Asymmetric information, security design, and the pecking (dis)order[*]

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Abstract

We study a security design problem under asymmetric information, in the spirit of Myers and Majluf (1984). We introduce a new condition on the right tail of the firm-value distribution that determines the optimality of debt versus equity-like securities. When asymmetric information has a small impact on the right-tail, risky debt is preferred for low capital needs, but convertible debt is optimal for larger capital needs. In addition, we show that warrants are the optimal financing instruments when the firm has already pre-existing debt in its capital structure. Finally, we provide conditions that generate reversals of the standard pecking order.

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1 Introduction

The problem of raising capital under asymmetric information is a classic question in corporate finance. Firms' insiders typically have access to better information than investors on the value of their firms' assets — a situation that leads to potential mispricing of the securities issued by a firm and, thus, to shareholders' value dilution. In these circumstances, firms of better-than-average quality may find it desirable to optimally choose the design of their security offerings in a way to minimize the adverse effect of asymmetric information. In a classic paper, Myers and Majluf (1984) suggest that firms can reduce dilution (i.e., mispricing) by issuing debt rather than equity, an intuition known as the pecking order theory. The rationale behind the pecking order, as argued in Myers (1984), is that the value of debt, by virtue of being a senior security, is less sensitive to private information.

Important deviations from the pecking-order theory, however, have emerged in several recent empirical studies. For example, Frank and Goyal (2003) and Fama and French (2005) document that small, high-growth firms, a class of firms which is presumably more exposed to the effects of asymmetric information, rely heavily on financing through outside equity, rather than debt. Leary and Roberts (2010) conclude that "the pecking order is never able to accurately classify more than half of the observed financing decisions."[1] This evidence has led researchers to conclude that asymmetric information may not be a first-order determinant of corporate capital structures.[2]

Failure of the pecking order theory in empirical tests may be due to the fact that asymmetric information is not a first-order driver of capital structure choices, but it may also be a sign that the circumstances under which the pecking order preference arises are not met. In this paper, we study the implications of different distributional properties of asymmetric information within a general security design model. We then apply these insights to the

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[1]Leary and Roberts (2010) also note that most of the empirical evidence is inconclusive, and write: "Shyam-Sunder and Myers (1999) conclude that the pecking order is a good descriptor of broad financing patterns; Frank and Goyal (2003) conclude the opposite. Lemmon and Zender (2010) conclude that a 'modified' pecking order—which takes into account financial distress costs—is a good descriptor of financing behavior; Fama and French (2005) conclude the opposite. Frank and Goyal (2003) conclude that the pecking order better describes the behavior of large firms, as opposed to small firms; Fama and French (2005) conclude the opposite. Finally, Bharath, Pasquariello, and Wu (2010) argue that firms facing low information asymmetry account for the bulk of the pecking order's failings; Jung, Kim, and Stulz (1996) conclude the opposite."

[2]For example, Fama and French (2005) suggest that violations of the pecking order theory imply that "asymmetric information problems are not the sole (or perhaps even an important) determinant of capital structures." In contrast, Gomes and Phillips (2012) argues that the pecking order reverses in private markets, where younger, growth firms issue private equity with a probability that increases with measures of asymmetric information.

question that is at the very heart of Myers and Majluf (1984): what is the relative dilution of debt and equity under asymmetric information? In other words, if firms of heterogeneous quality issue the same security, are firms of "better than average" quality better off by issuing debt or equity?

We study a general security design problem in the context of a capital raising game. We argue that the shape of the optimal security crucially hinges on the "location" of asymmetric information in the firm-value distribution. In particular, we introduce a new measure of the exposure to information in the right tail of the distribution, which we denote as Information Costs in the Right Tail (or ICRT). We show that when asymmetric information has a sufficiently small impact on the right tail of the value of a firm, relative to the rest of the distribution, straight (but risky) debt is optimal when the firm needs to raise low levels of capital, but equity-like securities — such as convertible debt — emerge as the optimal securities when the firm must raise larger amounts of capital. Furthermore, we find that warrants can be optimal securities when the firm has already pre-existing debt in its capital structure.

We then study economic environments that generate violations of the pecking order theory, and that allow us to obtain testable cross-sectional implications. We view a firm as a collection of assets, where firm insiders have varying degree of asymmetric information on each asset.[3] In particular, we assume the firm is endowed with both assets in place and a growth opportunity. The value of both assets in place and the growth opportunity are characterized by lognormal distributions, where the growth opportunity is riskier than the assets in place. We model asymmetric information by assuming that the firm insiders have private information on the means of the distributions, while their second moments are common knowledge.[4]

We show that equity financing can dominate debt financing when insiders are relatively better informed than investors on the firm's assets in place, rather than on its (riskier) growth opportunities. In other words, the pecking order can be reversed when a firm's assets

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[3]The idea of the firm as a collection of assets is a common one in the literature, see Berk, Green, and Naik (1999) for a recent example.

[4]Technically, in our paper we consider a lognormal model where private information orders firm-value distributions by first-order stochastic dominance, but where monotone-likelihood ratio properties and/or hazard rate orderings may not hold. Since the standard lognormal model implies the monotone likelihood ratio order, there is no room in a lognormal "single-asset" setting to violate the pecking order theory. By introducing a second source of uncertainty in a multiple-asset model, we are able to separate first-order stochastic dominance from monotone likelihood ratio properties of the firm-value distribution, which generates robust deviations from the pecking order. Our results reflect the rather limited closure properties of the conditional stochastic dominance order (see section 1.B.3 of Shaked and Shanthikumar (2007)).

in place are more exposed to asymmetric information relative to its new investments.[5] We show that equity is more likely to dominate debt for young firms that have greater investment needs, and that have access to riskier and more valuable growth options. Thus, our model can explain why young firms may initially prefer equity over debt, and then switch to debt financing as they mature. Intuitively, our results depend on the fact that the properties of the firm-value distribution for high realizations of firm value (that is, in the right-tail) are determined by the asset with higher volatility. It means that, contrary to the common intuition, the preference of debt versus equity financing is not driven by the *absolute amount* of asymmetric information, but rather by the *composition* of a firm's assets and their *relative exposure* to this asymmetric information.

Greater information asymmetry on a firm's assets in place relative to its growth opportunities may emerge in cases where a firm is exposed to substantial "learning-by-doing."[6] Consider a firm whose assets in place have been obtained by the exploitation of past investment opportunities, while the firm still has untapped growth options. In this situation it is plausible that the firm has accumulated relatively more accurate information on its assets in place relative to the still undeveloped growth opportunities. This is because more information has become privately available over time (for example, as the result of past R&D activities), rather than on the new potential investments, where critical information still has yet to be revealed.[7] If the new growth opportunities have greater volatility our model shows that the original Myers and Majluf's result may not hold.

The paper also studies the financing game that arises when the firm has pre-existing debt in its capital structure. We show that firms that already have debt outstanding are, all else equal, relatively more likely to prefer equity over debt financing, for reasons solely driven by information asymmetry considerations. This feature of our model suggests that (pre-existing) high leverage may lead to more equity financing, and vice versa. Thus, asymmetric information may in fact lead to a "mean reversion" in leverage levels, as it is often documented in the empirical literature on capital structure (see Leary and Roberts, 2005).

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[5]More generally, we show that the pecking order theory can be violated in the case of firms endowed with multiple asset classes, such as multidivisional firms. If the riskier division is less exposed to asymmetric information, a reversal of the pecking order arises. Thus, our model generates new predictions on the cross-sectional variation of firm capital structures of multidivisional firms.

[6]As in, for example, Berk, Green, and Naik (2004).

[7]An example of such situation is provided by a pharmaceutical company whose assets in place are formed by fully developed drugs as well as new drugs where substantial additional R&D is necessary to obtain a commercially exploitable product. The new R&D will privately reveal to the company valuable information to assess the true commercial value of the drug, thus increasing the extent of asymmetric information with outside investors with respect to the initial patent stage.

These predictions are novel within models based on informational frictions, and invite for further research.[8]

Our paper contributes to the ongoing research on the pecking order and, more generally, the security design literature.[9] The problem of the optimal design of securities issued by firms was originally studied by Diamond (1984) and Gale and Hellwig (1985) in situations where investors can verify future firm cash-flow only at a cost. In our model, we sidestep the issue of costly state verification by assuming that future cash flow is fully contractible. Innes (1990) considers a moral-hazard situation where the entrepreneur can take an action that affects the distribution of future cash flow. This paper shows that, in this case, the optimal security is non-monotonic in that it has a "live-or-die" feature, whereby the firm pays investors a constant share of output, when output is below a certain critical level, and then retains all the output otherwise. In our model, we sidestep the moral hazard problem by taking as given the set of distributions of future cash flow, and we restrict ourselves to monotonic securities.[10]

A paper closely related to our work is Nachman and Noe (1994). Like ours, they consider the optimal security design problem faced by an informed insider wishing to raise a fixed amount of required capital for her firm. The paper shows that standard debt is the solution to an optimal security design problem for all level of required capital (and, thus, the original Myers and Majluf's pecking order obtains) if and only if the private information held by the insider orders the (future) firm-value distribution by Conditional Stochastic Dominance (CSD), a condition that is considerably stronger that First Order Stochastic Dominance (FOSD).[11] The critical difference between our paper and theirs is that we maintain FOSD but we relax the assumption of CSD, and we characterize the optimal security design (and the debt-to-equity choice) when CSD fails. Interestingly, an important class of distributions that satisfies CSD is the lognormal distribution where the mean of the distribution is private information. We emphasize that while a collection of log-normally distributed assets may

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[8]Our model features a static capital structure choice, but it lends itself to a dynamic specification (in a similar framework, also Leland, 1994, allows for a static financing decision). Further research focusing on dynamic capital structure choices is suggested by the fact that the existing set of securities in a firm's balance sheet affects the optimal financing choice at later dates.

[9]Recent surveys of this literature can be found in Harris and Raviv (1992), Allen and Winton (1995), Fulghieri and Goldman (2008).

[10]The assumption of monotonic securities is common in the security design literature, and it is aimed at avoiding situations in which firm insiders can lower the payouts to outside investors by opportunistically contributing, ex-post, funds to the firm.

[11]Loosely speaking, CSD requires that private information orders the conditional distributions in the right tails by FOSD, for all possible truncations. The Statistics and Economics literature also often uses the term Hazard Rate Ordering to refer to CSD, and we shall use both terms interchangeably.

satisfy CSD when each asset is taken in isolation, it may fail to satisfy CSD when these assets are taken as a portfolio.

Subsequent research has focused on different aspects of the security design problem. DeMarzo and Duffie (1999) consider the "ex-ante" security design problem faced by a firm's insiders before learning their private information, rather then the interim security design problem (that is, after becoming informed) studied by Nachman and Noe (1994). This paper shows that debt is the ex-ante optimal security if the interim private information satisfies the "uniform worst case" condition, a property that is stronger than CSD.

Biais and Mariotti (2005) builds on DeMarzo and Duffie (1999) and study the interaction between adverse selection and liquidity provision. As in DeMarzo and Duffie (1999), unlike our paper, a firm's insiders design the security ex-ante, before becoming informed; different from DeMarzo and Duffie (1999) and our paper, Biais and Mariotti (2005) assumes that firm insiders privately observe the realization of the firm's cash flow at the interim stage, before the security is actually issued. Their paper shows that the ex-ante optimal security design problem results again in a standard debt contract, in which the choice of the face value of the debt allows the firms to reduce the rents extracted by the liquidity provider.

DeMarzo, Kremer, and Skrzypacz (2005) examine the security design problem in the context of auctions, where buyers (rather than sellers) have private information on the asset put up for sale. The paper finds that, in this case, sellers can capture more of the value held by the buyer's positive private information by selling an equity stake in the asset, that is a security with exposure to the right tail. It is worthwhile noting that these results obtain under the monotone likelihood ratio property (MLRP), a condition that is stronger than CSD.

Our paper differs from this literature in several ways. First, and foremost, in our paper, we only require FOSD and, thus, our distributions can violate conditions in the previously mentioned literature, i.e. the uniform worst case condition of DeMarzo and Duffie (1999), or MLRP in DeMarzo, Kremer, and Skrzypacz (2005), or perfect observation of true firm value at the time the security is issued, as in Biais and Mariotti (2005). Second, as in Myers and Majluf (1984) and Nachman and Noe (1994), we constrain the firm to raise a fixed amount of capital, which leads to pooling rather than separating equilibria. In contrast, in DeMarzo and Duffie (1999) issuers can separate in the interim security issuance stage by using retention as a signal (in the spirit of Leland and Pyle, 1977). In our paper, by design, we focus on pooling equilibria because we want to study the core issue of the pecking order theory, namely the relative dilution of debt and equity when firms of heterogeneous quality

pool and raise capital by issuing the same security.[12]

Chakraborty and Yılmaz (2009) show that when investors have access to noisy public information on the firm's private value after the security is issued, the dilution problem can be costlessly avoided by issuing securities having the structure of callable, convertible debt. A key difference between this paper and ours is that, different from our paper, Chakraborty and Yılmaz (2009) additional (noisy) public information becomes available over time, after the security is issued. Other related papers include Chemmanur and Fulghieri (1997) and Chakraborty, Gervais, and Yılmaz (2011), which argue that warrants may be part of the optimal security structure. Finally, a growing literature considers dynamic capital structure choice (Fischer, Heinkel, and Zechner, 1989; Hennessy and Whited, 2005; Strebulaev, 2007; Morellec and Schürhoff, 2011). We conjecture that the economic forces of our static framework will play a first-order role in a dynamic version of the model and we leave this study for future research.

There are several other papers that challenge Myers and Majluf (1984) and Myers (1984) by extending their framework in different ways.[13] These papers show that a wider range of financing choices, which allow for signaling with costless separation, can invalidate the pecking order (see, e.g., Brennan and Kraus, 1987; Noe, 1988; Constantinides and Grundy, 1989). However, Admati and Pfleiderer (1994) point out that the conditions for a fully revealing signaling equilibrium identified in these papers are rather restrictive. Cooney and Kalay (1993) relax the assumption that projects have a positive net present value. Fulghieri and Lukin (2001) relax the assumption that the informational asymmetry between a firm's insiders and outside investors is exogenous, and allow for endogenous information production. Dybvig and Zender (1991) study the effect of optimally designed managerial compensation schemes, and Edmans and Mann (2012) look at the possibility of asset sales for financing purposes. Hennessy, Livdan, and Miranda (2010) consider a dynamic model with asymmetric information and bankruptcy costs, with endogenous investment, dividends and share repurchases, where the choice of leverage generates separating equilibria. Bond and Zhong (2014) show that stock issues and repurchases are part of an equilibrium in a dynamic setting. In contrast to these papers, but in the spirit of Myers and Majluf (1984), we consider a pooling equilibrium of a static model where the only friction is asymmetric

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[12]DeMarzo (2005) considers both the ex-ante and the interim security design problems, and examines the question of whether to keep multiple assets in a single firm (pooling), and the priority structure of the securities issued by the firm (tranching).

[13]We focus our literature discussion on papers that study informational frictions. Moral hazard considerations are also important drivers of capital structure choices, i.e. DeMarzo and Fishman (2007), Biais, Mariotti, Plantin, and Rochet (2007).

information between insiders and outsiders.

The remainder of the paper is organized as follows. Section 2 presents a simple example that illustrates the basic results and intuition of our paper. Section 3 outlines the basic model of capital raising under asymmetric information. Section 4 considers the optimal security design problem. In Section 5 we study the choice of debt versus equity, and present our main empirical predictions. All the proofs are in the Appendix.

2 A simple example

The essence of the pecking order theory is typically illustrated via a pooling equilibrium with two types and a discrete state space. The basic results of our paper, and their intuition, can be shown with a simple numerical example, summarized in Table 1.[14]

We consider two types of firms: good type, $\theta = G$, and bad type, $\theta = B$, where a firm's type is private information to its insiders. We assume that the two types of firms are equally likely in the eyes of investors. At the beginning of the period, firms wish to raise capital I. When raising capital, the two types of firms pool and issue the same security, so that investors do not change their priors on the firms' type when seeing the security issuance decision.

For reasons that will become apparent below, we will assume that a firm's end-of-period firm value, Z, is characterized by a *trinomial* distribution with three possible outcomes $Z \in \{z_1, z_2, z_3\}$. To fix ideas, we assume that the states z_1 and z_2 are relevant for the value of assets in place, while state z_3 is relevant for the growth opportunity. In particular, we assume that the end-of-period value of the assets in place is given by $z_1 = 10$, $z_2 = z_3 = 100$. If the growth opportunity is exercised, firm value becomes $z_1 = 10$, $z_2 = 100$, $z_3 = 300$. Thus, exploitation of the growth opportunity adds value to the firm only in state z_3, increasing the end-of-period firm value in that state from 100 to 300. The firm's capital requirements are set to be equal to $I = 60$.

The probability of the three possible outcomes of Z depends on private information held by the firm's insiders, and is given by $f_\theta \equiv \{f_{\theta 1}, f_{\theta 2}, f_{\theta 3}\}$ for a firm of type θ, with $\theta \in \{G, B\}$. In our examples below, we will assume that $f_G = \{0.2, 0.4, 0.4\}$ and $f_B = \{0.3, 0.4 - x, 0.3 + x\}$, and we will focus in the cases $x = 0.02$ and $x = 0.08$ in the discussion.[15]

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[14]The numerical example presented in this section builds on the discussion in Nachman and Noe (1994), Section 4.3.

[15]Table 1 considers all cases $x \in (0, 0.1)$. We remark that $x \leq 0.1$ is necessary to maintain first-order stochastic dominance.

Note that the presence of the growth opportunity has the effect of changing the distribution of firm value in its right tail, and that the parameter x affects the probability on the high state z_3 relative to the middle state z_2 for the type-B firm.

Consider first the case where $x = 0.08$. The values of the firm for the good and bad types are given by $\mathbb{E}[Z_G] = 162$ and $\mathbb{E}[Z_B] = 149$, with a pooled value equal to 155.5. Firms can raise the investment of 60 to finance the growth opportunity by issuing a fraction of equity equal to $\lambda = 0.386 = 60/155.5$. This means that under equity financing the initial shareholders of a firm of type-G retain a residual equity value equal to $(1 - 0.386)162 = 99.5$. The firm could also raise the required capital by issuing debt, with face value equal to $K = 76.7$. In this case debt is risky, with payoffs equal to $\{10, 76.7, 76.7\}$, and it will default only in state z_1. The value of the debt security when issued by a type-G firm is $D_G = 63.3$, and when issued by a type-B firm is $D_B = 56.7$, with a pooled value of 60, since the two types are equally likely. This implies that under debt financing the shareholders of a type-G firm will retain a residual equity value equal to $\mathbb{E}[Z_G] - D_G = 98.7 < 99.5$, and equity is *less dilutive* than debt. Thus, the pecking order preference is reversed.

The role of the growth opportunity in reversing the pecking order can be seen by considering the following perturbation of the basic example. Set now $x = 0.02$, so that $f_B = \{0.30, 0.38, 0.32\}$. In the new example the growth opportunity is relatively less important for a type-B firm than in the base case. Note that this change does not affect debt financing, because debt is in default only in state z_1. Therefore the change in x affects equity dilution but not debt dilution. In the new case, $\mathbb{E}[Z_B] = 137$, lowering the pooled value to 149.5. This means that now the firm must issue a larger equity stake, $\lambda = 0.401 = 60/149.5$, and thus existing shareholders' value is now equal to $(1 - 0.401)162 = 97.0 < 98.7$. Thus, equity financing now is more dilutive than debt financing, restoring the pecking order.

The reason for the change in the relative dilution of debt and equity rests on the impact of asymmetric information on the right-tail of the firm-value distribution. In the base case, for $x = 0.08$, asymmetric information has a modest impact on the growth opportunity (since $f_{G3} - f_{B3} = 0.02$) relative to the "middle" of the distribution (since $f_{G2} - f_{B2} = 0.08$), which is determined by the exposure of the assets in place to asymmetric information. Thus, firms of type-G can reduce dilution by issuing a security that has greater exposure to the right-tail of the firm-value distribution, such as equity, rather than debt, which lacks such exposure. In contrast, in the case of $x = 0.02$, asymmetric information has a more substantial impact on the growth opportunity, and thus on the right tail relative to the middle of the distribution, since now we have $f_{G3} - f_{B3} = 0.08$ and $f_{G2} - f_{B2} = 0.02$, making equity more mispriced.

A second key ingredient of our example is that the firm is issuing (sufficiently) risky debt to make dilution a concern. If debt is (nearly) riskless, the pecking order would hold. We obtain this in our example by assuming $z_1 = 10$ and by setting $I = 60$. If the level of investment is reduced to $I = 10$, then the firm could issue riskless debt and avoid any dilution altogether. Similarly, for investment needs sufficiently close to $I = 10$ debt has little default risk and the potential mispricing will still be small. In contrast, for sufficiently large investment needs the firm will need to issue debt with non-trivial default risk, creating the potential for a reversal of the pecking order.

Finally note that in the special case in which $f_B \equiv \{0.3, 0.3, 0.4\}$ there is no asymmetric information at all in the right-tail (that is, for $z_3 = 300$). In this case, type-G firms would in fact be able to avoid dilution altogether by issuing securities that load only on cash flows in the right tail, such as warrants. We will exploit this feature in Section 4, where we study the security design problem, proving the optimality of securities with equity-like features.

In the rest of the paper we build models that generate a reversal of the pecking order, and we show that a reversal can emerge in many economically relevant situations. In Section 3 we introduce a condition, which we refer to as "low-information-costs-in-the-right-tail," that generalizes the parametric assumptions in the previous example. This condition is novel in the literature and it is critical to generate reversals of the pecking order. The decomposition of the firm-value distribution into three regions in Section 5 establishes formally that the trinomial structure of our example is necessary for our results, and it provides its key drivers. Section 5.2 considers a simple real options model which generates new cross-sectional predictions that can be used to test asymmetric information theories, with and without a pecking order.

3 The basic model

3.1 The capital raising game

We consider an one-period model with two dates, $t \in \{0, 1\}$. At the beginning of the period, $t = 0$, an all equity-financed firm with no cash wishes to raise a certain amount of capital, I, that needs to be invested in the firm at that time.[16] The firm's value at the end of the

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[16]By research design, we initially do not explicitly model the reason for this capital requirement, which we take as exogenous. The investment requirement of the firm I may reflect, for example, a new investment project that the firm wishes to undertake, as discussed in Section 5. Also note that, in the spirit of Myers and Majluf (1984), we rule out of the possibility that firms finances their growth opportunities separately from the assets in place, i.e., by "project financing."

period, $t = 1$, is given by a random variable Z_θ. There are two types of firms: "good" firms, $\theta = G$, and "bad" firms, $\theta = B$, which are present in the economy with probabilities p and $1 - p$, respectively. A firm of type θ is characterized by its density function $f_\theta(z)$ and by the corresponding cumulative distribution function $F_\theta(z)$, with $\theta \in \{G, B\}$. Because of limited liability, we assume that Z_θ takes values of the positive real line. For ease of exposition, we will also assume that the density function of Z_θ satisfies $f_\theta(z) > 0$ for all $z \in \mathbb{R}_+$. In addition, we assume type G firms dominate type B ones by first-order stochastic dominance, defined as follows.

Definition 1 (**FOSD**). *We will say that the distribution F_G dominates the distribution F_B by (strong) first-order stochastic dominance if $F_G(z) \leq (<)F_B(z)$ for all $z \in \mathbb{R}_{++}$.*

The stronger property of Conditional Stochastic Dominance, CSD, plays a crucial role in the security design problem, as argued in Nachman and Noe (1994).

Definition 2 (**CSD**). *We will say that the distribution F_G dominates the distribution F_B by conditional stochastic dominance if $F_G(z|z') \leq F_B(z|z')$ for all $z' \in \mathbb{R}_+$ and $z \geq z'$, where*

$$F_\theta(z|z') \equiv \frac{F_\theta(z + z') - F_\theta(z')}{1 - F_\theta(z')}.$$

By setting $z' = 0$, we see that CSD implies FOSD. We note that CSD can equivalently be defined by requiring that the truncated random variables $[Z_\theta | Z_\theta \geq \bar{z}]$, with distribution functions $(F_\theta(z) - F_\theta(\bar{z}))/(1 - F(\bar{z}))$, satisfy FOSD for all $\bar{z}$.[17] In addition, Nachman and Noe (1994) show that CSD is equivalent to the condition that the ratio $(1 - F_G(z))/(1 - F_B(z))$ is non-decreasing in z for all $z \in \mathbb{R}_+$ (see their Proposition 4). Thus, loosely speaking, CSD implies that the set of payoffs in the right tail of the firm-value distribution are always more likely to occur for a type-G firm relatively to a type-B firm.[18]

Firms raise the amount I to fund the investment project by seeking financing in capital markets populated by a large number of competitive, risk-neutral investors. Capital markets are characterized by asymmetric information in that a firm's type $\theta \in \{G, B\}$ is private information to its insiders. We assume that the firms will always find it optimal to issue securities and raise capital I, rather than not issuing any security and forgo the investment

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[17]We remark that the CSD (hazard-rate) ordering is weaker than the Monotone Likelihood Ratio order, which requires $[Z_G | Z_G \in (\underline{z}, \bar{z})] \overset{\text{fosd}}{\geq} [Z_B | Z_B \in (\underline{z}, \bar{z})]$ for all $\underline{z}$ and $\bar{z}$; see equation (1.B.7) and Theorem 1.C.5 in Shaked and Shanthikumar (2007).

[18]Referring back to the example in Section 2, it is easy to verify that if $x \leq 0.05$ the type-G distribution not only dominates the type-B in the first-order sense, but *also* in the CSD sense.

opportunity. We make this assumption to rule out the possibility of separating equilibria where type-B firms raise capital and invest I, while type-G firms separate by not issuing any security. By design, in this paper we explicitly focus on the properties of (pooling) equilibria where both types of firms issue securities and raise capital in equilibrium.

When insiders have private information, firms will typically issue securities at prices that diverge from their symmetric information values. Under these circumstances, firms will find it desirable to raise capital by issuing securities that reduce the adverse impact of asymmetric information. To fix ideas, let $\mathbb{S}$ be the set of admissible securities that the firm can issue to raise the required capital I. As is common in this literature (see, for example, Nachman and Noe (1994)), we let the set $\mathbb{S}$ be the set of functions satisfying the following conditions:

$$0 \leq s(z) \leq z, \qquad\qquad\qquad \text{for all } z \geq 0, \qquad\qquad (1)$$

$$s(z) \quad \text{is non-decreasing in } z \quad \text{for all } z \geq 0, \qquad\qquad (2)$$

$$z - s(z) \quad \text{is non-decreasing in } z \quad \text{for all } z \geq 0. \qquad\qquad (3)$$

Condition (1) ensures limited liability for both the firm and investors, while (2) and (3) are monotonicity conditions that ensure absence of risk-less arbitrage.[19] We define $\mathbb{S} \equiv \{s(z) : \mathbb{R}_+ \to \mathbb{R}_+ : s(z) \text{ satisfies (1), (2), and (3)}\}$ as the set of *admissible securities*.

We consider the following capital raising game. The firm moves first, and chooses a security $s(z)$ from the set of admissible securities $\mathbb{S}$. After observing the security $s(z)$ issued by the firm, investors update their beliefs on firm type θ, and form posterior beliefs $p(s) : \mathbb{S} \to [0, 1]$. Given their posterior beliefs on firm type, investors purchase the security issued by the firm at a price $V(s)$. The value $V(s)$ that investors are willing to pay for the security $s(z)$ issued by the firm is equal to the expected value of the security, conditional on the posterior beliefs $p(s)$, that is

$$V(s) = p(s)\mathbb{E}[s(Z_G)] + (1 - p(s))\mathbb{E}[s(Z_B)]. \qquad\qquad (4)$$

Condition (4) implies that securities are fairly priced, given investors' beliefs. If security s is issued, capital $V(s)$ is raised, and the investment project is undertaken, the payoff to the

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[19]See, for example, the discussion in Innes (1990). Note that, as pointed out in Nachman and Noe (1994), condition (2) is critical to obtain debt as an optimal security. In absence of (2), the optimal contract may have a "do or die" component, whereby outside investors obtain all of the firm cash flow when it falls below a certain threshold, and nothing otherwise.

initial shareholders for a firm of a type θ is given by

$$W(\theta, s, V(s)) \equiv \mathbb{E}[Z_\theta - s(Z_\theta)] + V(s) - I. \tag{5}$$

The firm will choose the security issued to finance the investment project by maximizing its payoff (5), subject to the constraint that the security is admissible and that it raises at least the required funds I. Let $s_\theta(z) \in \mathbb{S}$ be the security issued by a firm of type θ.

3.2 Equilibria

Following the literature, we will adopt the notion of Perfect Bayesian Equilibrium, PBE, as follows.

Definition 3 (Equilibrium). *A PBE equilibrium of the capital raising game is a collection $\{s_G^*(z), s_B^*(z), p^*(s), V^*(s)\}$ such that: (i) $s_\theta^*(z)$ maximizes $W(\theta, s, V^*(s))$ subject to the constraint that $s \in \mathbb{S}$ and $V^*(s) \geq I$, for $\theta \in \{G, B\}$, (ii) securities are fairly priced, that is $V^*(s) = p^*(s)\mathbb{E}[s(Z_G)] + (1 - p^*(s))\mathbb{E}[s(Z_B)]$ for all $s \in \mathbb{S}$, and (iii) posterior beliefs $p^*(s)$ satisfy Bayes rule whenever possible.*

We start with a characterization of the possible equilibria in our capital raising game.[20] The following Proposition mimics Proposition 1 of Nachman and Noe (1994).

Proposition 1. *Let F_θ satisfy strict FOSD. No separating equilibrium exists in the capital raising game. In addition, in a pooling equilibrium with $s_G^* = s_B^* = s^*$, the capital raising game is uninformative, $p(s^*) = p$, and the financing constraint is met with equality:*

$$I = p\mathbb{E}[s(Z_G)] + (1 - p)\mathbb{E}[s(Z_B)]; \tag{6}$$

this equilibrium is supported by the out-of-equilibrium belief that if investors observe the firm issuing a security $s' \neq s^*$ they believe that $p(s') = p$ (passive conjectures).

Proposition 1 follows from the fact that, with two types of firms only, a type-B firm has always the incentive to mimic the behavior of a type-G firm (i.e., to issue the same security). This happens because (2) and strict FOSD together imply that securities issued by a type-G firm are always priced better by investors than those issued by a type-B firm, and type-B firm is always better-off by mimicking a type-G one. This also implies that, in

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[20]We note that the strong form of FOSD is only necessary for Proposition 1. Our main results go through assuming only FOSD.

equilibrium, the type-G firm is exposed to dilution due to the pooling with a type-B firm, and the corresponding loss of value can be limited by issuing only the securities needed to raise the capital outlay I.

Proposition 1 allows us to simplify the exposition as follows. Since both types of firms pool and issue the same security s and the capital constraint is met as equality, (5) and (6) imply that the payoff to the original shareholders of firm of type G becomes

$$W(G, s, V(s)) = \mathbb{E}[Z_G] - I - (1-p)\mathcal{D}_s,$$

where the term

$$\mathcal{D}_s \equiv \mathbb{E}[s(Z_G)] - \mathbb{E}[s(Z_B)] \tag{7}$$

represents the mispricing when security $s \in S$ is used, which is the cause of the dilution suffered by a firm of type G.

Under these circumstances, firms of type G will find it optimal to finance the project by issuing a security that minimizes dilution $\mathcal{D}_s$, that is

$$\min_{s \in \mathbb{S}} \quad \mathcal{D}_s \tag{8}$$

subject to the financing constraint (6).

3.3 Information costs in the right-tail

In what follows we will be concerned with the asymmetric information costs in the right tail of the value distribution $F_\theta(z)$ for a firm of type G relative to a firm of type B. These asymmetric information costs are related to the function $H(z)$ defined as:

$$H(z) \equiv \frac{F_B(z) - F_G(z)}{1 - F(z)}, \tag{9}$$

where $F(z)$ denotes the mixture of the distributions of the good and bad types, that is,

$$F(z) = pF_G(z) + (1-p)F_B(z). \tag{10}$$

The function $H(z)$ plays a critical role in our analysis. First note that FOSD implies that $H(z) > 0$ for all $z \in \mathbb{R}_{++}$. In addition, and more importantly, monotonicity of $H(z)$ is

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equivalent to CSD, as it is established in the following proposition.[21]

Proposition 2. *The distribution F_G dominates F_B by (strong) conditional stochastic dominance if and only if the function $H(z)$ is (strictly) increasing in z for all $z \in \mathbb{R}_+$. This is equivalent to requiring that the hazard rates $h_\theta(z) \equiv f_\theta(z)/(1 - F_\theta(z))$ satisfy $h_G(z) \leq (<)h_B(z)$ for all $z \in \mathbb{R}_+$.*

The function $H(z)$ provides a measure of the extent of asymmetries of information, and for monotonic securities it is closely linked to the cost to a type-G firm of promising to investors an extra dollar in state z.[22] In what follows, it will be important to characterize properties the right tail of the firm-value distribution that are stronger than FOSD, but at the same time weaker than CSD. Note first that $H(0) = 0$ and that, from FOSD, we have $H(z) > 0$ for z in a right neighborhood of $z = 0$, which together imply that $H'(0) > 0$. We note that, while the monotonicity properties of $H(z)$ on the left-tail of the distribution of z are dictated by FOSD, this is not the case for the right-tail of the distribution.

To characterize the behavior of the information costs in the right-tail of the distribution, we introduce the following definition, which will play a key role in our analysis.

Definition 4 (h-ICRT). *We will say that distribution F_G has information costs in the right tail of degree h (h-ICRT) over distribution F_B if $\lim_{z\uparrow\infty} H(z) \leq h$.*

We will use the term NICRT (no-information-costs-in-the-right-tail) to refer to the case $h = 0$. The relationship between FOSD, CSD and h-ICRT may be seen by noting that for two distributions $\{F_G, F_B\}$ that satisfy FOSD, there may exist a sufficiently low $h \in \mathbb{R}_+$ such that the h-ICRT property holds, while conditional stochastic dominance fails. Thus, intuitively, distributions that satisfy the h-ICRT condition "fill" part of the space of distributions that satisfy FOSD but do not satisfy the CSD condition. In particular, all distributions that satisfy Definition 4 for $h = 0$ (NICRT) will fail to satisfy the CSD condition.

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[21]In the simple example of Section 2, the function H is increasing if $x \leq 0.05$. Thus a necessary condition for the distributions in the example to *not* satisfy CSD is that $x > 0.05$.

[22]This happens because, for monotonic securities, an extra dollar paid in state z means that investors will be paid an extra dollar also in all states $z' > z$. This interpretation will become apparent in Section 4 (see equation (14)).

4 Optimal security design

4.1 Base case

Following Nachman and Noe (1994), the optimal security design problem (8) can be expressed as:[23]

$$\min_{s \in \mathcal{S}} \int_0^\infty s'(z)(F_B(z) - F_G(z))dz, \tag{11}$$

subject to

$$\int_0^\infty s'(z)(1 - F(z))dz = I. \tag{12}$$

The Lagrangian to the above problem is

$$\mathcal{L}(s', \gamma) = \int_0^\infty s'(z)(F_B(z) - F_G(z) - \gamma(1 - F(z)))dz \tag{13}$$

$$= \int_0^\infty s'(z)(1 - F(z))(H(z) - \gamma)dz, \tag{14}$$

where $H(z)$ was defined in (9). Remember that the the function $H(z)$ measures, for any value z, the extent of the asymmetric information costs in the right tail of the firm-value distribution. The following is an immediate consequence of the linearity of the security design problem.

Proposition 3. *A solution s^* must satisfy, for some $\gamma \in \mathbb{R}_+$,*

$$(s^*)'(z) = \begin{cases} 1 & \text{if } H(z) < \gamma; \\ [0, 1] & \text{if } H(z) = \gamma; \\ 0 & \text{if } H(z) > \gamma. \end{cases} \tag{15}$$

Note that the value of the Lagrangian multiplier γ depends on the tightness of the financing constraint (12) and, thus, on the level of the required investment I, with $\partial \gamma / \partial I > 0$. From $H(0) = 0$ and FOSD we have that $H(z) < \gamma$, which implies that the optimal security must satisfy $(s^*)' = 1$ in a right neighborhood of $z = 0$. This means that an optimal security will always have a (possibly small) straight-debt component.[24] The importance of this straight-debt component (that is, the face value of the debt) will depend on the size of the investment

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[23]Remember that, in our setting, we only impose that the probability distributions of firm value satisfy FOSD; thus the only departure from Nachman and Noe (1994) is that we relax CSD.

[24]Note, however, that as Proposition 5 shows, this property hinges critically on the assumption that the firm has no pre-existing debt.

I (since it affects the Lagrangian multiplier γ), as well as on the particular functional form for $H(z)$. The shape of the optimal security for greater value z depends monotonicity properties of the function $H(z)$ and, thus, on the extent of asymmetric information in the right tail of the firm-value distribution, and it is characterized in the following proposition.

Proposition 4. *Consider the security design problem in (11)–(12).*

(a) *(Nachman and Noe, 1994) If the distribution F_G conditionally stochastically dominates F_B, then straight debt is the optimal security.*

(b) *If the problem satisfies the NICRT condition, and $H'(z^*) = 0$ for a unique $z^* \in \mathbb{R}_+$, then convertible debt is optimal for all investment levels I.*

(c) *Assume that there exists $\bar{z}$ such that $f_G(z) = f_B(z)$ for all $z \geq \bar{z}$, and $H'(0) > 0$. Further assume that $H'(z^*) = 0$ for a unique $z^* \in [0, \bar{z}]$. Then NICRT holds, and there exists a $\bar{I} > 0$ such that for all $I \leq \bar{I}$ warrants are optimal, whereas for all $I \geq \bar{I}$ convertible debt is optimal.*

(d) *If $\lim_{z \uparrow \infty} H(z) = \bar{h} > 0$ and there exists a unique $z^* \in \mathbb{R}_+$ such that $H'(z^*) = 0$, then there exists $\bar{I}$ such that for all $I \leq \bar{I}$ straight debt is optimal, whereas for all $I \geq \bar{I}$ convertible debt is optimal.*

Part (a) of Proposition 4 assumes CSD. In this case, monotonicity of the function $H'(z)$ implies that there is a z^* below which $(s^*)'(z) = 1$, for all $z \leq z^*$, with $(s^*)'(z) = 0$ otherwise, yielding straight debt as an optimal security (see Figure 2). The intuition for the optimality of straight debt can be seen as follows. As discussed in Nachman and Noe (1994), CSD (and thus monotonicity of $H(z)$) requires that the ratio of the measure of the right tails of the probability distribution for the two types, $(1 - F_G(z))/(1 - F_B(z))$, is monotonically increasing in z.[25] For monotonic securities, this ratio can be interpreted as measuring the marginal cost of increasing the payouts to investors by \$1 for a type-G firm relative to a type-B firm. When it becomes relatively more expensive for a firm of type-G to increase a payouts to investors as the firm value z becomes larger (that is, when information costs faced by a type-G firm are increasing in z) the proposition shows that the optimal security is debt. In this case, firms of better types prefer to have the maximum payout to investors for low realizations of z, that is in the (right) neighborhood of $z = 0$, and then to limit the payout

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[25]Equivalently, CSD requires that the hazard rate of the payoff distribution for a type-G is smaller than that for a type-B *for all* values of z.

to investors for high realizations of z. These considerations, together with the requirement that the security is monotonic, lead to the optimality of debt contracts.

The cases considered in parts (b) and (c) of Proposition 4 provide the conditions under which securities with equity-like components are optimal. The key driver of the optimal security choice is the size of the informational costs in the right tail of the payoff distribution, measured by $H(z)$. Under NICRT, we have that in the limit $H(z) = 0$ and, thus, that the information costs suffered by a type-G becomes progressively smaller as the firm-value z increases. Part (b) of Proposition (4) shows that in this case type-G firms can reduce their overall dilution by maximizing the payout to investors also in the right tail of the distribution, in addition to a neighborhood of $z = 0$. This happens because, by increasing the payoffs in the right tail, where information costs are now low because of NICRT, allows the firm to correspondingly reduce the (fixed) payout in the middle of the distribution, where the information costs are now relatively higher. This implies that the optimal security will initially have a unit slope, then a fixed payout, and then again a unit slope (see Figure 2). Thus the optimal security will have the shape of a convertible bond, where the bond is convertible into 100% of equity with lump-sum payment to original shareholders equal to κ, which we will refer to as the *"conversion price."*

Part (c) of the Proposition considers a particular (strong) form of NICRT. Specifically, in this case we have $H(z) = 0$ for all $z \geq \bar{z}$, and NICRT holds. Furthermore, the absence of information costs for $z \geq \bar{z}$ implies that, when the capital requirements are low, that is, for $I \leq \bar{I} \equiv E[\max(z - \bar{z}, 0)]$, a type-$G$ firm can raise financing without incurring in any dilution. For $I \geq \bar{I}$, the firm cannot fully finance the project by pledging the right-tail of the payoff distribution, and in that case the proposition shows that convertible debt is optimal.

In part (d) of Proposition (4), neither CSD nor NICRT hold, since we have both a non-monotone function H and the $\bar{h}$-ICRT condition holds for $\bar{h} > 0$. The proposition shows that the size of a project affects the financing choices of a firm: straight debt is optimal for low levels of I, while convertible debt becomes optimal for large levels of the investment I. This happens because when investment needs are low, the firm can finance the project by issuing only straight debt, a security that loads only in the left tail of the distribution, where the information costs are the lowest. For greater investment needs, under $\bar{h}$-ICRT the firm finds it again optimal to maximize its payout to investor in the right tail of the distribution, as discussed for part (b) of the proposition, by issuing convertible debt.

4.2 Optimal securities with pre-existing debt

We have described so far a firm that is all equity-financed ex-ante. In the paper we are also interested in studying the effect of prior financing on the optimal security design problem. In particular, we now extend the basic model and we assume that the firm has already issued straight debt with face value K_0 prior to the beginning of the period, $t = 0$, which is due at the end of the period, $t = 1$.[26] In accordance to anti-dilutive "me-first" rules that may be included in the debt covenants, we assume that this pre-existing debt is senior to all new debt that the firm may issue in order to finance the new project. We maintain the assumption that the firm needs to raise external capital at $t = 0$.[27]

The security design game is modified as follows. At the beginning of the period, the firm chooses a security $s \in \mathbb{S}$, where the set $\mathbb{S}$ satisfies (2)-(3), with the added constraints $s(z) = 0$ for all $z < K_0$, and

$$0 \leq s(z) \leq z - K_0, \text{for all } z \geq K_0. \tag{16}$$

The presence of pre-existing debt changes the structure of information costs in a non-trivial way, because cash flows in the left tail of the distribution cannot be pledged any longer to new investors. This makes equity-like securities relatively more attractive.

Proposition 5. *Consider the optimal security design problem when the firm has a senior debt security with face value K_0 outstanding. Assume that the NICRT condition holds, and that there exists a unique z^* such that $H'(z^*) = 0$.*

(a) If $H'(K_0) > 0$, then there exists $\bar{I}$ such that: (i) warrants are optimal for $I < \bar{I}$, and (ii) convertible debt is optimal for $I \geq \bar{I}$.

(b) If $H'(K_0) < 0$, then the optimal securities are warrants.

Proposition 5 provides conditions under which warrants arise as optimal financing instruments, in contrast to the case in which only straight debt or convertible debt is solution to the optimal security design problem that we discussed in Proposition 4. Intuitively, warrants are optimal securities when pre-existing debt has absorbed the information benefits in the

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[26]Pre-existing firm debt may have been issued, for example, to finance prior investment rounds in the firm. We emphasize, however, that the model we study is static, in that we do not study how this prior financing came to place.

[27]This assumption allows us to ignore a possible debt overhang problem in the sense of Myers (1977), whereby the presence of pre-existing debt may induce a firm not to raise new capital.

left tail of the distribution, that is in a right neighborhood of $z = 0$ and (which generates, as discussed in the previous section, the optimality of debt when $K_0 = 0$). When K_0 is moderate, so that $H'(K_0) > 0$, NICRT implies that the optimal security design is one that always loads in the right-tail, where information costs are the now the lowest (since now the left tail is already committed). In addition, when the financing needs are low, the firm is able to raise the required capital by issuing only warrants; when the financing needs are high, the firm raises the additional capital by issuing also (junior) debt, that is by using convertible debt. When K_0 is large, so that $H'(K_0) < 0$, the firm will always find it optimal to issue only warrants (since the firm now faces decreasing information costs). We note that warrants can emerge as optimal securities when the firm has pre-existing debt in its capital structure, even when the asymmetric information environment is such that straight debt would be optimal in the absence of pre-existing debt.

4.3   Characterizing NICRT and CSD

In this section we provide several numerical examples that will shed light on the drivers of the optimal security design problem (11)-(12). We begin with an extension of the simple example presented in Section 2 that illustrates the conditions under which the NICRT condition arises, as well as the optimality of different securities under CSD and NICRT established in Propositions 4 and 5. We then present parametric specifications that will provide plausible economic situations that will generate such cases.

The first numerical example builds on the simpler one we introduced in Section 2. We now assume a continuous distribution (as in the main body of the paper) rather than the discrete one we used earlier. Specifically, we assume that type-G and type-B densities are now given by:

$$
f_G(z) = \begin{cases} 0.2k & \text{for } z \in [0, 100] \\ 0.4k & \text{for } z \in [100, 200] \\ 0.4k & \text{for } z \in [200, 300] \end{cases}, \qquad f_B(z) = \begin{cases} 0.3k & \text{for } z \in [0, 100] \\ (0.4 - x)k & \text{for } z \in [100, 200] \\ (0.3 + x)k & \text{for } z \in [200, 300] \end{cases}, \quad (17)
$$

where $k = 0.01$ is a normalizing constant, and $x \in [0, 0.10]$ in order to have that a type-G firm dominates a type-B firm by FOSD. It is important to note that this example mirrors the one from Section 2 in that the type-B distribution has a higher probability in the left-tail of the payoff distribution, $z \in [0, 100]$, and the parameter x moves mass from the intermediate set of payoffs, $z \in [100, 200]$ to the right-tail, $z \in [200, 300]$. Figure 1 plots the densities for

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the type-G and type-B firms, as well as the $H(z)$ function, which, using Propositions 4 and 5, characterizes the optimal securities.

Consider first the basic case where the firm has no pre-existing debt, $K_0 = 0$, and the initial investment is $I = 75$. When $x \leq 0.05$ the function $H(z)$ is monotonic, and straight debt is the optimal security (see the top right graph in Figure 1). For example, at $x = 0.05$, a straight debt security with a face value of $F = 83.8$ is optimal. When $x = 0.10$, the function $H(z)$ achieves a maximum at $z = 100$ and then $H(z) = 0$ for $z \in [200, 300]$ (see middle right graph in Figure 1). In the $x = 0.10$ case, there is no asymmetric information in the right tail, so the NICRT condition holds. It is possible to verify that a convertible bond with a face value of $F = 24.9$, with a conversion price at $\kappa = 193.8$ is the optimal security that finances the investment of $I = 75$. In the first case, when $x \leq 0.05$ and the CSD condition holds, the firm issues optimally risky debt, a security that loads primarily on the left-tail of the distribution. In the second case, when $x = 0.10$ and NICRT holds, the firm issues first a debt tranche with a lower face value (and thus lower risk), and a warrants component that loads payouts to investors in the right-tail, where there is no asymmetric information.

Next, consider the intermediate case in the bottom of Figure 1, where $x = 0.09$, so that neither NICRT nor CSD hold. If $I = 40$, the optimal security is standard debt, with $F = 42.2$. For larger investment levels, namely for $I \geq 42$, the optimal security includes a convertibility provision. For example, for $I = 45$, the optimal security is convertible debt with face value of $F = 44.44$ and a conversion price $\kappa = 238.2$. Furthermore, it is easy to show that for values of $I \in (42.0, 82.1)$ the optimal security will involve a straight debt component with $F = 44.4$, and a warrants component with a conversion trigger κ that is a decreasing function of the investment requirements I.

Finally, note that when $x = 0.10$, condition (c) in Proposition 4 is satisfied and issuing only warrants may be optimal. For example, when $I = 40$ a warrant with a conversion price $\kappa = 223.5$ is an optimal security.[28] When $x = 0.10$, a type-G firm does not suffer any dilution for cash flows in the $[200, 300]$ range and the asymmetric information problem can be entirely avoided by restricting payouts to this range. When the required investment I is sufficiently small, the firm's cash needs can be entirely satisfied by issuing a security that loads *only* in the right-tail, and warrants become the optimal security. At larger investment requirements (as it was the case for $I = 75$), the firm exhausts the capacity to make payouts to investors out of the right tail, inducing the firm to issue also (straight) debt, making convertible bonds the optimal security.

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[28]Using the notation in Proposition 4, one can verify that $\bar{I} = 50$ in our example, that is, warrants are optimal for all $I \leq 50$, whereas for $I > 50$ convertible bonds are the optimal securities.

The main feature of the examples we discussed so far is to stress the key role of the exposure to asymmetric information in the right-tail of the payoff distribution. In particular, they show that once the CSD is violated, it may be "cheaper" to issue a security that concentrates payouts in the right-tail of the distribution, in contrast to the standard pecking order intuition. Our next step is study parametric specifications describing economic environments that are likely to generate such violations of the pecking order. As such, these parametric examples will be useful to generate sharper empirical implications.

We present three parametric specifications that can violate the CSD. The common feature of these examples is to represent the firm as a collection (a portfolio) of assets.[29] Specifically, we model the end-of-period firm value Z_θ as the combination of two lognormal random variables, X_θ and Y_θ. We will consider three alternative specifications: (a) $Z_\theta = \max(X_\theta, Y_\theta)$, whereby the firm has the option, at the end of the period, to exchange two assets, X_θ and Y_θ ("rainbow" or exchange option case);[30] (b) $Z_\theta = X_\theta + Y_\theta$, that is the firm is a multi-division firm, where firm value is the sum of the value of its divisions, X_θ and Y_θ ("multi-division firm" case); (c) $Z_\theta = X_\theta + \max(Y_\theta - I_T, 0)$, whereby the firm has the option at the future date T to invest in a "growth opportunity" by making the additional capital expenditure I_T ("real option" case), in addition to the initial investment I. The next Proposition summarizes our findings.

Proposition 6. *Let X_θ and Y_θ be two lognormal random variables with $E[\log(X_\theta)] = \mu_{\theta x}$, $E[\log(Y_\theta)] = \mu_{\theta y}$, $var[\log(X_\theta)] = \sigma_x^2$, $var[\log(Y_\theta)] = \sigma_y^2$, $cov[\log(X_\theta), \log(Y_\theta)] = \rho \sigma_x \sigma_y$. Without loss of generality, assume that $\sigma_y > \sigma_x$. Then:*

1. *If $Z_\theta = X_\theta$, that is, Z_θ has a lognormal distribution, then the distribution F_G dominates the distribution F_B by CSD.*

2. *If $\mu_{Gy} = \mu_{By}$, and the payoff from the project Z_θ satisfies either (a) $Z_\theta = \max(X_\theta, Y_\theta)$, (b) $Z_\theta = X_\theta + Y_\theta$, or (c) $Z_\theta = X_\theta + \max(Y_\theta - I_T, 0)$, then the NICRT holds.*

The intuition behind Proposition 6 is straightforward. For Gaussian random variables, such as lognormal distributions, second moments of the distributions dominate tail behavior. This implies that the joint assumptions that Y has higher volatility, $\sigma_y > \sigma_x$, and suffers no

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[29]There is a large literature in corporate finance that uses the type of parametric examples we introduce next. See Berk, Green, and Naik (1999) for an explicit model of a firm as a collection of different projects with exogenous investment timing, and the papers surveyed in Strebulaev and Whited (2012), which also feature endogenous investment timing.

[30]Stulz (1982) uses the exchange option specification in a real options framework, and motivates its appeal in a corporate context.

information costs, $\mu_{Gy} = \mu_{By}$, are sufficient to guarantee that NICRT holds. In these cases, as discussed in Propositions 4–5, depending on parameter values warrants or convertible debt can be optimal securities, leading to violations of the pecking order.

We conclude this section by providing three different constellations of parameter values that numerically illustrate the results of Proposition 4. We specialize the examples to the "real options" specification (c), which we will examine in more detail in Section 5.2. Table 2 presents three difference scenarios where, respectively, standard debt (Case A), convertible debt (Case B) and warrants (Case C) are optimal securities. For each of these cases, Figure 2 plots the $H(z)$ function in the left panels, and the optimal security in the right panels, each row corresponding to each of the cases in Table 2. In all cases we assume that $p = 0.5$, $\sigma_x = 0.3$, $\sigma_y = 0.6$, $\rho = 0.5$, $T = 5$, and that $I_T = 50$.

The first scenario (Case A) presents the case where the asymmetric information is concentrated entirely in the high volatility asset, Y. Namely, we set $\bar{X}_G = \bar{X}_B = 100$, $\bar{Y}_G = 250$, $\bar{Y}_B = 150$ and $I = 100$, where we define $\mathbb{E}[X_\theta] \equiv \bar{X}_\theta$ and $\mathbb{E}[Y_\theta] \equiv \bar{Y}_\theta$. In this case the $H(z)$ function is monotone over its whole domain (see the top left graph in Figure 2). This implies that the optimal security will have unit slope when $H(z) < \gamma$ (where γ is represented by the horizontal dotted line) and it will have zero slope when $H(z) \geq \gamma$. Thus, the optimal security is standard debt with a face value K, determined by $H(K) = \gamma$ (and which is equal to $K = 138.8$).

In the second scenario (Case B) the NICRT condition holds, since the asymmetric information is concentrated entirely in the low-volatility asset, X. Namely, we set $\bar{Y}_G = \bar{Y}_B = 200$, $\bar{X}_G = 150$, $\bar{X}_B = 50$, and $I = 120$. In this case, the $H(z)$ function is "hump-shaped," it is first an increasing and then a decreasing function of firm value z (see the middle left graph in Figure 2). This implies that the optimal security will have unit slope when for $z < K$, where $H(z) < \gamma$, and it will have zero slope for $K \leq z \leq \kappa$, where $H(z) \geq \gamma$, and will have unit slope when for $z \geq \kappa$, where again $H(z) < \gamma$. Thus, the optimal security is a convertible debt contract with face value $K = 69.5$ and conversion price $\kappa = 593.4$, where the values $\{K, \kappa\}$ satisfy $H(K) = H(\kappa) = \gamma$. As shown in Proposition 6 securities load in the lower end of the payoffs, due to the usual Myers and Majluf (1984) intuition, but also on upper end of the payoff distribution, because of the NICRT property introduced in our paper.

In the last numerical example, Case C, we consider the effect of pre-existing debt on the security design problem discussed in Proposition 5. We modify Case B by assuming that the firm has debt outstanding with $K_0 = 100$, and that the initial investment is 70 (see the lower left graph in Figure 2). It can be calculated that the value of the pre-existing debt is 79.3,

while total firm value (debt plus equity) is equal to 259.2. In this case, the $H(z)$ function is the same as in Case B, but now $H(K_0) \geq \gamma$, which means that the optimal security will have zero slope for $K_0 \leq z \leq \kappa$, where $H(z) \geq \gamma$, and will again have unit slope for $z \geq \kappa$, where $H(z) < \gamma$. Thus, the optimal security design is a warrant with an exercise price of $\kappa = 502.5$, where $H(\kappa) = \gamma$, as in the case (i) of part (a) of Proposition 5.

Finally, it can shown that if we change the initial investment from 70 and we set it to be equal to 120, as in the previous Case B, the optimal security will again be convertible debt, where the face value of the new (junior) debt is $K = 135.7$, and the conversion price becomes $\kappa = 317.2$, as in the case (ii) of part (a) of Proposition 5. It can also be shown that, given the parameters values of Case C, warrants will always be optimal if the pre-existing debt has a face value of $K_0 \geq 199$, as in part (b) of Proposition 5. This happens because, in this case, $H'(K_0) < 0$, and $H(K_0) \geq \gamma$, which means that the optimal security has zero slope for $K_0 \leq z \leq \kappa$, where $H(z) \geq \gamma$, and will have unit slope when for $z \geq \kappa$, where $H(z) < \gamma$.

We conclude this section by noting that the numerical examples presented in this section suggest the following observation: violations of the pecking order can arise when the asset with greater volatility, Y, also has smaller difference in the means, that is if $\mu_{Gy} - \mu_{By} < \mu_{Gx} - \mu_{Bx}$. Thus, reversal of the pecking order can be obtained when the asset with lower exposure to asymmetric information also has greater volatility. We develop this intuition further in the next sections, where we study debt and equity contracts as financing choices.

5 The debt-equity choice

We now restrict our attention to two classes of securities, debt and equity. We study this case explicitly because of the debt-equity choice problem — as opposed to the more general security design problem we have examined so far — has attracted so much attention in both the theoretical and the empirical corporate finance literature.

5.1 General results

From (7), the dilution costs associated with equity are given by

$$\mathcal{D}_E = \lambda \left(\mathbb{E}[Z_G] - \mathbb{E}[Z_B] \right), \tag{18}$$

with $\lambda = I/\mathbb{E}[Z]$, whereas those associated with debt

$$\mathcal{D}_D = \mathbb{E}[\min(Z_G, K)] - \mathbb{E}[\min(Z_B, K)], \tag{19}$$

where the parameter K represents the (smallest) face value of debt that satisfies the financing constraint $I = p\mathbb{E}[\min(Z_G, K)] + (1 - p)\mathbb{E}[\min(Z_B, K)]$. In what follows we will say the pecking order obtains if $\mathcal{D}_E > \mathcal{D}_D$, and the "reverse pecking order" holds if $\mathcal{D}_D > \mathcal{D}_E$. To obtain further insights on the factors that drive the relative dilution of debt and equity, note that the difference in the dilution costs of debt and equity can be written as:

$$\mathcal{D}_D - \mathcal{D}_E = \int_0^\infty \left(\min(z, K) - \lambda z\right) c(z) dz, \tag{20}$$

where $c(z) \equiv f_G(z) - f_B(z)$. Note that the density function $f_\theta(z)$ measures, loosely speaking, the (implicit) private valuation of a \$1 claim made by the insiders of a firm of type $\theta \in \{G, B\}$ if the final payoff of the firm is z. Thus, the function $c(z)$ can be interpreted as representing the private cost due to asymmetric information for a firm of type G, relative to a firm of type B, of issuing a security that has a payoff of \$1 if the final firm value is z. In particular, if $c(z) > 0$ we will say that the information costs for a type G are positive, and that these costs are negative if $c(z) < 0$.

We introduce next an additional regularity condition that will simplify the analysis and greatly streamline the presentation of some of the results.

Definition 5 (SCDP). *The distributions $F_\theta(z)$, for $\theta = G, B$, satisfy the single-crossing density property (SCDP) if F_G strictly first-order stochastically dominates F_B, and there exists a unique $\hat{z} \in \mathbb{R}_+$ such that $f_G(\hat{z}) = f_B(\hat{z})$.*

Note that the SCDP condition implies that for all $z \leq \hat{z}$ we have $f_B(z) \geq f_G(z)$, and for all $z \geq \hat{z}$ we have $f_B(z) \leq f_G(z)$. Intuitively, this means that cash flows above the critical cutoff $\hat{z}$ have a positive information cost for type G firms, $c(z) > 0$, whereas cash flows below that cutoff have negative information costs, $c(z) < 0$.[31]

Expression (20) can be further decomposed as follows. Define $\bar{z}(K, \lambda) \equiv K/\lambda$ and note that for $z < \bar{z}(K, \lambda)$ we have that $\min(z, K) > \lambda z$, which implies that the payoffs to debthold-

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[31] Note that FOSD alone only implies that there exists z_1 and z_2 such that $c(z) < 0$ for all $z < z_1$ and $c(z) > 0$ for all $z > z_2$, but it does not rule out other interior crossings; in contrast, SCDP ensures that $z_1 = z_2$. We will assume SCDP for ease of exposition. The discussion below could be adapted to take into account the presence of multiple crossings.

ers are greater than those to equity holders; the converse holds for $z > \bar{z}(K, \lambda)$. Under SCDP, the point $\hat{z}$ divides the positive real line into two disjoint sets: a first set at the lower end of the positive real line, $[0, \hat{z})$ where $c(z) < 0$, that is where a type-G firm enjoys "negative information costs" (effectively an information benefit), and a second set $[\hat{z}, \infty]$ where $c(z) \geq 0$, that is where a type-G firm faces "positive information costs." The point $\bar{z}(K, \lambda)$ divides the positive real line in two other subsets, depending on whether or not equity yield higher payoffs than debt to investors. We have the following.

Proposition 7. *Assume the SCDP holds. Then a necessary condition for the reverse pecking order is that $\bar{z} > \hat{z}$.*

If $\bar{z} > \hat{z}$, which we will refer to as the "un-pecking necessary condition" (or UNC), from (20) the reverse pecking order obtains if and only if

$$\mathcal{D}_D - \mathcal{D}_E = \int_{\hat{z}}^{\bar{z}} (\min(z, K) - \lambda z) c(z) dz - \int_0^{\hat{z}} (\lambda z - \min(z, K)) c(z) dz - \int_{\bar{z}}^{\infty} (\lambda z - K) c(z) dz > 0.$$
(21)

Under UNC and the maintained assumptions the three integrals in (21) are all positive. The first term of the r.h.s. of (21) measures the dilution cost of debt relative to equity in the *intermediate-value region* $[\hat{z}, \bar{z}]$, where debt has higher payouts than equity and type-G firms suffer a positive information cost, $c(z) > 0$. In this region dilution costs of equity are lower than those of debt because equity has lower payoff than debt precisely in those states in which type-G firms are exposed to positive information cost (since $c(z) > 0$). Note that existence of this region is guaranteed by UNC. It is the presence of this term that makes equity potentially less dilutive than debt.

The second term of the r.h.s. of (21) measures the benefits of debt financing for low realizations of firm value (i.e., for $z < \hat{z}$). In this *low-value region,* dilution costs are lower for debt than equity because debt gives a higher payoff than equity, but such payoff has negative information costs (i.e., $c(z) < 0$). The third and last term measures the dilution costs of equity relative to debt for high realizations of firm value (i.e., for $z > \bar{z}$). In this *high-value region*, equity payoffs are greater than debt in those states that are more likely to occur to a type-G firm, and thus carry positive information costs (i.e., $c(z) > 0$).

The relative importance of these three regions determines the optimality of debt versus equity choice. In particular, equity financing dominates debt financing when the advantages of equity financing originating from the intermediate region of firm value (for $z \in [\hat{z}, \bar{z}]$), that is, the first term on the r.h.s. of (21) dominate the disadvantages in the low (for $z < \hat{z}$) and the high (for $z > \bar{z}$) regions of firm value, that is, the second and the third term

on the r.h.s. of (21). Note that if UNC does not hold (so that $\bar{z}(K, \lambda) < \hat{z}$), equity has negative information costs (that is, $c(z) < 0$) precisely in the states where the payouts to equityholders are greater than those to debtholders, making it impossible for the inequality (21) to be satisfied. Thus, UNC is a necessary condition to reverse the pecking order.

Figure 3 displays the plots of the function $c(z)$ (top panel, solid line) and of the densities of firm value for both type of firms and their average, $\{f_G(z), f_B(z), f(z)\}$ (bottom panel). Note that in this numerical example, built around one of the specifications from Proposition 6, the region in which debt has a disadvantage over equity, the intermediate region of (21)) is relatively large, the interval $[76.4, 470.7]$. In addition, the bottom panel of Figure 3 plots the distributions of $Z_\theta \equiv X_\theta + \max(Y_\theta - I_T, 0)$ for $\theta \in \{B, G\}$. By direct inspection, it is easy to verify that the distribution of firm value Z_θ closely resembles a lognormal distribution, with the important difference that the asymmetric information loads in the "middle" of the distribution, and to a lesser extent in its right tail.

The presence of pre-existing debt will change our analysis as follows. We restrict again the choice of security to equity or (junior) debt. We assume that the firm can raise the necessary capital either by sale of junior debt with face value K, or by sale of a fraction λ of total (levered) equity of the firm to outside investors. Following an argument similar to the one that yields (21), the relative dilution of debt versus equity is now given by:

$$\mathcal{D}_D - \mathcal{D}_E = \int_{K_0}^{\infty} \left[(1 - \lambda) \max(z - K_0, 0) - \max(z - (K_0 + K), 0) \right] c(z) dz. \qquad (22)$$

Note that the main difference of (22) relative to the corresponding expression in (20) is the fact that all payoffs below K_0 are allocated to the pre-existing senior debt. This implies that only the probability mass located in the interval $[K_0, \infty)$ is relevant for the determination of the relative dilution costs of debt and equity and, thus, for the choice of financing of the new project. Recall from (21) that the two regions located at the left and the right tails of the probability distribution favor debt financing, while the intermediate region favors equity financing. Intuitively, the presence of pre-existing debt in a firm's capital structure, by reducing the importance of the left-tail region, makes equity more likely to be the less dilutive source of financing.

5.2   Empirical predictions

In this section, we study in more detail the real options specification introduced in Section 4.3, which will serve as the basis for our main cross-sectional predictions. We adopt a standard

specification of a real option framework, such as the one studied in Berk, Green, and Naik (1999), and modify the basic model of Section 3 as follows. A firm of type θ is endowed at the beginning of the period, $t = 0$, with both assets in place and a growth opportunity. Assets in place of a type-θ firm are denoted as X_θ. In addition, by making at $t = 0$ the investment I, the firm generates a new "growth option" that can be exercised at the future date T. To exercise the growth opportunity the firm must make at $t = T$ an additional investment I_T. Thus, the end of period firm value, Z_θ, for a firm of type $\theta \in \{B, G\}$ is given by

$$Z_\theta \equiv X_\theta + \max(Y_\theta - I_T, 0),$$

where X_θ represent the value at T of the firm's assets in place, and $\max(Y_\theta - I_T, 0)$ represents the value of the growth opportunity.[32] We again assume that the NPV of growth opportunity is sufficiently large that firms will always find it optimal to issue securities and invest, rather than not issuing any security and abandon the project. For simplicity, we also assume that the growth option is of the European type (see, e.g., Morellec and Schürhoff, 2011, for a model with endogenous investment timing). As in section 4.3, we assume that both X_θ and Y_θ follow a lognormal process, that is, both $\log(X_\theta)$ and $\log(Y_\theta)$ are normally distributed with means $\mu_{\theta x} T$ and $\mu_{\theta y} T$ and with variances $\sigma_x^2 T$ and $\sigma_y^2 T$. Let ρ be the correlation coefficient between $\log(X_\theta)$ and $\log(Y_\theta)$.

In the spirit of Myers and Majluf (1984), we model asymmetric information by assuming that the firm insiders have private information on the means of the distributions, while their variances are common knowledge. We let $\mathbb{E}[X_\theta] = \bar{X}_\theta$ and $\mathbb{E}[Y_\theta] = \bar{Y}_\theta$, and we assume $\bar{X}_G \geq \bar{X}_B$ and $\bar{Y}_G \geq \bar{Y}_B$, with at least one strict inequality. We define the average value of the assets in place and of the growth opportunity as $\bar{X} = p\bar{X}_G + (1-p)\bar{X}_B$ and $\bar{Y} = pY_G + (1-p)Y_B$, respectively, and let $c_x = X_G - X_B$ and $c_y = Y_G - Y_B$. Thus, c_x and c_y measure the exposure to asymmetric information of the assets in place and the growth opportunity. The assumptions ensure FOSD, and allow for scenarios in which the NICRT condition holds (see Proposition 6).

We now conduct a series of numerical examples centered on the base case reported in Table 3. In this leading example asymmetric information is more severe on assets in place, where $\bar{X}_G = 125$ and $\bar{X}_B = 75$, rather than the growth opportunity, where $\bar{Y}_G = 205$ and $\bar{Y}_B = 195$. Greater information asymmetry on a firm's assets in place relative to its growth opportunities may emerge in cases where a firm is exposed to substantial "learning-

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[32]Note that, by setting $I_T = 0$, this specification of nests the case of a multidivision firm, where $Z_\theta \equiv X_\theta + Y_\theta$.

by-doing." In addition, we assume that assets in place have lower volatility than the growth opportunities, as in Berk, Green, and Naik (2004), and we set $\sigma_x = 0.3$, $\sigma_y = 0.6$, $T = 15$ and $\rho = 0$. We let both types be equally likely, $p = 0.5$. In this base case specification, we set the initial investment amount to be $I = 100$, and the investment at exercise of the growth option to be $I_T = 50$.

The value of the firm post-investment for the two types is given by $\mathbb{E}[Z_G] = 307.9$ and $\mathbb{E}[Z_B] = 248.2$, so that $p\mathbb{E}[Z_G] + (1-p)\mathbb{E}[Z_B] = 278.1$. Without the project, the (average) status-quo firm value is the value of assets in place X, which is equal to $\bar{X} = 100$. Since the value of the post-investment is firm 278.1, and the investment is $I = 100$, the project has an (unconditional) positive NPV of 178.1. Note also that the efficient outcome is for *both* types of firms is to finance the project, since for a type-G we have that $\mathbb{E}[Z_G] - I = 307.9 - 100 = 207.9 > 125 = \bar{X}_G$, and for a type-$B$ we have that $\mathbb{E}[Z_B] - I = 248.2 - 100 = 148.2 > 75 = \bar{X}_B$.

It is easy to verify that issuing equity will require that the equity holders give up a stake of $\lambda = 0.360 = 100/278.1$. In order to finance the project with debt, the firm needs to promise bondholders a face value at maturity of $K = 218.4$. The dilution costs of equity are $\mathcal{D}_E = 0.36 \times (307.9 - 248.2) = 21.5$ whereas those of debt are $\mathcal{D}_D = 111.9 - 88.1 = 23.7$, with a relative dilution $\mathcal{D}_D/\mathcal{D}_E = 23.7/21.5 = 1.10$. Thus, the type-$G$ firm is exposed to lower dilution by raising capital with equity rather than debt.[33] Note that the implied credit spread for the risky debt is 5.3%, which in the recent years was roughly equivalent to spread on a non-investment grade bond, such as BB-rated debt.

The bottom portion of Table 3 examines the impact of changes of some of the key parameters in the base case on the relative dilution of debt an equity. The first set of examples focus of the exposure to asymmetric of the assets in place relative to the growth opportunity. Specifically, a decrease of the exposure to asymmetric information in the growth opportunity, by setting $Y_G = Y_B = 200$, has the effect of increasing the the dilution of debt relative to equity to 1.76. Conversely, an increase of the exposure to asymmetric information in the growth opportunity, by setting now $Y_G = 225$ and $Y_B = 175$, has the effect of reducing the dilution of debt relative to equity to 0.76, and making now debt less dilutive than equity. Similarly, an increase of the exposure to asymmetric information in the assets in place, by setting $\bar{X}_G = 150$ and $\bar{X}_B = 50$, has the effect of increasing the the dilution of debt relative

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[33]It is worthwhile to remark that the investment choices are individually rational when using either debt or equity. To see this, note that in the case of equity financing the residual equity value for a type-G firm is equal to $(1 - 0.36) \times 307.9 = 197.1 > 125 = \bar{X}_G$, and for a type-$B$ firm it is equal to $(1 - 0.36) \times 248.2 = 158.9 > 75 = \bar{X}_B$. In the case of debt financing, the residual equity value for a type-G firm is equal to $307.9 - 119.9 = 188 > 125 = \bar{X}_G$, and for a type-$B$ firm it is equal to $248.2 - 88.1 = 160.1 > 75 = \bar{X}_B$.

to equity to 1.26, and a decrease of their exposure to asymmetric information, by setting $\bar{X}_G = \bar{X}_B = 100$, has the effect of reducing the dilution of debt relative to equity to 0.20, making again debt less dilutive than equity. These results conform with the notion that reversals of the pecking order preference can occur when the asset with greater volatility is less exposed to asymmetric information relative to the asset with lower volatility.

We consider next the effect of the volatility parameters, σ_x and σ_y. An increase of the volatility of the assets in place to $\sigma_x = 0.40$ has the effect of reducing the dilution of debt relative to equity from $D_D/D_E = 1.10$ to 1.01, while an increase of the volatility of the growth opportunity to $\sigma_y = 0.80$ has the opposite effect of increasing the relative dilution of debt and equity to 1.53. As discussed in Proposition 6, these examples show that equity is less dilutive than debt when the volatility of the growth opportunities is sufficiently large relative to the volatility of the assets in place.

The impact of the subsequent investment I_T is as follows. Specifically, a decrease of the future investment requirement, from $I_T = 50$ to $I_T = 0$, reduces the dilution of debt relative to equity to 0.88, which makes debt overall less dilutive than equity restoring the pecking order. In contrast, an increase of the subsequent investment to $I_T = 100$ worsens the relative dilution of debt and equity, which is now equal to 1.18. These results depend on the fact that an increase of the subsequent investment requirements I_T increases the "exercise price" of the growth option, which has the same effect as an increase of the volatility σ_y.

In the last set of examples we examine the impact of pre-existing debt on the relative dilution debt and equity. In the spirit of Proposition 5, the presence of a pre-existing debt with face value $K_0 = 20$ in our base-case parameter constellation as the effect of increasing the relative dilution debt to equity to 1.28, increasing the advantage to equity relative to debt financing. This effect is further reinforced at greater levels of pre-existing debt, where for $K_0 = 40$ the relative dilution of debt to equity becomes 1.47.[34]

The dilution effects presented in 3 are further studied in Figures 4, 5 and 6, which present more general comparative static exercises based on the numerical examples from Table 3. The top graph in Figure 4 displays indifference lines of $\mathcal{D}_D = \mathcal{D}_E$, as a function of the exposure to asymmetric information of the assets in place, c_x, and the growth opportunity, c_y, for three levels of the volatility of the growth opportunity, $\sigma_y \in \{0.6, 0.7, 0.8\}$. In the region above the lines, we have that $\mathcal{D}_D > \mathcal{D}_E$ and hence equity is less dilutive than debt

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[34] Note that the default spreads implicit in the cases in which equity is less dilutive than debt range from 5.3% to 9.8%. These default spreads are currently associated with bonds with credit ratings ranging from BB to C. See, for example, `http://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/ratings.htm`.

and the reverse pecking order obtains. In the region below the lines, we have that $\mathcal{D}_D < \mathcal{D}_E$ and hence equity is more dilutive than debt, and the usual pecking order obtains. Note that the slope of the indifference lines declines when the volatility of the growth opportunity rises. These graphs reveal that equity is more likely to be less dilutive than debt when the exposure to asymmetric information on the less volatile assets in place, c_x, is larger, when the exposure to asymmetric information of the more volatile growth opportunities, c_y, is smaller. In addition, the parameter region where equity dominates debt becomes larger when the volatility of the growth opportunity increases.

The bottom graph in Figure 4 charts indifference lines of $\mathcal{D}_D = \mathcal{D}_E$, as a function of the time horizon, T, and the investment cost, I, for three levels of the average value of assets in place, $\bar{X} \in \{95, 100, 105\}$. For pairs of (I, T) below the lines debt is optimal, whereas equity is optimal above the lines. These graphs reveal that equity is more likely to be less dilutive than debt for higher investment costs I, and longer time horizons T (i.e., for younger firms). In addition, the parameter region where equity dominates debt becomes larger when the (average) values of assets in place, X, is lower (i.e., smaller firms).

The top graph of Figure 5 displays the pairs of the average value of assets in place and the average value of the growth option, $(\bar{X}, \bar{Y})$, for which the dilution costs of equity and debt are the same, i.e. $\mathcal{D}_E = \mathcal{D}_D$, for different level of asymmetric information on asset $c_x \in \{10, 25, 40\}$. For pairs of $(\bar{X}, \bar{Y})$ below the lines debt is optimal, whereas equity is optimal above the lines. These graphs reveal that equity is more likely to be less dilutive than debt when the growth opportunities represent a larger component of firm value. In addition, the parameter region where equity dominates debt becomes larger when the exposure to asymmetric information of assets in place, c_x, increases.

The bottom graph of Figure 5 plots the pairs of volatilities, (σ_x, σ_y), such that the dilution costs of equity and debt are the same, i.e. $\mathcal{D}_E = \mathcal{D}_D$, for three levels of the investment cost $I \in \{100, 110, 120\}$. For pairs of volatilities, (σ_x, σ_y), below the lines debt is optimal, whereas equity is optimal above the lines. These graphs reveal that equity is more likely to be less dilutive than debt when the volatility of assets in place is low, and when the volatility of growth opportunities is large. In addition, the parameter region where equity dominates debt becomes larger when the firm's investment need, I, increases.

The top graph of Figure 6 examines the impact of the size of the investments needs on the form of financing. The graph reveals that equity financing is more likely to be less dilutive than debt when the firm has greater investment needs either at the time of the initial investment, $t = 0$, or at the time the growth option is exercised, $t = T$. These observations

imply that future capital needs of the firm will have an independent effect on the financing decisions.

Finally, the bottom graph of Figure 6 examines the impact of pre-existing debt on the form of financing. The graph reveals that, for a given level of assets in place $\bar{X}$, equity financing is more likely to be less dilutive than debt when the firm has greater amount of pre-existing debt, K_0. In addition, the graph suggests that firms are likely to switch from equity to debt financing as they accumulate assets in place, that is as $\bar{X}$ becomes larger. At the same time, firms that finance asset acquisitions through debt financing are likely to switch to equity financing as they increase the amount of debt in their capital structure, K_0. These considerations suggests asymmetric information may in fact lead to "mean reversion" in leverage levels, as it is often documented in the empirical literature on capital structure (see Frank and Goyal, 2003; Fama and French, 2005; Leary and Roberts, 2005).

In summary, the examples in Table 3, as well as Figures 4, 5, and 6 reveal a very consistent pattern: violations of the pecking order are likely to be optimal for young firms, endowed with valuable and risky growth opportunities and with large investment needs. In addition, equity is more likely to be less dilutive than debt when growth opportunities represent a greater proportion of firm value, when these growth opportunities are riskier, and when the firm has greater financing needs. Thus, our model can help explain the stylized fact that small and young firms with large financing needs and valuable growth opportunities (i.e. high growth firms) often prefer equity over debt financing, even in circumstances where asymmetric information is potentially severe.

6   Conclusion

In this paper, we revisit the pecking order of Myers and Majluf (1984) and Myers (1984) in the context of a general security design problem. We show that optimality of equity-like securities, such as convertible debt and warrants, depend crucially on the exposure to asymmetric information of the right tail of the firm-value distribution, which we characterize with a novel measure, the h-ICRT. We show that for h sufficiently low, the solution to the optimal security design problem is either straight debt, convertible debt, or warrants, depending on the amount of financing required, and the presence of pre-existing debt.

We then study the debt to equity choice within a parametric specification of our model, in which a firm consists of a portfolio of lognormal assets. We show that even if the distribution of each individual assets satisfies the conditional stochastic dominance condition,

the distribution of the combined firm value may not. This means that, contrary to common intuition, equity financing can dominate debt financing under asymmetric information, even in cases where individual assets would be financed by debt when taken in isolation. In addition, we show that the presence of existing debt makes equity less dilutive than debt. Taken together, these results suggest that the relationship between asymmetric information and choice of financing is more subtle than previously believed.

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Appendix

Proof of Proposition 1. In a separating equilibrium $\{s_G^*, s_B^*\}$ where we have that $s_G^* \neq s_B^*$, $p(s_G^*) = 1$, and $p(s_B^*) = 0$, which implies that $V^*(s_\theta^*) = \mathbb{E}[s_\theta^*(Z_\theta)]$ and that $W(\theta, s_\theta^*, V^*(s_\theta^*)) = \mathbb{E}[Z_\theta] - I$. This implies that $W(B, s_G^*, V(s_G^*)) - W(B, s_B^*, V(s_B^*)) = V(s_G^*) - \mathbb{E}[s_G^*(Z_B)] = \mathbb{E}[s_G^*(Z_G)] - \mathbb{E}[s_G^*(Z_B)] > 0$ by FOSD. Thus, the pair $\{s_G^*, s_B^*\}$ cannot be an equilibrium. Furthermore, if in a candidate pooling equilibrium where the security s^* is offered by both types of firms, we have that $V^*(s^*) > I$, consider the scaled down contract γs^* for $\gamma \in (0,1)$. Then, there is at least one value of $\gamma \in (0,1)$ such that $p(\gamma s^*) = p$, by passive beliefs, $V^*(\gamma s^*) \geq I$ and $W(G, \gamma s^*, V^*(\gamma s^*)) = \mathbb{E}[Z_G] - \gamma(\mathbb{E}[s^*(Z_G)] - V(s^*(Z_G))) - I > \mathbb{E}[Z_G] - (\mathbb{E}[s^*(Z_G)] - V(s^*(Z_G))) - I = W(G, s^*, V^*(s^*))$, a contradiction. Thus, any pooling equilibrium must satisfy the budget constraint with equality, $V(s) = I$. ∎

Proof of Proposition 2. From the definition of $H(z)$ in (9), we have:

$$
\begin{aligned}
\frac{dH(z)}{dz} &= \frac{(f_B(z) - f_G(z))(1 - F(z)) + (pf_G(z) + (1-p)f_B(z))(F_B(z) - F_G(z))}{(1 - F(z))^2} \\
&= \frac{f_B(z) - f_G(z) + F_B(z)f_G(z) - F_G(z)f_B(z)}{(1 - F(z))^2} \\
&= \frac{f_B(z)(1 - F_G(z)) - f_G(z)(1 - F_B(z))}{(1 - F(z))^2}.
\end{aligned}
$$

Thus $H'(z) > 0$ if and only if $f_B(z)(1 - F_G(z)) > f_G(z)(1 - F_B(z))$, which reduces to the CSD condition. ∎

Proof of Proposition 3. See Theorem 8 in Nachman and Noe (1994). From the Lagrangian in (14), we note that the objective function is linear in the choice variable $s'(z)$. Thus, only corner solutions are optimal. When $H(z) < \gamma$ the Lagrangian is minimized making $s'(z)$ be equal to its upper bound, $s'(z) = 1$, whereas for $H(z) > \gamma$, the minimization calls for setting $s'(z)$ to its lower bound, $s'(z) = 0$. ∎

Proof of Proposition 4. Since H is increasing in (a), there is a single crossing point z such that $H(z) = \gamma$, for any $\gamma \in \mathbb{R}_+$. The claim in (a) follows immediately from Proposition 3. Assuming NICRT, and that $H'(z^*) = 0$ at most once, it is immediate that there are two unique crossing points for $H(z^*) = \gamma$, for any $\gamma \in \mathbb{R}_+$. The claim in (b) is immediate from Proposition 3. Under the conditions of case (c), we have that NICRT holds. Since $H'(0) > 0$, for a sufficiently low $\bar{I}$ all investment levels $I \leq \bar{I}$ are associated with $s'(z) = 0$ for all $z \leq z^*$. This will be true up to the level $\bar{I}$ that is possible to finance pledging all residual cash flows

above z^*, namely $\bar{I} = \mathbb{E}[\max(Z - z^*, 0)]$. For $I > \bar{I}$, we have that the condition $H(z) = \gamma > 0$ defines two crossings, and the optimal securities are convertible bonds, as in (b). Case (d) is analogous to case (b), but noting that for $\gamma \leq \bar{\gamma}$ there is a single point satisfying $H(z^*) = \gamma$, but two such points for γ sufficiently large. ∎

Proof of Proposition 5. The proof is analogous to that of Proposition 3. The first-order conditions require $s'(z)$ to be either one (or zero) at points for which $H(z) < \gamma$ (or $H(z) > \gamma$). Under the conditions in (b), and the initial assumptions, there is only one crossing, and all mass of the security is concentrated in the right tail. This occurs for low values of γ, or equivalently of the investment I. The claim in (a) mirrors case (b) from Proposition 4. ∎

Proof of Proposition 6. In order to proof the first statement, we argue that the distribution of the good type dominates the distribution of the bad type in the likelihood ratio sense, namely $f_G(z)/f_B(z)$ is monotonically non-decreasing for all $z \in \mathbb{R}_+$. From basic principles we have:

$$
\begin{aligned}
\frac{f_G(z)}{f_B(z)} &= \frac{\frac{1}{z\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\log(z) - \mu_G}{\sigma}\right)^2}}{\frac{1}{z\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\log(z) - \mu_B}{\sigma}\right)^2}} \\
&= e^{-\frac{1}{2}\left(\frac{\log(z) - \mu_G}{\sigma}\right)^2 + \frac{1}{2}\left(\frac{\log(z) - \mu_B}{\sigma}\right)^2} \\
&= e^{-\frac{1}{2}\frac{(\mu_G^2 - \mu_B^2)}{\sigma^2} + \log(z)\left(\frac{\mu_G - \mu_B}{\sigma^2}\right)} \\
&= e^{-\frac{1}{2}\frac{(\mu_G^2 - \mu_B^2)}{\sigma^2}} z^{\left(\frac{\mu_G - \mu_B}{\sigma^2}\right)};
\end{aligned}
$$

which is monotonically increasing in z when $\mu_G > \mu_B$, as we set to prove. Since the likelihood ratio order implies conditional stochastic dominance (Shaked and Shanthikumar, 2007), this concludes the proof.

In order to prove the second statement, we start with case (a). Using l'Hopital's rule, one has

$$
\lim_{z\uparrow\infty} H(z) = \lim_{z\uparrow\infty} \frac{F_B(z) - F_G(z)}{1 - F(z)} \tag{23}
$$

$$
= \lim_{z\uparrow\infty} \frac{f_G(z) - f_B(z)}{p f_G(z) + (1 - p) f_B(z)}. \tag{24}
$$

From basic principles it is clear that:

$$
\mathbb{P}\left(Z_\theta = z\right) \equiv f_\theta(z) = f_{x\theta}(z) + f_{y\theta}(z)
$$

38

with

$$f_{x\theta}(z) = \frac{1}{z\sigma_x\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\log(z)-\mu_{x\theta}}{\sigma_x}\right)^2}N\left(\frac{\log(z)-\mu_{y\theta}}{\sigma_y\sqrt{1-\rho^2}}-\rho\frac{(\log(z)-\mu_{x\theta})}{\sigma_x\sqrt{1-\rho^2}}\right)$$

$$f_{y\theta}(z) = \frac{1}{z\sigma_y\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\log(z)-\mu_{y\theta}}{\sigma_y}\right)^2}N\left(\frac{\log(z)-\mu_{x\theta}}{\sigma_x\sqrt{1-\rho^2}}-\rho\frac{(\log(z)-\mu_{y\theta})}{\sigma_y\sqrt{1-\rho^2}}\right)$$

where $\mu_{x\theta} = \log(X_\theta)$ and $\mu_{y\theta} = \log(Y_\theta)$. The limit in (24) is easy to compute by factoring out leading terms. We note that when $\sigma_y > \sigma_x$ the right-tail behavior is determined by the piece of the densities $f_\theta(z)$ that corresponds to the density of Y. When $c_y = 0$, the limit of these densities is zero, as we set to prove.

Consider next case (b), in which $Z_\theta = X_\theta + Y_\theta$. Let $F_m(z)$ denote the distribution function of a lognormal random variable with log-mean μ_{Gy} and log-variance σ_y^2. Since $1 - F(z) = p(1 - F_B(z)) + (1 - p)(1 - F_G(z))$, we have that

$$\lim_{z\uparrow\infty}\frac{1-F(z)}{1-F_m(z)} = \lim_{z\uparrow\infty}p\frac{1-F_B(z)}{1-F_m(z)} + (1-p)\frac{1-F_G(z)}{1-F_m(z)}. \tag{25}$$

Using Theorem 1 from Asmussen and Rojas-Nandayapa (2008), we have that

$$\lim_{z\uparrow\infty}\frac{1-F_G(z)}{1-F_m(z)} = 1, \tag{26}$$

and that

$$\lim_{z\uparrow\infty}\frac{1-F_B(z)}{1-F_m(z)} = \begin{cases} 1 & \text{if} \quad \mu_{yB} = \mu_{yG}, \\ 0 & \text{if} \quad \mu_{yB} < \mu_{yG}. \end{cases} \tag{27}$$

Further note that

$$H(z) = \left(\frac{1-F_G(z)}{1-F_m(z)}\right)\left(\frac{1-F(z)}{1-F_m(z)}\right)^{-1} - \left(\frac{1-F_B(z)}{1-F_m(z)}\right)\left(\frac{1-F(z)}{1-F_m(z)}\right)^{-1}. \tag{28}$$

Using this last expression together with (25)-(27), we conclude that

$$\lim_{z\uparrow\infty} H(z) = \begin{cases} 0 & \text{if} \quad \mu_{yB} = \mu_{yG}, \\ (1-p)^{-1} & \text{if} \quad \mu_{yB} < \mu_{yG}. \end{cases} \tag{29}$$

This completes the proof of case (b).

In order to see case (c), note that

$$\mathbb{P}(X_\theta + \max(Y_\theta - I_T, 0) > z) > \mathbb{P}(X_\theta + Y_\theta > z + I_T) \tag{30}$$

and

$$\mathbb{P}(X_\theta + \max(Y_\theta - I_T, 0) > z) < \mathbb{P}(X_\theta + Y_\theta > z). \tag{31}$$

These two inequalities serve as a bound for the limit of the function $H(z)$ for the random variable $X_\theta + \max(Y_\theta - I_T, 0)$. The two bounds fall within the scope of the proof of case (b) of the Proposition, and therefore have the same limits, which coincide with those of case (c). This completes the proof. ∎

Proof of Proposition 7. From the definition of the reverse pecking order, we are set to prove, by contradiction, that $\mathcal{D}_D > \mathcal{D}_E$ cannot hold if $\hat{z} > \bar{z}$, i.e., if UNC does not hold. The reverse pecking order condition, if $\hat{z} > \bar{z}$, can be written as

$$\int_0^{\bar{z}} \underbrace{(\min(K, z) - \lambda z)}_{>0} \underbrace{c(z)}_{<0} \, dz + \int_{\bar{z}}^{\hat{z}} \underbrace{(K - \lambda z)}_{<0} \underbrace{c(z)}_{<0} \, dz + \int_{\hat{z}}^{\infty} \underbrace{(K - \lambda z)}_{<0} \underbrace{c(z)}_{>0} \, dz > 0. \tag{32}$$

We note that since g is the difference of two densities, it must be the case that

$$\int_0^{\infty} c(z) dz = 0; \quad \Rightarrow \quad -\int_0^{\hat{z}} c(z) dz = \int_{\hat{z}}^{\infty} c(z) dz$$

Further, we have

$$
\begin{aligned}
\int_{\hat{z}}^{\infty} (\lambda z - K) c(z) dz \;&>\; \int_{\hat{z}}^{\infty} (\lambda \hat{z} - K) c(z) dz \\
&=\; (\lambda \hat{z} - K) \int_{\hat{z}}^{\infty} c(z) dz \\
&=\; (K - \lambda \hat{z}) \int_0^{\hat{z}} c(z) dz \\
&>\; (K - \lambda \hat{z}) \int_{\bar{z}}^{\hat{z}} c(z) dz \\
&>\; \int_{\bar{z}}^{\hat{z}} (K - \lambda z) c(z) dz.
\end{aligned}
$$

We note that the last two inequalities follow from the fact that $\int_0^{\bar{z}} c(z) dz < 0$, and by our conjecture that $\bar{z} < \hat{z}$, which implies that $K - \lambda \hat{z} < 0$. Therefore, these inequalities imply

that the sum of the last two terms in (32) is negative, and since the first one is negative as well, it follows that (32) cannot hold, and thus $\mathcal{D}_D - \mathcal{D}_E < 0$, i.e., a reversal of the pecking order cannot obtain if UNC is not true. ∎

Table 1: A simple example

The table presents the parameter values and equilibrium outcomes of the capital raising problem discussed in Section 2. The payoff of the firm is given by a trinomial random variable $Z \in \{z_1, z_2, z_3\}$. The growth opportunity requires an investment of $I = 60$, and generates an extra cash flow of 200 in the high state. The payoff and the state probabilities are summarized below.

Payoffs	z_1	z_2	z_3
Assets in place	10	100	100
Growth opportunity	0	0	200
Total payoff	10	100	300

Distributions	z_1	z_2	z_3
Good-type, f_G	0.2	0.4	0.4
Bad-type, f_B	0.3	$0.4 - x$	$0.3 + x$

The column labelled "Pooled value" below computes the expected value of the firm, $\mathbb{E}[Z]$, where each type is assumed equally likely. The variable x can take values in $[0, 0.10]$, to guarantee that the distribution f_G first-order stochastically dominates f_B. The variable λ denotes the fraction of equity the firm needs to issue to finance the investment of $I = 60$. The column labelled $\mathcal{D}_E$ denotes the dilution costs of equity, namely $\lambda(\mathbb{E}[Z_G] - \mathbb{E}[Z_B])$. For all values of x, the firm can also finance the project with a debt security with a face value $K = 76.7$, for which the dilution costs, $\mathcal{D}_D \equiv \mathbb{E}[\min(Z_G, K)] - \mathbb{E}[\min(Z_B, K)]$, are 6.7 (last column).

x	$\mathbb{E}[Z_G]$	$\mathbb{E}[Z_B]$	Pooled value	λ	$\mathcal{D}_E$	$\mathcal{D}_D$
0.00	162	133	147.5	0.407	11.8	6.7
0.01	162	135	148.5	0.404	10.9	6.7
0.02	162	137	149.5	0.401	10.0	6.7
0.03	162	139	150.5	0.399	9.2	6.7
0.04	162	141	151.5	0.396	8.3	6.7
0.05	162	143	152.5	0.393	7.5	6.7
0.06	162	145	153.5	0.391	6.6	6.7
0.07	162	147	154.5	0.388	5.8	6.7
0.08	162	149	155.5	0.386	5.0	6.7
0.09	162	151	156.5	0.383	4.2	6.7
0.10	162	153	157.5	0.381	3.4	6.7

Table 2: Optimal security design problem

The table presents the parameter values and equilibrium outcomes of the security design problem discussed in Section 4. The payoff of the firm for type θ is given by $Z_\theta = X_\theta + \max(Y_\theta - I_T, 0)$, where both X_θ and Y_θ are lognormal, with $\mathbb{E}[X_\theta] = X_\theta$, $\mathbb{E}[Y_\theta] = Y_\theta$. We further denote $\mathrm{var}(\log(X_\theta)) = \sigma_x^2 T$, $\mathrm{var}(\log(Y_\theta)) = \sigma_y^2 T$, and $\mathrm{cov}(\log(X_\theta), \log(Y_\theta)) = \rho \sigma_x \sigma_y T$. The labels "Straight debt," "Convertibles," and "Warrants" refer to the functions $s(z) = \min(K, z)$, $s(z) = \min(K, z) + \max(z - \kappa, 0)$, and $s(z) = \max(z - \kappa, 0)$ respectively.

	Symbol	Case A	Case B	Case C
Primitives				
Value of assets in place type G	$\bar{X}_G$	100	150	150
Value of assets in place type B	$\bar{X}_B$	100	50	50
Value of new assets type G	$\bar{Y}_G$	250	200	200
Value of new assets type B	$\bar{Y}_B$	150	200	200
Time to maturity	T	5	5	5
Volatility of assets in place	σ_x	0.3	0.3	0.3
Volatility of new assets	σ_y	0.6	0.6	0.6
Probability of the good type	p	0.5	0.5	0.5
Correlation between assets	ρ	0.5	0.5	0.5
Pre-existing debt face value	K_0	0	0	100
Initial investment	I	100	120	70
Investment at exercise	I_T	50	50	50
Equilibrium outcomes				
Optimal security	$s(z)$	Straight debt	Convertibles	Warrants
Face value	K	138.8	69.5	–
Conversion trigger/exercise price	κ	–	593.4	502.5

Table 3: Optimal debt-equity choice

The table presents the parameter values and equilibrium outcomes of the capital raising problem discussed in Section 5.2. The payoff of the firm for type θ is given by $Z_\theta = X_\theta + \max(Y_\theta - I_T, 0)$, where both X_θ and Y_θ are lognormal, with $\mathbb{E}[X_\theta] = X_\theta$, $\mathbb{E}[Y_\theta] = Y_\theta$. We further denote $\mathrm{var}(\log(X_\theta)) = \sigma_x^2 T$, $\mathrm{var}(\log(Y_\theta)) = \sigma_y^2 T$, and $\mathrm{cov}(\log(X_\theta), \log(Y_\theta)) = \rho \sigma_x \sigma_y T$.

	Symbol	Value
Base case		
Value of assets in place for the good type	$\bar{X}_G$	125
Value of assets in place for the bad type	$\bar{X}_B$	75
Value of new assets for the good type	$\bar{Y}_G$	205
Value of new assets for the bad type	$\bar{Y}_B$	195
Good type firm value	$\mathbb{E}[Z_G]$	307.9
Bad type firm value	$\mathbb{E}[Z_B]$	248.2
Time to maturity	T	15
Volatility of assets in place	σ_x	0.30
Volatility of new assets	σ_y	0.60
Probability of the good type	p	0.50
Correlation between assets	ρ	0
Investment amount	I	100
Investment at maturity	I_T	50
Equilibrium outcomes		
Value of firm post-investment	$\mathbb{E}[Z_T]$	278.1
Equity fraction issued	λ	0.360
Face value of debt	K	218.4
Credit spread	$r_D = (K/D)^{1/T} - 1$	5.3%
Dilution costs of debt	$\mathcal{D}_D = \mathbb{E}[\min(Z_{GT}, K)] - \mathbb{E}[\min(Z_{BT}, K)]$	23.7
Dilution costs of equity	$\mathcal{D}_E = \lambda(\mathbb{E}[Z_{GT}] - \mathbb{E}[Z_{BT}])$	21.5
Relative dilution	$\mathcal{D}_D/\mathcal{D}_E$	1.10

Comparative statics

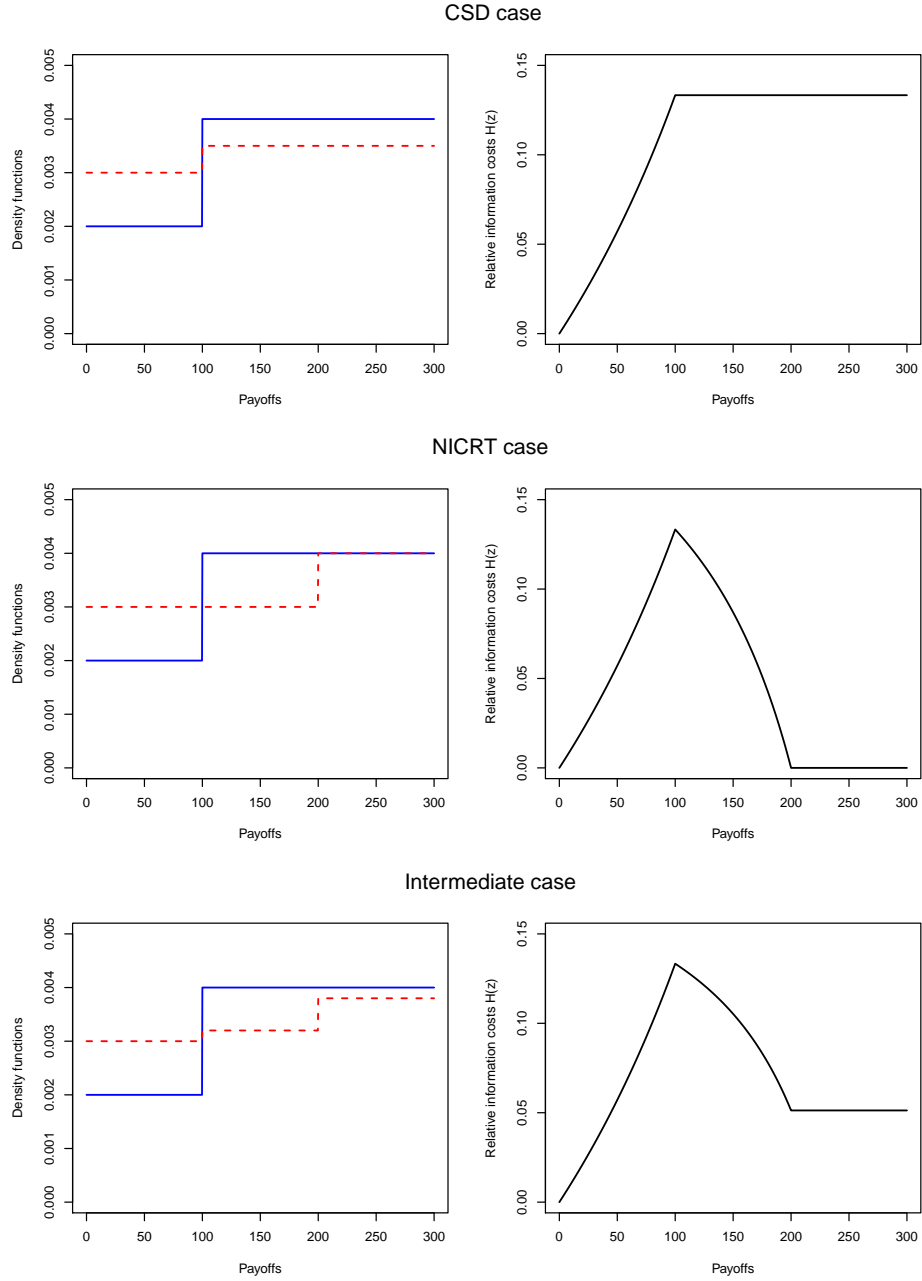| New parameter(s)	Equity share λ	Face value K	Spread r_D	Debt dilution $\mathcal{D}_D$	Equity dilution $\mathcal{D}_E$	Relative dilution $\mathcal{D}_D/\mathcal{D}_E$
$\bar{Y}_G = \bar{Y}_B = 200$	0.360	218.3	5.3%	22.9	18.0	1.27
$\bar{Y}_G = 225, \bar{Y}_B = 175$	0.359	219.4	5.4%	26.9	35.3	0.76
$\bar{X}_G = \bar{X}_B = 100$	0.360	213.9	5.2%	0.7	3.5	0.21
$\bar{X}_G = 150, \bar{X}_B = 50$	0.360	233.4	5.8%	49.7	39.5	1.26
$\sigma_x = 0.4$	0.360	290.6	7.4%	21.6	21.5	1.01
$\sigma_y = 0.8$	0.344	316.6	8.0%	31.6	20.6	1.53
$I_T = 0$	0.333	169.8	3.6%	17.6	20.0	0.88
$I_T = 100$	0.375	247.9	6.2%	26.5	22.3	1.19
$K_0 = 20$	0.386	303.3	7.7%	29.5	23.0	1.28
$K_0 = 40$	0.410	406.1	9.8%	34.5	23.4	1.47

Figure 1: The left panels plot the densities for the high-type (in blue) and the bad-type (in red), whereas the right panels plot the relative information costs, the function $H(z) = (F_B(z) - F_G(z))/(1 - F(z))$. The parameter values correspond to the example discussed in section 4.3, with $x = 0.05$ for the top two graphs ("CSD case"), with $x = 0.10$ for the middle graphs ("NICRT case"), and $x = 0.09$ for the bottom graphs ("Intermediate case").
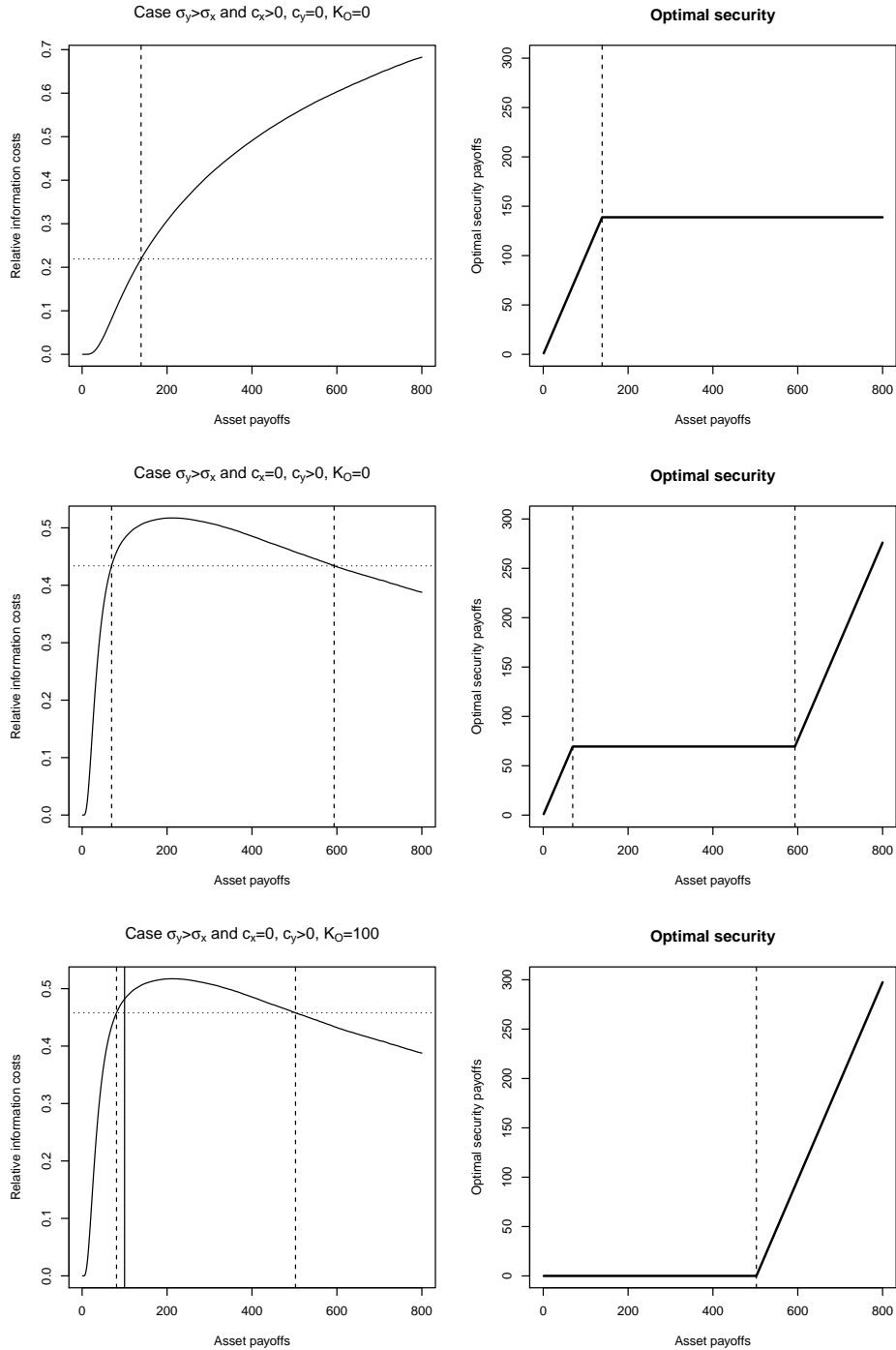
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Figure 2: The left panels plot the function $H(z) = (F_B(z) - F_G(z))/(1 - F(z))$, whereas the right panels plot the optimal securities. The parameter values correspond to the cases listed in Table 2: Case A is depicted in the top two graphs, Case B corresponds to the middle figure, and Case C to the bottom plots. The vertical dashed lines mark the points z for which $H(z) = \gamma$, where γ is given by the dotted horizontal line in the left panels. The vertical solid line in the bottom left graph shows the value of existing debt in Case C, namely $K_0 = 100$.
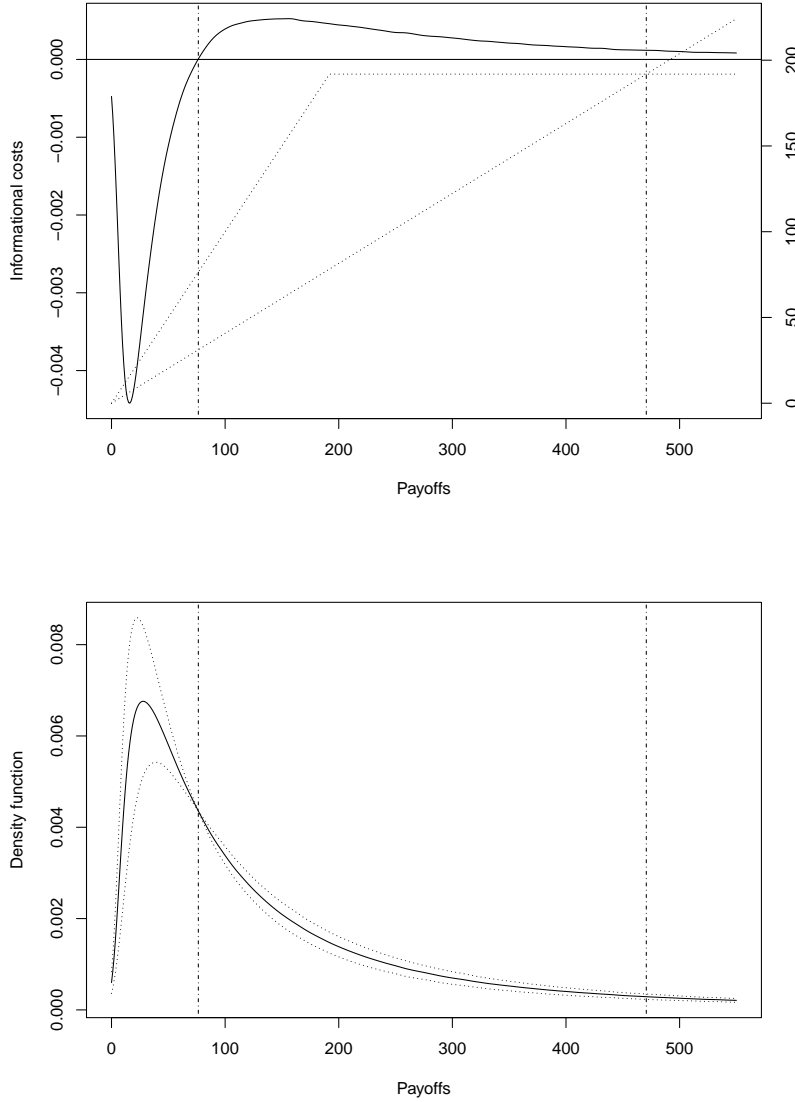
Figure 3: The top graph plots on the x-axis the payoffs from the firm at maturity, and in the y-axis it plots as a solid line the difference in the densities of the good and bad type firms, $f_G(z) - f_B(z)$ (y-axis labels on the left), and as dotted lines the payoffs from debt and equity (y-axis labels on the right). The left-most vertical dashed line is the point $\hat{z}$ for which $f_G(\hat{z}) = f_B(\hat{z})$, so points to the right of that line have positive information costs. The right-most vertical dashed line is the point $\bar{z}$ for which $K = \lambda\bar{z}$, so for payoffs to the right of that line equityholders receive more than debtholders. The bottom graph plots the densities of the good and bad types (dotted lines), as well as the joint density (integrated over types). The payoff of the firm for type θ is given by $Z_\theta = X_\theta + \max(Y_\theta - I_T, 0)$, where both X_θ and Y_θ are lognormal, as discussed in Section 4.3. The parameter values used in the figures are $\bar{X}_G = 125$, $\bar{X}_B = 75$, $\bar{Y}_G = \bar{Y}_B = 200$, $\sigma_x = 0.3$, $\sigma_y = 0.6$, $\rho = 0$, $T = 10$, $p = 0.5$, $I = 110$, $I_T = 50$. The dilution costs of debt for these parameters are $\mathcal{D}_D = 22.6$, whereas those of equity are $\mathcal{D}_E = 20.3$.
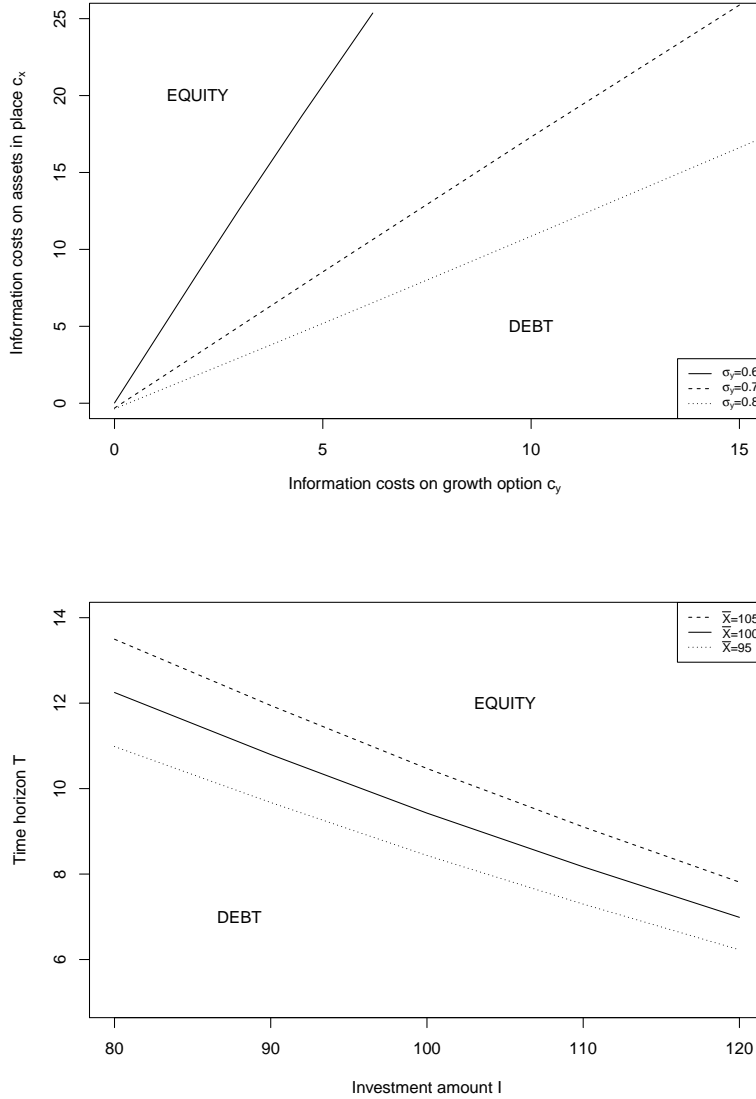
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Figure 4: The top graph plots the set of points (c_y, c_x) for which the dilution costs of equity and debt are the same, i.e. $\mathcal{D}_E = \mathcal{D}_D$. We consider the following parameter values: $\bar{X} = 100$, $\bar{Y} = 200$, $\sigma_x = 0.3$, $I = 120$, $I_T = 50$, $T = 10$, $\rho = 0$ and $p = 0.5$. Recall we set $\bar{X}_G = \bar{X} + c_x$ and $\bar{X}_B = \bar{X} - c_x$, and similarly $\bar{Y}_G = \bar{Y} + c_y$ and $\bar{Y}_B = \bar{Y} - c_y$. The solid line corresponds to the case where $\sigma_y = 0.6$, whereas the other two lines correspond to $\sigma_y = 0.7$ and $\sigma_y = 0.8$. For pairs of (c_y, c_x) below the lines debt is optimal, whereas equity is optimal above the lines. The bottom graph plots the set of points (I, T) for which the dilution costs of equity and debt are the same, i.e. $\mathcal{D}_E = \mathcal{D}_D$. We consider the following parameter values: $\bar{Y} = 200$, $\sigma_x = 0.3$, $I_T = 50$, $T = 10$, $c_x = 25$, $c_y = 0$, $\rho = 0$ and $p = 0.5$. The solid line corresponds to the case where $\bar{X} = 100$, whereas the other two lines correspond to $\bar{X} = 105$ and $\bar{X} = 95$. For pairs of (I, T) below the lines debt is optimal, whereas equity is optimal above the lines.
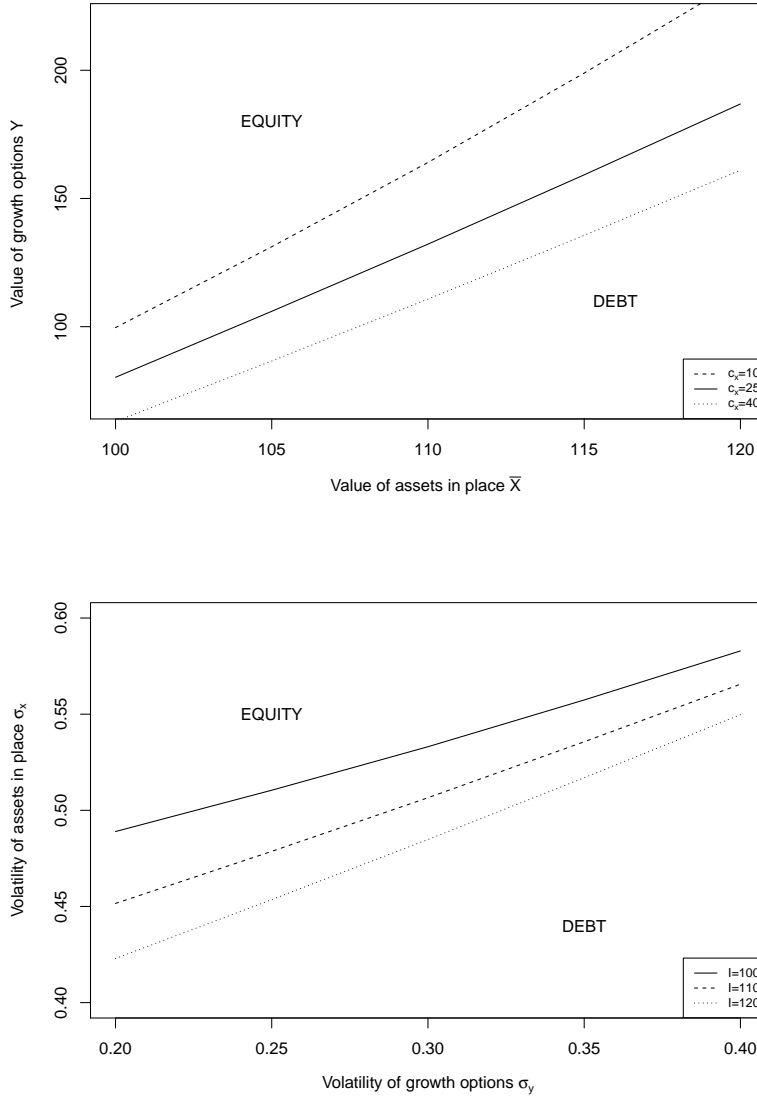
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Figure 5: The top graph plots the set of points $(\bar{X}, \bar{Y})$ for which the dilution costs of equity and debt are the same, i.e. $\mathcal{D}_E = \mathcal{D}_D$. We consider the following parameter values: $c_x = 25$, $c_y = 0$, $\sigma_x = 0.3$, $\sigma_y = 0.6$, $I = 110$, $T = 15$, $I_T = 50$, $\rho = 0$ and $p = 0.5$. The solid line corresponds to the case $c_x = 25$, whereas the other two lines correspond to $c_x = 10$ and $c_x = 40$. For pairs of $(\bar{X}, \bar{Y})$ below the lines debt is optimal, whereas equity is optimal above the lines. The bottom graph plots the set of points (σ_x, σ_y) for which the dilution costs of equity and debt are the same, i.e. $\mathcal{D}_E = \mathcal{D}_D$. We consider the following parameter values: $c_x = 25$, $c_y = 0$, $\bar{X} = 100$, $\bar{Y} = 150$, $T = 15$, $I_T = 50$, $\rho = 0$ and $p = 0.5$. The solid line corresponds to the case $I = 110$, whereas the other two lines correspond to $I = 100$ and $I = 120$. For pairs of (σ_x, σ_y) below the lines debt is optimal, whereas equity is optimal above the lines.
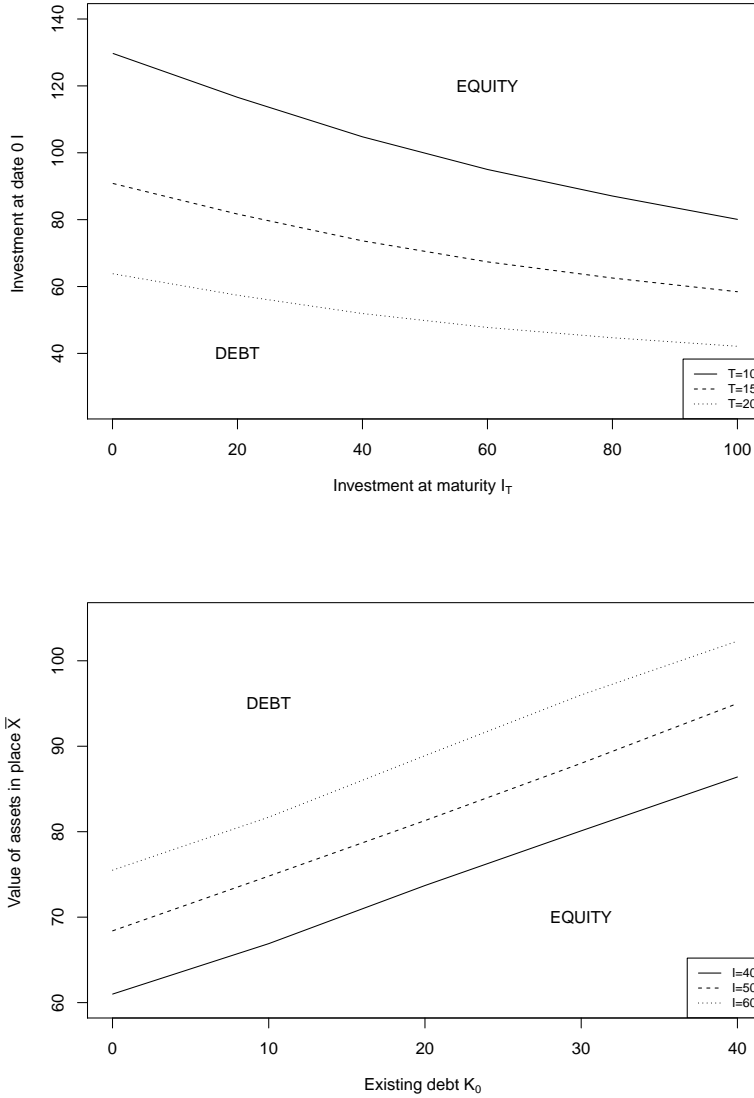
Figure 6: The top graph plots the set of points (I_T, I) for which the dilution costs of equity and debt are the same, i.e. $\mathcal{D}_E = \mathcal{D}_D$. We consider the following parameter values: $c_x = 25$, $c_y = 0$, $\sigma_x = 0.3$, $\sigma_y = 0.6$, $Y_G = Y_B = 175$, $\bar{X} = 100$, $\rho = 0$ and $p = 0.5$. The solid line corresponds to the case $T = 10$, whereas the other two lines correspond to $T = 15$ and $T = 20$. For pairs of (I_T, I) below the lines debt is optimal, whereas equity is optimal above the lines. The bottom graph plots the set of points $(K_0, \bar{X})$ for which the dilution costs of equity and debt are the same, i.e. $\mathcal{D}_E = \mathcal{D}_D$. We consider the following parameter values: $c_x = 25$, $c_y = 0$, $\sigma_x = 0.3$, $\sigma_y = 0.6$, $\bar{Y} = 175$, $I_T = 0$, $T = 10$, $\rho = 0$ and $p = 0.5$. The solid line corresponds to the case $I = 40$, whereas the other two lines correspond to $I = 50$ and $I = 60$. For pairs of $(K_0, \bar{X})$ below the lines equity is optimal, whereas debt is optimal above the lines.