Macroeconomic Fundamentals and Exchange Rates Dynamics:
A No-Arbitrage Multi-Country Model

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Abstract

This paper investigates the joint dynamics of multiple nominal exchange rates under a multi-country framework. Using a no-arbitrage macro-finance approach, information of macroeconomic fundamentals is employed to model exchange rate dynamics. Meanwhile, macroeconomic fundamentals are assumed to be determined by global (common) factors as well as by country-idiosyncratic factors. To do the empirical study, I mainly focus on an open economy including four countries, i.e. Germany, the UK, Japan and the US (the US dollar being the numeraire currency). The empirical results show that this model is able to explain 57%, 66% and 33% variations of the observed movements of the USD/DEM(EUR), the USD/GBP and the USD/JPY, respectively. This model implies foreign risk premiums satisfy the Fama condition (1984) and they are counter cyclical with respect to the US economy. Moreover, global and country-idiosyncratic macroeconomic factors do exist and play very different roles in driving exchange rate dynamics and foreign risk premiums.

Keywords: Multi-Country Model, Exchange Rate Dynamics, Macroeconomic Fundamentals, Global and Country-idiosyncratic Factors

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I. Introduction

The floating nominal exchange rate is the market price of one currency converted into another. It is one of the most important factors in international economic activities, such as international trade and international investment. But the question about whether exchange rate dynamics are driven by macroeconomic fundamentals or not has puzzled a great number of researchers after the seminal work by Meese and Rogoff (1983).

Large amount of models have been proposed to try to explain exchange rate dynamics by macroeconomic fundamentals. For instance, monetary models (Frenkel (1976, 1979), Mussa (1976), Bilson (1978), Dornbusch (1976)) state that the existence of a long-run equilibrium for the nominal exchange rate is a function of the differentials of money supplies and income levels between home and foreign countries. Recent studies, which adopt new open economy macroeconomic models (Obstfeld and Rogoff (2003)), investigate exchange rate movements by solving optimization problems from the stochastic dynamic general equilibrium approach in an open economy framework. However, these models cannot find empirical evidence on a close relationship between short-run exchange rates movements and macroeconomic fundamentals (Meese (1990), Frankel and Rose (1995), Engel and West (2005)). It is also worth mentioning that most of the studies on nominal exchange rates are under a two-country model setting which is the simplest setting in open economy studies.

The model presented in this paper has three main differences with respect to the traditional exchange rate models. First of all, this paper investigates exchange rates movements using macroeconomic fundamental information under a no-arbitrage macro-finance approach. Under this approach, the bilateral nominal exchange rate changes are endogenously determined by the ratio of stochastic discount factors between the two countries. The stochastic discount factor, also named intertemporal marginal utilities substitution, is modeled by a factor representation under the no-arbitrage condition. Outputs, inflations and short-term interest rates represent the macroeconomic fundamentals. Real output growth directly influences the aggregate consumption of a country and thus is a key element in the stochastic discount factor. Inflation can also enter into the stochastic discount factor via its dynamic interactions with real production (Piazzesi and Schneider (2006)). The short-term interest rate is typically viewed as a macro variable reflecting monetary policy (Duffee (2007)). This paper adopts the common specification for the stochastic discount factor used in macro-finance term structure models (Ang and Piazzesi (2003), Diebold, Rudebusch, and Aruoba (2005), Ang, Dong, and Piazzesi (2007)) and extends it into a multi-country framework. Term structure information on interest rates is included in order to help identify the time-varying market prices of risk, which in turn determine the foreign risk premium and amplify the role played by macroeconomic innovations on exchange rate changes. This is important since ignoring foreign risk premium or assuming it is constant may mislead to the conclusion that exchange rate dynamics are not linked to macroeconomic fundamentals.

Secondly, the model is built under a multi-country setting, thus it is able to investigate the dynamics of multiple exchange rates simultaneously. Dollar exchange rates are positively correlated according to the data. Hodrick and Vassalou (2002) point out that multi-country models can better explain the dynamics of exchange rates compared to two-country models under affine term structure models framework. In contrast, two-country models are only able to study single exchange rate movements. Moreover, in order to study more than one exchange rate at a time, each exchange rate for each two-country case has to be separately analyzed. Therefore, inconsistency issues concerning the parameters related to the numeraire country may potentially arise.

Thirdly, in this paper, global and country-idiosyncratic macroeconomic factors setting are used to model the correlated macroeconomic fundamentals across countries. The global and local factors have been used by Ahn (2004) in the study of exchange rates dynamics as well, but these factors are latent and do not have any economic meaning. The importance of the existence of global factors in modeling exchange rate dynamics has been mentioned by Litterman and Scheinkman (1991), Hodrick and Vassalou (2002), and Sarno, Schneider and Wagner (2011). Additionally, since the prices of risks are notoriously difficult to estimate from a statistical standpoint, through this setting the number of risk price parameters in the model can be significantly shrank so that the estimation becomes tractable when compared to the setting involving country-level macroeconomic factors. Moreover, this setting allows to distinguish the different roles played by global and country-idiosyncratic macroeconomic factors in driving exchange rate dynamics and foreign risk premiums.

There are some recent studies on multiple exchange rates using international term structure models with different focuses or adopting different methodologies. Sarno, Schneider and Wagner (2011) and Ang and Chen (2010) focus on the properties and yield curve predictors of the endogenous foreign exchange risk premiums arising from the no-arbitrage condition. Graveline and Joslin (2011) concentrate on the returns of currencies as a portfolio. Bauer and Diez (2011) assume that the law of motion of exchange rates movements is exogenous.

In this paper, the empirical study focuses on an open economy including four countries, i.e. Germany, the UK, Japan and the US, where the US is taken as the home country. This multi-country no-arbitrage term structure model is able to explain 57%, 66% and 33% of the variation of the observed exchange rate changes of the USD/DEM(EUR), the USD/GBP and the USD/JPY, respectively. The model-implied foreign risk premiums satisfy the Fama condition (1984) and they are counter cyclical to the US economy. The innovations, or 'news', are important in determining the exchange rate dynamics. Moreover, global and country-idiosyncratic macroeconomic factors do exist and play very different roles in driving exchange rate dynamics and foreign risk premiums. Global factors drive foreign risk premiums almost exclusively and account for more than half of the forecast error variance of exchange rate dynamics. In the short-run the dominant factor is global interest rate, while in the long-run the global output becomes dominant in driving both exchange rate dynamics and foreign risk premiums. Even through country-idiosyncratic macroeconomic factors are less important comparing to global ones, they do play some role in determing the short-run exchange rate dynamics, especially the US and German interest rate, and the UK and Japanese output.

The rest of this paper is organized as follows. Section II presents the data. Section III introduces a multi-country no-arbitrage exchange rate model under the macro-finance literature. Section IV proposes the econometric methodology, the likelihood-based estimation combined with the unscented Kalman filter. Section V presents the empirical results and discusses their economic implications. Section VI concludes.

II. Data and Preliminary Analysis

A. Data

Consider a multi-country world, with $(N+1)$ countries. The last country (the $(N+1)^{th}$ country) is the domestic country and the first N countries are the foreign countries. A $(3+1)$-country open economy case will be analyzed in the empirical study of this paper. The countries are Germany/euro area, the UK, Japan and the US. The first three are taken as the foreign countries, while the US is taken as the home country. The data is in monthly time frequency and the sample period goes from January 1985 to May 2009.

The macroeconomic fundamentals taken into account are output growths, inflation rates and short term interest rates. The nominal exchange rates are the end-of-period market rates. Both the exchange rate and macroeconomic data are coming from the International Financial Statistics (IFS) database, provided by the International Monetary Fund (IMF).

Output growth rates and inflation rates are the one-year percentage changes of seasonal adjusted Industrial Production Indexes (line *66*) and Consumer Price Indexes (line *64*), respectively. Exchange rate data are the US Dollar per National Currency (line *ag*). The exchange rate for the German Mark after 1999 is replaced by the exchange rate of the Euro, following Corte, Sarno, and Tsiakas (2009).

— Figure 1 around here —

Moreover, yield data is included in order to better identify the parameters that determine the market prices of risk, since the market prices of risk are important in modeling exchange rate dynamics. The zero-coupon bond yield data for German, the UK, Japan and the US are from the International Zero Coupon Yield Curve Dataset used by Wright (2011). We take yields with different maturities: 3 months (the shortest one in this dataset), 24 months and 60 months, which stand for the short, medium and long yields, respectively. These three yields are commonly used to get the empirical 'level', 'slope' and 'curvature' components, which are sufficient to capture the term structure of interest rates. In addition, short-term interest rates are proxied by 3-month zero-coupon bond yields. In order to match the unit of monthly exchange rates movements, both the macroeconomic and yield data are divided by 12 into monthly equal quantities.

B. Preliminary Analysis

It is important to clarify the cross-country relationships of macroeconomic fundamentals. The correlation matrix of these variables is presented in Table 8 by the correlation matrix.

— Table 8 around here —

By focusing on the diagonal sub matrix of this correlation matrix, relevant correlations among the three groups of macroeconomic variables (output growths, inflations and short-term interest rates) arise. The first 4 × 4 triangle matrix on the diagonal of this correlation matrix shows that the output growths are all positively correlated across these four countries. Among them, the highest two correlations are between Germany and Japan (53%), and between the UK and the US (53%). The second 4 × 4 triangle matrix on the diagonal shows that the inflation rates are positively correlated. The highest correlation is between the UK and the US (78%), followed by the one between the UK and Japan (70%). The third 4 × 4 triangle matrix on the diagonal of the same matrix shows that short-term interest rates are positively correlated as well, with the highest two correlations equal to 84% and 80%, between the UK and Japan, and the UK and the US, respectively. The fact that macroeconimic variables are positively correlated across countries suggests that there may exist some global macroeconomic factors driving output growths, inflations and short-term interest rates, which may determine the comovement of macroeconomic fundamentals across countries.

— Table 9 around here —

The above result is a starting point to explore deeper the question of whether there exist some common factors driving the comovement in macroeconomic fundamentals across countries or not. Principle components analysis is conducted for each group of macroeconomic fundamentals (output growths, inflations and short-term interest rates). The result is reported in Table 9. In each group of macroeconomic fundamentals, the first principle component associated with the highest eigenvalue is able to explain 76%, 71% and 84% of the variation, respectively. This result implies that there should exist a global (common) factor in each group, determining the comovement of these macroeconomic variables across countries. This is the evidence of adopting the global and country-idiosyncratic macroeconomic factors setting in the building up the model later on.

III. A Multi-Country No-Arbitrage Exchange Rate Dynamic Model

Consider a (*N+1*)-country world, with N foreign countries and *1* domestic country. Because of the important role of the US economy and its currency in the global economy activities after the Bretton Woods system collapse, the US is chosen as home country and correspondingly the US dollar is the numerical currency. Among these *N+1* countries' currencies, the minimal number of bilateral nominal exchange rates relationships is N, cause the rest can be induced by the triangle relationship from these N bilateral exchange rates. Hence, this paper focuses on the N bilateral exchange rates, which are the rates of the N foreign currencies against the US dollar.

Under the no-arbitrage assumption and the law of one price, the exchange rate dynamic is determined by the ratio of stochastic discount factors related to these two countries. In this section, firstly discuss the global and country-specific factor setting in a multi-country economy in subsection *A*. Then introduce how to model stochastic discount factors related to macroeconomic fundamentals in subsection *B*. After that, proceed to model the exchange rate dynamics in subsection *C*. In the end, present the recursive relationship of the bond pricing for each country under the affine term structure modeling framework in subsection *D*.

A. A Global and Country-Idiosyncratic Macro Factor Setting in a Multi-Country Economy

The choice of the state dynamics in driving a multi-country economy system is a trade-off. It is better to include as much macroeconomic information as possible. In the mean while, it is necessary to control the amount of the parameters as low as possible in order to enable to carry out the estimation. Hence we introduce the macroeconomic global and country-idiosyncratic factor setting, which is able to well balance these two points.

In a (*N+1*)-country open economy, suppose there exists a global macroeconomic factors of output growth g_t^G, inflation π_t^G and short-term interest rate r_t^G, which drive the co-movement of macroeconomic fundamentals across countries. Write them into a global factor vector G_t, where $G_t = \left(g_t^G,\ \pi_t^G,\ r_t^G\right)^T$.

For each economy i ($i = 1, 2, ..., N+1$), we assume that its underlying macroeconomic fundamental vector $X_{i,t} = \left(\tilde{g}_{i,t},\ \tilde{\pi}_{i,t},\ \tilde{r}_{i,t}\right)^T$ loads on the global factor vector $G_t = \left(g_t^G,\ \pi_t^G,\ r_t^G\right)^T$, as well as on its country-idiosyncratic factor vector $F_{i,t} = \left(f_{i,t}^g,\ f_{i,t}^\pi,\ f_{i,t}^r\right)^T$. Note that the tilde is used to distinguish the unobserved underlying fundamentals from the observed data, with the difference of measurement error between them. Hence for country i, its underlying macroeconomic fundamentals $\tilde{g}_{i,t},\ \tilde{\pi}_{i,t}$ and $\tilde{r}_{i,t}$ are,

$$\begin{aligned} \tilde{g}_{i,t} &= \alpha_i^g + \beta_i^g g_t^G + f_{i,t}^g, \\ \tilde{\pi}_{i,t} &= \alpha_i^\pi + \beta_i^\pi \pi_t^G + f_{i,t}^\pi, \\ \tilde{r}_{i,t} &= \alpha_i^r + \beta_i^r r_t^G + f_{i,t}^r, \end{aligned} \tag{1}$$

where $\{\alpha_i^g, \alpha_i^\pi, \alpha_i^r\}_{i=1,...,N+1}$ are constant terms, and $\{\beta_i^g, \beta_i^\pi, \beta_i^r\}_{i=1,...,N+1}$ are loadings on global factors ($g_t^G,\ \pi_t^G,\ r_t^G$) for country i; and $\{f_{i,t}^g, f_{i,t}^\pi, f_{i,t}^r\}_{i=1,...,N+1}$ are country-idiosyncratic factors in country i. Write above equations into a matrix equation,

$$X_{i,t} = \alpha_i + \beta_i G_t + F_{i,t}, \tag{2}$$

where $\{\alpha_i\}_{i=1,...,N+1}$ are constant 3×1 vectors, and $\{\beta_i\}_{i=1,...,N+1}$ are diagonal matrices of the loading on global factor G_t.

About the two types of state vectors determining the underlying macroeconomic fundamentals, firstly of all, assume that this global factor vector G_t follows a Gaussian vector autoregression process,

$$G_t = \Phi^G G_{t-1} + \Sigma^G \upsilon_t^G, \tag{3}$$

where Φ^G is a constant 3×3 matrix; υ_t^G is an i.i.d Gaussian white noise, with mean zeros and identity variance-covariance matrix; and Σ^G is a diagonal matrix. Besides, in order to identify the global factors, two sets of assumptions are needed. First, since the magnitudes of global factors and their loadings cannot be separately identified, hence we assume that the innovations to global factors have standard deviation of 0.001, which means $\Sigma^G = 0.001 \times I_3$. Second, to identify the signs of global factors and their loadings, we assume that the US loadings on the global factors are positive.

Secondly, assume that the country-idiosyncratic factor vector $F_{i,t}$ has a Gaussian vector autoregression process,

$$F_{i,t} = \Phi^{F_i} F_{i,t-1} + \Sigma^{F_i} \upsilon^F_{i,t}, \tag{4}$$

where Φ^{F_i} is a constant 3×3 diagonal matrix; $\upsilon^F_{i,t}$ is country-idiosyncratic shock vector, with mean zero and identity variance-covariance matrix; and we assume shocks in this equation are independent, hence the variance-covariance matrix of $\Sigma^{F_i}(\Sigma^{F_i})^T$ is diagonal. A similar setting of global and country-idiosyncratic factors for yield is adopted in the paper of Diebold, Li, and Yue (2008) on investigating the global yield curve under a multi-country economy.

B. Relating Macroeconomic Fundamentals to Stochastic Discount Factors

In this multi-country world, assume that no-arbitrage holds. Then there exists at least one almost surely positive process M_t with $M_0 = 1$ dominated in each currency such that the discounted gains process associated with any admissible trading strategy dominated in that currency is a martingale (Harrison and Kreps (1979)). M_t is called the stochastic discount factor (SDF). We denote the country i's SDF as $M_{i,t}$, for $i = 1, 2, ..., N+1$.

Without a generally accepted equilibrium model for asset pricing, many studies use flexible factor models under the no-arbitrage condition (Cochrane, 2004) from a partial equilibrium in the financial market. In this paper, I also use a factor representation for the SDF's, based on which exchange rates and term structures of interest rates are modeled. Under the complete market assumption, there exists one unique stochastic discount factor $M_{i,t}$, associated with each country i's currency, for $i = 1, 2, ..., N+1$. Given that the dynamics of country i's economy are jointly determined by the global factor as well as by its country-idiosyncratic factor, I assume that the SDF for country i has the following exponential form

$$\begin{aligned} M_{i,t+1} &= \exp(m_{i,t+1}) \\ &= \exp\Big(-\tilde{r}_{i,t} - \frac{1}{2}(\lambda^G_{i,t})^T \lambda^G_{i,t} - \frac{1}{2}(\lambda^F_{i,t})^T \lambda^F_{i,t} - (\lambda^G_{i,t})^T \upsilon^G_{t+1} - (\lambda^F_{i,t})^T \upsilon^F_{i,t+1}\Big), \end{aligned} \tag{5}$$

where $\tilde{r}_{i,t}$ is the short-term interest rate of country i, $\lambda_{i,t}^G$ and $\lambda_{i,t}^F$ are the time-varying market prices of global and country-idiosyncratic risks assigned by investors for assets dominated in country i's currency, and υ_{t+1}^G and $\upsilon_{i,t+1}^F$ are the global and country-idiosyncratic 'uncertainties' related to the country i's economy at time t, from equation (3) and (4).

This specification for SDF process is similar as the commonly used one in macro finance term structure literatures, such as Ang and Piazzesi (2003), Duffee (2002) and Duffee (2007). The only difference from the standard ones is that there are two types of market prices of risks and innovations associated with two types of state vectors in this paper, global and country-idiosyncratic ones. In a Lucas-type exchange economy (Lucas (1982)), the stochastic discount factor is also named the intertemporal marginal rate of substitution from the representative agent's optimization problem.

Note that the market prices of global and country-idiosyncratic risks related to country i's currency are $\lambda_{i,t}^G$ and $\lambda_{i,t}^F$, respectively. The country i's state vectors G_t and $F_{i,t}$ summarize uncertainties in country i's economy and assume that the market prices of global and country-idiosyncratic risks related to each country i's currency are affine functions of their corresponding state vectors, G_t and $F_{i,t}$, for each country $i = 1, ..., N+1$, (Dai and Singleton (2002); Duffee (2002))

$$\begin{aligned} \lambda_{i,t}^G &= \lambda_{i,0}^G + \lambda_{i,1}^G G_t, & (6) \\ \lambda_{i,t}^F &= \lambda_{i,0}^F + \lambda_{i,1}^F F_{i,t}, & (7) \end{aligned}$$

where $\lambda_{i,0}^G$ and $\lambda_{i,0}^F$ are constant 3×1 vectors, and $\lambda_{i,1}^G$ and $\lambda_{i,1}^F$ are constant 3×3 matrices. It is crucial to make some reasonable restriction on the coefficients of $\lambda_{i,1}^G$ and $\lambda_{i,1}^F$ to make the model capable to be estimated. Here we simplify $\lambda_{i,1}^G$ and $\lambda_{i,1}^F$ to be diagonal matrices. This is able to reduce the amount of parameters in this multi-country model and without loss generalization and efficiency on modeling market prices of risk by macroeconomic information.

C. Exchange Rate Dynamics

Let $\mathcal{S}_{j,t}$ $(j = 1, ..., N)$ be the exchange rate between the foreign country j and the US, and it is defined as the price of the US dollar per one unit of the foreign country j's currency. No-arbitrage and law of one price dictate that the ratio of the stochastic discount factors between the home and foreign countries determines the dynamics of their exchange rate (Bachus, Foresi, and Telmer (2001); Bekaert (1996); Brandt and Santa-Clara (2002); Brandt, Cochrane, and Santa-Clara (2006)). Thus we have

$$\frac{\mathcal{S}_{j,t+1}}{\mathcal{S}_{j,t}} = \frac{M_{j,t+1}}{M_{N+1,t+1}}. \tag{8}$$

The above relation formally defines the link between the stochastic discount factors of two economies and the exchange rate movements between them. In complete markets, the stochastic discount factors in both economies are unique, therefore they uniquely determine the dynamics of their exchange rate.

Taking natural logarithms for both sides of equation (8) and using specifications of the SDF's (5), we obtain the following exchange rate changes equation,

$$\begin{aligned} \Delta s_{j,t+1} \;\; = \;\; & \Big(\tilde{r}_{N+1,t} - \tilde{r}_{j,t}\Big) + \frac{1}{2}\Big((\lambda^{G}_{N+1,t})^{T}\lambda^{G}_{N+1,t} - (\lambda^{G}_{j,t})^{T}\lambda^{G}_{j,t} + (\lambda^{F}_{N+1,t})^{T}\lambda^{F}_{N+1,t} - (\lambda^{F}_{j,t})^{T}\lambda^{F}_{j,t}\Big) \\ & + \Big((\lambda^{G}_{N+1,t})^{T}\upsilon^{G}_{t+1} - (\lambda^{G}_{j,t})^{T}\upsilon^{G}_{t+1} + (\lambda^{F}_{N+1,t})^{T}\upsilon^{F}_{N+1,t+1} - (\lambda^{F}_{j,t})^{T}\upsilon^{F}_{j,t+1}\Big), \end{aligned} \tag{9}$$

which shows that the global factor G and the two country-idiosyncratic factors F_j and F_{N+1} are imparted to the exchange rate changes $\Delta s_{j,t+1}$, via market prices of risk, with nonlinear form. This is in contrast to the traditional models that often give a linear relation between the exchange rate dynamics and macroeconomic fundamentals or these only use latent factors and do not have any economically meaningful interpretations.

The exchange rate changes can be divided into two parts, the expected and unexpected ones. The expected foreign exchange rate changes,

$$\begin{aligned} \Delta s^{exp.}_{j,t+1} \;\; \equiv \;\; & E_t\Big(\Delta s_{j,t+1}\Big) \qquad\qquad (10) \\ = \;\; & \Big(\tilde{r}_{N+1,t} - \tilde{r}_{j,t}\Big) + \frac{1}{2}\Big((\lambda^{G}_{N+1,t})^{T}\lambda^{G}_{N+1,t} - (\lambda^{G}_{j,t})^{T}\lambda^{G}_{j,t}\Big) + \frac{1}{2}\Big((\lambda^{F}_{N+1,t})^{T}\lambda^{F}_{N+1,t} - (\lambda^{F}_{j,t})^{T}\lambda^{F}_{j,t}\Big) \end{aligned}$$

which captures predictable variation of returns in foreign exchange markets. We can see that market prices of risks are important in modeling the expected part of exchange rate changes.

The uncovered interest rate parity does not hold for this model. Because the expected exchange rate changes are determined not only by the interest rate differentials between the two countries $(\tilde{r}_{N+1,t} - \tilde{r}_{j,t})$, but also by a time varying foreign exchange risk premium term, $rp_{j,t+1}$,

$$rp_{j,t+1} \equiv \frac{1}{2}\Big((\lambda^G_{N+1,t})^T \lambda^G_{N+1,t} - (\lambda^G_{j,t})^T \lambda^G_{j,t}\Big) + \frac{1}{2}\Big((\lambda^F_{N+1,t})^T \lambda^F_{N+1,t} - (\lambda^F_{j,t})^T \lambda^F_{j,t}\Big), \tag{11}$$

The above equation shows the foreign exchange risk premium is determined by two parts, the one governed by global factors and the other governed by country-idiosyncratic factors.

The unexpected exchange rate changes,

$$\begin{aligned} \Delta s^{unexp.}_{j,t+1} &\equiv \Delta s_{j,t+1} - E_t\Big(\Delta s_{j,t+1}\Big) \qquad (12) \\ &= \Big((\lambda^G_{N+1,t})^T \upsilon^G_{t+1} - (\lambda^G_{j,t})^T \upsilon^G_{t+1}\Big) + \Big((\lambda^F_{N+1,t})^T \upsilon^F_{N+1,t+1} - (\lambda^F_{j,t})^T \upsilon^F_{j,t+1}\Big), \end{aligned}$$

which implies that the unexpected exchange rate changes are constructed by the products of state vector shock and its corresponding market price of risk. The same as the foreign risk premium in equation (11), it also has two parts, the global part and the country-idiosyncratic part. Note that the market price of risk is time-varying according to its linear relationship with macroeconomic variables. This implies the exchange rate changes are heteroskedastic.

In summary, the exchange rate dynamic equation (9) can be write in following way as well,

$$\begin{aligned} \Delta s_{j,t+1} &= \Big(\tilde{r}_{N+1,t} - \tilde{r}_{j,t}\Big) + rp_{j,t+1} + \Delta s^{unexp.}_{j,t+1}, \qquad (13) \\ &= \Delta s^{exp.}_{j,t+1} + \Delta s^{unexp.}_{j,t+1}, \qquad (14) \end{aligned}$$

D. Bond Pricing

In the last part of our model, we give the recursive relationship of the bond pricing for each country i $(i = 1, ..., N+1)$ under the affine term structure modeling framework.

For each country i, having specified the stochastic discount factor $M_{i,t}$ (equation (5)) and its state dynamics (equation (3) and (4)), then we can price its zero-coupon bonds. Introduction of bonds in our modeling framework is important for identifying market prices of risks.

Because each country's short rate $\tilde{r}_{i,t}$ is a function of the global factor as well as its country-idiosyncratic factor, as in equation (1). We can write the short rate equation as an affine function of the global factor G_t and its country-idiosyncratic factor $F_{i,t}$,

$$\tilde{r}_{i,t} = \delta_{i,0} + (\delta_{i,1}^G)^T G_t + (\delta_{i,1}^F)^T F_{i,t}, \tag{15}$$

with $\delta_{i,0} = \alpha_i^r$, $\delta_{i,1}^G = \Big(0, 0, \beta_i^r\Big)^T$, and $\delta_{i,1}^F = \Big(0, 0, 1\Big)^T$.

In each country i, no-arbitrage guarantees that a zero-coupon bond with n-period-maturity at time t can be priced by using the following Euler equation

$$\tilde{P}_{i,t}^{(n)} = E_t\Big[M_{i,t+1}\tilde{P}_{i,t+1}^{(n-1)}\Big] \tag{16}$$

with the initial condition $\tilde{P}_{i,t}^{(0)} = 1$. Again, tilde indicates the true value. Under specifications of the country state factors dynamics (equation (3) and (4)), the short rate (15) and the SDF (5), we can show that the country i's bond price is an exponential linear function of the glabal factor G_t as well as its country-idiosyncratic factors $F_{i,t}$,

$$\tilde{P}_{i,t}^{(n)} = \exp\Big(A_{i,n} + (B_{i,n})^T G_t + (C_{i,n})^T F_{i,t}\Big), \tag{17}$$

where $A_{i,n}, B_{i,n}$ and $C_{i,n}$ solve the following difference equations,

$$\begin{aligned} A_{i,n+1} &= A_{i,n} - (B_{i,n})^T \Sigma^G \lambda_{i,0}^G - (C_{i,n})^T \Sigma^{F_i} \lambda_{i,0}^F + \frac{1}{2}(B_{i,n})^T \Sigma^G (\Sigma^G)^T B_{i,n} + \frac{1}{2}(C_{i,n})^T \Sigma^{F_i} (\Sigma^{F_i})^T C_{i,n} - \delta_{i,0}, \\ B_{i,n+1} &= \Big(\Phi^G - \Sigma^G \lambda_{i,1}^G\Big)^T B_{i,n} - \delta_{i,1}^G, \\ C_{i,n+1} &= \Big(\Phi^{F_i} - \Sigma^{F_i} \lambda_{i,1}^F\Big)^T C_{i,n} - \delta_{i,1}^F, \end{aligned} \tag{18}$$

with $A_{i,1} = -\delta_{i,0}$, $B_{i,1} = -\delta_{i,1}^G$, and $C_{i,1} = -\delta_{i,1}^F$ being the initial conditions. Accordingly, the yield is also an affine function of the state

$$\tilde{y}_{i,t}^{(n)} \equiv -\frac{\log P_{i,t}^{(n)}}{n} = a_{i,n} + (b_{i,n})^T G_t + (c_{i,n})^T F_{i,t}, \tag{19}$$

where $a_{i,n} = -A_{i,n}/n$, $b_{i,n} = -B_{i,n}/n$, and $c_{i,n} = -C_{i,n}/n$.

From difference equations in (18), we can see that the constant market price of risk parameters $\lambda^G_{i,0}$ and $\lambda^F_{i,0}$ only affect the constant yield coefficient $a_{i,n}$, whereas the coeffient parameters $\lambda^G_{i,1}$ and $\lambda^F_{i,1}$ affect the loadings on global and country-idiosyncratic factors, $b_{i,n}$ and $c_{i,n}$, repectively. This implies that the parameters $\lambda^G_{i,0}$ and $\lambda^F_{i,0}$ affect average term spreads and average expected bond returns, whereas the parameters $\lambda^G_{i,1}$ and $\lambda^F_{i,1}$ govern time variation in term spreads and expected bond returns.

IV. Econometric Methodology

Since we assume that the macroeconomic factors $X_{i,t}$, yields $y_{i,t}$, and exchange rate changes $\Delta s_{j,t}$, are unobservable and that the econometrician observe the corresponding ones, $X^{obs.}_{i,t}$, $y^{obs.}_{i,t}$, and $\Delta s^{obs.}_{j,t}$, with measurement errors, $\eta^X_{i,t}$, $\eta^y_{i,t}$ and $\eta^{\Delta s}_{j,t}$. We first transform the model into a state-space representation and then use a Bayesian filtering approach to estimate the model.

A. State-Space Model Representation

At each period t, we can observe the exchange rate changes, macroeconomic variables, and zero-coupon bond data. We assume that each of these variables is collected with normal i.i.d measurement errors. Thus, we have the following measurement equations

$$\begin{aligned}
\Delta s^{obs.}_{j,t} &= \Big(\tilde{r}_{N+1,t-1} - \tilde{r}_{j,t-1}\Big) + \frac{1}{2}\Big((\lambda^G_{N+1,t-1})^T\lambda^G_{N+1,t-1} - (\lambda^G_{j,t-1})^T\lambda^G_{j,t-1} \\
&\quad +(\lambda^F_{N+1,t-1})^T\lambda^F_{N+1,t-1} - (\lambda^F_{j,t-1})^T\lambda^F_{j,t-1}\Big) + (\lambda^G_{N+1,t-1} - \lambda^G_{j,t-1})^T(\Sigma^G)^{-1}(G_t - \Phi^G G_{t-1}) \\
&\quad +\Big((\lambda^F_{N+1,t-1})^T(\Sigma^{F_{N+1}})^{-1}(F_{N+1,t} - \Phi^{F_{N+1}}F_{t-1}) - (\lambda^F_{j,t-1})^T(\Sigma^{F_j})^{-1}(F_{j,t} - \Phi^{F_j}F_{t-1})\Big) + \eta^{\Delta s}_{j,t}, \\
&\quad for\ j = 1,\ \ldots\ , N; \qquad (20) \\
X^{obs.}_{i,t} &= \alpha_i + \beta_i G_t + F_{i,t} + \eta^X_{i,t},\ for\ i = 1, ..., N+1;, \qquad (21) \\
y^{obs.}_{i,t} &= a_i + (b_i)^T G_t + (c_i)^T F_{i,t} + \eta^y_{i,t},\quad for\ i = 1,\ \ldots\ , N+1. \qquad (22)
\end{aligned}$$

where in the exchange rate changes equation (20), we use $\upsilon^G_t = (\Sigma^G)^{-1}(G_t - \Phi^G G_{t-1})$ and $\upsilon_{i,t} = (\Sigma^{F_i})^{-1}(F_{i,t} - \Phi^{F_i}F_{t-1})$ (for $i = 1, ..., N+1$), from equation (3) and (4); the market prices of risks are linear functions of the state vectors, $\lambda^G_{i,t-1} = \lambda^G_0 + \lambda^G_1 G_t$ and $\lambda^F_{i,t-1} = \lambda^F_0 + \lambda^F_1 F_{i,t}$.

η_t's capture measurement errors with distinct variances for different variables/series and are assumed to be mutually independent.

For the state vector in this multi-country system, we have the global factor G_t and country-idiosyncratic factors $F_{i,t}$ (for $i = 1, 2, 3, 4$), following a first-order VAR with their dynamics in equation (3) and (4), respectively. From the measurement equations, we notice that observations depend on both current and lagged values of global and country-idiosyncratic factors. Hence all of them should be taken as states and the state equations are,

$$\begin{pmatrix} G_t \\ G_{t-1} \end{pmatrix} = \begin{pmatrix} \Phi^G & 0_{3\times3} \\ I_3 & 0_{3\times3} \end{pmatrix} \begin{pmatrix} G_{t-1} \\ G_{t-2} \end{pmatrix} + \begin{pmatrix} I_3 \\ 0_{3\times3} \end{pmatrix} \Sigma^G \upsilon_t^G, \tag{23}$$

$$\begin{pmatrix} F_{i,t} \\ F_{i,t-1} \end{pmatrix} = \begin{pmatrix} \Phi^{F_i} & 0_{3\times3} \\ I_3 & 0_{3\times3} \end{pmatrix} \begin{pmatrix} F_{i,t-1} \\ F_{i,t-2} \end{pmatrix} + \begin{pmatrix} I_3 \\ 0_{3\times3} \end{pmatrix} \Sigma^{F_i} \upsilon_{i,t}^F, \; for \; i = 1, ..., N+1. \tag{24}$$

Therefore, the set of parameters needed to estimate in the multi-Country model is,

$$\Theta = \Big(\{\alpha_i, \, \beta_i; \, \Phi^{F_i}, \, \Sigma^{F_i}; \, \lambda_{i,0}^G \, \lambda_{i,1}^G, \;\; \lambda_{i,0}^F \, \lambda_{i,1}^F; \, \Sigma^{\eta^{X_i}}, \, \Sigma^{\eta^{y_i}}\}_{i=1,2,3,4}; \, \{\sigma^{\eta^{\Delta s_j}}\}_{j=1,2,3}; \, \Phi^G, \, \Sigma^G \Big), \tag{25}$$

B. Quasi-Maximum Likelihood Estimation and Unscented Kalman Filter

Given the state-space model representation (equations (20) to (24)) with Gaussian noises, we can implement model estimation using Bayesian filtering approaches. We have noted that the exchange rate dynamic equations are highly non-linear functions of states, which makes the standard Kalman filter inapplicable. Instead, we can use the nonlinear Kalman filters. The usually used nonlinear Kalman filter is the extended Kalman filter, which linearizes the nonlinear system around the current state estimate using a Taylor approximation. However, for the highly nonlinear system, the extended Kalman filter is computationally demanding and performs very poorly. An alternative is the unscented Kalman filter (UKF), recently developed in the field of engineering (Julier and Ulman (1997, 2004)). The idea behind this approach is that in order to estimate the state information after a nonlinear transformation, it is favorable to approximate the probability distribution directly instead of linearizing the nonlinear functions. The unscented

Kalman filter overcomes to a large extent pitfalls inherent to the extended Kalman filter and improves estimation accuracy and robustness without increasing computational cost.

To implement the unscented Kalman filter, we firstly concatenate the state variables $x_{t-1} = [G_{t-1}, F_{1,t-1}, ..., F_{4,t-1}, G_{t-2}, F_{1,t-2}, ..., F_{4,t-2},]'$, the observation noises η_{t-1} and the state noises $\varepsilon_{t-1} = [v^G_{t-1}, v^F_{1,t-1}, ..., v^F_{4,t-1}]'$ at time $t-1$

$$x^e_{t-1} = \begin{bmatrix} x'_{t-1} & \eta'_{t-1} & \varepsilon'_{t-1} \end{bmatrix}', \tag{26}$$

whose dimension is $L = L_x + L_\eta + L_\varepsilon$ and whose mean and covariance are

$$\hat{x}^e_{t-1} = \Big[E[x_{t-1}] \quad 0 \quad 0\Big]', \qquad P^e_{t-1} = \begin{bmatrix} P^x_{t-1} & 0 & 0 \\ 0 & \Sigma^2_\eta & 0 \\ 0 & 0 & I_{15} \end{bmatrix}.$$

We then form a set of $2L+1$ sigma points

$$\chi^e_{t-1} = \begin{bmatrix} \hat{x}^e_{t-1} & \hat{x}^e_{t-1} + \sqrt{(L+\lambda)P^e_{t-1}} & \hat{x}^e_{t-1} - \sqrt{(L+\lambda)P^e_{t-1}} \end{bmatrix} \tag{27}$$

and the corresponding weights

$$w^{(m)}_0 = \frac{\lambda}{L+\lambda}, \quad w^{(c)}_0 = \frac{\lambda}{L+\lambda} + (1-\alpha^2+\beta), \tag{28}$$

$$w^{(m)}_i = w^{(c)}_i = \frac{1}{2(L+\lambda)}, \ \ i = 1, 2, \ldots, 2L, \tag{29}$$

where superscripts (m) and (c) indicate that weights are for construction of the posterior mean and covariance, respectively, $\lambda = \alpha^2(L+\bar{\kappa}) - L$ is a scaling parameter, the constant α determines the spread of sigma points around $\bar{x}$ and is usually set to be a small positive value, $\bar{\kappa}$ is a second scaling parameter with value set to 0 or $3-L$, and β is a covariance correction parameter and is used to incorporate prior knowledge of the distribution of x.

With these sigma points, we implement the UKF as follows: for the time update

$$\begin{aligned}
\chi^x_{t|t-1} &= F(\chi^x_{t-1}, \chi^\varepsilon_{t-1}), \quad \hat{x}_t^- = \sum_{i=0}^{2L} w_i^{(m)} \chi^x_{i,t|t-1}, \\
P^-_{x_t} &= \sum_{i=0}^{2L} w_i^{(c)} (\chi^x_{i,t|t-1} - \hat{x}_t^-)(\chi^x_{i,t|t-1} - \hat{x}_t^-)',
\end{aligned}$$

and for the measurement update

$$\begin{aligned}
\mathcal{Y}_{t|t-1} &= H(\chi^x_{t|t-1}, \chi^\eta_{t|t-1}), \quad \hat{Y}_t^- = \sum_{i=0}^{2L} w_i^{(m)} \mathcal{Y}_{i,t|t-1}, \\
P^-_{Y_t} &= \sum_{i=0}^{2L} w_i^{(c)} (\mathcal{Y}_{i,t|t-1} - \hat{Y}_t^-)(\mathcal{Y}_{i,t|t-1} - \hat{Y}_t^-)', \\
P_{x_t Y_t} &= \sum_{i=0}^{2L} w_i^{(c)} (\chi^x_{i,t|t-1} - \hat{x}_t^-)(\mathcal{Y}_{i,t|t-1} - \hat{Y}_t^-)', \\
\hat{x}_t &= \hat{x}_t^- + P_{x_t Y_t}(P^-_{Y_t})^{-1}(Y_t - \hat{Y}_t^-), \\
P_{x_t} &= P^-_{x_t} - (P_{x_t Y_t}(P^-_{Y_t})^{-1}) P^-_{Y_t} (P_{x_t Y_t}(P^-_{Y_t})^{-1})',
\end{aligned}$$

where Y_t is the observation vector containing all the observed variables, $\hat{Y}_t^-$ its predicted values, $P^-_{Y_t}$ its conditional variance-covariance matrix, $\hat{x}_t$ the filtered state vector, and P_{x_t} its variance-covariance matrix.

Assuming that the predictive errors are normally distributed, we can construct the log likelihood function at time t as follows

$$\mathcal{L}_t(\Theta) = -\frac{1}{2}\ln|P^-_{Y_t}| - \frac{1}{2}(Y_t - \hat{Y}_t^-)'(P^-_{Y_t})^{-1}(Y_t - \hat{Y}_t^-), \tag{30}$$

where Θ is a vector of model parameters. Parameter estimates can be obtained by maximizing the joint log likelihood

$$\hat{\Theta} = \arg\max_{\Theta \in \Xi} \sum_{t=1}^{T} \mathcal{L}_t(\Theta), \tag{31}$$

where Ξ is a compact parameter space, and T is the length of total observations of the data. Because the log likelihood function is misspecified for the non-Gaussian model, a robust estimate of the variance-covariance matrix of parameter estimates can be obtained using the approach proposed by White (1982)

$$\hat{\Sigma}_{\Theta} = \frac{1}{T}\Big[AB^{-1}A\Big]^{-1}, \tag{32}$$

where

$$A = -\frac{1}{T}\sum_{t=1}^{T}\frac{\partial^2 \mathcal{L}_t(\hat{\Theta})}{\partial\Theta\partial\Theta'}, \qquad B = \frac{1}{T}\sum_{t=1}^{T}\frac{\partial \mathcal{L}_t(\hat{\Theta})}{\partial\Theta}\frac{\partial \mathcal{L}_t(\hat{\Theta})}{\partial\Theta'}. \tag{33}$$

With these parameter estimates $\hat{\Theta}$, the latent global and country-idiosyncratic factors, $\hat{G}_t$ and $\hat{F}_{i,t}$ $(i = 1, 2, 3, 4)$, can be extracted using the unscented Kalman filter.

The number of parameters in our model is large. Maximization of the likelihood (30) may involve a large number of likelihood evaluations. Therefore, we adopt a sophisticated quasi-Newton approach with the inverse Hessian of the likelihood function updated by the BFGS algorithm. The initial values are carefully selected by the following way. We first run the Nelder-Mead optimization algorithm for 100 feasible sets of starting values and stop them after 100 iterations. Then the best 10 parameter estimate sets (in terms of the likelihood) are selected among these 100 runnings as the initial values for the quasi-Newton algorithm. The parameter estimates are those resulting in the largest likelihood among these 10 runnings of the quasi-Newton method.

V. Empirical Results and Discussions

A. Model Performance on Exchange Rate Dynamics

Exchange rates dynamics are the main focuses of this paper. The model performance for exchange rate dynamics is reported in Table 4 with the summary of statistics from the observed data and the model-implied data. It shows that the model is able to capture the statistic moments of the observed exchange rate movements. However model-implied exchange rate changes are with less volatilities comparing to the observed ones.

— Table 4 around here —

The above results also can be seen from Figure 2, where both the model implied exchange rates dynamics and the observed ones are plotted. Generally speaking, the model implied exchange rates dynamics are able to mimic the dynamics of the observed ones very well along the sample period. From the first panel in Table 5, model-implied exchange rate dynamics is able to capture 57%, 66% and 33% of the variations of the observed exchange rate dynamics for the USD/DEM(EUR), the USD/GBP and the USD/JPY, repectively. Comparing with linear models on exchange rate daynamics by using macroeconomic fundamental information, this no-arbitrage multi-country model makes a big improvement.

— Figure 2 around here —

B. Macroeconomic Shocks and the Exchange Rate Dynamics

Previous studies find that exchange rate movements are largely disconnected to macroeconomic fundamentals. In monetary models and/or new open economy macroeconomic models, the exchange rate is a linear function of contemporaneous macroeconomic variables. Since the residuals are usually serially correlated in these models, the estimation is implemented using the first-order differences of relevant variables,

$$\Delta s_t = \beta_0 + \beta_1^{(h)}\Delta r_t^{(h)} + \beta_1^{(f)}\Delta r_t^{(f)} + \beta_2^{(h)}\Delta g_t^{(h)} + \beta_2^{(f)}\Delta g_t^{(f)} + \beta_3^{(h)}\Delta \pi_t + \beta_3^{(f)}\Delta \pi_t^{(f)} + u_t, \quad (34)$$

where u_t is a noise term. In these models, coefficients are typically constrained by $\beta_k^{(h)} = -\beta_k^{(f)}$, for $k = 1, 2, 3$. When estimating this linear model for the three types of exchange rate changes used in this paper, we find R^2 of 3.3%, 5.7%, and 4.5% for the unconstrained regressions and R^2 of 1.4%, 1.2%, and 1.5% for the constrained regressions. Even though global and country-idiosyncratic macroeconomic factors in our model is able to account for 57%, 66% and 33% of the variation of exchange rate movements for three types of exchange rate dynamics repectively, the linear model in equation (34) cannot capture this link between macroeconomic fundamentals and exchange rates.

What exact roles do macroeconomic fundamentals play in our model? Recall the exchange rate dynamic equation (13),

$$\Delta s_{j,t+1} \quad = \quad \Big(\tilde{r}_{N+1,t} - \tilde{r}_{j,t}\Big) + rp_{j,t+1} + \Delta s_{j,t+1}^{unexp.},$$

where we decompose the exchange rate dynamics into three components, the short-term interest differential, the foreign risk premium and the unexpected exchange rate changes.

— Figure 3 around here —

Figure 3 presents these three components of the exchange rate changes as well as their sum, the model-implied exchange rate changes. For each exchange rate changes, the first component, $(\tilde{r}_{N+1,t} - \tilde{r}_{j,t})$, which is the only concern in the UIP model, is very smooth for all the three currencies. The second one, foreign risk premium $rp_{j,t+1}$, becomes volatile in comparison to the first term, but it still has much smaller variation than model-implied exchange rate changes. This implies that the third component, $\Delta s_{j,t+1}^{unexp.}$, must be more volatile and should play more important role on explaining exchange rate movements. This is true from the figure that $\Delta s_{j,t+1}^{unexp.}$ is very volatile and mimics fluctuations of exchange rate changes for each exchange rates dynamics. The regression of the data on the unexpected exchange rate changes and a constant results in R^2 of 48%, 57% and 21%, taking 84%, 86% and 64% of the total explained variance, for the USD/DEM, the USD/GDP and the USD/JPY, repectively.

From equation (12), the unexpected exchange rate change $\Delta s_{j,t+1}^{unexp.}$ has two parts, the one driving by global innovations $(\lambda_{N+1,t}^G)^T \upsilon_{t+1}^G - (\lambda_{j,t}^G)^T \upsilon_{t+1}^G$ and the other driving by country-idiosyncratic innovations $(\lambda_{N+1,t}^F)^T \upsilon_{N+1,t+1}^F - (\lambda_{j,t}^F)^T \upsilon_{j,t+1}^F$, both of which are macro-dependent. When we regress the data on the model-implied macroeconomic innovations $\hat{\upsilon}_{t+1}^G, \hat{\upsilon}_{N+1,t+1}^F, \hat{\upsilon}_{j,t+1}^F$ with a constant, the R^2 is 30%, 36%, and 13%. In our model, the role of the macro innovations is further amplified by the time-varying market prices of risks, and hence the exchange rate dynamic is heteroskedastic. Macroeconomic innovations are always regarded as ‘news’ on macroeconomic fundamentals. Their importance has also been investigated by Engel, Mark and West (2007) and Andersen et al. (2003).

C. Model Performance on Macroeconomic and Yield Variables

This no-arbitrage macro-finance model is able to not only model exchange rate dynamics but also macroeconomic and financial variables as well. Table 6 and Table 7 show the observed and model-implied statistic summaries for macroeconomic and yield data. It is different with the results for exchange rate dynamics. The model-implied macroeconomic and yield variables capture very well the statistics of observed ones, such as the mean, the standard error, the skewness, the kurtosis as well as the autocorrelation.

— Table 6 and 7 around here —

Moreover, from Figure 6 and 7, they show both model implied macroeconomic and financial variables move tightly with the observed ones. The good model performance on these two types of variables also can be seen from the estimates of the standard deviation of measurement errors in Table 3. They are very small with values between 0.2 and 10.8 basis points.

— Figure 6 and 7 around here —

D. Foreign Exchange Risk Premium and Forward Premium Anomaly

One of the most notable puzzles in foreign exchange markets is the forward premium anomaly, which finds the tendency for high interest rate currencies to appreciate. Fama (1984) attributes this departure from uncovered interest parity (UIP) to a time-varying risk premium. Our model also suggests that the expected exchange rate change is equal to the sum of the interest rate differential and the time-varying foreign risk premium, which is constructed by the market prices of risk.

— Table 2 around here —

Table 2 provides the estimates of market prices of risks, where more than half of parameters are statistically significant. Most of estimates in $\lambda_{US,0}$ and $\lambda_{GM/UK/JP,0}$ and in $\lambda_{US,1}$ and $\lambda_{GM/UK/JP,1}$ not only have the same signs, but also have very close values with each other. This implies that the SDFs of the three foreign currency should be highly correlated with the US SDF. Indeed, the correlations of the model-implied SDFs are as high as 99%. Brandt et al. (2006) show that volatility of the exchange rate and volatility of the SDF's based on asset markets imply that SDF's must be highly correlated across countries. The constant parameters $\lambda_{i,0}^{G}$ and $\lambda_{i,0}^{F}$ are negative, which is consist with previous findings (Backus et al., 1998). The estimates of coefficients $\lambda_{i,1}^{G}$ are much larger than $\lambda_{i,1}^{F}$. The resulting two parts of foreign exchange risk premiums (driving by global factor and by country-idiosyncratic factor) can been seen in Figure 4. The sum of these two parts can also been seen in Figure 3.

— Figure 4 around here —

In Figure 4, it shows that for these three foreign risk premiums, the global factor driven parts are the dominant ones, since their magnitudes are 100 times to the country-idiosyncratic factor driven parts. The global factor driven parts have similar pattens among the three types of exchange rates. They have three positive peaks around three monetary/financial crises, i.e., European monetary mechanism crisis in 1992, Asian financial crisis in 1997 and the recently financial crisis started at 2008. Along the sample period, European monetary mechanism crisis generates the biggest effect on Germany mark and British pound against the US dollar, while Asian financial crisis creates the highest peak of risk premium for Japanese yen against the US dollar. On the contrary, the parts of foreign risk premiums forced by country-idiosyncratic factor have very idiosyncratic dynamic patterns among these three exchange rates.

Fama (1984) argues that the implied risk premium should be negatively correlated with and have larger variance than the interest rate differential. They are usually termed as Fama's conditions. For each exchange rates of the USD/DEM, the USD/GDP and the USD/JPY, our model implied risk premium (rp_t) does negatively correlate with the interest rate differential ($r^{(h)} - r^{(f)}$) with correlations -9%, -58%, and -22%, and have a larger variance (0.82 vs. 0.04, 0.62 vs. 0.02, and 0.84 vs. 0.03). These results are presented in Panel B, Table 5.

— Table 5 around here —

Moreover, the estimated foreign risk premiums are counter-cyclical to the US economy. From the last panel in Table 5, it shows foreign risk premiums are negatively correlated with output growth differentials between the US and foreign countries. This negative correlation implies that when the foreign output growth is higher than the domestic one, people in the market anticipate the foreign currency to appreciate while the domestic currency to depreciate. When one country is in a better economic situation than the other, the market becomes more confident to that country's currency and thus people would like to hold it, leading to its currency to appreciate. In addition, the foreign risk premium and the inflation rate differential ($\pi^{(h)} - \pi^{(f)}$) are negatively correlated. If the current inflation of the foreign country is high, people may expect the central bank to increase its interest rate in the future. This results in a decreased interest rate differential and an increased risk premium.

E. What Drive Exchange Rate Dynamics and Foreign Risk Premiums, Global or Country-Idiosyncratic Factors?

In order to know which factors are important in driving exchange rate dynamics and foreign risk premiums, we implement the variance decomposition for this nonlinear exchange rate dynamics model. According to Harris and Yu (2010), given that $\Delta s_{i,t}$, $rp_{i,t}$, and $\Delta s_{i,t}^{unexp.}$ are nonlinear of the state vectors G_t, $F_{i,t}$, ..., $F_{N+1,t}$, the variance decompositions can be computed by using Monte Carlo simulation conditional on filtered state factors in the sample period. First, simulate the model by drawing random shocks v_{t+h}^{G}, $v_{i,t+h}^{F}$, ..., $v_{N+1,t+h}^{F}$, (for $h = 1, 2, ..., 60$) from $N(0, I)$. Then the evolution of the state vectors can be computed by state dynamic equation (23) and (24), and the corresponding values of Δs_i, rp_i and $\Delta s_i^{unexp.}$ can be obtained by equation (9), (11) and (12). Last, compute variances of forecast errors numerically according to Harris and Yu (2010) with repeated 1000 times of the this process to get the nonlinear variance decompositions.

The result of the variance decomposition for each type of foreign exchange rate is reported in Table 10, 11 and 12, respectively.

— Table 10, 11 and 12 around here —

Among all these three tables, it implies that global factors are much more important comparing to country-idiosyncratic factors in driving dynamics of exchange rates, foreign risk premiums as well as the unexpected exchange rate changes.

For exchange rate changes Δs of all the three types of foreign exchange rate changes (Panel A in Table 10, 11 and 12), the global factors take around 60% to 70% of the short-run (1-month) forecast error variances. Their importance increase as the forecast horizon increasing, with more than 90% of forecast error variances in the long-run (60-month). Among the global factors, the output growth and the interest rate are the two most important ones. In the short-run, the global interest rate is absolutely dominant factor for the USD/GBP and the USD/JPY, and is almost equally important as the global output growth for the USD/DEM(EUR). However, in the long-run, the global output growth is the most important factor for all the three exchange rates.

Even through country-idiosyncratic factors are less important, they do play certain roles in the short-run, accounting around 30% to 40% of forecast error variances. For the USD/DEM(EUR), the interest rate factors from both the US and Germany are the most important factors, while the US one explains twice the forecast variance to German one. For the USD/GBP (USD/JPY), the US interest rate factor as well as the UK (Japanese) output growth factors are the most important ones.

Foreign risk premiums are almost exclusively explained by three global factors, from the results in Panel B of Table 10, 11 and 12. This consists with the information provided from Figure 4, which shows that the magnitude of the global factor driving part of foreign risk premium is about 100 times larger than the part driving by country-idiosyncratic factor. For each of the three types of foreign risk premiums, more than two thirds of the short-run forecast error variance is driven by the global interest rate factor. However, the role of the global interest rate factor decreases while the role of global output and inflation factors increases as forecast horizon increasing. Almost half of the forecast variance is explained by output factor and one fourth by inflation factor in the long-run. This finding is consistent with other studies as well. For instance, Bauer and Diez (2011) finds global output growth and inflation account for about 40% of variation in USD/EUR risk premium in 1-year horizon, while the value is around 50% in this paper.

The variance decompositions for unexpected exchange changes $\Delta s^{unexp.}$ have similar patterns as the ones for exchange rate changes. This consists with the above finding that the macroeconomic shocks play very important role in driving exchange rate dynamics. Even through foreign risk premiums are exclusively driven by global factors, the roles of the country-idiosyncratic factors on exchange changes cannot be ignored at all.

F. Global and Country-Idiosyncratic Macroeconomic Factors

The above section shows that global and country-idiosyncratic macroeconomic factors play very different roles in driving exchange rate dynamics and foreign risk premiums. Hence it is worthy to investigate on them.

— Table 1 around here —

Table 1 reports the estimates of parameters related to the global and country-idiosyncratic factors. In the upper panel, the factor loadings for underlying macroeconomic fundamentals on global factors are reported. Most coefficients of global loadings are significant from zero. The middle panel reports the global factor dynamics. From the coefficients in matrix Φ^G, we can see that each global macroeconomic factor is highly persistent, with the diagonal values close to one. The statistic t-ratios in these two panels implies that global macroeconomic factors do exit.

The bottom panel in Table 1 presents the dynamics of country-idiosyncratic factors. Diagonal values in matrix Φ^{F_i} $(i = 1, 2, 3, 4)$ are significant from zero. Hence the country-idiosyncratic factors are not ignorable in determining the underlying macroeconomic fundamentals X_i. In addition, these values are close to one, especially coefficients for output growth. This implies that country-idiosyncratic factors are very persistent.

— Figure 8 around here —

Figure 8 draws three global macroeconomic factors, i.e. the output growth, the inflation and the interest rate. There are three big slumps for global output growth factor around the years of 1992, 1997 as well as 2008, when there were monetary/financial crises, for instance, European monetary mechanism crisis, Asian financial crisis and the recently crisis from the US. The global inflation factor is positive along the sample period except the period around 2009. The global short-term interest rate factor has the highest peak around the year of 1991, which is the same time period for peak of short-term interest rates in Figure 1.

— Figure 9 around here —

Figure 9 plots country-idiosyncratic output growth, inflation, interest rate factors in the top, middle and bottum penal, respectively. In each penal, there are four country-idiosyncratic factors for Germany, the UK, Japan and the US, respectively. Comparing to the macroeconomic fundamentals in Figure 1, country-idiosyncratic factors show less cross-country comovement in Figure 9.

VI. Conclusion

This paper investigates the dynamics of multiple bilateral nominal exchange rates simultaneously under a multi-country framework. Macroeconomic fundamental information is introduced to model exchange rate dynamics by adopting a no-arbitrage macro-finance approach. Macroeconomic fundamentals are assumed to be determined by both global (common) factors and country-idiosyncratic factors.

The empirical study focuses an open economy including four countries, i.e. Germany, the UK, Japan and the US, where the US is taken as the home country. The empirical results show that this multi-county model can capture 57%, 66% and 33% variations of the observed changes of the USD/DEM(EUR), the USD/GBP and the USD/JPY, respectively. Model-implied foreign risk premiums satisfy Fama conditions (1984) and they are counter cyclical with respect to the US economy. The macroeconomic innovations, or 'news' are important in determining the exchange rate dynamics. Moreover, global and country-idiosyncratic macroeconomic factors do exist and play very different roles in driving exchange rate dynamics and foreign risk premiums. Global factors drive foreign risk premiums almost exclusively and account for more than half of the forecast error variance of exchange rate dynamics. The dominant factor in the short-run is global interest rate, while in the long-run is global output, in driving both exchange rate dynamics and foreign risk premiums. Even through country-idiosyncratic macroeconomic factors are less important, they do play some roles in the short-run exchange rate dynamics, especially the US and German interest rate, and the UK and Japanese output growth.

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Table 1: **Estimates of the Country, Global and Country-Idiosyncratic Factor Parameters**

Factor Loadings ($X_{i,t} = \alpha_i + \beta_i G_t + F_{i,t}$)						
	$\alpha_i (\times 10^3)$			β_i		
	α_i^g	α_i^π	α_i^r	β_i^g	β_i^π	β_i^r
GM	4.59	-0.14	-0.08	0.62	0.16	0.72
	(2.26)	(2.83)	(2.20)	(1.86)	(1.98)	(4.46)
UK	-0.10	0.18	0.14	0.31	0.26	0.85
	(1.81)	(2.68)	(2.74)	(2.61)	(3.74)	(4.01)
JP	0.15	-0.59	0.23	0.94	0.22	0.57
	(3.09)	(2.99)	(3.28)	(2.96)	(2.59)	(5.27)
US	0.46	0.26	0.01	0.18	0.25	0.76
	(1.94)	(2.28)	(3.02)	(3.67)	(2.70)	(4.03)
Global Factor Dynamics ($G_t = \Phi^G G_{t-1} + \Sigma^G \upsilon_t^G$)						
	Φ^G			$\Sigma^G (\times 10^3)$		
	g^G	π^G	r^G	g^G	π^G	r^G
g^G	0.98	-0.01	-0.02	1	0	0
	(48.45)	(3.64)	(2.82)	–	–	–
π^G	0.15	0.90	0.07	0	1	0
	(2.40)	(27.40)	(1.64)	–	–	–
r^G	-0.16	0.08	0.89	0	0	1
	(2.53)	(4.90)	(16.16)	–	–	–
Country-Idiosyncratic Factor Dynamics ($F_{i,t} = \Phi^{F_i} F_{i,t-1} + \Sigma^{F_i} \upsilon_{i,t}^F$)						
	Φ^{F_i} (*diagonal*)			Σ^{F_i} ($\times 10^3$, *diagonal*)		
	f_i^g	f_i^π	f_i^r	f_i^g	f_i^π	f_i^r
GM	0.98	0.99	0.94	1.07	0.26	0.25
	(16.70)	(73.16)	(160.39)	(3.74)	(7.47)	(6.43)
UK	0.99	0.99	0.87	0.73	0.33	0.42
	(54.54)	(66.79)	(62.35)	(5.37)	(9.31)	(6.39)
JP	0.98	0.96	0.99	1.34	0.28	0.15
	(52.73)	(32.75)	(145.85)	(1.85)	(5.99)	(5.14)
US	0.99	0.98	0.93	0.65	0.26	0.33
	(93.88)	(38.83)	(133.83)	(3.80)	(2.66)	(5.90)

Note: This Table reports the estimates of the country, global and country-idiosyncratic factor parameters. In parentheses, the absolute value of t-ratio of each estimate is reported. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 2: **Estimates of Market Prices of Risk Parameters**

	$\lambda^G_{i,0}$ $(\times 10^2)$			$\lambda^F_{i,0}$ $(\times 10^2)$		
	g^G	π^G	r^G	f^g_i	f^π_i	f^r_i
GM	-4.45	-7.81	-6.09	-0.39	-0.14	-0.82
	(2.76)	(4.45)	(3.13)	(1.92)	(1.22)	(1.29)
UK	-4.37	-9.51	-2.95	-1.10	-0.14	0.48
	(3.35)	(3.90)	(2.85)	(1.84)	(3.00)	(3.12)
JP	-3.84	-9.55	-2.04	-1.84	-0.41	-0.25
	(5.45)	(2.77)	(2.03)	(5.09)	(1.42)	(3.30)
US	-1.61	-7.75	-4.56	-0.60	-0.40	-1.33
	(2.28)	(2.42)	(3.83)	(4.20)	(2.27)	(4.14)
	$\lambda^G_{i,1}$ $(diagonal)$			$\lambda^F_{i,1}$ $(diagonal)$		
	g^G	π^G	r^G	f^g_i	f^π_i	f^r_i
GM	39.37	-25.65	-34.44	1.46	1.98	-6.62
	(7.30)	(5.96)	(8.68)	(1.74)	(1.83)	(2.69)
UK	39.89	-23.63	-37.71	3.63	-1.12	-6.90
	(7.90)	(5.17)	(9.49)	(2.38)	(1.36)	(1.70)
JP	38.53	-22.79	-39.11	2.69	-1.79	-4.44
	(7.27)	(3.98)	(7.68)	(2.68)	(1.32)	(3.41)
US	42.18	-24.10	-40.96	1.44	-2.86	-2.77
	(7.59)	(6.16)	(9.35)	(1.89)	(2.02)	(1.96)

Note: This Table reports the estimates of market prices of risk parameters. In parentheses, the absolute value of t-ratio of each estimate is reported. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 3: **Estimates of Standard Deviation of Measurement Errors Parameters ($\times 10^4$)**

	g_i	π_i	r_i	$y_i^{(24)}$	$y_i^{(60)}$	Δs_j
GM	10.81	0.34	3.75	0.26	1.77	212.27
	(4.65)	(2.87)	(7.84)	(3.49)	(3.10)	(2.48)
UK	6.56	0.67	6.62	0.09	0.88	202.52
	(3.70)	(2.59)	(4.60)	(1.91)	(1.95)	(3.64)
JP	9.70	1.31	1.56	0.79	0.99	271.85
	(2.12)	(3.73)	(2.82)	(3.60)	(4.52)	(7.58)
US	0.55	0.28	5.64	0.66	2.80	
	(2.89)	(3.57)	(6.73)	(2.18)	(2.94)	

Note: This Table reports the estimates of market prices of risk parameters. In parentheses, the absolute value of t-ratio of each estimate is reported. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 4: **Model fit: Exchange Rate Dynamics**

	Mean(%)	*Std. Dev.(%)*	*Skewness*	*Kurtosis*	*Autocorr.*
		1. USD/GEM(EUR)			
Data	0.28	3.22	-0.24	3.92	0.07
Model	0.02	2.59	-0.27	4.93	0.09
		2. USD/GBP			
Data	0.12	3.05	-0.28	5.95	0.11
Model	-0.09	2.54	-0.41	5.89	0.14
		3. USD/JPY			
Data	0.33	3.24	0.30	4.43	0.08
Model	0.17	2.49	-0.97	8.57	0.13

Note: This Table reports model fitting for exchange rate dynamics. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 5: **Model-implied Exchange Rate Dynamics and Foreign Risk Premia**

Panel A. Δs and $\hat{\Delta s}$			
	USD/DEM	*USD/GDP*	*USD/JPY*
Explained Variation (R^2,%)	57	66	33
Corr(Δs,$\hat{\Delta s}$)(%)	75	81	58
Panel B. Fama Conditions			
	USD/DEM	*USD/GDP*	*USD/JPY*
Corr(rp,$r^{(h)} - r^{(f)}$)(%)	-9	-58	-22
Var(rp) ($\times 10^4$)	0.82	0.62	0.84
Var($r^{(h)} - r^{(f)}$) ($\times 10^4$)	0.04	0.02	0.03
Panel C. Foreign Risk Premia and Macro Differentials			
	USD/DEM	*USD/GDP*	*USD/JPY*
Corr(rp,$g^{(h)} - g^{(f)}$)(%)	-11	-32	-4
Corr(rp,$\pi^{(h)} - \pi^{(f)}$)(%)	-13	-59	-27

Note: This Table reports model fitting for exchange rate dynamics. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 6: **Model Fit: Macro Data**

		Mean(%)	*Std. Dev.(%)*	*Skewness*	*Kurtosis*	*Autocorr.*
1. Germany						
output growth	Data	0.14	0.42	-2.07	10.51	0.86
	Model	0.15	0.40	-2.24	10.65	0.92
inflation	Data	0.16	0.11	0.85	4.34	0.96
	Model	0.16	0.11	0.85	4.34	0.96
interest rate	Data	0.38	0.18	0.84	2.93	0.98
	Model	0.38	0.18	1.02	3.21	0.99
2. UK						
output growth	Data	0.07	0.27	-1.02	6.85	0.88
	Model	0.07	0.26	-1.09	6.99	0.93
inflation	Data	0.31	0.17	1.24	4.79	0.97
	Model	0.31	0.17	1.24	4.79	0.97
interest rate	Data	0.61	0.28	0.76	2.73	0.98
	Model	0.59	0.25	0.57	2.65	0.97
3. Japan						
output growth	Data	0.07	0.54	-2.32	11.82	0.92
	Model	0.07	0.53	-2.37	12.01	0.93
inflation	Data	0.06	0.10	0.73	2.71	0.95
	Model	0.06	0.10	0.72	2.67	0.96
interest rate	Data	0.18	0.21	0.93	2.49	0.99
	Model	0.17	0.20	0.90	2.41	0.99
4. US						
output growth	Data	0.19	0.29	-1.41	6.76	0.94
	Model	0.19	0.29	-1.41	6.76	0.94
inflation	Data	0.25	0.10	0.01	3.63	0.93
	Model	0.25	0.10	0.01	3.63	0.93
interest rate	Data	0.37	0.17	-0.21	2.41	0.98
	Model	0.41	0.19	-0.15	2.47	0.97

Note: This Table reports model-implied and observed macro data statistic summary. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 7: **Model Fit: Yield Data**

Maturities		*Mean(%)*	*Std. Dev.(%)*	*Skewness*	*Kurtosis*	*Autocorr.*
1. Germany						
24-m	Data	0.40	0.16	0.79	2.88	0.98
	Model	0.40	0.16	0.78	2.88	0.98
60-m	Data	0.44	0.14	0.50	2.48	0.98
	Model	0.44	0.15	0.49	2.31	0.98
2. UK						
24-m	Data	0.57	0.22	0.44	2.32	0.98
	Model	0.57	0.22	0.44	2.32	0.98
60-m	Data	0.58	0.20	0.33	1.86	0.98
	Model	0.59	0.20	0.31	1.86	0.98
3. Japan						
24-m	Data	0.19	0.19	0.84	2.34	0.99
	Model	0.19	0.19	0.81	2.28	0.99
60-m	Data	0.23	0.18	0.61	1.98	0.99
	Model	0.23	0.18	0.66	2.08	0.99
4. US						
24-m	Data	0.44	0.18	-0.12	2.41	0.97
	Model	0.44	0.18	-0.11	2.42	0.97
60-m	Data	0.49	0.16	0.18	2.49	0.97
	Model	0.49	0.17	-0.03	2.07	0.98

Note: This Table reports model-implied and observed yield data statistic summary. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 8: **Data Correlations: Macro and Exchange Rate Data**

	01	*02*	*03*	*04*	*05*	*06*	*07*	*08*	*09*	*10*	*11*	*12*	*13*	*14*	*15*
01	1.00														
02	0.15	1.00													
03	0.53	0.32	1.00												
04	0.14	0.53	0.30	1.00											
05	-0.25	-0.19	-0.22	-0.16	1.00										
06	0.49	0.01	0.37	-0.17	0.20	1.00									
07	0.20	-0.07	0.11	-0.08	0.55	0.70	1.00								
08	0.34	0.04	0.49	-0.12	0.35	0.78	0.58	1.00							
09	-0.03	-0.15	-0.12	-0.24	0.76	0.56	0.73	0.56	1.00						
10	0.36	0.12	0.24	-0.18	0.19	0.88	0.72	0.68	0.65	1.00					
11	0.08	-0.01	0.09	-0.27	0.44	0.71	0.77	0.61	0.82	0.84	1.00				
12	0.48	0.35	0.36	0.14	-0.04	0.67	0.47	0.60	0.42	0.80	0.54	1.00			
13	0.03	0.03	-0.02	-0.05	-0.02	0.07	0.03	-0.04	0.03	0.10	0.10	-0.00	1.00		
14	0.02	0.01	0.03	0.02	-0.06	0.01	-0.01	-0.03	-0.06	0.02	0.01	-0.02	0.72	1.00	
15	-0.05	0.10	-0.01	0.00	-0.03	0.01	0.03	-0.02	0.03	0.05	0.12	-0.01	0.54	0.44	1.00

Note: This Table reports the correlations of original monthly macroeconomic variables and exchange rate changes. There are four time series of output growth rates (index of *1-4*), inflation rates (index of *5-8*), and short-term interest rates (index of *9-12*), and are for Germany, UK, Japan, and US, repectively. Exchange rate changes data (index of *13-15*) are respectively Germany Mark/Euro, British Proud, Japanese Yen, against US Dollar. The change of exchange rates is the one-month change of log nominal exchange rate, where monthly nominal exchange rates are the end of period values. The output growth rates, and inflation rates are the 12-month changes of IP and CPI, respectively. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 9: **Principal Components Analysis for Macroeconomic Fundamentals**

I. Output Growth				
	PC1	*PC2*	*PC3*	*PC4*
Variance prop.	76	12	8	4
Cumulative prop.	76	88	96	100
II. Inflation Rates				
	PC1	*PC2*	*PC3*	*PC4*
Variance prop.	71	18	7	3
Cumulative prop.	71	89	97	100
III. Short-Term Interest Rates				
	PC1	*PC2*	*PC3*	*PC4*
Variance prop.	84	10	4	2
Cumulative prop.	84	94	98	100

Note: This table reports the preliminary analysis of principle component analysis for macroeconomic fundamentals. For each group of output growth rates, inflation rates, and short-term interest rates, I report the variance proportions and cumulative variance proportions in percentage associated with the four principal components, which is positioned with a descending order according to associated eigenvalues. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 10: **Variance Decompositions: USD/DEM(EUR)**

	Global: G			*US*:F_4			*Germany*: F_1		
N	g^G	π^G	r^G	f_4^g	f_4^π	f_4^r	f_1^g	f_1^π	f_1^r
Panel A. Exchange rate changes, Δs									
1	31.47	4.41	27.81	2.29	4.35	15.80	3.80	0.39	9.67
3	31.82	4.36	29.57	2.28	4.15	14.95	3.77	0.41	8.68
12	39.33	4.35	28.64	2.32	3.20	11.58	3.74	0.41	6.42
24	48.44	5.46	29.87	1.83	1.55	6.60	2.70	0.30	3.25
60	51.69	10.33	30.83	0.98	0.62	2.80	1.26	0.14	1.36
Panel B. Foreign risk premium, rp									
1	22.59	4.41	72.98	0.00	0.00	0.00	0.00	0.00	0.01
3	25.69	3.26	71.04	0.00	0.00	0.00	0.00	0.00	0.01
12	41.89	11.47	46.63	0.00	0.00	0.00	0.00	0.00	0.00
24	49.43	18.07	32.50	0.00	0.00	0.00	0.00	0.00	0.00
60	49.59	21.28	29.13	0.00	0.00	0.00	0.00	0.00	0.00
Panel C. Unexpected changes, $\Delta s^{unexp.}$									
1	31.46	4.42	27.64	2.30	4.37	15.88	3.82	0.39	9.71
3	31.83	4.38	29.22	2.30	4.19	15.10	3.80	0.42	8.76
12	38.85	4.09	28.20	2.42	3.35	12.05	3.91	0.44	6.69
24	48.20	2.26	29.76	2.24	1.90	8.01	3.28	0.37	3.98
60	52.74	1.59	33.54	1.66	1.05	4.76	2.13	0.24	2.31

Note: This Table reports variance decompositions of forecast variance for model-implied exchange rate changes of USD/DEM(EUR), and its risk premium and unexpected changes. The forecast horizons (N) are in months. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 11: **Variance Decompositions: USD/GBP**

	Global: G			*US*:F_4			*UK*: F_2		
N	g^G	π^G	r^G	f_4^g	f_4^π	f_4^r	f_2^g	f_2^π	f_2^r
				Panel A. Exchange rate changes, Δs					
1	20.22	11.22	47.11	1.29	2.55	8.88	5.69	0.50	2.53
3	20.69	11.28	46.70	1.35	2.58	8.81	5.85	0.50	2.24
12	24.61	11.12	42.36	1.54	2.28	8.09	7.46	0.50	2.04
24	36.37	11.13	32.80	1.65	1.45	5.92	8.37	0.35	1.96
60	45.64	13.70	27.81	1.22	0.76	3.40	6.17	0.19	1.12
				Panel B. Foreign risk premium, rp					
1	20.24	1.95	77.78	0.00	0.00	0.00	0.02	0.00	0.01
3	23.09	1.58	75.30	0.00	0.00	0.00	0.02	0.00	0.01
12	37.81	14.90	47.25	0.00	0.00	0.00	0.03	0.00	0.00
24	47.68	20.43	31.87	0.00	0.00	0.00	0.01	0.00	0.00
60	48.90	22.74	28.35	0.00	0.00	0.00	0.00	0.00	0.00
				Panel C. Unexpected changes, $\Delta s^{unexp.}$					
1	20.23	11.23	47.06	1.29	2.56	8.89	5.71	0.50	2.52
3	20.67	11.32	46.61	1.36	2.59	8.83	5.88	0.51	2.24
12	24.03	11.10	42.49	1.58	2.33	8.26	7.64	0.51	2.06
24	34.37	9.47	33.81	1.88	1.67	6.73	9.47	0.40	2.21
60	44.41	7.57	29.00	1.80	1.14	5.08	9.08	0.28	1.64

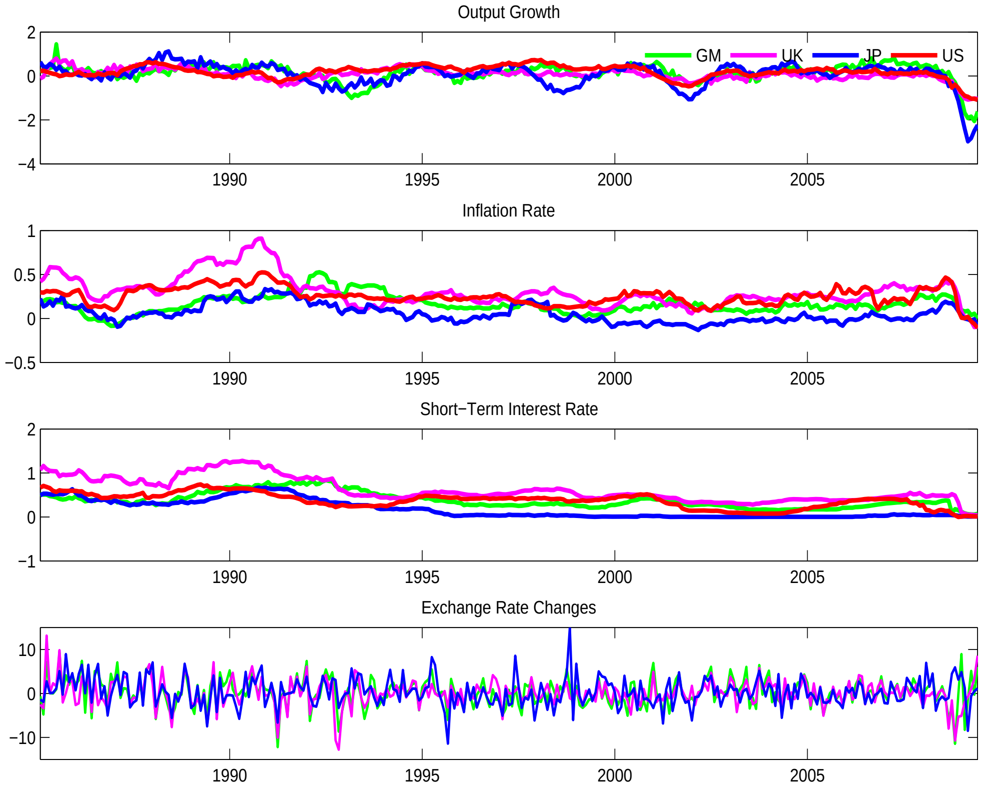
Note: This Table reports variance decompositions of forecast variance for model-implied exchange rate changes of USD/GBP, and its risk premium and unexpected changes. The forecast horizons (N) are in months. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 12: **Variance Decompositions: USD/JPY**

	Global: G			*US*:F_4			*Japan*: F_3		
N	g^G	π^G	r^G	f_4^g	f_4^π	f_4^r	f_3^g	f_3^π	f_3^r
	Panel A. Exchange rate changes, Δs								
1	18.10	6.77	51.50	1.09	2.30	8.25	9.76	0.89	1.34
3	19.51	6.57	50.31	1.13	2.28	7.95	10.12	0.88	1.25
12	27.51	6.79	42.02	1.29	1.87	6.83	11.87	0.77	1.05
24	44.26	9.08	28.66	1.20	1.02	4.29	10.44	0.46	0.59
60	54.18	13.42	22.23	0.75	0.48	2.15	6.29	0.23	0.28
	Panel B. Foreign risk premium, rp								
1	30.01	0.60	69.33	0.00	0.00	0.00	0.06	0.00	0.00
3	32.36	1.46	66.12	0.00	0.00	0.00	0.06	0.00	0.00
12	45.55	15.55	38.82	0.00	0.00	0.00	0.07	0.00	0.00
24	51.28	21.23	27.47	0.00	0.00	0.00	0.02	0.00	0.00
60	50.57	24.00	25.42	0.00	0.00	0.00	0.00	0.00	0.00
	Panel C. Unexpected changes, $\Delta s^{unexp.}$								
1	18.07	6.79	51.44	1.10	2.31	8.28	9.78	0.89	1.35
3	19.40	6.60	50.25	1.14	2.30	7.98	10.18	0.89	1.26
12	26.99	6.58	42.06	1.33	1.93	7.02	12.23	0.80	1.07
24	44.27	5.96	28.42	1.42	1.23	5.09	12.36	0.55	0.69
60	60.30	4.57	18.84	1.19	0.77	3.46	10.05	0.38	0.44

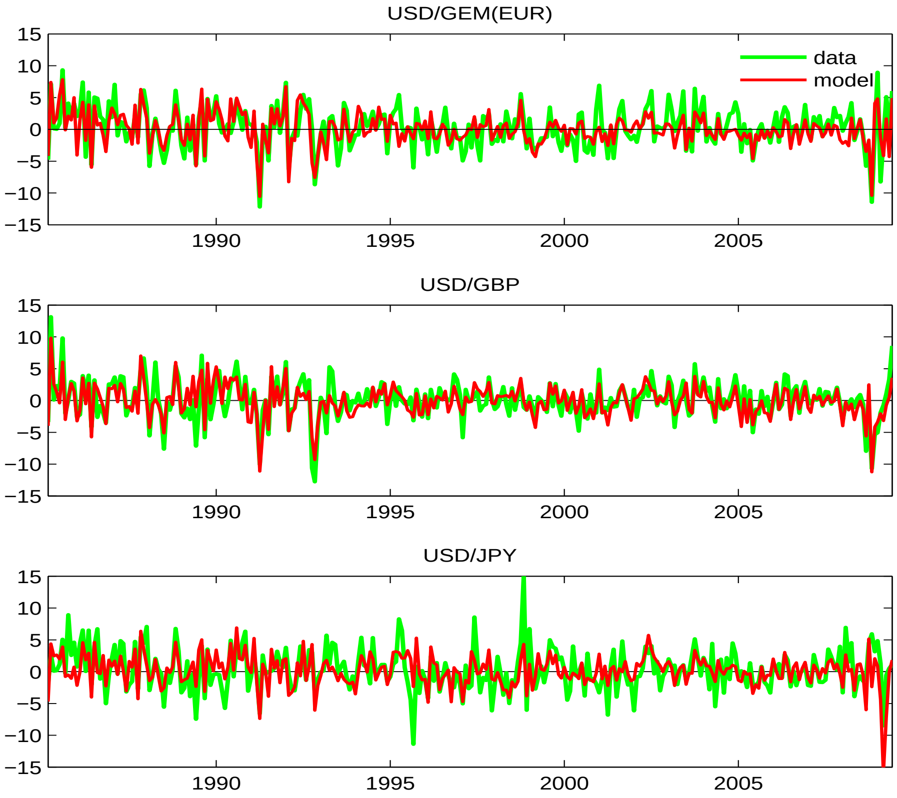
Note: This Table reports variance decompositions of forecast variance for model-implied exchange rate changes of USD/JPY, and its risk premium and unexpected changes. The forecast horizons (N) are in months. The sample period is from 1985m01 to 2009m05 (293 observations).

Figure 1: Macroeconomic Data and Exchange Rates Changes



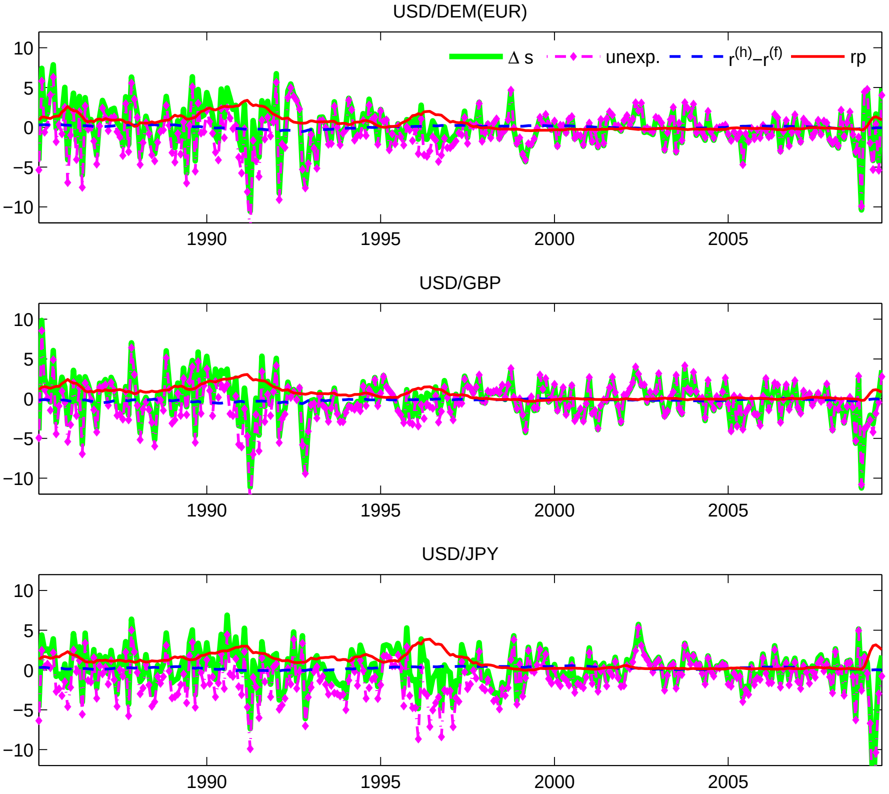
Note: This figure plots the macroeconomic fundamentals and monthly changes of exchange rates used in the estimation, for Germany, UK, Japan, and US. The upper, middle and bottom panels are for output growth rates, inflation rates, short-term interest rates and exchange rate changes, respectively. I plot annualized quantities for macroeconomic differentials, and monthly changes of exchange rates. The sample period is from 1985m01 to 2009m05 (293 observations).

Figure 2: Exchange Rate Dynamics



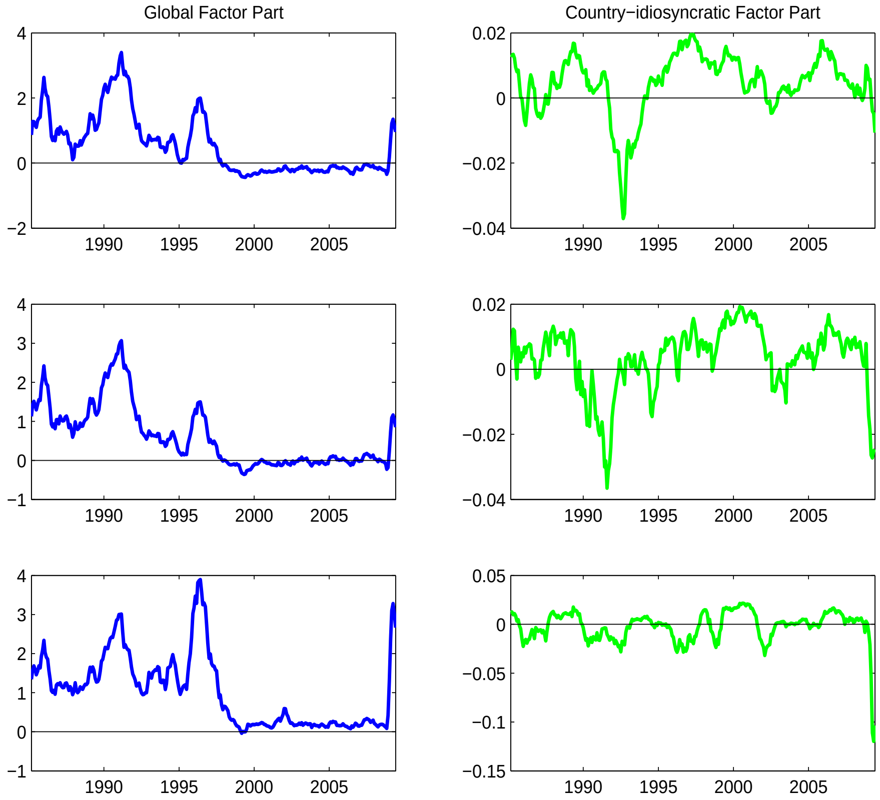
Note: This figure plots the monthly exchange rate changes in percentage of the foreign currencies, such as, Germany, the UK, and Japan, against USD. The thick lines are observed data, while the thin lines are model implied ones. The model cam explain 57%, 66% and 33% of the observed exchange rates variances for USD/GEM(EUR), USD/GBP and USD/JPY, respectively. The sample period is from 1985m01 to 2009m05 (293 observations).

Figure 3: Model-implied Exchange Rate Dynamics and Their Components



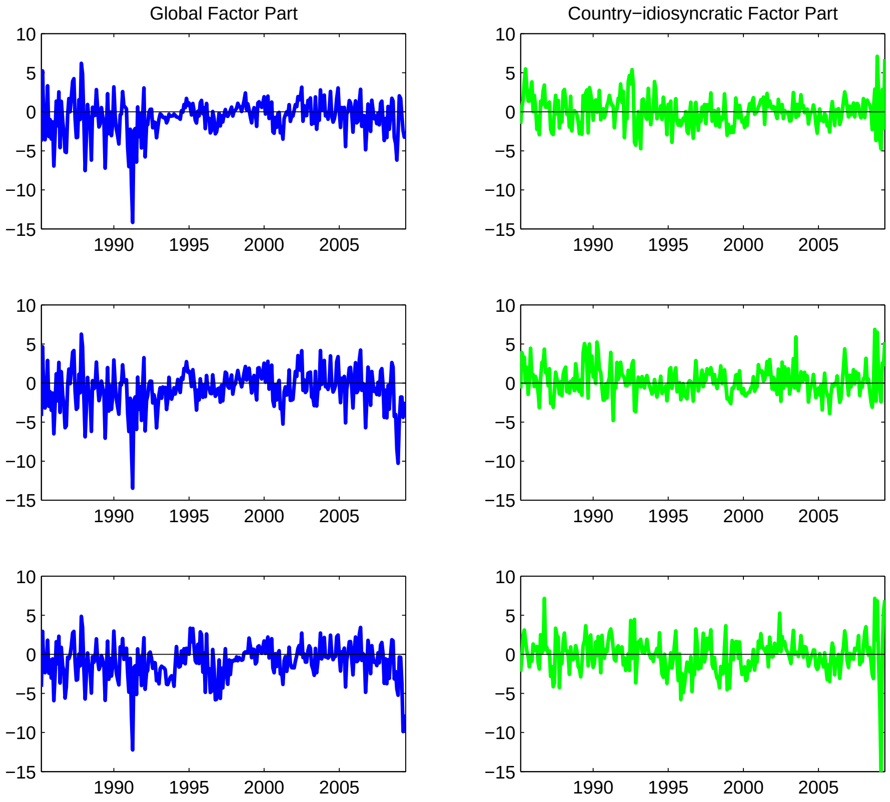
Note: This figure plots the model-implied monthly exchange rate changes, unexpected exchange rate changes, short-term interest rate differentials as well as foreign risk premia, in percentage for the foreign currencies, such as, Germany, the UK, and Japan, against USD. The thick light lines are model-implied exchange rate changes, the thin dark lines are model-implied foreign risk premia, while the dash-dot lines are short-term interest rate differentials. The sample period is from 1985m01 to 2009m05 (293 observations).

Figure 4: Model-implied Foreign Risk Premiums by Global and Country-idiosyncratic Factors



Note: This figure plots the model-implied foreign risk premia by global factors (on the right panels) and country-idiosyncratic factors (on the right panels) in percentage. Pictures from the top to the bottom panels are for USD/DEM(EUR), USD/GBP and USD/JPY. The sample period is from 1985m01 to 2009m05 (293 observations).

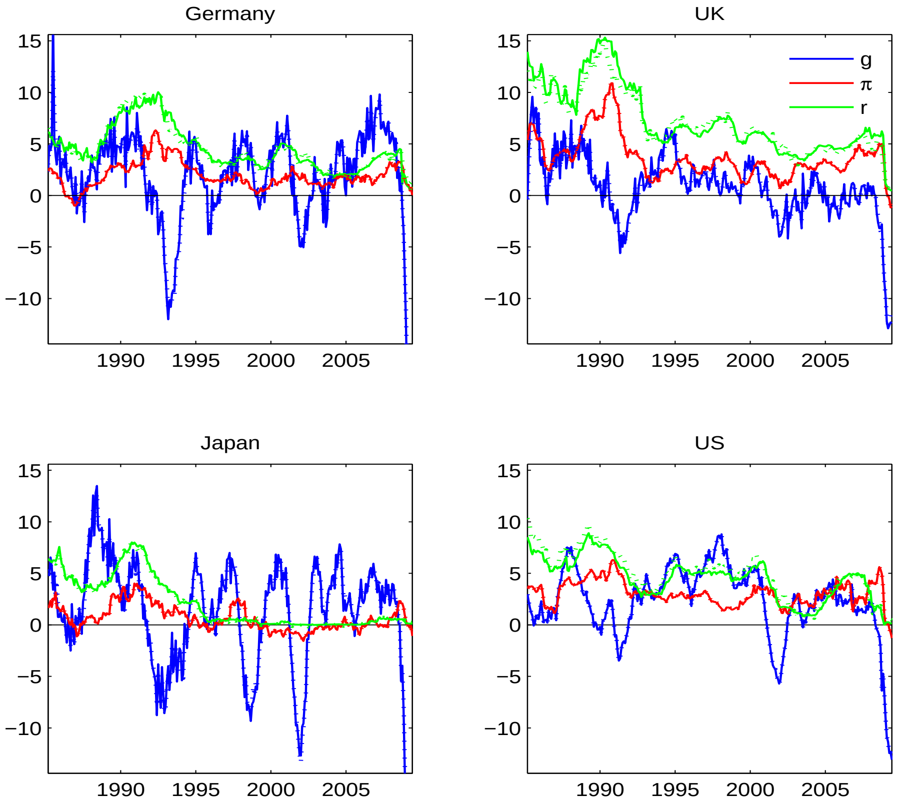
Figure 5: Model-implied Unexpected Exchange Rate Dynamics by Global and Country-idiosyncratic Factors


Global Factor Part

Country−idiosyncratic Factor Part

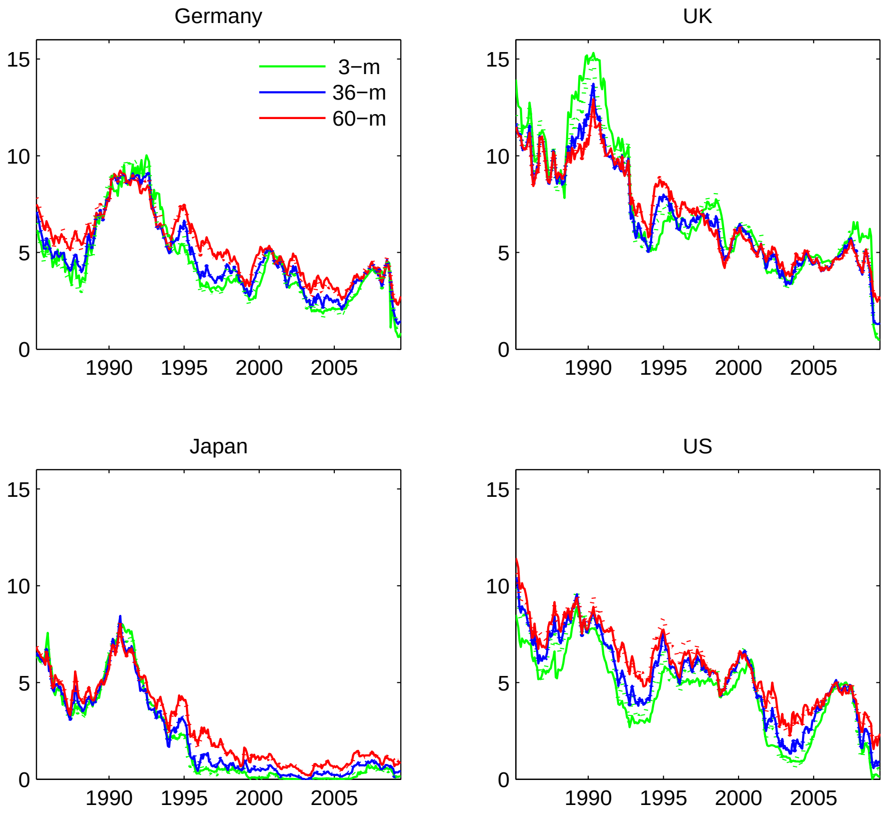
Note: This figure plots the model-implied unexpected exchange rate Dynamics by global factors (on the right panels) and country-idiosyncratic factors (on the right panels) in percentage. Pictures from the top to the bottom panels are for USD/DEM(EUR), USD/GBP and USD/JPY. The sample period is from 1985m01 to 2009m05 (293 observations).

Figure 6: Macroeconomic Fundamentals



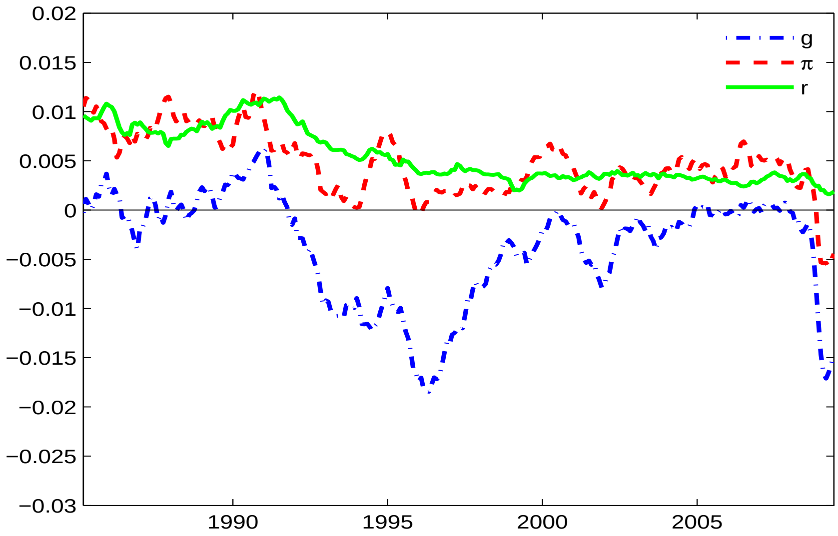
*Note:*This figure plots the monthly percent macroeconomic fundamentals, i.e., output growth, inflation rate, short-term interest rate, for Germany, the UK, Japan and the US. The solid lines are for observed data, while the dotted lines are for model-implied data. The sample period is from 1985m01 to 2009m05 (293 observations).

Figure 7: Yield Data



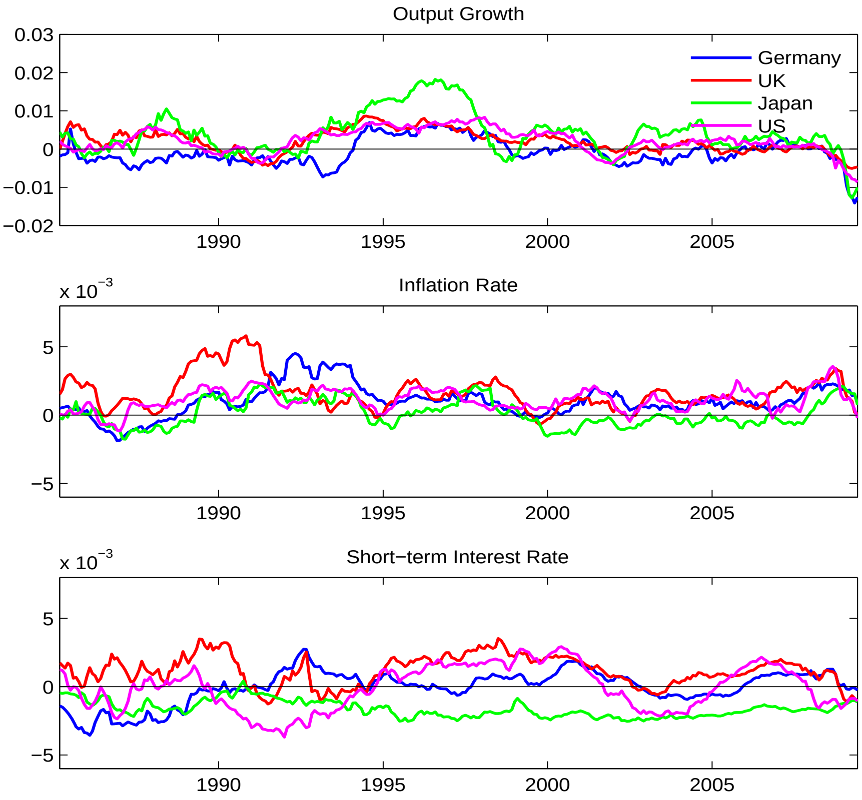
*Note:*This figure plots the monthly percent yield data, with maturities of 3-month, 24-month and 60-month, for Germany, UK, Japan and US. The solid lines are for observed data, while the dotted lines are for model-implied data. The sample period is from 1985m01 to 2009m05 (293 observations).

Figure 8: Global Macroeconomic Factors



Note: This figure plots the global macroeconomic factors filtered from the no-arbitrage multi-country model. The sample period is from 1985m01 to 2009m05 (293 observations).

Figure 9: Country-Idiosyncratic Macroeconomic Factors



*Note:*I plot the country-idiosyncratic macroeconomic factors filtered from the no-arbitrage multi-country model, for Germany, UK, Japan and US, respectively. The sample period is from 1985m01 to 2009m05 (293 observations).