a target has a higher growth rate or higher volatility of assets (or interest rate is lower), the difference between initiation thresholds of cash and stock bidders is passed quicker (or affects the present value of target revenues less). As a result, the effect of extra delay is less important for the present value of high-growth targets, which leads to their preference for battles in stock.[20]

This result suggests that in a dynamic setting, most targets (including targets with "standard"

[19] The results remain the same for any X_0 below the lowest initiation threshold of the bidders: expected target revenue takes the form $\alpha + \gamma_j X_0^\beta$, where $j \in \{c, s\}$ corresponds to the case of cash and stock bids. This also means that as long as X_t stays below the lowest initiation threshold, the target does not have incentives to attempt and change the security design in this region.

[20] If the target can choose whether to restrict the type of bids at any point of time, learning about bidder valuations from X_t strengthens its incentives to commit to restricting bids. This is because, as the support of possible bidder valuations shrinks, stock bids extract an increasingly higher proportion of revenues from the bidders.

characteristics that are similar to an average COMPUSTAT firm) can have aligned incentives with the bidders: both the bidders and the target can prefer cash deals. This is in line with the observation that there are very few (if any) practical cases in which the target attempts to restrict the type of bids. However, a small fraction of firms with either high growth or high volatility of assets can have misaligned incentives with the bidders. If there is any evidence regarding the target's attempts to restrict the type of bids in takeover contests, it is likely to be found among high-μ, high-σ targets.[21]

VII Concluding Remarks

This paper presents a theoretical analysis of the timing of acquisitions, takeover premiums, and means of payment in the setting in which firms' fortunes and ability to pay cash are affected by the technological change and cash constraints. Optimal choices of each bidder and, in turn, takeover outcomes are affected not only by synergies of this bidder with the target but also by financing constraints of both the bidder and its competitors. The results of our general model are consistent with a variety of empirical findings and provide further implications. In particular, they provide an explanation, alternative to previous studies, why cash deals feature higher average takeover premiums than non-cash cash deals. In our model, high-synergy bidders approach their targets earlier, before they grow large enough to make bidders' cash constraints binding, and thus have enough cash to finalize the deal. We also propose several novel testable predictions that relate the timing of an acquisition and its outcomes to key characteristics of competing bidders. Importantly, these predictions explicitly recognize the effect of private information and selection on decision making.

A potential direction of future research is to understand targets' motives to initiate takeover contests by themselves. We abstract from this issue because our focus is on strategic acquisitions, and they are usually bidder-initiated (Fidrmuc et al., 2012). However, target-initiated deals are also common, especially among private equity deals. Another direction for future research is to test predictions of our model. In particular, it can be interesting to quantify the relative importance of our dynamic selection mechanism and other theories of means of payment (e.g., Fishman, 1989) on takeover outcomes.

[21] In contests for growth targets and targets from hi-tech industries, it is common that target managers are major stockholders in their company and have much control over its decision making, including the ability to negotiate terms of a potential takeover. In many cases, they eventually become large stockholders of the combined company, which is consistent with our result.

Appendix A Proofs

Proof of Proposition 1. Suppose that bidder i wins the auction at price y. Paying $b \leq y$ in cash requires the bidder to pay $\alpha(b, y)$ in stock. The value of this payment is

$$\begin{aligned} & \frac{p-b}{\Pi_o + p}\left(v_i X_t + \Pi_b\right) + b \\ = & \frac{p\left(v_i X_t + \Pi_b\right) + b\left(p - v_i X_t - \Delta\right)}{\Pi_o + p} \end{aligned}$$

The value of the bid is increasing in b if $v_i < p^{-1}(y)$, decreasing in b if $v_i > p^{-1}(y)$, and does not depend on b if $v_i = p^{-1}(y)$.

Consider the decision of a bidder to drop out at a price different from (5). Suppose that it follows the strategy of dropping out at a price above (5). If the bidder wins at price $y > p(v_i)$, its payoff from winning is

$$\begin{aligned} & \max_{b \leq C_i}\left\{\left(1 - \alpha(b, y)\right)\left(v_i X_t + \Pi_b\right) - b\right\} \\ = & \frac{\Pi_o}{\Pi_o + y}\left(v_i X_t + \Pi_b\right) \\ < & \frac{\Pi_o}{\Pi_o + v_i X_t + \Pi_b - \Pi_o}\left(v_i X_t + \Pi_b\right) = \Pi_o, \end{aligned} \tag{A1}$$

where the first equality follows from the optimality of paying in stock when $y > p(v_i)$, and the inequality follows from $y > p(v_i)$. Therefore, dropping out at a price above (5) is suboptimal. Similarly, suppose that bidder i follows the strategy of dropping out at a price p below (5). Dropping out at (5) instead leads to winning when the other bidder drops out at prices y between p and $p(v_i)$. The payoff of a bidder from winning in such events is

$$\begin{aligned} & \max_{b \leq C_i}\left\{\left(1 - \alpha(b, y)\right)\left(v_i X_t + \Pi_b\right) - b\right\} \\ \geq & \frac{\Pi_o}{\Pi_o + y}\left(v_i X_t + \Pi_b\right) \\ \geq & \Pi_o. \end{aligned} \tag{A2}$$

Here, the first inequality holds because paying the bid using only stock is a feasible strategy for the bidder, and the second inequality follows from $y \leq p(v_i)$. Therefore, dropping out at a price below (5) is also suboptimal. Thus, it is a weakly dominant strategy for a bidder with valuation v_i to drop out at price (5).

Because (5) is strictly increasing in v_i, on equilibrium path, the valuation of the winning bidder is always greater or equal than $p^{-1}(y)$, where y is the price at which the other bidder drops out. Hence, in equilibrium $b = \max\{y, C_i\}$, i.e., the winning bidder pays as much cash as possible.

Proof of Proposition 2. Taking the first-order condition (9) and dividing both sides by X_0^β yields

$$\begin{aligned} 0 &= -\beta \frac{1}{\bar{X}^{\beta+1}} \int_{\underline{v}}^{\bar{X}_c^{-1}(\bar{X})} \left(\bar{X} \max\{v-w,0\} - \Delta\right) dF(w) \\ &\quad + \frac{1}{\bar{X}^{\beta}} \int_{\underline{v}}^{\bar{X}_c^{-1}(\bar{X})} \max\{v-w,0\}\, dF(w). \end{aligned} \tag{A3}$$

In equilibrium, the maximum is reached at $\bar{X}_c(v)$. Plugging in and multiplying both sides by $\bar{X}_c(v)^{\beta+1}$, we get

$$\bar{X}_c(v)(\beta-1)\int_{\underline{v}}^{v}(v-w)\, dF(w) = \beta \Delta F(v). \tag{A4}$$

Hence,

$$\bar{X}_c(v) = \frac{\beta}{\beta-1}\frac{\Delta}{v-\mathbb{E}[w|w\leq v]}. \tag{A5}$$

By assumption, $v-\mathbb{E}[w|w\leq v]$ is increasing in v. Therefore, $\bar{X}_c(v)$ is indeed decreasing in v.

Proof of Proposition 3. Taking the first-order condition (12) and dividing both sides by X_0^β yields

$$\begin{aligned} 0 &= -\beta \frac{1}{\bar{X}^{\beta+1}} \int_{\underline{v}}^{\bar{X}_s^{-1}(\bar{X})} \left(\Pi_o \frac{\Pi_b+\bar{X}v}{\Pi_b+\bar{X}w} - \Pi_b\right) dF(w) \\ &\quad + \frac{1}{\bar{X}^{\beta}} \int_{\underline{v}}^{\bar{X}_s^{-1}(\bar{X})} \Pi_o \left[\frac{\Pi_b+\bar{X}v}{\Pi_b+\bar{X}w}\right]' dF(w). \end{aligned} \tag{A6}$$

The derivative is equal to

$$\left[\frac{\Pi_b+\bar{X}v}{\Pi_b+\bar{X}w}\right]' = \frac{(v-w)\Pi_b}{\left(\Pi_b+\bar{X}w\right)^2}. \tag{A7}$$

Plugging it into (A6), dividing by $F(v)$, and using the fact that in equilibrium the maximum is reached at $\bar{X}_s(v)$, we obtain

$$\begin{aligned} 0 &= -\beta\Pi_o\mathbb{E}\left[\frac{\Pi_b+v\bar{X}_s(v)}{\Pi_b+w\bar{X}_s(v)}|w\leq v\right] + \beta\Pi_b \\ &\quad + \Pi_o\Pi_b\mathbb{E}\left[\frac{(v-w)\bar{X}_s(v)}{\left(\Pi_b+w\bar{X}_s(v)\right)}|w\leq v\right]. \end{aligned} \tag{A8}$$

Rewriting, we obtain (13).

Proof of Proposition 4. We need to compare

$$\mathbb{E}[v-w|w\leq v] \text{ and } \mathbb{E}\left[\frac{\Pi_o\left(\Pi_b+\frac{\beta}{\beta-1}w\bar{X}\right)}{\left(\Pi_b+w\bar{X}\right)^2}(v-w)|w\leq v\right]. \tag{A9}$$

Consider the following difference:

$$1-\frac{\Pi_o\left(\Pi_b+\frac{\beta}{\beta-1}w\bar{X}\right)}{\left(\Pi_b+w\bar{X}\right)^2} = \frac{\Pi_b^2+2\Pi_b w\bar{X}+w^2\bar{X}^2-\Pi_o\Pi_b-\frac{\beta}{\beta-1}\Pi_o w\bar{X}}{\left(\Pi_b+w\bar{X}\right)^2}$$
$$= \frac{\Pi_b\left(\Pi_b-\Pi_o\right)+\left(2\Pi_b-\frac{\beta}{\beta-1}\Pi_o\right)w\bar{X}+w^2\bar{X}^2}{\left(\Pi_b+w\bar{X}\right)^2}. \quad \text{(A10)}$$

The first term in the numerator is positive because $\Pi_b > \Pi_o$. The second term in the numerator is positive because of (17). Therefore, (A10) is positive for all w and $\bar{X}$. Consequently,

$$\mathbb{E}\left[v-w|w\leq v\right] > \mathbb{E}\left[\frac{\Pi_o\left(\Pi_b+\frac{\beta}{\beta-1}w\bar{X}\right)}{\left(\Pi_b+w\bar{X}\right)^2}\left(v-w\right)|w\leq v\right]. \quad \text{(A11)}$$

Because of this and monotonicity of the left-hand side of (13) with respect to $\bar{X}$, the unique solution of (13), $v > v^*$ is higher than the unique solution of (10).

Proof of Proposition 5. First, we maximize (18) with respect to threshold $\bar{X}$. Analogously to the proof of proposition 1, we obtain (19). Second, we maximize (20) with respect to threshold $\bar{X}$:

$$0 = -\frac{\beta}{\bar{X}^{\beta+1}}\int_{\underline{v}}^{\bar{X}_1^{-1}(\bar{X})}\left(\Pi_o\frac{\left(v-w\right)\bar{X}}{\Pi_b+w\bar{X}}-\Delta\right)f\left(w\right)dw$$
$$+\frac{1}{\bar{X}^{\beta}}\int_{\underline{v}}^{\bar{X}_1^{-1}(\bar{X})}\Pi_o\left[\frac{\left(v-w\right)\bar{X}}{\Pi_b+w\bar{X}}\right]'f\left(w\right)dw. \quad \text{(A12)}$$

Equivalently,

$$0 = -\beta\int_{\underline{v}}^{\bar{X}_1^{-1}(\bar{X})}\Pi_o\frac{\left(v-w\right)\bar{X}}{\Pi_b+w\bar{X}}f\left(w\right)dw+\beta\Delta F\left(\bar{X}_1^{-1}\left(\bar{X}\right)\right)$$
$$+\bar{X}\int_{\underline{v}}^{\bar{X}_1^{-1}(\bar{X})}\Pi_o\frac{\left(v-w\right)\Pi_b}{\left(\Pi_b+w\bar{X}\right)^2}f\left(w\right)dw. \quad \text{(A13)}$$

Dividing by $F\left(\bar{X}_1^{-1}\left(\bar{X}\right)\right)$:

$$0 = -\beta\Pi_o\mathbb{E}\left[\frac{\left(v-w\right)\bar{X}}{\Pi_b+w\bar{X}}|w\leq\bar{X}_1^{-1}\left(\bar{X}\right)\right]+\beta\Delta$$
$$+\Pi_o\mathbb{E}\left[\frac{\left(v-w\right)\bar{X}\Pi_b}{\left(\Pi_b+w\bar{X}\right)^2}|w\leq\bar{X}_1^{-1}\left(\bar{X}\right)\right]. \quad \text{(A14)}$$

Equivalently,

$$\mathbb{E}\left[\beta\frac{v-w}{\Pi_b+w\bar{X}}-\Pi_b\frac{v-w}{\left(\Pi_b+w\bar{X}\right)^2}|w\leq\bar{X}_1^{-1}\left(\bar{X}\right)\right]\bar{X}=\beta\frac{\Delta}{\Pi_o}. \quad \text{(A15)}$$

Rewriting yields (21). Finally, we need to determine valuation v^* such that bidder 2 never approaches the target if $v \leq v^*$. Consider $\bar{X} \to \infty$. Because $\bar{X}_1(\underline{v})$ is finite as $\underline{v} > 1$, $\bar{X}_1^{-1}\left(\bar{X}\right) = \underline{v}$. Therefore, the left-hand side of (A15) is

$$\mathbb{E}\left[\beta\frac{v-w}{w}|w \leq \underline{v}\right] = \beta\frac{v-\underline{v}}{\underline{v}}. \tag{A16}$$

Point v^* is such that

$$\beta\frac{v^*-\underline{v}}{\underline{v}} = \beta\frac{\Delta}{\Pi_o}, \tag{A17}$$

which yields

$$v^* = \frac{\Pi_b}{\Pi_o}\underline{v}. \tag{A18}$$

Proof of Proposition 6. Proposition 3 establishes that $\bar{X}_s(v) > \bar{X}_c(v)$ for all v when $\frac{\beta}{\beta-1} < 2\frac{\Pi_b}{\Pi_o}$. Suppose that $\bar{X}_1(\tilde{v}) = \bar{X}_2(\tilde{v})$ for some $\tilde{v}$. Then, $\Psi(\tilde{v}) = 1$, $\Omega(\tilde{v}) = v$. As a result, $\bar{X}_1(\tilde{v}) = \bar{X}_c(\tilde{v})$; $\bar{X}_2(\tilde{v}) = \bar{X}_s(\tilde{v})$ and, under the assumption $\bar{X}_1(\tilde{v}) = \bar{X}_2(\tilde{v})$, all four strategies have to be equal at $\tilde{v}$ – a contradiction with the result of Proposition 3. Hence $\bar{X}_2$ and $\bar{X}_1$ cannot cross.

Assume that $\bar{X}_1(\hat{v}) > \bar{X}_2(\hat{v})$ for some $\hat{v}$. From Proposition 4, as $v \downarrow v^*$, $\bar{X}_2(v) \to \infty$ while $\bar{X}_1(v)$ remains finite. Hence, there exists $\epsilon > 0$ such that $\bar{X}_2(v^*+\epsilon) > \bar{X}_1(v^*+\epsilon)$. This, together with the assumption $\bar{X}_1(\hat{v}) > \bar{X}_2(\hat{v})$ and continuity of both $\bar{X}_1(v)$ and $\bar{X}_2(v)$ in v, implies that $\bar{X}_1(\tilde{v}) = \bar{X}_2(\tilde{v})$ for some $\tilde{v} \in (v^*+\epsilon, \hat{v})$. By earlier proof, however, $\bar{X}_2$ and $\bar{X}_1$ cannot cross. Hence, $\bar{X}_2(v) > \bar{X}_1(v)$ for all v.

The final step is to show that $\bar{X}_s(v) > \bar{X}_2(v)$ and $\bar{X}_1(v) > \bar{X}_2(v)$ for all v. Both inequalities follow from the fact that, when $\bar{X}_2(v) > \bar{X}_1(v)$ for all v, then $\Psi(v) > 1$ and $\Omega(v) < v$.

Proof of Proposition 7. The first-order condition of (23) is

$$\begin{aligned} 0 &= -\frac{\beta}{\bar{X}^{\beta+1}}\int_{\underline{v}}^{\bar{X}_{-i}^{-1}(\bar{X})}\left(\min\left\{\frac{\Pi_o+C_i}{\Pi_b+w\bar{X}},1\right\}\bar{X}\max\{v-w,0\}-\Delta\right)dF(w) \\ &\quad +\frac{1}{\bar{X}^{\beta}}\int_{\underline{v}}^{\bar{X}_{-i}^{-1}(\bar{X})}\left[\min\left\{\frac{\Pi_o+C_i}{\Pi_b+w\bar{X}},1\right\}\bar{X}\max\{v-w,0\}\right]'dF(w). \end{aligned} \tag{A19}$$

Equivalently,

$$\begin{aligned} 0 &= -\beta\int_{\underline{v}}^{\bar{X}_{-i}^{-1}(\bar{X})}\left(\min\left\{\frac{\Pi_o+C_i}{\Pi_b+w\bar{X}},1\right\}\bar{X}\max\{v-w,0\}\right)dF(w) \\ &\quad +\beta\Delta F\left(\bar{X}_{-i}^{-1}\left(\bar{X}\right)\right)+\bar{X}\int_{\underline{v}}^{\bar{X}_{-i}^{-1}(\bar{X})}\left[\min\left\{\frac{\Pi_o+C_i}{\Pi_b+w\bar{X}},1\right\}\bar{X}\max\{v-w,0\}\right]'dF(w). \end{aligned} \tag{A20}$$

Applying the equilibrium condition that the maximum is reached at $\bar{X}_i(v)$ and dividing by $F(\Omega(v))$ yields

$$\begin{aligned} & \mathbb{E}\left[\beta \min\left\{\frac{\Pi_o + C_i}{\Pi_b + w\bar{X}_i(v)}, 1\right\}(v-w)\,|w \leq \Omega(v)\right]\bar{X}_i(v) \\ & -\mathbb{E}\left[\left[\min\left\{\frac{\Pi_o + C_i}{\Pi_b + w\bar{X}_i(v)}, 1\right\}(v-w)\,\bar{X}_i(v)\right]'|w \leq \Omega(v)\right]\bar{X}_i(v) \\ = \; & \beta\Delta\Psi(v). \end{aligned} \tag{A21}$$

Let us decompose this expression into two intervals:

- if $w < (C_i - \Delta)/\bar{X}_i(v)$, then the expression under the expectation operator is

$$\beta(v-w) - \left[(v-w)\,\bar{X}_i(v)\right]' = (\beta - 1)(v-w); \tag{A22}$$

- if $w > (C_i - \Delta)/\bar{X}_i(v)$, then the expression under the expectation operator is

$$\begin{aligned} & (\Pi_o + C_i)\left(\frac{\beta(v-w)}{\Pi_b + w\bar{X}_i(v)} - \left[\frac{(v-w)\,\bar{X}_i(v)}{\Pi_b + w\bar{X}_i(v)}\right]'\right) \\ = \; & (\Pi_o + C_i)\left(\frac{\beta(v-w)}{\Pi_b + w\bar{X}_i(v)} - \frac{(v-w)\,\Pi_b}{\left(\Pi_b + w\bar{X}_i(v)\right)^2}\right) \\ = \; & (\beta - 1)\frac{(\Pi_o + C_i)(v-w)}{\Pi_b + w\bar{X}_i(v)} + (\beta - 1)\frac{(\Pi_o + C_i)(v-w)}{\Pi_b + w\bar{X}_i(v)}\frac{\frac{1}{\beta-1}w\bar{X}_i(v)}{\Pi_b + w\bar{X}_i(v)}. \end{aligned} \tag{A23}$$

Hence, we can rewrite (A21) as (24).

Similar to Section II.B, equations (24) do not have solutions for low enough v. Let v_i^* be such that $\lim_{v \to v_i^*} \bar{X}_i(v) = \infty$. Rewriting (24) at this point yields

$$\mathbb{E}\left[\frac{v_i^* - w}{w}|w \leq \Omega(v_i^*)\right] = \frac{\Delta\Psi(v_i^*)}{\Pi_o + C_i}. \tag{A24}$$

In the case of symmetric cash constraints, $C_1 = C_2 = C$ and $v_1^* = v_2^* = v^*$, given by

$$\mathbb{E}\left[\frac{v^* - w}{w}|w \leq v^*\right] = \frac{\Delta}{\Pi_o + C_i}. \tag{A25}$$

It is easy to see that in the special cases of $C \to \infty$ and $C = 0$ and , we obtain $\underline{v}$ and v^* from Section II.B, respectively.

Proof of Proposition 8. Let $v_i(x) := \bar{X}_i^{-1}(x)$ be the type of bidder $i \in \{1, 2\}$ that approaches the target at threshold x. We can re-write (A21) in terms of $v_1(x)$ and $v_2(x)$:

$$\mathbb{E}\left[\left(\beta\min\left\{\frac{\Pi_o+C_i}{\Pi_b+wx},1\right\}-\left[\min\left\{\frac{\Pi_o+C_i}{\Pi_b+wx},1\right\}x\right]'\right)(v_i(x)-w)\,|w\leq\min_{j\in\{1,2\}}v_j(x)\right]x$$
$$-\beta\Delta\frac{F\left(\max_{j\in\{1,2\}}v_j(x)\right)}{F(v_i(x))}=0. \tag{A26}$$

Denote the left-hand side by $\delta_i(x,v_i,v_{-i},\Theta)$, where Θ is the set of comparative statics parameters, and where the suppress the dependence of v_i and v_{-i} on x for notational simplicity. The system of equations is thus $\delta_i(x,v_i(x),v_{-i}(x),\Theta)=0$, $i\in\{1,2\}$.

The following auxiliary result will be useful to prove the proposition.

Lemma 1. $\frac{\partial\delta_1}{\partial v_1}\frac{\partial\delta_2}{\partial v_2}-\frac{\partial\delta_1}{\partial v_2}\frac{\partial\delta_2}{\partial v_1}>0$ *at the equilibrium.*

Proof of Lemma 1. Taking the full derivatives of these equations around the solution x everywhere where the derivatives exist yields

$$\frac{\partial\delta_1}{\partial x}+\frac{\partial\delta_1}{\partial v_1}v_1'(x)+\frac{\partial\delta_1}{\partial v_2}v_2'(x) = 0, \tag{A27}$$
$$\frac{\partial\delta_2}{\partial x}+\frac{\partial\delta_2}{\partial v_2}v_2'(x)+\frac{\partial\delta_2}{\partial v_1}v_1'(x) = 0. \tag{A28}$$

Combining these equations, we obtain:

$$\left(\frac{\partial\delta_1}{\partial v_1}\frac{\partial\delta_2}{\partial v_2}-\frac{\partial\delta_1}{\partial v_2}\frac{\partial\delta_2}{\partial v_1}\right)v_i'(x)=\frac{\partial\delta_i}{\partial v_{-i}}\frac{\partial\delta_{-i}}{\partial x}-\frac{\partial\delta_i}{\partial x}\frac{\partial\delta_{-i}}{\partial v_{-i}}, \tag{A29}$$

where $i\in\{1,2\}$. Because $\bar{X}_i(v)$ maximizes the bidder's value function and not minimizes it, $\frac{\partial\delta_i(x,v_i,v_{-i},\Theta)}{\partial x}>0$, $i\in\{1,2\}$.[22]

Fix x. Without loss of generality, assume $v_i(x)\geq v_{-i}(x)$. Then, $\min_{j\in\{1,2\}}v_j(x)=v_{-i}(x)$ and $\max_{j\in\{1,2\}}v_j(x)=v_i(x)$. First, consider bidder i. In the neighborhood of the equilibrium,

$$\delta_i(x,v_i,v_{-i},\Theta)=\mathbb{E}\left[\left(\beta\min\left\{\frac{\Pi_o+C_i}{\Pi_b+wx},1\right\}-\left[\min\left\{\frac{\Pi_o+C_i}{\Pi_b+wx},1\right\}x\right]'\right)(v_i-w)\,x-\beta\Delta|w\leq v_{-i}\right].$$

Hence,

$$\frac{\partial\delta_i}{\partial v_i}=\mathbb{E}\left[\left(\beta\min\left\{\frac{\Pi_o+C_i}{\Pi_b+wx},1\right\}-\left[\min\left\{\frac{\Pi_o+C_i}{\Pi_b+wx},1\right\}x\right]'\right)|w\leq v_{-i}\right]x>0.$$

Let

$$d_i(x,v_i,w,\Theta)\equiv\left(\beta\min\left\{\frac{\Pi_o+C_i}{\Pi_b+wx},1\right\}-\left[\min\left\{\frac{\Pi_o+C_i}{\Pi_b+wx},1\right\}x\right]'\right)(v_i-w)\,x-\beta\Delta.$$

[22]This follows from the second derivative of the bidder's value function with respect to the threshold at $\bar{X}_i(v)$ being $-\frac{\partial\delta\left(\bar{X}_i(v),v,v_{-i}\left(\bar{X}_i(v)\right),\Theta\right)}{\partial x}/\bar{X}(v)^{\beta+1}$. It must be negative for any v.

be the integrand under the expectation sign in $\delta_i(x, v_i, v_{-i}, \Theta)$. Let us show that $d_i(x, v_i(x), v_{-i}(x), \Theta) < 0$. Consider $d_i(x, v_i, w, \Theta)$ as a function of w. Clearly, it is strictly decreasing in w in the range $w < (C_i - \Delta)/x$, as $d_i(x, v_i, w, \Theta) = (\beta - 1)(v_i - w)$. Consider $w > (C_i - \Delta)/x$. Differentiating with respect to w,

$$\begin{aligned}
\frac{\partial d_i}{\partial w}(x, v_i, w, \Theta) &= -\frac{(\Pi_o + C_i)x}{(\Pi_b + wx)^2}\left((\Pi_b + xv_i)\left(\beta - \frac{\Pi_b}{\Pi_b + wx}\right) - \frac{\Pi_b x(v_i - w)}{\Pi_b + wx}\right) \\
&< -\frac{(\Pi_o + C_i)x}{(\Pi_b + wx)^2}\left((\Pi_b + xv_i)\frac{\Pi_b + 2wx}{\Pi_b + wx} - \frac{\Pi_b x(v_i - w)}{\Pi_b + wx}\right) \\
&= -\frac{(\Pi_o + C_i)x}{(\Pi_b + wx)^3}\left(\Pi_b^2 + 3wx\Pi_b + 2wx^2 v_i\right) < 0,
\end{aligned}$$

where the intermediate inequality follows, because $\frac{\beta}{\beta-1} < 2\frac{\Pi_b}{\Pi_o}$ implies $\beta > 2$. Because either $C_i < \Delta$ or $C_i \to \infty$, $d_i(x, v_i, w, \Theta)$ never jumps from one region to the other as w changes. Therefore, $d_i(x, v_i, w, \Theta)$ is strictly decreasing in w. Thus, $\mathbb{E}[d_i(x, v_i(x), w, \Theta) | w \leq v_{-i}(x)] = 0$ implies $d_i(x, v_i(x), v_{-i}(x), \Theta) < 0$. Therefore,

$$\begin{aligned}
\frac{\partial \delta_i}{\partial v_{-i}} &= (d_i(x, v_i, v_{-i}, \Theta) - \delta_i(x, v_i, v_{-i}, \Theta))\frac{f(v_{-i})}{F(v_{-i})} \\
&= d_i(x, v_i, v_{-i}, \Theta)\frac{f(v_{-i})}{F(v_{-i})} < 0.
\end{aligned}$$

Second, consider bidder $-i$. In the neighborhood of the equilibrium,

$$\begin{aligned}
&\delta_{-i}(x, v_{-i}, v_i, \Theta) \\
&= \mathbb{E}\left[\left(\beta \min\left\{\frac{\Pi_o + C_{-i}}{\Pi_b + wx}, 1\right\} - \left[\min\left\{\frac{\Pi_o + C_{-i}}{\Pi_b + wx}, 1\right\}x\right]'\right)(v_{-i} - w)x | w \leq v_{-i}\right] - \beta\Delta\frac{F(v_i)}{F(v_{-i})}.
\end{aligned}$$

Hence,

$$\frac{\partial \delta_{-i}}{\partial v_i} = -\beta\Delta\frac{f(v_i)}{F(v_{-i})} < 0 \text{ for all } v_i \in [\underline{v}, \bar{v}];$$

$$\begin{aligned}
\frac{\partial \delta_{-i}}{\partial v_{-i}} &= \int_{\underline{v}}^{v_{-i}}\left(\beta \min\left\{\frac{\Pi_o + C_{-i}}{\Pi_b + wx}, 1\right\} - \left[\min\left\{\frac{\Pi_o + C_{-i}}{\Pi_b + wx}, 1\right\}x\right]'\right)x\frac{dF(w)}{F(v_{-i})} - \frac{f(v_{-i})}{F(v_{-i})}\delta_{-i}(x, v_{-i}, v_i, \Theta) \\
&= \int_{\underline{v}}^{v_{-i}}\left(\beta \min\left\{\frac{\Pi_o + C_{-i}}{\Pi_b + wx}, 1\right\} - \left[\min\left\{\frac{\Pi_o + C_{-i}}{\Pi_b + wx}, 1\right\}x\right]'\right)x\frac{f(w)}{F(v_{-i})}dw > 0.
\end{aligned}$$

Because in the neighborhood of the equilibrium $\partial\delta_i/\partial x > 0$, $\partial\delta_i/\partial v_i > 0$, and $\partial\delta_i/\partial v_{-i} < 0$, where $i \in \{1, 2\}$, the right-hand side of (A29) is negative. Because $v_i'(x) < 0$ in equilibrium with strictly decreasing strategies, $\frac{\partial\delta_1}{\partial v_1}\frac{\partial\delta_2}{\partial v_2} - \frac{\partial\delta_1}{\partial v_2}\frac{\partial\delta_2}{\partial v_1} > 0$ at the equilibrium.

Using this lemma, we can prove comparative statics. Consider the derivative of $\delta_i(x, v_i, v_{-i}, \Theta)$ with respect to $\theta \in \Theta$ at the equilibrium. Combining the equations for $i \in \{1,2\}$, we obtain:

$$\left(\frac{\partial \delta_1}{\partial v_1}\frac{\partial \delta_2}{\partial v_2} - \frac{\partial \delta_1}{\partial v_2}\frac{\partial \delta_2}{\partial v_1}\right)\frac{\partial v_i}{\partial \theta} = \frac{\partial \delta_i}{\partial v_{-i}}\frac{\partial \delta_{-i}}{\partial \theta} - \frac{\partial \delta_i}{\partial \theta}\frac{\partial \delta_{-i}}{\partial v_{-i}}. \tag{A30}$$

Lemma 1 implies that the sign of $\partial v_i/\partial\theta$ coincides with the sign of the right-hand side of (A30). As shown above, $\frac{\partial \delta_i}{\partial v_{-i}} < 0$ and $\frac{\partial \delta_{-i}}{\partial v_{-i}} > 0$. In addition, because $v_i(x)$ is the inverse function of $\bar{X}_i(v)$ and $\bar{X}_i'(v) < 0$, the sign of $\partial v_i(x)/\partial\theta$ coincides with the sign of $\partial \bar{X}_i(v)/\partial\theta$. This can be seen from the full derivative of $\bar{X}_i(v)$ with respect to θ:

$$\bar{X}_i'(v)\frac{\partial v_i}{\partial \theta} + \frac{\partial \bar{X}_i(v)}{\partial \theta} = 0.$$

Therefore, a sufficient condition for $\partial \bar{X}_i(v)/\partial\theta$ to be positive (negative) is that $\partial\delta_i/\partial\theta < 0$ $(\partial\delta_i/\partial\theta > 0)$ for both $i \in \{1,2\}$.

First, consider $\theta = \beta$:

$$\begin{aligned}\frac{\partial \delta_i(x, v_i, v_{-i}, \Theta)}{\partial \beta} &= \mathbb{E}\left[\min\left\{\frac{\Pi_o + C_i}{\Pi_b + wx}, 1\right\}(v_i - w)\,|w \le \min_{j\in\{1,2\}} v_j(x)\right]x - \Delta\frac{F\left(\max_{j\in\{1,2\}} v_j(x)\right)}{F(v_i(x))} \\ &= \frac{1}{\beta}\mathbb{E}\left[\left[\min\left\{\frac{\Pi_o + C_i}{\Pi_b + wx}, 1\right\}x\right]'(v_i(x) - w)\,|w \le \min_{j\in\{1,2\}} v_j(x)\right]x > 0,\end{aligned}$$

where the second equation sign holds by the first-order condition. Hence, $\partial \bar{X}_i(v)/\partial\beta < 0$. Because $\partial\beta/\partial\mu < 0$, $\partial\beta/\partial\sigma < 0$, and $\partial\beta/\partial r > 0$, we obtain $\partial\bar{X}_i/\partial\mu > 0$, $\partial\bar{X}_i/\partial\sigma > 0$, and $\partial\bar{X}_i/\partial r < 0$.

Second, consider $\theta = \Delta$, keeping Π_b fixed. If $C_i \to \infty$,

$$\frac{\partial \delta_i(x, v_i, v_{-i}, \Theta)}{\partial \Delta} = -\beta\frac{F\left(\max_{j\in\{1,2\}} v_j\right)}{F(v_i)} < 0.$$

If $C_i < \Delta$,

$$\frac{\partial \delta_i(x, v_i, v_{-i}, \Theta)}{\partial \Delta} = -\mathbb{E}\left[\frac{1}{\Pi_b + wx}\left(\beta - \frac{\Pi_b}{\Pi_b + wx}\right)(v_i - w)\,|w \le \min_{j\in\{1,2\}} v_j\right]x - \beta\frac{F\left(\max_{j\in\{1,2\}} v_j\right)}{F(v_i)} < 0.$$

Hence, $\partial\bar{X}_i(v)/\partial\Delta > 0$.

Finally, consider $\theta = \Pi_b$, keeping Δ fixed. If $C_i \to \infty$, $\partial\delta_i(x, v_i, v_{-i}, \Theta)/\partial\Pi_b = 0$. If $C_i < \Delta$,

$$\begin{aligned}\frac{\partial \delta_i(x, v_i, v_{-i}, \Theta)}{\partial \Pi_b} &= \mathbb{E}\left[\frac{(wx + \Delta - C_i)(\beta(\Pi_b + wx) - \Pi_b) - wx(\Pi_b - \Delta + C_i)}{(\Pi_b + wx)^3}(v_i - w)\,|w \le \min_{j\in\{1,2\}} v_j\right] \\ &> \mathbb{E}\left[\frac{2w^2x^2 + (\Delta - C_i)(\Pi_b + 3wx)}{(\Pi_b + wx)^3}(v_i - w)\,|w \le \min_{j\in\{1,2\}} v_j\right] > 0,\end{aligned}$$

where the first inequality follows from $\beta > 2$. Hence, $\partial\bar{X}_i(v)/\partial\Pi_b \le 0$.

Appendix B Asymmetric Initiation: Numerical Procedure

For illustrative purposes, consider the case of cash versus stock bidder, $C_1 \to \infty$, $C_2 = 0$. The case of endogenous means of payment is numerically solved in the same fashion, using equations (A21). We use substitution of variables to express the first order conditions for the two asymmetrically constrained bidders in terms of $\bar{X}_1^{-1}(x)$, $\bar{X}_2^{-1}(x)$ for a given initiation threshold x. Specifically, let

$$\begin{aligned} x_1 &\equiv \bar{X}_1(v_1) \Rightarrow v_1 = \bar{X}_1^{-1}(x_1),\ \bar{X}_2^{-1}(\bar{X}_1(v_1)) = \bar{X}_2^{-1}(x_1); \\ x_2 &\equiv \bar{X}_2(v_1) \Rightarrow v_2 = \bar{X}_2^{-1}(x_2),\ \bar{X}_1^{-1}(\bar{X}_2(v_2)) = \bar{X}_1^{-1}(x_2). \end{aligned} \tag{B1}$$

Then, the system of equations (19), (21) becomes

$$x_1 = \frac{\beta}{\beta-1}\frac{\Delta}{\bar{X}_1^{-1}(x_1) - \int_{\underline{v}}^{\bar{X}_1^{-1}(x_1)} w\frac{f(w)}{F\left(\bar{X}_1^{-1}(x_1)\right)}dw}\frac{F(\bar{X}_2^{-1}(x_1))}{F(\bar{X}_1^{-1}(x_1))}, \tag{B2}$$

$$x_2\int_{\underline{v}}^{\bar{X}_1^{-1}(x_2)} \frac{\Pi_o\left(\Pi_b + \frac{\beta}{\beta-1}wx_2\right)}{(\Pi_b + wx_2)^2}(\bar{X}_2^{-1}(x_2) - w)\frac{f(w)}{F(\bar{X}_1^{-1}(x_2))}dw = \frac{\beta}{\beta-1}\Delta. \tag{B3}$$

We have two equations and four different combinations of functions and arguments as unknowns. We consider the interior case ($\bar{X}_i^{-1}(x) \in (\underline{v}, \bar{v})$ for $i \in \{1,2\}$, $x \in \{x_1, x_2\}$). Assume that both boundaries are equal, $x_1 = x_2 = x$, for some $v = \bar{X}_1^{-1}(x), w = \bar{X}_2^{-1}(x)$. This allows to simplify the system to two non-linear equations and two functions of one argument as unknowns, which can be easily solved with a mathematical package.

Note that the above algorithm does not provide corner solution for $v > \tilde{v} = \bar{X}_1^{-1}(X_2(\bar{v}))$. Observe, however, that (B2) in this case can be rewritten as

$$x = \frac{\beta}{\beta-1}\frac{\Delta}{\bar{X}_1^{-1}(x) - \int_{\underline{v}}^{\bar{X}_1^{-1}(x)} w\frac{f(w)}{F\left(\bar{X}_1^{-1}(x)\right)}dw}\frac{1}{F(\bar{X}_1^{-1}(x))}, \tag{B4}$$

and does not depend on $\bar{X}_2^{-1}(x)$. As a result, a single non-linear equation with a single unknown is easily solved numerically. Combinations $(\bar{X}_1^{-1}(x), x)$ and $(\bar{X}_2^{-1}(x), x)$ constitute pairs of valuations and equilibrium initiation strategies for the two bidders.

As an example, when bidder valuations are uniformly distributed on $[\underline{v}, \bar{v}]$, in the interior case

$$x = \frac{\beta}{\beta-1}\frac{\Delta}{\left(\bar{X}_1^{-1}(x) - \underline{v}\right)/2}\frac{\bar{X}_2^{-1}(x) - \underline{v}}{\bar{X}_1^{-1}(x) - \underline{v}}, \tag{B5}$$

$$x\int_{\underline{v}}^{\bar{X}_1^{-1}(x)} \frac{\Pi_o\left(\Pi_b + \frac{\beta}{\beta-1}wx\right)}{(\Pi_b + wx)^2}\frac{\bar{X}_2^{-1}(x) - w}{\bar{X}_1^{-1}(x) - w}dw = \frac{\beta}{\beta-1}\Delta. \tag{B6}$$

The integral in (B6) has a closed form representation.

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Table I: Benchmark model parameters

This table reports the benchmark parametrization of the model.

Variable	Description	Value
r	Risk-free rate	0.05
μ	Growth rate of target value	0.01
σ	Volatility of growth rate of target value	0.25
Π_b	Initial value of bidders	100
Π_o	Post-takeover value of the losing bidder	95
Δ	Value loss of the losing bidder	5
$\underline{v}$	Lowest value of the acquired target	110%
$\bar{v}$	Highest value of the acquired target	150%
$F(v)$	Distribution of valuations	Uniform
$D(v)$	Dispersion of valuations*	11.55%

$*$ Note: Dispersion of valuations for the uniform distribution is $D(v) = \sqrt{(\bar{v} - \underline{v})^2/12}$.