THE PARADOX OF PLEDGEABILITY*

Jason Roderick Donaldson† Denis Gromb‡ Giorgia Piacentino§

October 19, 2016

Abstract

In this paper, we develop a model in which collateral serves to protect creditors from the claims of competing creditors. We find that borrowers rely most on collateral when cash flow pledgeability is high, because this is when it is easy to take on new debt, diluting existing creditors. Creditors thus require collateral for protection against being diluted. This causes a collateral rat race that results in all borrowing being collateralized. But collateralized borrowing has a cost: it encumbers assets, constraining future borrowing and investment, i.e. there is a collateral overhang. Our results suggest that increasing the supply of collateral can have adverse effects.

*For valuable comments we thank Andrea Attar, Bo Becker, Nittai Bergman, Bruno Biais, Jesse Davis, Paolo Fulghieri, Piero Gottardi, Mina Lee, Yaron Leitner, Andres Liberman, Nadya Malenko, Christine Palour, Cecilia Parlatore, George Pennacchi, Paul Pfleiderer, Uday Rajan, Adriano Rampini, Valdimir Vladimirov, Jeffrey Zwiebel and audiences at the 2016 FTG Meeting at Imperial, the 2016 IDC Summer Finance Conference, UNC, Stanford GSB (FRILLS), and Washington University in St. Louis.

†Washington University in St. Louis; j.r.donaldson@wustl.edu.

‡HEC Paris, CEPR, and ECGI; gromb@hec.fr.

§Washington University in St. Louis and CEPR; piacentino@wustl.edu.

1 Introduction

Collateral matters.[1] By pledging collateral, a borrower mitigates enforcement frictions and loosens his financial constraints. In other words, "collateral pledging makes up for a lack of pledgeable cash" (Tirole (2006), p. 169). This suggests that collateral should matter most when cash flow pledgeability is low. However, some of the world's most developed debt markets rely heavily on collateral. Notably, upwards of five trillion dollars of securities are pledged as collateral in US interbank markets,[2] where strong creditor rights, effective legal enforcement, intense regulatory supervision, and developed record-keeping technologies ensure that cash flow pledgeability is high. Why does collateral matter in these markets?

Whereas the finance literature has focused on how collateral can mitigate enforcement problems between a borrower and his creditor, in this paper we focus on how collateral can mitigate enforcement problems among creditors. We find that borrowers rely most on collateral when cash flow pledgeability is high, because this is when it is easy to take on new debt, diluting existing creditors. Creditors thus require collateral for protection against being diluted. This causes a collateral rat race that results in all borrowing being collateralized. But collateralized borrowing has a cost: it encumbers assets, constraining future borrowing and investment, i.e. there is a collateral overhang.

Model preview. In the model, a borrower, called B, has two projects, called Project 0 and Project 1, to finance sequentially. B finances Project 0 by borrowing from one creditor, called C_0, and, after Project 0 is underway, B finances Project 1 by borrowing from another creditor, called C_1. Both projects are riskless, but the payoff of Project 1 is revealed only after Project 0 is underway. Project 0 has positive NPV, but Project 1 may have either positive or negative NPV. Thus, it is efficient for B always to undertake Project 0 and to undertake Project 1 only in the event that it has positive NPV.

The amount that B can borrow is constrained by two frictions. First, cash flow *pledgeability* is limited. Specifically, the total repayment that B makes to his creditors cannot exceed a fixed fraction θ of the projects' terminal cash flows. Second, contracts are *non-exclusive* in the sense that when B borrows from one creditor, he cannot commit not to borrow from another creditor.[3] However, collateral mitigates this friction. By borrowing collateralized, B "fences off" a project from the claims of competing creditors. This ring-

[1]See, e.g., Benmelech and Bergman (2009, 2011), Rampini and Viswanathan (2013), and Rampini, Sufi, and Viswanathan (2014) for empirical evidence on the importance of collateral for borrowing.

[2]See Homquist and Gallin (2014).

[3]Note that this assumption rules out covenants by which a borrower commits contractually to one creditor not to borrow from new creditors in the future. As we discuss in detail in Subsection 6.2, such covenants sometimes do mitigate the non-exclusive-contracting friction in reality. However, their effectiveness is limited in circumstances in which the borrower can use collateral to borrow secured from new creditors. As Bolton and Oehmke (2015) put it:

fencing involves a proportional cost $1 - \mu$, where we refer μ to as the collateralizability of a project.[4] I.e. collateralization is "the protection...against the claims of competing creditors" (Kronman and Jackson (1979)), as is emphasized in the law literature, rather than the compensation for a lack of pledgeable cash, as emphasized in the finance literature. To be clear, collateralization does not affect pledgeability θ in our baseline model (we relax this in Subsection 6.6).

To finance a project, B can borrow via either secured (or "collateralized") debt or unsecured debt.[5] If B borrows via secured debt, the secured creditor has an exclusive claim over the project's pledgeable cash flow. If B borrows via unsecured debt instead, the creditor still has a claim on B's pledgeable cash flow. This claim is senior to any new unsecured debt B takes on. However, it is effectively junior to any new secured debt that B takes on.[6] This is because a collateralized project is protected from the claims of existing creditors.

Results preview. We now explain our two main results, that (i) if pledgeability θ is sufficiently high, then B can borrow from C_0 only via secured debt and, as a result, that (ii) if B borrows via secured debt and collateralization is costly ($\mu < 1$), then B may not undertake positive NPV projects due to a "collateral overhang" problem.

To see why B can borrow from C_0 only via secured debt for high pledgeability, suppose that B finances Project 0 by borrowing from C_0 via *unsecured* debt. Because unsecured contracts are non-exclusive, B can borrow from another creditor, C_1, to finance Project 1. If B borrows from C_1 via secured debt, then C_1 is prioritized over C_0—the new secured debt dilutes the existing unsecured debt.[7] As a result, C_0 will not lend to B via unsecured debt in the first place. However, this dilution occurs only if B is not too constrained to borrow from C_1, i.e. if the repayment that he can credibly promise to C_1 exceeds the cost of Project

> an important question is whether the firm can commit ex ante not to collateralize...ex post, for example via covenants that restrict such collateralization.... Under current U.S. bankruptcy law this is difficult: If a breach of such a covenant is discovered in bankruptcy, the collateral has already left the firm and...cannot be recovered by lenders (p. 2368).

[4]Following Kiyotaki and Moore (2001), we assume that this cost of collateralization reflects the fact that ring-fences are costly to "build" (cf. Subsection 2.2). However, the cost has other interpretations as well; for example, it could represent the cost of monitoring a borrower to ensure he maintains possession of the collateral or the cost of storing physical collateral in a warehouse or financial collateral with a tri-party custodian. It is also effectively equivalent to exogenous "over-collateralization," by which the value of posted collateral exceeds the promised repayment, i.e. to an exogenous haircut.

[5]In the baseline model we restrict attention to debt contracts for simplicity. In Subsection 6.3, we allow for more general borrowing instruments and show that our main results are robust.

[6]In Subsection 6.1, we show that these specific assumptions about seniority are not strictly necessary for our results. What is necessary is that secured debt is protected against some form of dilution.

[7]This prioritization of secured creditors is consistent with the legal treatment of secured debt as described by Listokin (2008): "Late-arriving secured creditors can leapfrog earlier unsecured creditors, redistributing value to the benefit of the issuer and the secured creditor but to the detriment of unsecured creditors" (p. 1039).

1. Since secured debt is effectively senior, B can promise all of his *pledgeable* cash flow to C_1. Hence, C_1 lends whenever the pledgeable fraction θ of B's cash flow exceeds the cost of investment—when θ is high. In summary, B dilutes C_0's unsecured debt only if pledgeability θ is sufficiently high and, as a result, C_0 lends to B only via secured debt. Paradoxically, high cash flow pledgeability undermines unsecured credit.

If B borrows from C_0 via secured debt, he must pay the cost of collateralizing Project 0. This cost constitutes a haircut on the value of Project 0 as collateral.This haircut "uses up" pledgeable cash flow, constraining B's debt capacity. This makes it difficult for B to borrow to finance Project 1, even if it has positive NPV. Indeed, collateralization effectively encumbers B's assets, in the sense that it limits B's ability to use them to raise liquidity and invest in Project 1. This is a collateral overhang problem: if B borrows collateralized, it prevents him from undertaking efficient investments later on. Our model thus reflects practitioners' intuition that "asset encumbrance not only poses risks to unsecured creditors...but also has wider...implications since encumbered assets are generally not available to obtain...liquidity" (Deloitte Blogs (2014)).

Due to the collateral overhang problem, secured borrowing from C_0 can lead to inefficient investment: if B borrows from C_0 via secured debt, he "uses up" pledgeable cash flow. This prevents him from borrowing to invest in Project 1, even if it has positive NPV. There is underinvestment. But, unsecured borrowing from C_0 can also lead to inefficient investment: if B borrows from C_0 via unsecured debt, he can "reuse" pledgeable cash flow from Project 0 to borrow from C_1. This subsidizes B's investment in Project 1, giving him the incentive to invest in it, even if it has negative NPV. There is over-investment. In this case, unsecured debt and secured debt may coexist, with B borrowing from C_0 via unsecured debt at a high interest rate and then borrowing from C_1 via secured debt at a low interest rate, diluting C_0 to make an inefficient investment in Project 1. However, this inefficiency may be so severe that it makes unsecured borrowing from C_0 infeasible for high θ, as discussed above. In contrast, these inefficiencies are not present when pledgeability is sufficiently low. In this case, B may finance Project 0 by borrowing from C_0 via unsecured debt and may finance Project 1 by borrowing from C_1 via junior unsecured debt only when Project 1 has positive NPV.[8] I.e., increasing pledgeability may decrease efficiency.

Role of collateral. In reality, borrowers use collateral for at least two reasons, (i) collat-

[8] In our model, decreasing pledgeability increases efficiency because it mitigates the non-exclusive contracting friction. In general, however, decreasing pledgeability has the direct effect of decreasing efficiency by inhibiting borrowing. When we set up the model, we restrict parameters in such a way that this countervailing force is effectively "switched off." This is because we wish to focus on the interaction between pledgeability and non-exclusive contracting (which, to the best of our knowledge, has not been studied before), rather than on the direct effect of pledgeability on borrowing and efficiency (which has been well-studied; see, e.g., Holmström and Tirole (1997, 1998) or Kiyotaki (1998)).

eral mitigates enforcement problems between a borrower and his creditor and (ii) collateral mitigates enforcement problems among creditors. These two roles of collateral correspond to the two components of property rights, (i) the "right of access" and (ii) the "right of exclusion" (see Segal and Whinston (2012)). Whereas the corporate finance literature is largely focused on the first role of collateral—as reflected by the quote from Tirole's textbook above—in this paper we are focused on the second.[9] This second role of collateral is also emphasized by practitioners and lawyers—as reflected by the *definition* of a secured transaction in Kronman and Jackson (1979): "a secured transaction is the protection...against the claims of competing creditors" (p. 1143). Thus, "borrowers that cannot make credible promises to comply with financial covenants may protect lenders against dilution by issuing secured debt" (Schwartz (1997), p. 1397), as we discuss further in Subsection 6.2. This view of collateral is also in line with Parlour and Rajan's (2001) view that "collateral can be interpreted as a commitment on the part of a consumer to accept only one contract" (p. 1322). Empirical support for our assumption that collateral mitigates the friction of non-exclusive contracting is in Degryse, Ioannidou, and von Schedvin (2016).

We examine these two roles of collateral jointly (in Subsection 6.6) and we find that the first role of collateral dominates when pledgeability is low. This is consistent with the intuition that collateral is necessary to create pledgeability in environments with weak contractual enforceability. However, we find that the second role of collateral dominates when pledgeability is high. This is consistent with the pervasive use of collateral in interbank markets. This may not be explained by the classical theory—i.e. that pledging collateral makes up for a lack of pledgeable cash—for two reasons. (i) In interbank markets, pledging collateral may not be necessary to make up for a lack of pledgeable cash. In fact, in the securities lending market, cash itself *is* the collateral—borrowers pledge cash to borrow securities. Further, even in the repo market, the securities used as collateral are typically so liquid that they are referred to as "cash equivalents." (ii) Relatedly, in the repo market, borrowers often buy securities "on margin"—i.e. a borrower uses a small amount of initial capital as a down payment to buy assets on credit, using the assets themselves as collateral. In this case, the borrowed assets coincide with the collateralized assets. This is the case in our model, but typically not in models in which collateral makes up for a lack of pledgeable cash. In these models, a borrower typically posts a "tangible" or "illiquid" asset as collateral to borrow cash.

Policy. Our model casts light on the ongoing policy debate about the supply of collateral in financial markets. Recently, central banks have been "manufacturing quality collateral"

[9] In Subsection 6.6, we include the first role of collateral in a simplified version of our model. We show how the two roles of collateral interact with pledgeability differently.

because "there's still not enough of the quality stuff to go around...as quality collateral becomes impossible to find.... The crunch has further been heightened by the general trend towards collateralised lending and funding" (Kaminska (2011)). Our analysis suggests that expanding the supply of collateral may backfire by making creditors less willing to lend unsecured, thus tightening credit constraints. The reason is that when collateral supply is high, it is easy to borrow via secured debt. This makes it easy for a borrower to dilute unsecured creditors by taking on new secured debt. This induces a *collateral rat race* in which creditors require collateral for protection against future collateralization. In fact, in our model reducing the supply of collateral can restore efficiency.

Applications. In our baseline model, a borrower can use collateral to take on new senior debt, leapfrogging existing creditors.[10] This is the case in the repo market, since a repo is formally a sale and repurchase of securities: a borrower sells securities to a creditor and other creditors have no recourse to the securities if the borrower defaults—indeed the securities are exempt from the automatic stay in bankruptcy. In repo markets, the collateralization cost $1 - \mu$ corresponds to the repo haircut (as formalized in Subsection 6.5).

Leasing provides another way for new secured creditors to leapfrog existing creditors. A lease is effectively a super-senior secured loan: like repo collateral, leased assets are not stayed in bankruptcy, so a lessor can repossess leased assets even before other secured creditors in the event of a borrower's default. A borrower can dilute his existing creditors by taking on new debt in the form of a lease. For leases, the collateralization cost may correspond to the inefficiencies arising from the separation of ownership and control, as in Eisfeldt and Rampini (2009).

Related literature. Our paper makes three main contributions relative to the literature. First, we provide an explanation for the pervasive use of collateral in high pledgeability environments, such as US interbank markets, which is arguably a challenge for received theories. Second, we provide a formal analysis of the role of collateral in mitigating conflicts of interest among creditors, which has not yet been explored in the corporate finance literature. Third, we show that the ability to provide exclusivity selectively can be a friction. This gives a new perspective on the problem of sequential borrowing with non-exclusive contracts explored in Admati, DeMarzo, Hellwig, and Pfleiderer (2013), Bizer and DeMarzo (1992), Brunnermeier and Oehmke (2013), and Kahn and Mookherjee (1998).

Our paper is also related to papers that argue that decreasing credit market frictions can have perverse effects. Notably, Myers and Rajan (1998) argue that increasing asset liquidity decreases efficiency because it reduces a borrower's ability to commit to future actions. Donaldson and Micheler (2016) suggest that increasing cash flow pledgeability can increase

[10] We relax this assumption in Subsection 6.1.

systemic risk, because it leads borrowers to favor non-resaleable debt instruments, such as repos, over resaleable debt instruments, such as bonds.

The collateral rat race in our model is reminiscent of the eponymous "maturity rat race" in Brunnermeier and Oehmke (2013). This is because in that paper short maturity plays a similar role to collateral in our model: it serves to establish priority, protecting creditors against the claims of competing creditors—by definition, short-term creditors are repaid before long-term creditors. However, Brunnermeier and Oehmke (2013) do not study the effects of limited pledgeability of cash flows. Further, our other main results are independent of this rat race, as we show in the analysis of pari passu debt in Subsection 6.1.

Our paper also relates to the literature on non-exclusive contracts in finance, such as Acharya and Bisin (2014), Attar, Casamatta, Chassagnon, and Décamps (2015), Bisin and Gottardi (1999, 2003), Bisin and Rampini (2005), Faure-Grimaud and Gromb (2004), Leitner (2012), and Parlour and Rajan (2001). Our incremental contribution relative to this literature is to study how collateral can work to mitigate—and, in equilibrium, amplify—the effects of the non-exclusive-contracting friction. We show that exclusive contracts have a dark side when they coexist with non-exclusive contracts. In particular, in our model collateral serves to grant a creditor an exclusive claim on a project's cash flow, potentially undercutting existing creditors. In other words, collateral allows contracting parties to enter into exclusive relationships selectively, at the expense of other parties—exclusive contracts might not be better than non-exclusive contracts if other non-exclusive contracts are already in place. This suggests a caveat to papers that emphasize how non-exclusive contracts can undermine efficiency in credit markets, such as Bolton and Scharfstein (1990), Petersen and Rajan (1995), and Donaldson, Piacentino, and Thakor (2016). Also, we study the interaction of limited pledgeability and non-exclusive contracts, which these papers do not.

By analyzing secured debt in a corporate finance model with multiple creditors, we also relate to the literature on collateral, covenants, and property rights in law and corporate finance, such as Ayotte and Bolton (2011), Bebchuk and Fried (1996), Kronman and Jackson (1979), Schwarcz (1997), Schwartz (1984), and Stulz and Johnson (1985). The idea of investing in a multi-lateral commitment by ring-fencing, i.e. "collateralizing," a project builds on Kiyotaki and Moore (2000, 2001), who focus on the macroeconomic effects of such multilateral commitments. Bolton and Scharfstein (1996) present a contrasting view of multiple creditors and commitment. In their model, having more creditors allows a firm to commit not to renegotiate debt repayments.

Bhattacharya and Faure-Grimaud (2001) argue that when a firm's investments are non-contractible, renegotiation between borrowers and creditors may not resolve the debt-overhang problem. Relatedly, we find the "collateral overhang" of secured credit cannot be resolved by renegotiation (given the limited pledgeability friction).[11]

Our paper is related to the literature on a possible shortage of collateral in funding markets, such as Caballero (2006) and Di Maggio and Tahbaz-Salehi (2015). We offer a new perspective by studying the role of collateral in mitigating non-exclusive contracting.

Layout. The paper is organized as follows. In Section 2, we present the model. In Section 3, we analyze two benchmarks: the first-best outcome and the outcome with exclusive contracting. In Section 4, we solve the model. In Section 5, we discuss welfare and policy. In Section 6, we analyze a number of extensions and robustness issues. In Section 7, we conclude. Appendix A contains all proofs.

2 Model

In this section, we present the model.

2.1 Players and Projects

There is one good called cash, which is the input of production, the output of production, and the consumption good. A risk-neutral borrower B lives for three dates, $t \in \{0, 1, 2\}$, and consumes at Date 2. B has no cash, but has access to two investment projects, Project 0 at Date 0 and Project 1 at Date 1. Both projects are riskless and payoff at Date 2, but the payoff of Project 1 is revealed only at Date 1. Specifically, Project 0 costs I_0 at Date 0 and pays off X_0 at Date 2 and Project 1 costs I_1 at Date 1 and pays off X_1 at Date 2, where $X_1 \in \left\{X_1^L, X_1^H\right\}$ is a random variable realized at Date 1 with $X_1^L < X_1^H$ and $p := \mathbb{P}\left[X_1 = X_1^H\right]$.

B can fund his projects by borrowing I_0 at Date 0 and I_1 at Date 1 from competitive credit markets: we assume that B makes a take-it-or-leave-it offer to borrow from a risk-neutral creditor C_t at Date $t \in \{0, 1\}$.

2.2 Pledgeability and Collateralizability

B must promise to repay his creditors out of his projects' cash flows under two frictions. First, the pledgeability of cash flows is limited in that B may divert a fraction $1 - \theta$ of cash flows, leaving only a fraction θ for his creditors. We refer to θ as the *pledgeability* of cash flows. Second, contracts are non-exclusive in that if B borrows from one creditor, he cannot commit not to borrow from another creditor, potentially diluting the initial creditor's claim.

[11] This is result of the analysis in the extension in Subsection 6.3.

In other words, when B borrows from C_0 at Date 0, B cannot commit not to borrow from C_1 at Date 1.

The role of collateral in our model is to mitigate the effects of non-exclusive contracting: if a creditor's claim is collateralized (or "secured") by a project, then the creditor has the exclusive right to the project if the borrower defaults—no other creditor has a claim on the project.[12] To collateralize a project with cash flow X, B must "fence it off" from the claims of competing creditors, which costs $(1-\mu)X$.[13] We refer to μ as the *collateralizability* of projects. In the modern economy, "ring-fencing" is the legal analog of physical fence-building: a borrower's ring-fenced assets are legally insulated from its other obligations. The idea that costly ring-fencing is necessary to protect claims from a third party follows Kiyotaki and Moore (2001).[14] Recall that we abstract from the role of collateral in making up for a lack of pledgeable cash (except in Subsection 6.6)—i.e. collateralization does not affect θ.

2.3 Borrowing Instruments

At the crux of the model is B's choice to borrow via unsecured or secured (or "collateralized") debt. At Date t, B borrows I_t from C_t in exchange for the promise to repay the fixed face value F_t at Date 2. (Our restriction to two-period debt contracts is for simplicity: in Subsection 6.3 and Subsection 6.4, we expand the analysis to consider contingent contracts and short-term contracts, respectively, and the main results are unchanged.)

To borrow secured, B must collateralize his project. If B collateralizes a project with cash flows X to borrow secured from a creditor C, then C has priority over X. In particular, X cannot be collateralized and used to borrow secured from another creditor.

We assume that if B borrows unsecured from multiple creditors then the creditor that lent first is senior. In the model, this just says that C_0's unsecured debt is senior to C_1's unsecured debt. It could also be reasonable to assume that B's unsecured debt is all treated equally, and we discuss this case of pari passu debt in Subsection 6.1. However, we rule out the possibility that seniority is a contracting variable.

[12]Note that we assume for simplicity that collateralization is a binary decision—B either collateralizes a project or does not, he cannot collateralize only a fraction of a project.

[13]In Subsection 6.5, we show that it is equivalent to assume that to borrow via secured debt B must post a haircut or a margin $(1-\mu)/\mu$ rather than pay the cost $1-\mu$. Further, ring-fencing is not the only interpretation for the cost of securing a project away from the claim of a third-party. For example, B could pay a custodian or warehouse to hold the securities. In this case, the cost $1-\mu$ represents the collateral management fee that many custodians charge in practice, for example in the tri-party repo market. Other microfundations of the cost $1-\mu$ include lawyer's fees, ex post monitoring to ensure that collateral stays with the borrower, and ex ante auditing to ensure that collateral is unencumbered.

[14]They say that a borrower "ring-fences his project in a way that limits the potential for asset-stripping" to a third party (p. 24).

2.4 Timeline

The timeline is as follows.

Date 0	B borrows I_0 from C_0 via secured debt or unsecured debt or does not borrow
	If B has borrowed from C_0, he invests in Project 0
Date 1	The payoff of Project 1 is observed, $X_1 = X_1^H$ or $X_1 = X_1^L$
	B borrows I_1 from C_1 via secured debt or unsecured debt or does not borrow
	If B has borrowed from C_1, he invests in Project 1
Date 2	Projects payoff, repayments are made, and players consume

If B undertakes both projects, then total payoff is given by

$$W := \begin{cases} X_0 + X_1 & \text{if neither project is collateralized,} \\ \mu X_0 + X_1 & \text{if only Project 0 is collateralized,} \\ X_0 + \mu X_1 & \text{if only Project 1 is collateralized,} \\ \mu(X_0 + X_1) & \text{if both projects are collateralized.} \end{cases} \tag{1}$$

If B has debt F_0 to C_0 and F_1 to C_1, his payoff is the equity value

$$\text{B's equity } = \max\left\{W - F_0 - F_1, (1-\theta)W\right\}. \tag{2}$$

If B does not default, each creditor C_t gets F_t. If B does default, C_0 and C_1 divide θW according to priority.

2.5 Parameter Restrictions

We impose several restrictions on parameters. These restrict attention to cases of interest, i.e. in which non-exclusivity alone causes the outcome to be inefficient.

Parameter Restriction 1. Net of the cost $1-\mu$ of collateralization, Project 0 has positive NPV and Project 1 has positive NPV if and only if $X_1 = X_1^H$:

$$0 < I_0 < \mu X_0 \quad \text{and} \quad 0 < X_1^L < I_1 < \mu X_1^H. \tag{3}$$

Parameter Restriction 2. The pledgeable cash flow from Project 0 exceeds its cost of investment net of the cost of collateralization, but the pledgeable cash flow from Project 1 does not:

$$I_0 \le \theta\mu X_0 \quad \text{and} \quad \theta X_1^H < I_1. \tag{4}$$

Parameter Restriction 3. The pledgeable cash flow from the "portfolio" of Project 0 and Project 1 exceeds the cost of investment if and only if $X_1 = X_1^H$:

$$\theta(X_0 + X_1^L) < I_0 + I_1 < \theta(X_0 + X_1^H). \tag{5}$$

The two parameter restrictions below are less economically important. They rule out cases that complicate the analysis but do not enrich it.[15]

Parameter Restriction 4. This technical restriction ensures that the payoff of Project 1 is always large enough that B has the incentive to undertake it. Specifically, it ensures that if B can fund Project 1 by taking on new debt which dilutes existing debt, he will always do so.[16]

$$X_1^L > \frac{\big(1 - \mu(1-\theta)\big)X_0 - I_0}{\mu(1-\theta)}. \tag{6}$$

Parameter Restriction 5. This is another somewhat more technical restriction. It simplifies the analysis by ensuring that the cost of Project 1 is not so large that B can never borrow from C_1 to invest in it.

$$I_1 < \theta\mu\big(X_0 + X_1^H\big). \tag{7}$$

3 Benchmarks

In this section, we present two benchmarks. We first solve for the first-best outcome, in which the total surplus is maximized. We then solve for the outcome of the model with exclusive contracting, which corresponds to $C_0 = C_1$ in our model. The main result is that both outcomes coincide.

3.1 First Best

In this subsection, we describe the first-best outcome of the model. This is the outcome in which all positive NPV projects are undertaken. It follows immediately from Parameter Restriction 1 that the first-best outcome is to undertake Project 0 at Date 0 and Project

[15] Both restrictions matter only for the proof of Proposition 3.

[16] Note that it might also be reasonable to assume that B gets private benefits from empire building and, therefore, always has the incentive to undertake Project 1, regardless of its NPV (cf. footnote 26). In that case this assumption is unnecessary.

1 at Date 1 if and only if $X_1 = X_1^H$. The next proposition gives the associated first-best expected surplus.

Proposition 1. (First-best outcome and expected surplus.) *In the first-best outcome, B undertakes Project 0 and undertakes Project 1 if and only if $X_1 = X_1^H$. The expected surplus is*

$$X_0 - I_0 + p\left(X_1^H - I_1\right). \tag{8}$$

3.2 Exclusive Contracts

In this subsection, we describe the outcome of the model if B borrows via an exclusive contract. In our environment, this is the outcome of the model in which B can borrow exclusively from a single creditor, i.e. $\mathrm{C}_1 = \mathrm{C}_0$ (and everything else is as described in Section 2).

Proposition 2. (Exclusive contracts implement the first best.) *With exclusive contracts the first-best outcome obtains.*

The key to understanding this result is to see that with exclusive contracts B borrows at the fair price to fund each project he undertakes. This is because when B takes on new debt, he borrows from the same creditor, C_0, that holds his existing debt and, thus, the interest rate that C_0 charges on the new debt reflects its effect on the value of existing debt. As a result, B chooses to undertake only positive NPV projects, which leads to the first-best outcome.[17]

4 Model Solution

In this section, we solve the model. First, we solve the subgames in which B borrows via unsecured debt at Date 0 and in which B borrows via secured debt at Date 0. Then we compare B's payoffs in each of these subgames to find B's equilibrium choice of borrowing instrument at Date 0.

4.1 Unsecured Debt to C_0

We now solve for the equilibrium of the subgame in which B borrows from C_0 at Date 0 via unsecured debt with face value F_0^u. We focus on the case in which $F_0^u \geq I_0$ without loss of

[17] This intuition that with exclusive contracts B wants to undertake all and only positive NPV projects is a general feature of our environment, but the fact that the first-best outcome is achieved is not. In general, limited pledgeability alone could constrain B's borrowing, as we discuss further in Subsection 6.6. However, the parameter restrictions in Subsection 2.5 rule this out, allowing us to focus on the inefficiencies induced by the non-exclusivity of contracts.

generality, since C_0 must recoup I_0 in expectation.

Given B has unsecured debt F_0^u to C_0, we ask, first, when B can borrow from C_1 via unsecured debt and, second, when he can borrow from C_1 via secured debt.

Unsecured debt to C_0 and unsecured debt to C_1. If B borrows from C_1 via unsecured debt, this new debt is junior to the existing debt F_0^u. Thus, C_1 will lend to B via unsecured debt only if B's portfolio of projects $X_0 + X_1$ generates sufficient pledgeable cash flow to repay I_1 to C_1 after having repaid F_0^u to C_0, or if $I_1 \leq \theta(X_0 + X_1) - F_0^u$. This implies that B can never borrow from C_1 via unsecured debt when the return on Project 1 is low, $X_1 = X_1^L$.

LEMMA 1. *If B has unsecured debt to C_0, then B can never borrow unsecured from C_1 if $X_1 = X_1^L$.*

The result follows from Parameter Restriction 3 and the fact that $F_0^u \geq I_0$: if $X_1 = X_1^L$, the pledgeable cash flow that B has left after repaying C_0 is less than I_1.

Unsecured debt to C_0 and secured debt to C_1. If B borrows from C_1 via secured debt at Date 1, this new debt is effectively senior to the existing debt F_0^u. This is because to borrow via secured debt, B collateralizes his projects, protecting C_1's claim to its cash flow. Thus, C_1 will lend to B via secured debt as long as B's portfolio of collateralized projects $\mu(X_0 + X_1)$ generates sufficient pledgeable cash flow to repay I_1 (independently of B's unsecured debt F_0^u to C_0), or if

$$I_1 \leq \theta\mu(X_0 + X_1). \tag{9}$$

By borrowing from C_1 via secured debt at Date 1, B can dilute his existing debt to C_0. This gives B the incentive to borrow and invest in Project 1 even when it has negative NPV.[18] Thus, B borrows at Date 1 whenever C_1 is willing to lend to him, i.e. whenever his pledgeable cash flow is sufficiently high.

LEMMA 2. *If B has unsecured debt to C_0 and pledgeability is above a threshold*

$$\theta^* := \frac{I_1}{\mu\left(X_0 + X_1^L\right)}, \tag{10}$$

then B borrows from C_1 via secured debt if and only if $X_1 = X_1^L$.

This corollary implies that higher cash-flow pledgeability loosens B's borrowing constraint at Date 1.

[18] Parameter Restriction 4 ensures that the payoff X_1^L is large enough that B always wishes to dilute C_0 to do Project 1. See the proof of Lemma 2 for the formal argument.

Equilibrium borrowing and payoff with unsecured debt to C_0. We now turn to the equilibrium face value F_0^u of B's unsecured debt to C_0. We consider the cases of low pledgeability and high pledgeability separately.

If pledgeability θ is low, then B cannot borrow from C_1 via secured debt if $X_1 = X_1^L$ (Lemma 2). B does not dilute C_0's debt by collateralizing his projects to C_1. Without the risk of being diluted, C_0 lends to B at the risk-free rate and B undertakes Project 1 only when it is efficient; he finances it by borrowing from C_1 via unsecured debt.

If pledgeability θ is high, then B can borrow from C_1 via secured debt (Lemma 2). B can dilute C_0's debt by collateralizing his projects to C_1. When $X_1 = X_1^H$, Project 1 has positive NPV and the portfolio $X_0 + X_1$ generates enough pledgeable cash flow to cover the costs of both projects $I_0 + I_1$, so C_0 is likely to be repaid even if his debt is diluted by new debt to C_1. When $X_1 = X_1^L$, in contrast, Project 1 has negative NPV. However, B still undertakes it because he benefits from diluting his debt to C_0 (Lemma 2). Given that C_0 risks being diluted when the payoff of Project 1 is low, C_0 lends unsecured only if B will repay with interest when the payoff of Project 1 is high. Indeed, if the probability p that $X_1 = X_1^H$ is high, then B borrows from C_0 via risky debt—the expected gain of surplus in the event that $X_1 = X_1^H$ offsets the expected loss of surplus when $X_1 = X_1^L$. In contrast, if p is low, then B cannot borrow from C_0 via unsecured debt—the surplus gained when $X_1 = X_1^H$ does not offset the surplus lost when $X_1 = X_1^L$; B cannot promise enough interest when the return on Project 1 is high to make C_0 break even in expectation.

The next proposition summarizes B's equilibrium borrowing behavior, given that he borrows from C_0 via unsecured debt.

PROPOSITION 3. (EQUILIBRIUM BORROWING WITH UNSECURED DEBT TO C_0.)

Assume B can only borrow unsecured from C_0 and define

$$\theta^{**} := \frac{I_1}{\mu X_0}, \tag{11}$$

$$p^* := \frac{I_0 + I_1 - \theta\mu\left(X_0 + X_1^L\right)}{\theta\left(X_0 + X_1^H\right) - \theta\mu\left(X_0 + X_1^L\right)} \in (0,1), \tag{12}$$

$$p^{**} := \frac{I_0 + I_1 - \theta\left(\mu X_0 + X_1^L\right)}{\theta\left(X_0 + X_1^H\right) - \theta\left(\mu X_0 + X_1^L\right)} \in (0,1). \tag{13}$$

- *If $\theta \leq \theta^*$, then B borrows from C_0 via unsecured risk-free debt with face value $F_0^u = I_0$; B borrows from C_1 via unsecured risk-free debt if $X_1 = X_1^H$ and does not borrow from C_1 if $X_1 = X_1^L$.*

- *If $\theta^* < \theta < \theta^{**}$ and $p \geq p^*$, then B borrows from C_0 via unsecured risky debt with face* *value*

$$F_0 = \frac{I_0 - (1-p)\left(\theta\mu\left(X_0 + X_1^L\right) - I_1\right)}{p}; \tag{14}$$

B borrows from C_1 via risk-free unsecured debt if $X_1 = X_1^H$ and borrows from C_1 via risk-free secured debt if $X_1 = X_1^L$.

- *If $\theta \geq \theta^{**}$ and $p \geq p^{**}$, then B borrows from C_0 via unsecured risky debt with face value*

$$F_0 = \frac{I_0 - (1-p)\left(\theta\left(\mu X_0 + X_1^L\right) - I_1\right)}{p}; \tag{15}$$

B borrows from C_1 via risk-free unsecured debt if $X_1 = X_1^H$ and borrows from C_1 via risk-free secured debt if $X_1 = X_1^L$.

- *Otherwise, B does not borrow from C_0 or C_1.*

We can now write B's expected payoff at Date 0. Since C_0 and C_1 break even in expectation, B captures the NPVs of the projects he undertakes, which depend on the pledgeability θ of cash flows and the probability p that the return on Project 1 is high, as described in Proposition 3 above. Given B borrows from C_0 via unsecured debt, his payoff Π_{B}^u is given by the following expression:

$$\Pi_{\mathrm{B}}^u = \begin{cases} X_0 - I_0 + p\left(X_1^H - I_1\right) & \text{if } \theta \leq \theta^*, \\ p\left(X_0 + X_1^H\right) + (1-p)\left(\mu(X_0 + X_1^L)\right) - I_0 - I_1 & \text{if } \theta^* < \theta < \theta^{**} \text{ and } p \geq p^*, \\ p\left(X_0 + X_1^H\right) + (1-p)\left(\mu X_0 + X_1^L\right) - I_0 - I_1 & \text{if } \theta \geq \theta^{**} \text{ and } p \geq p^{**}, \\ 0 & \text{otherwise.} \end{cases} \tag{16}$$

4.2 Secured Debt to C_0

We now solve for the equilibrium of the subgame in which B borrows from C_0 via secured debt with face value F_0^s. We focus on the case in which $F_0^s \geq I_0$ without loss of generality, since C_0 must be repaid at least as much as it lends. We maintain the assumption that $F_0^s \leq \mu X_0$, and we verify that it holds in equilibrium later.

Given B has secured debt F_0^s to C_0, we ask, first, when B can borrow from C_1 via unsecured debt and, second, when he can borrow from C_1 via secured debt.

Secured debt to C_0 and unsecured debt to C_1. If B borrows from C_1 via unsecured debt, this new debt is junior to the existing debt F_0^s. Thus, C_1 will lend to B via unsecured debt only if B's portfolio of projects $\mu X_0 + X_1$ generates sufficient pledgeable cash flow to repay I_1 to C_1 after having repaid F_0^s to C_0, or if

$$I_1 \leq \theta(\mu X_0 + X_1) - F_0^s. \tag{17}$$

This implies that B can never borrow from C_1 via unsecured debt when the return on Project 1 is low, $X_1 = X_1^L$.

LEMMA 3. *If B has secured debt to C_0, then B can never borrow unsecured from C_1 if $X_1 = X_1^L$.*

The result follows from Parameter Restriction 3 and the fact that $F_0^u \geq I_0$: if $X_1 = X_1^L$, the pledgeable cash flow that B has left after collateralizing Project 0 and repaying C_0 is less than I_1.

Secured debt to C_0 and secured debt to C_1. B's ability to borrow from C_1 via secured debt at Date 1 is limited, because B has already collateralized Project 0 to C_0, protecting C_0's claim to its cash flows. Thus, C_1 will lend to B via secured debt only if B's portfolio of collateralized projects $\mu(X_0 + X_1)$ generates sufficient pledgeable cash flow to repay I_1 to C_1 after having repaid F_0^s to C_0, or $I_1 \leq \mu\theta(X_0 + X_1) - F_0^s$. Observe that this condition for B to borrow from C_1 via secured debt is more restrictive then the condition for B to borrow from C_1 via unsecured debt in equation (17) above. As a result, B will never borrow from C_1 via secured debt if he has already borrowed from C_0 via secured debt.

LEMMA 4. *If B has secured debt to C_0, then B does not borrow secured from C_1.*

This is a result of the fact that if B borrows from C_0 via secured debt, then all new debt, both secured and unsecured, is effectively junior to C_0's debt. As a result, it is better for B to borrow from C_1 via unsecured debt than to pay the the cost $(1 - \mu)X_1$ of collateralizing Project 1 to borrow from C_1 via secured debt. In other words, when B borrows from C_0 via secured debt, he "uses up" $(1 - \mu)X_0$ of pledgeable cash flow, tightening his borrowing constraint at Date 1.

Equilibrium borrowing and payoff with secured debt to C_0. We now turn to the equilibrium face value F_0^s of B's secured debt to C_0. If B borrows from C_0 via secured debt, C_0 does not bear any risk. This is because, as a secured creditor, C_0 has priority over Project 0's pledgeable cash flow and this cash flow is sufficient to cover its cost of investment: $I_0 < \mu\theta X_0$, by Parameter Restriction 2. Thus, B can always borrow from C_0 via secured debt at the risk-free rate, $F_0^s = I_0$. And, as a result, B can borrow from C_1 via unsecured debt whenever inequality (17) is satisfied with $F_0^s = I_0$, or

$$I_1 \leq \theta\left(\mu X_0 + X_1\right) - I_0. \tag{18}$$

We can rewrite this condition in terms of collateralizability μ: B can borrow from C_1 via unsecured debt if collateralizability is above a threshold as follows:

$$\mu \geq 1 - \frac{\theta\left(X_0 + X_1\right) - I_0 - I_1}{\theta X_0}. \tag{19}$$

Given that B never borrows from C_1 via secured debt (Lemma 4) and never borrows from C_1 if the payoff of Project 1 is low (Lemma 3), we can fully characterize B's Date-1 borrowing.

LEMMA 5. *If B has secured debt to C_0 with face value I_0, B borrows from C_1 if and only if $X_1 = X_1^H$ and collateralizability is above a threshold μ^*, given by*

$$\mu^* := 1 - \frac{\theta\left(X_0 + X_1^H\right) - I_0 - I_1}{\theta X_0}. \tag{20}$$

The next proposition summarizes B's equilibrium borrowing behavior, given that he borrows from C_0 via secured debt.

PROPOSITION 4. (EQUILIBRIUM BORROWING WITH SECURED DEBT TO C_0.) *Assume B can only borrow secured from C_0.*

- *If $\mu \geq \mu^*$, B borrows from C_0 via risk-free secured debt with face value $F_0 = I_0$; B borrows from C_1 via risk-free unsecured debt if $X_1 = X_1^H$ and does not borrow from C_1 if $X_1 = X_1^L$.*
- *If $\mu < \mu^*$, B borrows from C_0 via risk-free secured debt with face value $F_0 = I_0$; B does not borrow from C_1.*

We can now write B's expected payoff at Date 0. Since C_0 and C_1 break even in expectation, B captures the NPVs of the projects he undertakes. Given B borrows from C_0 via secured debt, his payoff Π_B^s is given by the following expression:

$$\Pi_{\mathrm{B}}^s = \begin{cases} \mu X_0 - I_0 + p\left(X_1^H - I_1\right) & \text{if } \mu \geq \mu^*, \\ \mu X_0 - I_0 & \text{otherwise.} \end{cases} \tag{21}$$

4.3 Equilibrium Debt Instrument

In the preceding subsections, we solved for B's equilibrium payoffs Π_{B}^u and Π_{B}^s from borrowing from C_0 via unsecured debt and secured debt, respectively. In equilibrium, B borrows from C_0 via unsecured debt whenever $\Pi_{\mathrm{B}}^u \geq \Pi_{\mathrm{B}}^s$ and borrows from C_0 via secured debt otherwise. The next proposition characterizes B's equilibrium choice of debt instrument. It follows immediately from comparing the expression for Π_{B}^{u} in equation (16) with the expression for Π_{B}^{s} in equation (21).

Proposition 5. (Equilibrium debt instrument.) *Recall the thresholds θ^*, θ^{**}, p^*, p^{**} and μ^* from equations (10), (11), (12), (13), and (20) above.*

The equilibrium Date-0 debt instrument is determined as follows.

- *If $\theta \leq \theta^*$, then B borrows from C_0 via unsecured debt.*
- *If either $\theta^* < \theta < \theta^{**}$ and $p < p^*$ or $\theta \geq \theta^{**}$ and $p < p^{**}$, then B borrows from C_0 via secured debt.*
- *Otherwise, whether B borrows from C_0 via unsecured debt or secured debt depends on the relative inefficiencies of unsecured and secured debt: B borrows from C_0 via unsecured debt if and only if*

$$p(1-\mu)X_0+pX_1^H+(1-p)\left[1-(1-\mu)\mathbb{1}_{\{\theta^*<\theta<\theta^{**}\}}\right]X_1^L-I_1 \geq \mathbb{1}_{\{\mu\geq\mu^*\}}p\left(X_1^H-I_1\right). \quad (22)$$

This proposition implies that unsecured debt and secured debt may coexist in equilibrium.

Corollary 1. *Suppose that either $\theta^* < \theta < \theta^{**}$ and $p < p^*$ or $\theta \geq \theta^{**}$ and $p < p^{**}$.*

If the inequality in equation (22) holds, then secured debt and unsecured debt coexist in equilibrium: B borrows from C_0 via risky unsecured debt and borrows from C_1 via riskless secured debt when $X_1 = X_1^L$.

If the inequality in equation (22) is violated and $\mu \geq \mu^*$, then secured debt and unsecured debt coexist in equilibrium: B borrows from C_0 via riskless secured debt and borrows from C_1 via riskless unsecured debt when $X_1 = X_1^H$.

5 Welfare and Policy

In this section, we analyze welfare and policy in the model. We first show that the first-best surplus is attained in equilibrium if and only if pledgeability is sufficiently *low*—there is a "paradox of pledgeability." We then show that borrowing via unsecured debt leads to over-investment and borrowing via secured debt leads to under-investment—there is a "collateral overhang" problem. Finally, we suggest that expanding the supply of collateral may have adverse effects, because it can induce a "collateral rat race."

5.1 The Paradox of Pledgeability

Having solved for the equilibrium of the model, we can now compare the equilibrium surplus with the first-best surplus. Given that the creditors C_0 and C_1 are competitive, the borrower B captures all of the surplus. Thus, B's equilibrium payoff $\Pi_B = \max\{\Pi_B^u, \Pi_B^s\}$ coincides with the equilibrium surplus. Comparing this with the expression for the first-best surplus in equation (8), we see that the equilibrium is efficient—i.e. the first-best surplus is attained—only if pledgeability θ is sufficiently *low*.

PROPOSITION 6. (PARADOX OF PLEDGEABILITY.) *The first-best level of surplus is attained if and only if pledgeability is low, or $\theta \le \theta^*$, where θ^* is as defined in equation (10).*

The intuition behind this result is as follows. An increase in pledgeability θ allows B to pledge more of his cash flows to C_1, making C_1 more willing to lend. This makes it easier for B to take on new debt to C_1. However, this new debt may dilute B's existing debt to C_0. Thus, C_0 becomes less willing to lend. In other words, increasing pledgeability makes it easier to borrow at Date 1 and, hence, paradoxically, makes it harder to borrow at Date 0.

This result follows from the friction of non-exclusive contracts: when B borrows from C_0, he cannot commit not to borrow from C_1. When pledgeablity is low, this friction does not induce an inefficiency because B is too constrained to borrow from C_1 when $X_1 = X_1^L$—low pledgeability makes B's contract with C_0 effectively exclusive, by allowing B to commit not to borrow from C_1 to dilute C_0's debt. When pledgeablity is high, this friction does induce an inefficiency: B either over-invests in negative NPV projects or underinvests in positive NPV projects, as discussed in the next subsections.

5.2 Collateral Rat Race

We now turn to the inefficiency of borrowing via unsecured debt, which arises for high pledgeability. If B borrows from C_0 via unsecured debt and pledgeability is high, B can dilute C_0's debt by collateralizing his projects and borrowing from C_1 via secured debt (Lemma 2). B borrows cheaply from C_1, because, as a secured creditor, C_1 does not bear the default costs associated with B's increased debt. These default costs are transferred to B's existing creditor, C_0, whose debt is now effectively junior. As a result, B's investment in Project 1 is subsidized, since B funds it via secured debt to a new creditor, C_1, at the expense of his old creditor, C_0. In other words, undertaking Project 1 is a way for B to syphon off cash flows from C_0. This subsidy distorts B's incentives, inducing B to undertake Project 1 when $X_1 = X_1^L$, even though it has negative NPV. This incentive to over-invest in negative NPV projects is the main inefficiency of unsecured debt in the model, as summarized in the next proposition.

Proposition 7. (Over-investment with existing secured debt.) *Suppose* $\theta > \theta^*$ *as defined in equation (10). If B borrows from* C_0 *via unsecured debt, B over-invests in Project 1 when* $X_1 = X_1^L$.

This proposition is a result of the friction of non-exclusive contracts: B cannot commit not to dilute existing unsecured debt with new secured debt. The resulting inefficiency may be so severe that C_0 is unwilling to lend to B unsecured to fund Project 0, even though Project 0 is riskless and its pledgeable cash flow exceeds its investment cost—$\theta X_0 > I_0$ by Parameter Restriction 2. This is the next corollary, which follows from the characterization in Proposition 3.

Proposition 8. (Collateral rat race.) *Suppose either* $\theta^* < \theta < \theta^{**}$ *and* $p < p^*$ *or* $\theta \geq \theta^{**}$ *and* $p < p^{**}$. C_0 *will not lend to B via unsecured debt.*

This is due to a "collateral rat race," by which collateralization is required to protect against future collateralization.

The intuition behind this result is as follows. When pledgeability is high, B funds the low-return Project 1 by borrowing from C_1 via secured debt to undercut his unsecured debt to C_0. B repays C_1 in full, but defaults on his debt to C_0. C_0 requires collateral to protect against this: if C_0 is a secured creditor, he is effectively senior in the event that B defaults. In other words, collateralization is required at Date 0 to protect against collateralization at Date 1: there is a collateral rat race. This finding suggests that the ability to use collateral can create a friction when it allows a borrower to *selectively* enter into an exclusive contract. This rat race can lead to inefficient underinvestment, as we discuss in the next subsection.

Investment Efficiency (for $\theta^{**} > 1$)

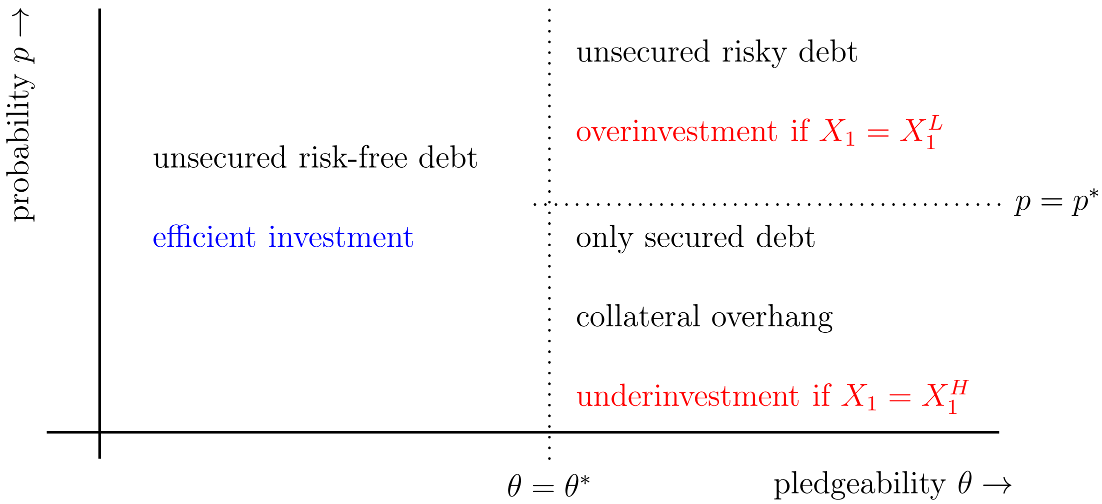

Figure 1: The figure above illustrates B's investment decisions as a function of θ and p. For illustrative purposes, we restrict attention to the case in which $\theta^{**} > 1$. For $\theta < \theta^*$, B takes the efficient action. For $\theta \geq \theta^*$, B over-invests in X_1 if $p \geq p^*$ and underinvests in Project 1 if $p < p^*$ (cf. Proposition 7 and Proposition 9).

5.3 Collateral Overhang

We now turn to the inefficiency of borrowing via secured debt, which arises for high pledgeability. If B borrows from C_0 via secured debt, B pays the cost $(1-\mu)X_0$ of collateralization. This cost is a deadweight loss and hence decreases the surplus to a level below the first-best. You might imagine that this decrease in surplus is relatively small. However, it can be amplified in equilibrium. This is because by collateralizing his project to C_0, B uses up his pledgeable cash flow and thus makes it more difficult to borrow from C_1. In other words, there is a *collateral overhang*, by which collateralizing his project at Date 0 prevents B from borrowing at Date 1. As a result, B may not undertake Project 1, even when it is efficient to do so. Figure 1 depicts which inefficiency arises for different values of the parameters θ and p.

Proposition 9. (Collateral overhang: underinvestment with existing secured debt.) *If B borrows from C_0 via secured debt, he can undertake Project 1 when $X_1 = X_1^H$ only if collateralizability is above the threshold μ^* defined in equation (20). In other words, collateralizing Project 0 at Date 0 can prevent B from undertaking an efficient investment at Date 1.*

Observe that this collateral overhang kicks in only when collateralizability is below the threshold μ^*. At first blush, this suggests that a policy maker should increase collateralizability to prevent this distortion. Indeed, this would prevent the distortion at Date 1. However, the analysis in the next section shows that when Date-0 borrowing is taken into account, the opposite may be true: decreasing collateralizability can increase the surplus.

5.4 Collateral Shortage or Collateral Glut?

We now turn to the effects of varying the collateralizability μ on the surplus.

PROPOSITION 10. (SURPLUS-INCREASING COLLATERAL BAN.) *If collateralization is banned, i.e. $\mu = 0$, the first-best surplus is attained in equilibrium.*

The intuition behind this result is as follows. If μ is sufficiently low, then B cannot collateralize his projects to borrow from C_1 via secured debt. As a result, B cannot undercut C_0's debt and B's contract with C_0 is effectively exclusive. This leads to the first-best outcome (as in Proposition 2).

This result may cast light on some aspects of the policy debate about which financial assets may be used as collateral in interbank markets as well as how such collateral should be treated in bankruptcy. Within our model, we view an increase in μ as corresponding to an increase in the ease with which assets can be collateralized or as an increase in the total supply of assets that can be used as collateral.[19] Notably, the special bankruptcy treatment of repo collateral, which makes it effectively super-senior in bankruptcy, corresponds to an increase in μ, since the special treatment makes each collateralized asset more valuable to creditors. The set of assets eligible for special treatment was expanded in 2005, effectively increasing the supply of repo collateral. Despite this effective increase in the supply of collateral, markets perceived a shortage of collateral. As Caballero (2006) puts it, "The world has a shortage of financial assets. Asset supply is having a hard time keeping up with the global demand for...collateral" (p. 272). In our model, an increase in μ can also lead to a high-dependence on collateral. It makes it easier for B to collateralize his projects and borrow secured at Date 1, which triggers the collateral rat race, so he must borrow collateralized at Date 0. However, although B may perceive a collateral shortage, lowering the supply of collateral—decreasing μ—may make markets function better. The proposition above shows that this can actually prevent the collateral rat race from starting in the first place. In fact, if collateralization is banned completely, i.e. $\mu = 0$, the first-best surplus is

[19] We view the supply of collateral in the model as the total cash flow that can used to borrow secured. This is $\mu\theta(X_0 + X_1)$. This is increasing in μ, suggesting an increase in μ corresponds to an increase in the supply of collateral.

attained in equilibrium.

6 Extensions and Robustness

In this section, we analyze a number of extensions of our model and confirm the robustness of our main results. First, we relax the assumption that existing unsecured debt is senior to new unsecured debt. We include a formal analysis of pari passu debt. Second, we include a detailed discussion of covenants. We argue that covenants restricting borrowing from third parties—i.e. attempting to circumvent the non-exclusivity friction—may be ineffective, especially for banks. Third, we study how security design might affect our results, allowing for contingent contracts as well as simple debt. Fourth, we show that the cost of ring-fencing has an equivalent interpretation as an exogenous haircut on secured debt. Fifth, we analyze a simple model in which collateral plays both of the roles discussed in the Introduction, it both (i) mitigates enforcement problems between a borrower and his creditors and (ii) mitigates enforcement problems among creditors. We show that whereas increasing pledgeability increases the importance of the second role of collateral (as in the paradox of pledgeability above), it decreases the importance of the first role (in line with the textbook intuition).

6.1 Pari Passu Debt

In this subsection, we argue that our main result that increasing pledgeability leads to more collateralized borrowing—the paradox of pledgeability—does not depend on the assumption that new unsecured debt is effectively senior to old unsecured debt. Increasing pledgeability can increase the use of collateral even if collateral cannot be used to establish priority over existing debt. The result obtains as long as taking on new debt has some negative effect on old debt.[20] We show this by considering the case of pari passu debt in detail.

Here we focus on the case in which all unsecured debt is treated equally (pari passu). Consider the following twist on the baseline model. At Date 1, B cannot borrow from C_1 via secured debt, for example because it is too late to collateralize assets or because secured debt is not legally prioritized over existing debt.[21] But B can borrow from C_1 via pari passu unsecured debt, i.e. if B defaults with unsecured debt to C_0 with face value F_0 and unsecured debt to C_1 with face value F_1, each creditor is repaid a pro rata fraction of B's

[20]Even so, we argue in Subsection 6.2 below, that our results apply most pertinently in the baseline case, in which new secured debt does have priority over old unsecured debt.

[21]In many circumstances, such as the interbank market, this assumption may not be realistic. Secured debt typically has legal priority, as in the baseline model. See, e.g., Bjerre (1999), as well as the other legal literature cited in the Introduction and the discussion of covenants below.

pledgeable cash flows. Thus, if B defaults after undertaking both Project 0 and Project 1, the repayment to $C_t \in \{C_0, C_1\}$ is as follows:

$$\text{repayment to } C_t \; = \frac{F_t}{F_0+F_1}\theta(X_0+X_1). \tag{23}$$

If B undertakes Project 1 when $X_1 = X_1^L$, B's portfolio of projects X_0+X_1 does not generate sufficient pledgeable cash flow to cover the costs of the projects I_0+I_1 (Parameter Restriction 3), so B must default. However, B may still be able to borrow from C_1 via unsecured debt by diluting his debt to C_0. Specifically, B can borrow from C_0 whenever the repayment to C_1 in the event of default is greater than I_1. Using equation (23) above, this says that given $X_1 = X_1^L$, B can borrow I_1 from C_1 via debt with face value F_1 as long as

$$\theta \geq \frac{F_0+F_1}{F_1}\frac{I_1}{X_0+X_1}. \tag{24}$$

Here, B promises C_1 a high face value F_1 to dilute C_0's claim, effectively subsidizing B's investment in Project 1, just as in the baseline case with secured borrowing. This is feasible if B can offer C_1 a sufficiently high face value F_1 to ensure C_1 is repaid in full even in the event of default (even though C_1's debt is not prioritized in bankruptcy). In other words, despite the fact that C_0 is supposedly on equal footing with C_1 in bankruptcy, C_1's debt has diluted C_0's debt so severely that C_1's debt is in fact risk free. Mathematically, B can borrow from C_1 as long as F_1 is sufficiently high to satisfy inequality (24). Since $(F_0+F_1)/F_1 \to 1$ as $F_1 \to \infty$, inequality (24) is satisfied if and only if pledgeability θ is sufficiently large, or

$$\theta > \theta^{\text{p.p.}} := \frac{I_1}{X_0+X_1^L}. \tag{25}$$

Thus, if pledgeablity is sufficiently high, B borrows from C_1 via unsecured debt. We can solve for the face value by setting the repayment to C_1 equal to I_1 in equation (23):

$$F_1 = \frac{I_1 F_0}{\theta\left(X_0+X_1^L\right)-I_1}. \tag{26}$$

Observe that B defaults on his debt to C_1 and repays $I_1 < F_1$. However, the debt is still "risk free" in the sense that C_1 has a deterministic return equal to the risk-free rate (zero).

Now turn to B's debt to C_0. Since C_1 is always repaid I_1, the repayment to C_0 if $X_1 = X_1^L$ is given by the total pledgeable cash flow $\theta(X_0+X_1^L)$ less the repayment I_1 that is made to C_1, i.e.

$$\text{repayment to } C_0 = \theta\left(X_0+X_1^L\right) - I_1. \tag{27}$$

This is less than I_0 by Parameter Restriction 2. Thus, if B borrows from C_0 via unsecured debt, B repays C_0 less than I_0 whenever $X_1 = X_1^L$. Thus, if the probability $1-p$ that $X_1 = X_1^L$ is high, C_0 is rarely repaid. As a result, C_0 will not lend to B via unsecured debt, but only via secured debt. In other words, the paradox of pledgeability also holds with pari passu debt. This is the next proposition.

PROPOSITION 11. (PARADOX OF PLEDGEABILITY WITH PARI PASSU DEBT.) *Suppose that p is relatively small,*[22]

$$p < p^{\text{p.p.}} := \frac{I_0 + I_1 - \theta\left(X_0 + X_1^L\right)}{\theta\left(X_1^H - X_1^L\right)}. \tag{28}$$

- *If $\theta \le \theta^{\text{p.p.}}$ as defined in equation (25), then B borrows from C_0 via unsecured risk-free debt with face value $F_0^u = I_0$; B borrows from C_1 via risk-free unsecured debt if $X_1 = X_1^H$ and does not borrow from C_1 if $X_1 = X_1^L$.*

  The first-best surplus is attained in equilibrium.

- *If $\theta > \theta^{\text{p.p.}}$, then B cannot borrow from C_0 via unsecured debt.*

This result demonstrates that the driving force in our model is not the borrower's ability to use collateral to establish priority over existing debt, but rather the borrower's ability to take on new debt more generally, i.e. the fact that contracts are non-exclusive. However, in reality creditors take contractual measures to approximate exclusive relationships with their borrowers. Notably, they impose covenants in debt contracts that restrict future borrowing. These covenants offer limited protection against future *secured* borrowing, however, for reasons we discuss in the next subsection.

6.2 Covenants

In this subsection, we discuss the potential use of covenants in our model. We suggest that even though covenants may be effective to mitigate the friction of non-exclusive contracting in some circumstances, their ability to prevent a borrower from taking on new *secured* debt is limited.

The inefficiencies in our model come from the fact that the borrower cannot commit not to dilute its existing debt with new debt, i.e. that contracts are non-exclusive. In reality, debt contracts include covenants, called "negative pledge covenants," by which a borrower promises its creditor not to borrow from other creditors via secured debt. If such commitments were

[22]The cutoff $p^{\text{p.p.}}$ in equation (28) equals the cutoff p^* in equation (12) if collateralizability $\mu = 1$. This reflects the fact that pari passu debt allows B to effectively prioritize C_1's debt without bearing the cost of collateralization. It follows from Parameter Restriction 2 that the cutoff is always between zero and one.

binding, they could restore efficiency in our model. Unfortunately, however, the effectiveness of such negative pledge covenants is limited. This is because an unsecured creditor holds a claim against only the borrower, not against other creditors. Thus, an unsecured creditor cannot recover collateral that has been seized by a secured creditor. Bjerre (1999) describes these legal restrictions as follows:

> the negative pledge covenant [is a covenant] by which a borrower promises its lender that it will not grant security interests to other lenders. These covenants are common in unsecured loan agreements because they address one of the most fundamental concerns of the unsecured lender: that the borrower's assets will become unavailable to repay the loan, because the borrower will have both granted a security interest in those assets to a second lender and dissipated the proceeds of the second loan. Unfortunately, negative pledge covenants' prohibition of such conduct may be of little practical comfort, because as a general matter they are enforceable only against the borrower, and not against third parties who take security interests in violation of the covenant. Hence, when a borrower breaches a negative pledge covenant, the negative pledgee generally has only a cause of action against a party whose assets are, by hypothesis, already encumbered (pp. 306–307).

The potential for these negative pledge covenants to be effective in the event of bankruptcy is especially limited for repo and derivatives liabilities, since these contracts are exempt from automatic stays in bankruptcy—i.e. creditors can liquidate collateral without the approval of the bankruptcy court, making it difficult or impossible for any third party to enforce a claim to the collateral.

Whereas negative pledge covenants may be ineffective inside bankruptcy, they may still be useful outside bankruptcy. This is because a violation of such a covenant constitutes a default, and a borrower may adhere to the terms of covenants to avoid such a default. However, verifying that a solvent firm has violated a covenant can be difficult itself, especially for complicated firms like banks, which may have thousands of counterparties (e.g. depositors). Indeed, banks effectively do not have to disclose their short-term borrowing. Indeed:

> There are no specific MD&A requirements to disclose intra-period short-term borrowing amounts, except for [some] bank holding companies [that must] disclose on an annual basis the average, maximum month-end and period-end amounts of short-term borrowings (Ernst & Young (2010)).

There is a another reason that banks in particular may not be able to promise not to dilute existing debt with new debt: the very business of banking constitutes maturity and size transformation, which requires frequent short-term borrowing from many small creditors. If a bank agrees to covenants that restrict its ability to borrow in the future, it could undermine its ability to engage in these banking activities. As Bolton and Oehmke (2015) put it:

> debt covenants prohibiting the collateralization...are likely to be...costly to enforce...for financial institutions.... By the very nature of their business, financial institutions cannot assign...collateral to all depositors and creditors, because this would, in effect, erase their value added as financial intermediaries (p. 2356).

This reinforces the idea that non-exclusive contracting is an especially important friction for banks and, therefore, it may add credibility to our thesis that non-exclusive contracting is the reason that interbank markets are heavily reliant on collateral.

6.3 Contingent Debt

So far we have restricted attention to debt contracts, viz. contracts in which the promised repayment is non-contingent. In this subsection, we show that our main results also hold for contingent contracts.[23,24] The inefficiencies in our model result from the fact that contracts are non-exclusive, not that they are incomplete (cf. the exclusive contracting benchmark in Subsection 3.2). We focus on debt contracts for simplicity and realism.

Now suppose that B borrows from C_0 via unsecured contingent debt, i.e. B borrows I_0 from C_0 in exchange for the contingent repayments F_0^H when $X_1 = X_1^H$ and F_0^L when $X_1 = X_1^H$. If the payoff of Project 1 is low, i.e. $X_1 = X_1^L$, then B has the incentive to borrow from C_1 via secured debt, diluting C_0's debt.[25] In this event, C_0 is not repaid in full (Parameter Restriction 2). In the baseline analysis, C_0 must require collateral to protect against being diluted. Now, with contingent debt, C_0 can protect against being diluted in another way: C_0 can lower the repayment F_0^L when $X_1 = X_1^L$. In particular, if F_0^L is sufficiently low, then the benefits of diluitng C_0—and thus avoiding repaying F_0^L—may not compensate for the costs of doing a negative-NPV investment. In other words, if B's promised repayment to C_0 is sufficiently low, then it may be incentive compatible for B not

[23]The results in this subsection imply not only that our results are robust to contingent contracts, but also that they are robust to renegotiable debt: any outcome of renegotiation between B and C_0 at Date 1 can be implemented via contracting contingent on Date-1 information (viz. on the realization of X_1).

[24]It may be worth noting that we analyze only contingent repayments here, not contingent collateralization. However, our results are also robust to content collateralization—i.e. B would collateralize his project to C_0 only if pledgeability is sufficiently high—but with the caveat that B collateralizes only when $X_1 = X_1^L$, not when $X_1 = X_1^H$. This is because C_0 is effectively never diluted when $X_1 = X_1^H$ and thus does not require collateral to protect against dilution.

[25]This will be feasible whenever pledgeability is high, $\theta \geq \theta^*$, as in Lemma 2, which holds independently of whether debt is contingent or not.

to borrow from C_1.[26] Formally, B's payoff from not borrowing from C_1 and repaying F_0^L to C_0 must exceed his payoff from borrowing from C_1, diverting the fraction $1-\theta$ of his cash flows, and defaulting, i.e. the following incentive constraint must be satisfied[27]

$$X_0 - F_0^L \geq (1-\theta)\mu\left(X_0 + X_1^L\right). \tag{29}$$

This constraint imposes an upper bound on the repayment F_0^L:

$$F_0^L \leq X_0 - (1-\theta)\mu\left(X_0 + X_1^L\right). \tag{30}$$

This expression is less than the cost I_0 of Project 0. This implies that for any incentive-compatible contract, C_0 is not repaid as much as it lent when $X_1 = X_1^L$ (given that θ is high enough that B can borrow from C_1). So C_0 is repaid in full only with probability p, i.e. in the event that $X_1 = X_1^H$. Hence, if p is sufficiently low, C_0 will not lend to B via unsecured contingent debt, but rather will require collateral. This implies that our main results are robust to allowing for contingent contracts, as the next proposition summarizes.

Proposition 12. (Paradox of pledgeability with contingent debt.) *Suppose that p is relatively small,*[28]

$$p < p^{\text{c.d.}} := \frac{I_0 - \Big(X_0 - (1-\theta)\mu\left(X_0 + X_1^L\right)\Big)}{\theta\Big(X_0 + X_1^H\Big) - \Big(X_0 - (1-\theta)\mu\left(X_0 + X_1^L\right)\Big)}. \tag{31}$$

- *If $\theta \leq \theta^*$ as defined in equation (10), then B borrows from C_0 via unsecured risk-free debt debt with face value $F_0^u = I_0$; B borrows from C_1 via risk-free unsecured debt if $X_1 = X_1^H$, and does not borrow from C_1 if $X_1 = X_1^L$.*

  The first-best surplus is attained in equilibrium.

- *If $\theta > \theta^*$ and $p < p^{\text{c.d.}}$, then B cannot borrow from C_0 via unsecured debt (even if the debt is contingent).*

[26] Observe that, with the current setup, contingent contracting can only help insofar as it decreases B's incentive to undertake Project 1 when $X_1 = X_1^L$. However, it might also be reasonable to assume that B always wants to undertake new projects, e.g. because he gets private benefits from new investments. This is the setup in Hart and Moore (1995), in which "management's empire-building tendencies are sufficiently strong that it will always undertake the new investment if it can, even if the investment has negative net present value" (p. 568). Under this alternative assumption, allowing for contingent debt does not change the baseline analysis at all.

[27] We restrict attention to the case in which $I_1 < \theta\mu X_0$, so that B must collateralize both projects to borrow from C_1. We do this just to keep the analysis streamlined and not consider two separate cases.

[28] The cutoff $p^{\text{c.d.}}$ in equation (12) is always between zero and one by Parameter Restriction 3 and Parameter Restriction 4.

6.4 Short-term Debt

Another possibility we have not considered so far is short-term debt: B could borrow from C_0 via one-period debt and roll over. In this subsection, we show that if debt is required to be renegotiation-proof, then short-term debt cannot improve upon the outcome of the baseline model.

Here we augment the model and suppose that C_0 can lend to B via short-term debt. We assume that short-term debt matures at the end of Date 1, i.e. after B has (potentially) borrowed from C_1 and invested in Project 1. Further, we assume that the debt is subject to renegotiation at Date 1. Specifically, after the debt matures, B can either repay C_0 or offer C_0 an alternative repayment, e.g. he can offer a rescheduling of the debt, so that he repays at Date 2 instead of Date 1.[29] If C_0 accepts B's offer to renegotiate the debt, then B continues his projects. If C_0 reject's B's offer, then C_0 has the right to liquidate. However, since B's projects generate cash flows only at Date 2, we assume that their liquidation value is zero.

PROPOSITION 13. (SHORT-TERM DEBT.) *Suppose B borrows from C_0 via short-term debt. If B borrows from C_1 via secured debt and invests in Project 1 at Date 1, then C_0 prefers to accept a rescheduling of his debt to Date 2 than to liquidate B's assets. I.e. renegotiation-proof short-term debt does not improve on the implementation of long-term contracts.*

6.5 Collateralization Cost as a Haircut

So far, we have interpreted the cost of collateralization as the cost of ring-fencing assets to protect them from a third party (Subsection 2.2). This cost is important for our collateral overhang result (Proposition 9): because B must pay the cost $(1-\mu)X$ to collateralize X, collateralization uses up B's pledgeable cash flow. This inhibits his ability to borrow in the future. However, this mechanism is not specific to our interpretation of collateralization as costly ring-fencing. One equivalent interpretation is that B must post a haircut on collateralized debt. To see this, suppose that, in order to borrow I, B must post collateral worth $(1+m)I > I$. Here, m corresponds to the "margin" and mI corresponds to the haircut. Thus, B can borrow I against a project with cash flow X if its collateral value θX exceeds I plus the haircut mI, i.e. if $\theta X \geq (1+m)I$ or

$$I \leq \frac{\theta X}{1+m}. \tag{32}$$

[29] If B and C_0 can also renegotiate *before* B borrows from C_1, then C_0 can write down B's debt to disincentivize dilution when $X_1 = X_1^L$. This can implement the outcome of contingent debt, as discussed in footnote 23.

This implies that having to post a haircut mI is equivalent to having to pay the cost of ring-fencing $1-\mu$. In fact, if the margin $m=(1-\mu)/\mu$, then the constraint becomes

$$I \leq \frac{\theta X}{1+m} = \mu\theta X, \tag{33}$$

which is just B's constraint to borrow via secured debt in the baseline model.

This analysis implies that posting a haircut leads to the collateral overhang problem just as costly ring-fencing does. Even though B does not pay a deadweight cost to post a haircut like he does to "build" a costly ring-fence, B uses up pledgeable cash flow to post the haircut mI, which tightens his borrowing constraints in the future, potentially leading to underinvestment.

6.6 The Two Roles of Collateral: Use vs. Exclude

As discussed in the Introduction, real-world borrowers pledge collateral for at least two reasons, (i) to mitigate enforcement problems between a borrower and his creditor and (ii) to mitigate enforcement problems among creditors. Whereas much of the finance literature has focused on the first role of collateral, we focus on the second. In this subsection, we briefly discuss a model in which both roles of collateral are present at once. Our analysis suggests that the first role of collateral dominates for low pledgeability, whereas the second role dominates for high pledgeability, so collateralization is a u-shaped function of θ.

Here we consider the following twist on the baseline model. If B borrows via secured debt, the proportion of pledgeable cash flows is $\theta^s := s\theta$, whereas if B borrows via unsecured debt, the proportion of pledgeable cash flows is $\theta^u := u\theta$. We assume not only that collateralization establishes exclusivity, as in the baseline model, but also that collateralization increases pledgeability, i.e. that $\mu\theta^s > \theta^u$ or $\mu s > u$.

We focus on the case in which B always has sufficient pledgeable cash flow to fund Project 0 via secured debt, i.e. $\mu\theta^s X_0 > I_0$. Further, for simplicity, we assume that $p=0$, so $X_1 = X_1^L$ for sure, but B wants to undertake Project 1 anyway, to benefit from diluting C_0.[30]

Proposition 14. (Two roles of collateral.) *B collateralizes Project 0 whenever θ is sufficiently small or sufficiently large, i.e.*

$$\theta < \frac{I_0}{uX_0} \quad \text{or} \quad \theta \geq \frac{I_1}{\mu s(X_0 + X_1^L)}. \tag{34}$$

[30]We take B's incentive to undertake Project 1 has an assumption here (cf. footnote 26). This is just for simplicity, however. A parameter restriction analogous to Parameter Restriction 4 would generate this endogenously, as in the baseline model (Proposition 3).

For low θ, B borrows with collateral to increase his pledgeable cash flow—otherwise he could not borrow from C_0 to get the Project 0 off the ground. For high θ, B borrows with collateral to offer protection against the claims of other creditors—otherwise he could borrow from C_1 with collateral, diluting C_0's debt, as in the baseline model.

7 Conclusion

In this paper, we considered a model in which collateral serves to protect creditors against the claims of competing creditors; in particular, collateral protects old creditors against dilution by new creditors. This role of collateral leads to a paradox of pledgeability: borrowers are more reliant on collateral when cash flow pledgeability is high. This is because increasing pledgeability makes it easier to borrow, and thus easier to dilute existing creditors. Existing creditors, in turn, require collateral to protect against this dilution. In other words, looser borrowing constraints tomorrow can lead to tighter borrowing constraints today.

This reliance on collateral leads to a collateral overhang problem, whereby collateralized assets are encumbered and cannot be used to raise liquidity. We find that reducing the supply of collateral may mitigate this problem. The reason is that decreasing the supply of collateral may prevent a collateral rat race, whereby new creditors use collateral to effectively dilute exiting creditors and existing creditors, in turn, use collateral to protect against being diluted. Our results cast doubt on the idea that policy makers should be focused on increasing the supply of high-quality collateral to combat a perceived collateral shortage.

A Proofs

A.1 Proof of Proposition 1

The argument is in the text. □

A.2 Proof of Proposition 2

To see the outcome of exclusive contracting, suppose that B borrows from C_0 at the risk-free rate, $F_0 = I_0$. Since the contract with C_0 is exclusive, B must borrow from C_0 at Date 1. By Parameter Restriction 2, B can borrow from $C_1 = C_0$ if and only if $X_1 = X_1^H$, since C_0 lends at Date 1 only if its total surplus from the *two* loans increases. Thus, he can undertake Project 1 if and only if $X_1 = X_1^H$. In summary, B invests in Project 0 and invests in Project 1 when $X_1 = X_1^H$. Since $X_1 = X_1^H$ with probability p, the expected surplus is

$$\text{expected surplus } = X_0 - I_0 + p\big(X_1^H - I_1\big), \tag{35}$$

as stated in the proposition. □

A.3 Proof of Lemma 1

The argument is in the text. □

A.4 Proof of Lemma 2

Suppose that B borrows from C_0 via unsecured debt with face value F_0. If $X_1 = X_1^L$, then B cannot borrow from C_1 via unsecured debt (by Lemma 1), but B can borrow from C_1 via secured debt whenever equation (9) holds, which is equivalent to $\theta > \theta^*$ as defined in the lemma. The lemma says that indeed B does borrow from C_1 via secured debt when $\theta > \theta^*$. To prove this, we compare the payoff that B gets from borrowing from C_1 via secured debt with the payoff B gets from not borrowing from C_1. We prove this lemma under the additional hypothesis that $F_0 \leq \theta X_0$ and we verify this in the proof of Proposition 3 below (cf. equations (51) and (**??**)).

If B borrows via secured debt, he does not have sufficient pledgeable cash flows to repay both C_0 and C_1, so he defaults (this follows from Parameter Restriction 3). If B does not borrow, he does have sufficient pledgeable cash flow to repay C_0, his only creditor, so he does not default (this follows from the fact that $F_0 \leq \theta X_0$). Thus, B prefers to borrow from C_1

via secured debt as long as

$$\mu(1-\theta)\big(X_0 + X_1^L\big) > X_0 - F_0 \tag{36}$$

or

$$X_1^L > \frac{\big(1-\mu(1-\theta)\big)X_0 - F_0}{\mu(1-\theta)}, \tag{37}$$

which is always satisfied by Parameter Restriction 4 and the fact that $F_0 \geq I_0$. So B borrows secured from C_0 and invests in the negative NPV project.

A.5 Proof of Proposition 3

Before we give the proof of the equilibrium, we prove a preliminary result that we employ later. The result says B always prefers to borrow via riskless unsecured debt than riskless secured debt.

LEMMA 6. *Suppose B has borrowed from C_0 via unsecured debt and that B can borrow from C_1 via unsecured debt and not default, i.e.*

$$\theta\big(X_0 + X_1\big) \geq F_0 + I_1. \tag{38}$$

B prefers to borrow from C_1 via unsecured debt than via secured debt.

Proof. Here we suppose that B has unsecured debt to C_0 with face value F_0 and we compare B's payoff from borrowing from C_1 via unsecured debt and via secured debt.

If B borrows from C_1 via unsecured debt, he does not default by assumption (equation (38)). Thus, his payoff is

$$\Pi_{\mathrm{B}}^{\text{unsec.}} = X_0 + X_1 - F_0 - I_1. \tag{39}$$

Observe that this is larger than the payoff if B defaults and diverts the fraction $1-\theta$ of his cash flow:

$$X_0 + X_1 - F_0 - I_1 = \theta\big(X_0 + X_1\big) + (1-\theta)\big(X_0 + X_1\big) - F_0 - I_1 \tag{40}$$

$$\geq (1-\theta)\big(X_0 + X_1\big), \tag{41}$$

since, by the no-default assumption, $\theta\big(X_0 + X_1\big) \geq F_0 + I_1$.

Now turn to the case in which B borrows from C_1 via secured debt. In this case, he may or may not default with C_0. Denoting the total final payoff by W, as in equation (1), B's payoff is

$$\Pi_{\mathrm{B}}^{\text{sec.}} = \max\left\{W - F_0 - I_1\,,\,(1-\theta)W\right\}. \tag{42}$$

We can see immediately that this is less than $\Pi_B^{\text{unsec.}}$ above as follows: if B borrows secured, then $W < X_0 + X_1$, since $\mu < 1$. Thus, first term in the max function is less than the expression in equation (39) and the second term in the max function is less than the expression in equation (41). □

We now proceed with the construction of the equilibrium, given that B borrows from C_0 via unsecured debt. We break the proof up for different regions of the parameter space: we analyze first the case in which θ is low, then the case in which θ is high and p is high, and finally the case in which θ is high and p is low.

Low pledgeability: $\theta \leq \theta^*$. For $\theta \leq \theta^*$, we proceed as follows. We assume that B borrows from C_0 via risk-free debt. We show that B borrows from C_1 via risk-free junior debt when $X_1 = X_1^H$ and does not borrow from C_1 when $X_1 = X_1^L$. We confirm that B's initial debt to C_0 is indeed risk free.

Suppose that $\theta \leq \theta^*$ and that B borrows from C_0 via risk-free debt, so that $F_0 = I_0$. If $X_1 = X_1^H$, then B has sufficient pledgeable cash flow to borrow from C_1 via unsecured risk-free debt by Parameter Restriction 3 which says $\theta(X_0 + X_1^H) \geq I_0 + I_1$. By Lemma 6 above, B indeed borrows via unsecured debt rather than secured debt.

If $X_1 = X_1^L$, B cannot borrow from C_1 via unsecured debt (by Lemma 1) or via secured debt (by Lemma 2).

We now show that B's debt to C_0 is indeed risk free. First observe that if $X_1 = X_1^H$, then B repays both C_0 and C_1 since $\theta(X_0 + X_1^H) \geq I_0 + I_1 = F_0 + F_1$, having used Parameter Restriction 3 and the fact that the risk-free rate is zero. Now observe that when $X_1 = X_1^L$, B repays C_0 since B does not borrow from C_1 (since θ is low) and $\theta X_0 > I_0$ by Parameter Restriction 2.

High pledgeability and high probability that $X_1 = X_1^H$. Recall that B borrows secured when $X_1 = X_1^L$ (Lemma 2). There are three ways to borrow secured: (i) collateralize only Project 1, (ii) collateralize only Project 0, and (iii) collateralize both projects. Case (i) is infeasible because $\mu\theta X_1 < I_1$. Case (ii) is preferable to case (iii) because the deadweight loss from collateralization is lower. We now show that case (ii) arises when $\theta \geq \theta^{**}$ and $p \geq p^{**}$ and that case (iii) arises when $\theta^* < \theta < \theta^{**}$ and $p \geq p^*$.

(ii) For $\theta > \theta^{**}$ and $p \geq p^{**}$, we proceed as follows. We assume that B borrows from C_0 via risky debt with face value F_0, where

$$I_0 < F_0 \leq \theta\big(X_0 + X_1^H\big) - I_1. \tag{43}$$

We then show that, given this condition, B borrows from C_1 via risk-free junior debt when $X_1 = X_1^H$ and borrows from C_1 via risk-free secured debt when $X_1 = X_1^L$. We confirm that the face value F_0 of B's initial debt to C_0 is indeed in the range specified in equation (43).

Suppose that B borrows from C_0 via risky debt, so $F_0 > I_0$. Suppose also that F_0 is lower than the upper bound in equation (43) above. Note that this implies that $F_0 < \theta X_0$ since, simply rearranging equation (43) implies that

$$F_0 < \theta X_0 + \big(\theta X_1^H - I_1\big) \tag{44}$$

and $\theta X_1^H < I_1$ by Parameter Restriction 2. If $X_1 = X_1^H$, then B has sufficient pledgeable cash flow to borrow from C_1 via unsecured risk-free debt by the hypothesis in equation (43). By Lemma 6 above, B indeed borrows via unsecured debt rather than secured debt. Thus, B is repaid in full if $X_1 = X_1^H$.

If $X_1 = X_1^L$, B borrows secured from C_1 (collateralizing only Project 0) and invests in the negative NPV project (by Lemma 2). Thus, B defaults on his debt to C_0 when $X_1 = X_1^L$. C_0 gets the pledgeable cash flow after B has repaid C_1:

$$\text{repayment to } \mathrm{C}_0 \text{ if } X_1^L \; = \theta\big(\mu X_0 + X_1^L\big) - I_1. \tag{45}$$

We now show that the face value F_0 of B's debt to C_0 is in the range given in equation (43). The fact that $F_0 > I_0$ follows from the fact that B defaults when $X_0 = X_1^L$, since $\theta\big(\mu X_0 + X_1^L\big) - I_1 < I_0$ by Parameter Restriction 2. We now show that F_0 is less than the upper bound in equation (43). Given the analysis above, C_0's break-even condition reads

$$I_0 = pF_0 + (1-p)\left(\theta\big(\mu X_0 + X_1^L\big) - I_1\right) \tag{46}$$

so

$$F_0 = \frac{I_0 - (1-p)\Big(\theta\big(\mu X_0 + X_1^L\big) - I_1\Big)}{p}. \tag{47}$$

Thus, F_0 is less than the required upper bound whenever

$$\frac{I_0 - (1-p)\Big(\theta\big(\mu X_0 + X_1^L\big) - I_1\Big)}{p} \leq \theta\big(X_0 + X_1^H\big) - I_1. \tag{48}$$

We can rewrite this condition as

$$p \geq \frac{I_0 + I_1 - \theta\big(\mu X_0 + X_1^L\big)}{\theta\big(X_0 + X_1^H\big) - \theta\big(\mu X_0 + X_1^L\big)} \equiv p^{**}, \tag{49}$$

which is satisfied by assumption.

(iii) For $\theta^* < \theta < \theta^{**}$ and $p \geq p^*$, we proceed as follows. We assume that B borrows from C_0 via risky debt with face value F_0, where

$$I_0 < F_0 \leq \theta\big(X_0 + X_1^H\big) - I_1. \tag{50}$$

We then show that, given this condition, B borrows from C_1 via risk-free junior debt when $X_1 = X_1^H$ and borrows from C_1 via risk-free secured debt when $X_1 = X_1^L$. We confirm that the face value F_0 of B's initial debt to C_0 is indeed in the range specified in equation (50).

Suppose that B borrows from C_0 via risky debt, so $F_0 > I_0$. Suppose also that F_0 is lower than the upper bound in equation (50) above. Note that this implies that $F_0 < \theta X_0$ since, simply rearranging implies that

$$F_0 < \theta X_0 + \big(\theta X_1^H - I_1\big) \tag{51}$$

and $\theta X_1^H < I_1$ by Parameter Restriction 2. If $X_1 = X_1^H$, then B has sufficient pledgeable cash flow to borrow from C_1 via unsecured risk-free debt by the hypothesis in equation (50). By Lemma 6 above, B indeed borrows via unsecured debt rather than secured debt. Thus, B is repaid in full if $X_1 = X_1^H$.

If $X_1 = X_1^L$, B borrows secured from C_1 (collateralizing both Project 0 and Project 1) and invests in the negative NPV project (by Lemma 2). Thus, B defaults on his debt to C_0 when $X_1 = X_1^L$. C_0 gets the pledgeable cash flow after B has repaid C_1:

$$\text{repayment to } C_0 \text{ if } X_1^L \ = \theta\mu\big(X_0 + X_1^L\big) - I_1. \tag{52}$$

We now show that the face value F_0 of B's debt to C_0 is in the range given in equation (50). The fact that $F_0 > I_0$ follows from the fact that B defaults when $X_0 = X_1^L$, since $\theta\mu\big(X_0 + X_1^L\big) - I_1 < I_0$ by Parameter Restriction 2. We now show that F_0 is less than the upper bound in equation (50). Given the analysis above, C_0's break-even condition reads

$$I_0 = pF_0 + (1-p)\left(\theta\mu\big(X_0 + X_1^L\big) - I_1\right) \tag{53}$$

so

$$F_0 = \frac{I_0 - (1-p)\Big(\theta\mu\big(X_0 + X_1^L\big) - I_1\Big)}{p}. \tag{54}$$

Thus, F_0 is less than the required upper bound whenever

$$\frac{I_0 - (1-p)\Big(\theta\mu\big(X_0 + X_1^L\big) - I_1\Big)}{p} \leq \theta\big(X_0 + X_1^H\big) - I_1. \tag{55}$$

We can rewrite this condition as

$$p \geq \frac{I_0 + I_1 - \theta\mu\big(X_0 + X_1^L\big)}{\theta\big(X_0 + X_1^H\big) - \theta\mu\big(X_0 + X_1^L\big)} \equiv p^*, \tag{56}$$

which is satisfied by assumption.

High pledgeability and low probability that $X_1 = X_1^H$. For high θ and low p, we proceed as follows. We first explain that the analysis above implies that B defaults when $X_1 = X_1^L$ and therefore B must repay $F_0 > \theta\big(X_0 + X_1^H\big) - I_1$ when $X_1 = X_1^H$, given that p is small. We then argue that this repayment is infeasible.

The analysis of cases (ii) and (iii) above implies that B borrows from C_1 when $X_1 = X_1^L$ and defaults on his debt to C_0, making a repayment less than I_0 (given in equations (45) and (52)).

C_0's break-even condition implies that F_0 must be larger than $\theta\big(X_0 + X_1^H\big) - I_1$ (this is implied by equation (49) and (56) and the analysis that precedes them). Thus, F_0 must be so high that B cannot borrow from C_1 via unsecured debt if $X_1 = X_1^H$. If B borrows via unsecured debt, B defaults on his debt to C_0 and C_0's break-even condition is violated. □

A.6 Proof of Lemma 3

The argument is in the text. □

A.7 Proof of Lemma 4

The argument is in the text. □

A.8 Proof of Lemma 5

The argument is in the text. □

A.9 Proof of Proposition 4

The proof proceeds as follows. We first recall that if B borrows from C_0 via secured debt, then (i) B's debt to C_0 is risk free and (ii) B does not borrow from C_1 when $X_1 = X_1^L$. We then analyze what happens when $X_1 = X_1^H$, which depends on the collateralizability μ.

If B borrows from C_0 via secured debt, C_0 is effectively always a senior clamant on the pledgeable collateralized cash flows from Project 0, $\theta\mu X_0$. Since this is greater than the cost I_0 of Project 0 (by Parameter Restriction 2), B can borrow from C_0 risk free.

Recall also that Lemma 3 says that if B borrows from C_0 via risk-free secured debt, then B does not borrow from C_1 when $X_1 = X_1^L$.

We now analyze what happens when $X_1 = X_1^H$. Given that B has borrowed from C_0 via secured debt, he never borrows from C_1 via secured debt (by Lemma 3). If B borrows from C_1 via unsecured debt, C_1 is effectively junior to C_0. Thus, C_0 lends only if B's pledgeable cash flow *net repayment to* C_0 exceeds the cost of Project 1, or

$$I_1 \leq \theta\big(\mu X_0 + X_1\big) - I_0 \tag{57}$$

or

$$\mu \geq 1 - \frac{\theta\big(X_0 + X_1^H\big) - I_0 - I_1}{\theta X_0} \equiv \mu^*. \tag{58}$$

Thus, when $\mu \geq \mu^*$, B borrows from C_1 and invests in Project 1 when $X_1 = X_1^H$, but when $\mu < \mu^*$, B is constrained and does not invest in Project 1 when $X_1 = X_1^H$. □

A.10 Proof of Proposition 5

The result follows immediately from comparing the expression for Π_B^u in equation (16) with the expression for Π_B^s in equation (21).

A.11 Proof of Corollary 1

The statement follows immediately from Proposition 5.

A.12 Proof of Proposition 6

The argument is in the text. □

A.13 Proof of Proposition 7

The argument is in the text. □

A.14 Proof of Proposition 8

The argument is in the text. □

A.15 Proof of Proposition 9

The argument is in the text. □

A.16 Proof of Proposition 10

The argument is in the text. □

A.17 Proof of Proposition 11

Much of the argument is already in the text. However, there are a few gaps to fill in. Most importantly, we argued that C_0 does not lend unsecured if the probability p that Project 1 has the high payoff is sufficiently small. It remains to show that any $p < p^{\text{p.p.}}$ is indeed "sufficiently small." Below we complete the proof. We first summarize the case in which $\theta < \theta^{\text{p.p.}}$ (as defined in equation (25)) and then proceed to analyze the case in which $\theta \geq \theta^{\text{p.p.}}$.

Low pledgeability: $\theta < \theta^{\text{p.p.}}$. When $\theta < \theta^{\text{p.p.}}$, B cannot borrow when $X_1 = X_1^L$, as shown in analysis leading up to equation (25). In contrast, when $X_1 = X_1^H$, B borrows via risk-free unsecured debt. To see this, note that B prefers to borrow via unsecured debt than via secured debt (by Lemma 6) and that B has sufficient pledgeable cash flow to borrow (by Parameter Restriction 2). Thus, C_0 and C_1 both lend via risk-free unsecured debt, as stated in the proposition.

High pledgeability: $\theta \geq \theta^*$. For $\theta \geq \theta^*$, we proceed as follows. We first analyze B's repayments to C_0 and C_1 when $X_1 = X_1^L$. We show that B does not repay C_0 in full. We then ask under what circumstances B can promise C_0 a high enough repayment when $X_1 = X_1^H$ to offset this loss when $X_1 = X_1^L$. This analysis gives the threshold $p^{\text{p.p.}}$ given in the proposition.

When $X_1 = X_1^L$, B can borrow from C_1 via secured debt, as shown in analysis leading up to equation (25). Further, recall that B cannot borrow via unsecured debt (by Parameter Restriction 2) and, further, that B prefers to borrow than not to borrow (by Parameter Restriction 4). Given that C_1 breaks even, B's repayment to C_0 when $X_1 = X_1^L$ is given by B's total pledgeable cash flow minus the repayment I_1 to C_1:

$$\text{repayment to } C_0 \text{ if } X_1^L \;= \theta\left(X_0 + X_1^L\right) - I_1 \tag{59}$$

as shown in the text (equation (27)). This is less than I_0 by Parameter Restriction 2. Thus, it constitutes a default on C_0's debt. We now ask whether B can promise to repay C_0 enough when $X_1 = X_1^H$ to compensate C_0 for this loss when $X_1 = X_1^L$.

When $X_1 = X_1^H$, B makes the repayment F_0 to C_0. F_0 must satisfy two conditions (i) C_0's break-even condition and (ii) B's limited liability constraint if $X_1 = X_1^H$ (where by "limited liability constraint" we mean that B's total repayment to all his creditors cannot exceed his pledgeable cash flow). C_0's break-even condition reads:

$$I_0 = pF_0 + (1-p)\Big(\theta\left(X_0 + X_1^L\right) - I_1\Big), \tag{60}$$

having substituted in from equation (59) above. B's limited liability constraint if $X_1 = X_1^H$ reads:[31]

$$\theta\big(X_0 + X_1^H\big) \geq F_0 + F_1 = F_0 + I_1. \tag{61}$$

Substituting the expression for F_0 implied by the break-even condition in equation (60) into this the limited liability constraint above implies that we must have

$$\theta\big(X_0 + X_1^H\big) \geq \frac{I_0 - (1-p)\Big(\theta\big(X_0 + X_1^L\big) - I_1\Big)}{p} + I_1 \tag{62}$$

which can be re-written as

$$p \geq \frac{I_0 + I_1 - \theta\big(X_0 + X_1^L\big)}{\theta\big(X_1^H - X_1^L\big)} \equiv p^{\text{p.p.}}, \tag{63}$$

where $p^{\text{p.p.}}$ is defined in equation (28). Thus, for $\theta \geq \theta^{\text{p.p.}}$ and $p < p^{\text{p.p.}}$, B cannot borrow from C_0, as stated in the proposition. □

A.18 Proof of Proposition 12

In this proof we argue that for high θ the incentive constraint puts an upper bound on B's repayment to C_0 when $X_1 = X_1^L$. This upper bound is less than the size of C_0's loan I_0, so if C_0 lends unsecured, it must take a loss when $X_1 = X_1^L$. If the probability $1-p$ that $X_1 = X_1^L$ is sufficiently large than C_0 will not lend to B via unsecured debt.

First observe that this incentive constraint can bind only if pledgeability is high. If $\theta \leq \theta^*$ as in Proposition 3, then B cannot borrow from C_1, so C_0 does not risk dilution.

For high pledgeability, $\theta > \theta^*$, in contrast, the incentive constraint in equation (29) puts an upper bounds on B's repayment if $X_1 = X_1^L$.

$$F_0^L \leq X_0 - (1-\theta)\mu\left(X_0 + X_1^L\right). \tag{64}$$

[31] Here we have tacitly assumed that B undertakes Project 1 when $X_1 = X_1^H$. This is implied by Parameter Restriction 5. See the the proof of Proposition 3 for further explanation.

Thus, for any feasible repayment $F_0^H \geq I_0$,[32] we can substitute this upper bound into C_0's break-even condition to find a necessary condition for C_0 to lend to B via unsecured debt:

$$\begin{aligned} I_0 &= pF_0^H + (1-p)F_0^L && (65)\\ &\leq pF_0^H + (1-p)\Big[X_0 - (1-\theta)\mu\left(X_0 + X_1^L\right)\Big]. && (66) \end{aligned}$$

Observe that Parameter Restriction 4 says that the term in square brackets above is less than I_0,

$$I_0 - \Big[X_0 - (1-\theta)\mu\left(X_0 + X_1^L\right)\Big] > 0. \tag{67}$$

Thus, we can rewrite the necessary condition as

$$p \geq \frac{I_0 - \Big[X_0 - (1-\theta)\mu\left(X_0 + X_1^L\right)\Big]}{F_0^H - \Big[X_0 - (1-\theta)\mu\left(X_0 + X_1^L\right)\Big]}. \tag{68}$$

Observe that the right-hand side above is positive. Thus, p must be sufficiently large in order for C_0 to lend to B via unsecured debt. In other words, given that $\theta > \theta^*$, for small p C_0 lends only via secured debt, as desired.

The expression for the cutoff $p^{\text{c.d.}}$ in equation (31) comes from considering the loosest lower bound in equation (68) above. This follows by considering the largest feasible repayment $F_0^H = \theta(X_0 + X_1^H)$. □

A.19 Proof of Proposition 13

The result follows immediately from the fact that B has no cash flows at Date 0, so C_0 has zero recovery value in the event of liquidation. Thus, C_0 always prefers to accept a rescheduling to Date 2 than to liquidate at Date 1. Hence, renegotiation-proof one-period contracts do not improve on the two-period contracts we focus on in the baseline model.[33]

A.20 Proof of Proposition 14

B can finance Project 0 only if his pledgeable cash flow exceeds I_0. B borrows from C_0 via unsecured debt if (i) Project 0's unsecured pledgeable cash flows are sufficient to cover the investment and (ii) C_0 is not at risk of dilution by the new debt to C_1. Condition (i) says

[32] It is without loss of generality to restrict attention to the case in which $F_0^H \geq I_0$. Otherwise, C_0 is repaid less than I_0 not only when $X_1 = X_1^L$ but also when $X_1 = X_1^H$ and C_0 will not lend unsecured.

[33] This result is subject to the caveats about the timing of renegotiation in footnote 23 and about contingent debt in Subsection 6.3.

that

$$\theta^u X_0 > I_0 \tag{69}$$

and condition (ii) says that

$$\mu\theta^s(X_0 + X_1^L) \leq I_1. \tag{70}$$

Substituting $\theta^u = u\theta$ and $\theta^s = s\theta$ gives the conditions in the proposition.

References

Acharya, V. and A. Bisin (2014). Counterparty risk externality: Centralized versus over-the-counter markets. *Journal of Economic Theory 149*(C), 153–182.

Admati, A. R., P. M. DeMarzo, M. F. Hellwig, and P. Pfleiderer (2013, August). The Leverage Ratchet Effect. Working paper series of the Max Planck Institute for Research on Collective Goods, Max Planck Institute for Research on Collective Goods.

Attar, A., C. Casamatta, A. Chassagnon, and J.-P. Décamps (2015). Multiple lenders, strategic default, and the role of debt covenants. Technical report.

Ayotte, K. and P. Bolton (2011). Optimal property rights in financial contracting. *Review of Financial Studies 24*(10), 3401–3433.

Bebchuk, L. A. and J. M. Fried (1996). The uneasy case for the priority of secured claims in bankruptcy. *The Yale Law Journal 105*(4), 857–934.

Benmelech, E. and N. K. Bergman (2009, March). Collateral pricing. *Journal of Financial Economics 91*(3), 339–360.

Benmelech, E. and N. K. Bergman (2011). Bankruptcy and the collateral channel. *The Journal of Finance 66*(2), 337–378.

Bhattacharya, S. and A. Faure-Grimaud (2001). The debt hangover: Renegotiation with noncontractible investment. *Economics Letters 70*(3), 413–419.

Bisin, A. and P. Gottardi (1999). Competitive equilibria with asymmetric information. *Journal of Economic Theory 87*, 1–48.

Bisin, A. and P. Gottardi (2003). Competitive markets for non exclusive contracts with adverse selection: The role of entry fees. *Review of Economic Theory 6*(2), 313–338.

Bisin, A. and A. A. Rampini (2005). Exclusive contracts and the institution of bankruptcy. *Economic Theory 27*(2), 277–304.

Bizer, D. S. and P. M. DeMarzo (1992, February). Sequential Banking. *Journal of Political Economy 100*(1), 41–61.

Bjerre, C. S. (1999). Secured transactions inside out: Negative pledge covenants, property and perfection. *Cornell Law Review*.

Bolton, P. and M. Oehmke (2015). Should derivatives be privileged in bankruptcy? *Journal of Finance 70*(6), 2352–2394.

Bolton, P. and D. S. Scharfstein (1990, March). A theory of predation based on agency problems in financial contracting. *American Economic Review 80*(1), 93–106.

Bolton, P. and D. S. Scharfstein (1996, February). Optimal Debt Structure and the Number of Creditors. *Journal of Political Economy 104*(1), 1–25.

Brunnermeier, M. and M. Oehmke (2013). The maturity rat race. *The Journal of Finance 68*(2), 483–521.

Caballero, R. (2006). On the macroeconomics of asset shortages.

Degryse, H., V. Ioannidou, and E. von Schedvin (2016). On the Non-Exclusivity of Loan Contracts: An Empirical Investigation. *Management Science*.

Deloitte Blogs (2014). Asset encumbrance: The elephant in the room?

Di Maggio, M. and A. Tahbaz-Salehi (2015). Collateral shortages and intermediation networks. Technical report, Columbia University.

Donaldson, J. R. and E. Micheler (2016). Resaleable debt and systemic risk. Technical report, Washington University.

Donaldson, J. R., G. Piacentino, and A. Thakor (2016). Intermediation variety. Technical report, Washington University in St Louis.

Eisfeldt, A. L. and A. A. Rampini (2009). Leasing, ability to repossess, and debt capacity. *Review of Financial Studies 22*(4), 1621–1657.

Ernst & Young (2010). SEC proposal: Short-term borrowings and disclosures in MD&A. `http://www.ey.com/Publication/vwLUAssets/HotTopic_CC0303_SEC_22September2010/$FILE/Hot`

Faure-Grimaud, A. and D. Gromb (2004). Public trading and private incentives. *Review of Financial Studies 17*(4), 985–1014.

Hart, O. and J. Moore (1995, June). Debt and Seniority: An Analysis of the Role of Hard Claims in Constraining Management. *American Economic Review 85*(3), 567–85.

Holmström, B. and J. Tirole (1997, August). Financial Intermediation, Loanable Funds, and the Real Sector. *The Quarterly Journal of Economics 112*(3), 663–91.

Holmström, B. and J. Tirole (1998). Private and public supply of liquidity. *Journal of Political Economy 106*(1), 1–40.

Homquist, E. and J. Gallin (2014). Repurchase agreements in the financial accounts of the United States. http://www.federalreserve.gov/econresdata/notes/feds-notes/2014/repurchase-agreements-in-the-financial-accounts-of-the-united-states-20140630.html.

Kahn, C. M. and D. Mookherjee (1998). Competition and incentives with nonexclusive contracts. *The RAND Journal of Economics 29*(3), 443–465.

Kaminska, I. (2011, November 25). Manufacturing quality collateral. *Financial Times: FTAlphaville*.

Kiyotaki, N. (1998). Credit and business cycles. *Japanese Economic Review 49*, 19–36.

Kiyotaki, N. and J. Moore (2000). Inside money and liquidity. working paper.

Kiyotaki, N. and J. Moore (2001). Evil is the root of all money. Clarendon Lectures, Oxford.

Kronman, A. T. and T. H. Jackson (1979). Secured financing and priorities among creditors. *Yale Law Journal 88*.

Leitner, Y. (2012). Inducing agents to report hidden trades: A theory of an intermediary. *Review of Finance 16*, 1013–1042.

Listokin, Y. (2008). Is secured debt used to redistribute value from tort claimants in bankruptcy? an empirical analysis. *Duke Law Journal 57*(4), 1037–1079.

Myers, S. C. and R. G. Rajan (1998). The paradox of liquidity. *The Quarterly Journal of Economics 113*(3), 733–771.

Parlour, C. A. and U. Rajan (2001). Competition in loan contracts. *American Economic Review 91*(5), 1311–1328.

Petersen, M. A. and R. G. Rajan (1995). The effect of credit market competition on lending relationships. *The Quarterly Journal of Economics 110*(2), 407–43.

Rampini, A. A., A. Sufi, and S. Viswanathan (2014). Dynamic risk management. *Journal of Financial Economics 111*(2), 271 – 296.

Rampini, A. A. and S. Viswanathan (2013). Collateral and capital structure. *Journal of Financial Economics 109*(2), 466 – 492.

Schwarcz, S. L. (1997). The Easy case for the priority of secured claims in bankruptcy. *Duke Law Journal*.

Schwartz, A. (1984, October). The continuing puzzle of secured debt. *Vanderbilt Law Review 37*(5).

Schwartz, A. (1997). Priority contracts and priority in bankruptcy. Yale school of management working papers, Yale School of Management.

Segal, I. and M. D. Whinston (2012). *Property Rights
[The Handbook of Organizational Economics]*. Introductory Chapters. Princeton University Press.

Stulz, R. and H. Johnson (1985, December). An analysis of secured debt. *Journal of Financial Economics 14*(4), 501–521.

Tirole, J. (2006). *The Theory of Corporate Finance*. Princeton University Press.