Does Monetary Policy Matter for a Long-Term Investor?*

Abraham Lioui[†]and Andrea Tarelli[‡]

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Abstract

This work studies the impact of central bank activity on the optimal portfolio choice of a long-term investor. The setting proposed incorporates a Taylor rule into an extended term structure model, accounting for macroeconomic risks and equity dynamics. Empirical evidence shows that an actively conservative monetary policy (higher weight on price stability relative to real economic activity) provides a better hedge of inflation, increases nominal bond volatility and leads to a reduction of the positions in risky assets, as well as to an overall increase of welfare. Furthermore, when the investor derives utility over real balances, a hedging demand covering instantaneous variations in relative risk aversion appears in the optimal strategy, causing bond positions to be reduced.

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1 Introduction

Following the seminal contribution of Vasicek (1977) and Cox et al. (1985), Duffie and Kan (1996) offered a comprehensive analysis of bond pricing within affine dynamic term structure models. Dai and Singleton (2000) and Duffee (2002) show how to take such affine models to the data and offer ample evidence that dynamic term structure models

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[†]EDHEC Business School, abraham.lioui@edhec.edu
[‡]Bocconi University, andrea.tarelli@unibocconi.it

featuring (predictable) time varying bond risk premia are necessary to fit yield dynamics. Dai and Singleton (2002) show that a Gaussian three-factor dynamic term structure model, estimated with maximum likelihood, captures well the dynamics of the nominal term structure, resolves the Campbell-Shiller expectation puzzle and overperforms other affine specifications with stochastic volatilities.[1]

A lively literature studies the implications of the stochastic nature of interest rates and bond risk premia for long-term investors. Campbell and Viceira (2001) and Brennan and Xia (2002) focus on the role of inflation risk on dynamic asset allocation and assume constant bond risk premia. On the other hand, Sangvinatsos and Wachter (2005) and Koijen et al. (2010) account explicitly for time-varying bond risk premia and show that this time variation generates a significant intertemporal hedging demand for long-term bonds, which, if taken into account, leads to a significant welfare gain.

While early work on dynamic term structure models relied mainly on nominal bond data, the recent decade witnessed a flourishing literature, reviewed in Section 2, concerning monetary policy and the relation between macro variables and the term structure of bond yields. Accounting for monetary policy and its role in the equilibrium of the bond market turned out to be a natural setting to bring about a role for macroeconomic variables into dynamic term structure models. As put forward by Taylor (1993), the Central Bank adjusts the lending rate according to two main objectives: guaranteeing the stability of prices and favouring economic growth. This can be modelled through a Taylor rule, where the short-term rate depends, among other factors, on variables related to past, current or expected inflation and variables directly linked with real economy, such as the output gap. The relative importance of these two components determines whether the priority of the Central Bank is respectively price stability or economic growth.

The aim of this paper is determining the impact of monetary policy on the optimal portfolio choice of a long-term investor. The short-term rate is adjusted according to a forward-looking Taylor rule, hence relatively to the levels of expected inflation and expected output gap. We show that the optimal allocation is particularly sensitive to the weight given to expected inflation and, only secondarily, to the weight associated to the expected output gap. A more conservative policy, that is when the weight given to expected inflation is high, tends to increase bond volatility, reduces total bond and stock positions, shifts the bond exposure towards longer maturities and increases investor's wealth. We support these results by giving an interpretation based on the impulse responses of the model to macroeconomic shocks, as well as with static comparative analyses. Differently from most asset allocation works, we establish an affine one-to-one relation between latent and macroeconomic variables, thus being able to express all our results in terms of economic quantities rather than of "hidden" states.

In parallel with the study of the effects of monetary policy on asset allocation, our

[1] See also the reviews in Dai and Singleton (2003) and Duffee (2013).

work makes several other contributions. First of all, the dynamic term structure literature incorporating macroeconomic risks focuses in general on the bond market and ignores the stock market. Our asset universe includes both the bond and the equity markets, which parameters are estimated simultaneously and, although the presence of the stock does not significantly affect the goodness of fit of the term structure, in terms of asset allocation we show that monetary policy shifts have a strong impact on the stock/bond mix. Particular attention has been put on the estimation of bond and equity risk premia. Sangvinatsos and Wachter (2005) and Koijen et al. (2010) assume a constant equity premium when estimating their models. A plethora of papers insisted on the time variation in the equity premium and its impact on portfolio choice. To make the model realistic, we thus allow for such time variation in two distinct settings. In a first setting, similarly to the existing literature, we consider a canonical term structure model driven by three yield-based factors, where inflation and equity premium also depend on the yield factors. This setting is widespread in the literature, but has at least two drawbacks: i) since traded bonds perfectly hedge the latent factors, the stock market is useless for the intertemporal hedging of shifts in the opportunity set, tilting the portfolio positions towards bonds and ii) in a multiple-bond universe (three or more), the positions in the bonds are huge and unrealistic, due to the strong correlations amongst observed yields. We thus propose a different specification, with two yield-based factors and a third filtered state variable that, together with the information carried by the yield factors, contributes to the determination of the equity premium and expected inflation. Differently from Wachter (2002), the equity premium depends on information carried both by fixed-income and equity markets and its innovation is not forced to have a perfect negative correlation with unexpected equity returns. Furthermore, the economic variables and the equity premium are not perfectly spanned by bond yields.[2] This allows to better describe the dynamics of macroeconomic variables and risk premia, as well as their co-movements. In particular, in this setting we recover a negative correlation between the dynamics of the expected output gap and that of equity and bond risk premia, which is consistent with the empirical findings on return predictability in Cooper and Priestley (2009) and Campbell et al. (2013).

In terms of portfolio choice, differently from Brennan and Xia (2002) and Sangvinatsos and Wachter (2005), the presence of a state variable carrying information about the equity premium, and independent from the nominal bond market, allows the intervention of the stock, with a strong horizon effect, in the intertemporal hedging component of the portfolio allocation. Another advantage of this setting is also that the optimal strategy can be computed, in nominal market completeness, when the investor has access to two bonds only and the equity index. This prevents the need of introducing a third bond in the investable portfolio, which would cause the bond positions to be huge, because of the

[2] Joslin et al. (2013) show how imposing the spanning condition of macroeconomic variables by bond yields can significantly reduce the goodness of fit of the macro variables.

multicollinearity of nominal bond yields. With respect to the literature considering variable equity premium, since the stock premium is not assumed to be perfectly negatively correlated with stock returns, as it is the case in Wachter (2002), the stock allocation is not strongly increasing with the investment horizon, but it is instead decreasing with the horizon. Although the alternative estimation framework we propose has several advantages over the more standard three-factor term structure model augmented with inflation and stock dynamics, the specific conclusions we draw on the impact of monetary policy on the optimal allocation prove to be robust to the choice of the estimation setting. In both cases, indeed, we obtain the same qualitative pattern for the sensitivity of the key risk/return parameters of the investable universe with respect to shifts in monetary policy variables, which is also reflected in the results we obtain in terms of portfolio strategy and welfare.

A final innovation of our work with respect to the existing asset allocation literature is that we introduce money in the utility function, which has the consequence that relative risk aversion is time-varying. The presence of money in the utility function, as in the model originally developed by Sidrauski (1967) and as considered for example in Bakshi and Chen (1996), Lioui and Poncet (2004) or Lioui and Maio (2013), is a way to take into account the utility derived from the flow of services per unit of time derived from holdings of real cash balances. In other words, the presence of money in the utility function represents an element of friction introduced into preferences in order to take into account the fact that investors do not allocate their total wealth into financial assets, but they instead keep part of their wealth uninvested, as they need to satisfy their short-term spending needs. When the investor benefits from real cash balances, we show that the optimal portfolio strategy is affected, both in terms of instantaneous and intertemporal hedging demands, especially for what concerns bond positions. A substantial instantaneous hedging demand appears due to the time-variation of relative risk aversion introduced by the presence of money in preferences. This component hedges the instantaneous change in the cost of holding real balances, represented by the short-term interest rate. The intertemporal hedging demand is also affected, with a slight decrease of long-maturity bonds exposure for long horizons, as there is a substitution effect between consumption and money and a lower fraction of utility derives from future wealth consumption. As in Aoki et al. (2012), who develop a life-cycle model with shopping costs increasing in consumption and decreasing in real money balances, we conclude that money positions can be substantial. The amount of uninvested capital decreases with the interest rate and increases sharply when the short-term rate approaches zero.

The remainder of the paper is organised as follows. Section 2 provides a review of the literature concerning term structure models accounting for macroeconomic factors and monetary policy. In Section 3 we describe the financial market specification, the investor's preferences and we derive the optimal portfolio strategy. Section 4 presents the dataset used for the estimation, the two different settings considered and comments the estimation results. In Section 7 we discuss the results of the optimal portfolio problem for different investor's preferences and market specifications. We present our conclusions in Section 8. Finally, technical details and mathematical derivations are relegated to an appendix.

2 Related literature

While early works on dynamic term structure models relied mainly on nominal bond data, the recent decade witnessed a flourishing literature concerning monetary policy and the relation between macro variables and the term structure of bond yields. Ang and Piazzesi (2003) develop a no-arbitrage Gaussian model of the yield curve, including both macroeconomic and latent variables, finding that macro factors, mostly inflation, can explain up to 85% of the short- and middle-duration parts of the yield curve. Ang et al. (2007) estimate different monetary policy rules, such as the original Taylor rule, as well as backward- and forward-looking Taylor rules, in a no-arbitrage framework accommodating for time-varying bond premia. They find that inflation and output gap shocks have a large explanatory power of movements of bond excess returns, especially for what concerns the term spread. Bikbov and Chernov (2010) develop a VAR model of the nominal yield curve, with macro and latent factors, proposing a decomposition of the total variation of the yields in a macro component and a residual component of latent factors, orthogonal to the first. They show that inflation and real activity explain the 80% of the variation of the short-term nominal rate and the $54\% - 68\%$ of the variation of term premia. Baele et al. (2010) try to explain the co-movements of bond and stock markets by developing a dynamic factor model in a regime switching framework, distinguishing macroeconomic, risk premium and liquidity factors. Macroeconomic factors play a key role in explaining bond return volatility, which is a relation captured by our model, whilst equity volatility is mostly explained by non-macroeconomic factors (liquidity and option-implied variance premium). Ang et al. (2011) study the effect of monetary policy shifts, by estimating a quadratic term structure model that embeds a Taylor-rule with time-varying weights on inflation and output gap. Chun (2011) analyses the impact of the introduction of survey expectations of macroeconomic variables in a macro-only no-arbitrage dynamic term structure model, concluding that considering forward-looking forecasts significantly enhances the predictive power of the Taylor rule. Joslin et al. (2014) develop an arbitrage-free Gaussian dynamic term structure model where macroeconomic variables are included into the state vector, but they are not perfectly spanned by contemporaneous bond yields. They conclude that macro variables, such as inflation and output, carry additional information related to bond risk premia with respect to the information included in the current shape of the yield curve, allowing to significantly increase predictability of future bond excess returns. Barillas (2010) implements their model and tests the importance of incorporating information from unspanned macro variables into the determination of bond premia, by quantifying its impact on the optimal allocation of a long-term investor. Joslin et al. (2013) show that no-arbitrage macro term structure models, when they do not allow for observation errors of spanned macroeconomic variables, behave in a similar fashion whether or not yield factors are observed with errors. These models have poor bond pricing properties and have predictive properties similar to unconstrained VAR models. However, when the term structure model allows for macroeconomic variables to be observed with errors, the bond pricing properties increase sharply, as the latent factors replicate very accurately the first principal components of bond yields, but the model loses accuracy with respect to the replication of macroeconomic variables. Campbell et al. (2013) study the impact of monetary policy shifts and macroeconomic shocks on nominal bond and stock risk/return behaviours, identifying three subperiods with different monetary regimes and relating the increase of nominal bond volatility and correlation with equity to the higher weight over the inflation component registered in anti-inflationary monetary regimes. Hau and Lai (2013) analyse the cross-country variation of monetary policy within the eurozone, measured by country-based Taylor rule residuals and short-term real rates, finding that a relative decrease in the real interest rate entails an incremental equity fund inflow and an outflow from money market funds.

3 The Optimal Portfolio choice

We first describe the financial market and the asset universe, then the investor's preferences and the budget constraint, and finally we derive the optimal portfolio strategy.

3.1 The financial market

The long-term investor has access to a frictionless and arbitrage-free financial market. We assume the existence of a nominal Stochastic Discount Factor (SDF) with the following dynamics:

$$\frac{\mathrm{d}\Phi_t}{\Phi_t} = -R_t \mathrm{d}t - \mathbf{\Lambda}_t' \mathrm{d}\mathbf{z}_t, \tag{3.1}$$

where R_t stands for the nominal interest rate, $\mathbf{\Lambda}_t$ is a $n_B \times 1$ vector of market prices of risk and $\mathbf{z}_t$ is a $n_B \times 1$ dimensional standard Wiener process. For tractability, we follow the bulk of the dynamic term structure literature[3] and consider an affine setting in which the dynamics of the market prices of risk are affine function of n_X state variables. More

[3] See Dai and Singleton (2003) and Duffee (2013) for references.

precisely, we assume that the time varying vector of market prices of risk has the form:

$$\mathbf{\Lambda}_t = \mathbf{\Lambda}_0 + \mathbf{\Lambda}_1 \mathbf{X}_t, \tag{3.2}$$

where $\mathbf{\Lambda}_0$ is a column vector of constants with n_B elements and $\mathbf{\Lambda}_1$ is an $n_B \times n_X$ matrix where n_X is the number of state variables in the economy and thus $\mathbf{X}_t$ is a $n_X \times 1$ dimensional vector. The vector of state variables has the following mean reverting dynamics:

$$\mathrm{d}\mathbf{X}_t = \mathbf{\Theta}\left(\bar{\mathbf{X}} - \mathbf{X}_t\right)\mathrm{d}t + \mathbf{\Sigma}_X^{'}\mathrm{d}\mathbf{z}_t, \tag{3.3}$$

where $\mathbf{\Theta}$ is a $n_X \times n_X$ matrix of the parameters of speed of mean reversion of the state variables, $\bar{\mathbf{X}}$ is the $n_X \times 1$ vector of long run means and $\mathbf{\Sigma}_X$ the $n_B \times n_X$ volatility matrix.

To complete the description of the nominal bond market, we need to postulate a process for the dynamics of the nominal short-term rate, R. Following Taylor (1993) and the strand of theoretical as well as empirical literature it spawned, we assume that:

$$R_t = R_0 + \eta\left(\pi_t - \pi_0\right) + \xi\left(\chi_t - \chi_0\right) + \upsilon_t \tag{3.4}$$

where R_0, η, ξ, π_0 and χ_0 are constants. π_t stands for the expected inflation and π_0 its long-run target. Similarly, χ_t is the expected output gap and χ_0 the long-run target. Without loss of generality, we assume that the long run targets are equal to the long-run means of the processes π_t and χ_t. υ_t stands for the monetary policy shock.

The specification (3.4) calls for the following precisions. The original Taylor rule (Taylor, 1993) relates the nominal short term rate to current inflation and current real activity (output gap). While some authors extended the original version by including lagged values of the two economic variables (backward looking rules, see Eichenbaum and Evans (1995) and Clarida et al. (1998) for example), others included expected values of inflation and output gap (forward looking rules, see Clarida et al. (2000) for example). For our purposes, having expected inflation instead of realised inflation is of prime importance since it allows us to guaranty that realised inflation is not perfectly spanned by traded assets and thus the real market will be incomplete. The same reasoning is true for the output gap although its innovation do not enter explicitly into investor's wealth dynamics. The second term of the Taylor rule is related to price stabilization and the coefficient η measures the activeness of the Central Bank with respect to this objective. Finally, the last term of the Taylor rule is related to real activity through the expected output gap.

In order to keep the model affine, thus tractable, we assume that η and ξ are constant[4] and specify monetary policy shock, expected inflation and expected output gap as follows:

$$\upsilon_t = \boldsymbol{\upsilon}_1'\mathbf{X}_t, \qquad \pi_t = \pi_0 + \boldsymbol{\pi}_1'\mathbf{X}_t, \qquad \chi_t = \chi_0 + \boldsymbol{\chi}_1'\mathbf{X}_t, \tag{3.5}$$

[4]Ang et al. (2011) advocate time varying monetary policy parameters.

where $\boldsymbol{\upsilon}_1$, $\boldsymbol{\pi}_1$ and $\boldsymbol{\chi}_1$ are $n_X \times 1$ vectors of constants.[5]

Combining the previous assumptions in (3.5) with the Taylor rule (3.4) one realizes that the nominal interest rate R_t can be written as an affine function of the state variables, making it easier both to price bonds and solve for dynamic asset allocation:

$$\begin{aligned} R_t &= R_0 + (\boldsymbol{\upsilon}_1 + \eta \boldsymbol{\pi}_1 + \xi \boldsymbol{\chi}_1)' \mathbf{X}_t && (3.6) \\ &\equiv R_0 + \mathbf{R}_1' \mathbf{X}_t. && (3.7) \end{aligned}$$

In fine, our nominal term structure model is affine and Gaussian, being thus similar to the one used by Brennan and Xia (2002), Sangvinatsos and Wachter (2005) or Koijen et al. (2010). Beyond tractability, Dai and Singleton (2002) and Duffee (2002) have shown that Gaussian dynamic term structure models fit reasonably well the behaviour of observed nominal bond yields. The incorporation of a Taylor rule allows to relate the latent factors of the model to economic variables. For example, the 3-dimensional vector of υ_t, π_t and χ_t, can be written as an affine function of the state variables:

$$\begin{bmatrix} \upsilon_t \\ \pi_t \\ \chi_t \end{bmatrix} = \begin{bmatrix} 0 \\ \pi_0 \\ \chi_0 \end{bmatrix} + \begin{bmatrix} \boldsymbol{\upsilon}_1' \\ \boldsymbol{\pi}_1' \\ \boldsymbol{\chi}_1' \end{bmatrix} \mathbf{X}_t. \tag{3.8}$$

As this 3×3 system is invertible, given a shift $\Delta\upsilon_t$, $\Delta\pi_t$ or $\Delta\chi_t$, the corresponding shift $\Delta\mathbf{X}_t$ can be determined by inverting equation (3.8).[6]

The last component of our description of the nominal sector of the economy is the general price level, denoted with P_t, which allows one to convert nominal values into real values. It is assumed to have the following dynamics:

$$\frac{\mathrm{d}P_t}{P_t} = \pi_t \mathrm{d}t + \boldsymbol{\sigma}_P' \mathrm{d}\mathbf{z}_t. \tag{3.9}$$

Our setting is compatible with the possibility that the innovation in the price index is only partially spanned by the assets available for trade, which means allowing for an incompleteness of the real market.

The long-term investor can trade a nominal risk-less asset, a bank account, earning the instantaneous nominal rate R_t and a set of n_Y non-redundant risky assets, stocks and

[5] In Ang et al. (2011), equation (3) page 433, it is shown that the assumption for the monetary policy shock could be seen as a reduced form for accounting for time varying monetary policy parameters.

[6] In our setting, the set of economic variables used for switching from latent factors to economic variables is clearly not observable. This is trivially true for expected inflation and expected output gap, this is not less true for the short term rate. Although Collin-Dufresne et al. (2008) used the short term rate and its moments instead of latent factors as observables, this assumption is not reasonable in a continuous time setting, where the short term is indeed instantaneous and not observable. As advocated by Duffie and Kan (1996), some imperfections like market (il)liquidity may alter the short term yields and prevent them from mimicking the nominal instantaneous interest rate. For a lucid discussion, see Joslin et al. (2011).

bonds. For a generic nominal zero-coupon bond of maturity τ, which price is denoted with $B_{t,\tau}$, the dynamics can be written as follows (see Appendix A):

$$\frac{\mathrm{d}B_{t,\tau}}{B_{t,\tau}} = \left[R_t + \mathbf{A}_1(\tau)\,\mathbf{\Sigma}_X'\mathbf{\Lambda}_t\right]\mathrm{d}t + \mathbf{A}_1(\tau)\,\mathbf{\Sigma}_X'\mathrm{d}\mathbf{z}_t, \tag{3.10}$$

where $\mathbf{A}_1(\tau)$ is a $1 \times n_X$ vector deterministic function solving a system of ordinary differential equations, given in Appendix A. The stock price dynamics we retain is:

$$\frac{\mathrm{d}S_t}{S_t} = \left(R_t + \boldsymbol{\sigma}_S'\mathbf{\Lambda}_t\right)\mathrm{d}t + \boldsymbol{\sigma}_S'\mathrm{d}\mathbf{z}_t. \tag{3.11}$$

The specification (3.11) embeds another important innovation of our setting. Relative to the dynamic term structure literature, it incorporates information from the stock market into the estimation process beyond the one in macro variables and the bond yields. This extension has already been undertaken by Sangvinatsos and Wachter (2005) or Koijen et al. (2010). Our incremental contribution is by letting the equity premium to be time-varying, while Sangvinatsos and Wachter (2005)[7] or Koijen et al. (2010)[8] assume it is constant.

For the remaining, and to simplify notations, we will denote the nominal price of an arbitrary risky asset at time t by Y_t^i. Its dynamics, that will be helpful to write the budget constraint, takes the following form:

$$\frac{\mathrm{d}Y_t^i}{Y_t^i} = \left(R_t + \boldsymbol{\sigma}_{Y^i}'\mathbf{\Lambda}_t\right)\mathrm{d}t + \boldsymbol{\sigma}_{Y^i}'\mathrm{d}\mathbf{z}_t. \tag{3.12}$$

3.2 The budget constraint

Long-term investors allocate their wealth to consumption and real balances holdings, as well as positions in n_Y risky assets and the bank account. The nominal wealth dynamics can thus be written as:

$$\frac{\mathrm{d}W_t}{W_t} = \sum_{i=1}^{n_Y}\omega_t^i\frac{\mathrm{d}Y_t^i}{Y_t^i} + \left(1 - \sum_{i=1}^{n_Y}\omega_t^i - \frac{M_t}{W_t}\right)R_t\mathrm{d}t - \frac{C_t}{W_t}\mathrm{d}t, \tag{3.13}$$

where ω_t^i stands for the proportion of wealth invested in the risky asset i. Using (3.12), this dynamics could be written as follows:

$$\frac{\mathrm{d}W_t}{W_t} = \left[\boldsymbol{\omega}_t'\mathbf{\Sigma}_Y'\mathbf{\Lambda}_t + \left(1 - \frac{M_t}{W_t}\right)R_t - \frac{C_t}{W_t}\right]\mathrm{d}t + \boldsymbol{\omega}_t'\mathbf{\Sigma}_Y'\mathrm{d}\mathbf{z}_t, \tag{3.14}$$

[7] See equations (46) and (47), page 194.

[8] See equation (5), page 747.

where $\boldsymbol{\omega}_t$ is the $n_Y \times 1$ vector of weights and $\boldsymbol{\Sigma}_Y$ is the volatility matrix of the risky assets which i-th column is $\boldsymbol{\sigma}_{Y^i}$. The dynamics of real wealth obtains by applying Ito's lemma to $W_t P_t^{-1} \equiv w_t$:

$$\begin{aligned}\frac{\mathrm{d}w_t}{w_t} &= \left[\boldsymbol{\omega}_t' \boldsymbol{\Sigma}_Y' \left(\boldsymbol{\Lambda}_t - \boldsymbol{\sigma}_P\right) + \left(1 - \frac{m_t}{w_t}\right) R_t - \pi_t - \frac{c_t}{w_t} + \boldsymbol{\sigma}_P' \boldsymbol{\sigma}_P\right] \mathrm{d}t \\ &\quad + \left(\boldsymbol{\omega}_t' \boldsymbol{\Sigma}_Y' - \boldsymbol{\sigma}_P'\right) \mathrm{d}\mathbf{z}_t. \qquad (3.15)\end{aligned}$$

An alternative way of writing the budget constraint, useful for the application of the martingale approach to dynamic asset allocation, is to proceed as follows. If markets were complete, that is if the assets available for trade were spanning all the uncertainty in the economy, then the SDF in (3.1) could be reverse engineered out of the dynamics of the traded assets. However, when markets are incomplete, knowing the dynamics of the traded assets does not allow to completely identify the market prices of risk. Hence, market prices of risk can be decomposed into two components, the first corresponding to their projection onto the assets available for trade and the second, orthogonal to the first, the remaining component, not spanned by the traded assets. Formally speaking, the market prices of risk can be therefore written as:

$$\boldsymbol{\Lambda}_t = \underbrace{\boldsymbol{\Sigma}_Y \left(\boldsymbol{\Sigma}_Y' \boldsymbol{\Sigma}_Y\right)^{-1} \boldsymbol{\Sigma}_Y' \boldsymbol{\Lambda}_t}_{\boldsymbol{\Lambda}_t^*} + \underbrace{\left(\mathbf{I}_{n_B} - \boldsymbol{\Sigma}_Y \left(\boldsymbol{\Sigma}_Y' \boldsymbol{\Sigma}_Y\right)^{-1} \boldsymbol{\Sigma}_Y'\right) \boldsymbol{\Lambda}_t}_{\boldsymbol{\nu}_t}, \qquad (3.16)$$

where $\boldsymbol{\Lambda}_t^*$ belongs to the column space of the volatility matrix of traded assets and $\boldsymbol{\nu}_t$, orthogonal to $\boldsymbol{\Lambda}_t^*$, belongs to the null space. Each choice of the vector $\boldsymbol{\nu}_s$, for $s \in [0, t]$, characterizes an admissible SDF at time t, for $t \in [0, T]$. Since there is an infinite number of such vectors, the long-term investor faces an infinity of SDFs compatible with the dynamics of the assets available for trade. This family of SDFs can be denoted as $\Phi_t^{\boldsymbol{\nu}}$ defined as follows:

$$\frac{\mathrm{d}\Phi_t^{\boldsymbol{\nu}}}{\Phi_t^{\boldsymbol{\nu}}} = -R_t \mathrm{d}t - \left(\boldsymbol{\Lambda}_t^* + \boldsymbol{\nu}_t\right)' \mathrm{d}\mathbf{z}_t. \qquad (3.17)$$

Applying Ito's lemma to $\Phi_t^{\boldsymbol{\nu}} W_t$, using equations (3.14) and (3.17):

$$\frac{\mathrm{d}\left(W_t \Phi_t^{\boldsymbol{\nu}}\right)}{W_t \Phi_t^{\boldsymbol{\nu}}} = \left[-\frac{M_t}{W_t} R_t - \frac{C_t}{W_t}\right] \mathrm{d}t + \left[\boldsymbol{\omega}_t' \boldsymbol{\Sigma}_Y' - \left(\boldsymbol{\Lambda}_t^* + \boldsymbol{\nu}_t\right)'\right] \mathrm{d}\mathbf{z}_t. \qquad (3.18)$$

Integrating this stochastic differential equation from time t to time T:

$$\begin{aligned}W_T \Phi_T^{\boldsymbol{\nu}} - W_t \Phi_t^{\boldsymbol{\nu}} &= \int_t^T W_s \Phi_s^{\boldsymbol{\nu}} \left[-\frac{M_s}{W_s} R_s - \frac{C_s}{W_s}\right] \mathrm{d}s \\ &\quad + \int_t^T W_s \Phi_s^{\boldsymbol{\nu}} \left[\boldsymbol{\omega}_s' \boldsymbol{\Sigma}_Y' - \left(\boldsymbol{\Lambda}_s^* + \boldsymbol{\nu}_s\right)'\right] \mathrm{d}\mathbf{z}_s. \qquad (3.19)\end{aligned}$$

As there is no utility from bequest, at the optimum it must be verified that $W_T = 0$. Taking the conditional expectation at time t of (3.19) therefore yields:

$$W_t = \mathrm{E}_t \left[\int_t^T \frac{\Phi_s^{\boldsymbol{\nu}}}{\Phi_t^{\boldsymbol{\nu}}} \left[M_s R_s + C_s \right] \mathrm{d}s \right]. \tag{3.20}$$

This provides us with an alternative expression for the budget constraint. It shows that current wealth should finance the future consumption plan as well as the wealth losses associated to real balance holdings, that are equal to the instantaneous opportunity cost of a unit of uninvested money, $R_s \mathrm{d}s$, times the amount of money held, M_s. This expression can be converted in real terms as follows:

$$w_t = \mathrm{E}_t \left[\int_t^T \frac{\phi_s^{\boldsymbol{\nu}}}{\phi_t^{\boldsymbol{\nu}}} \left[m_s R_s + c_s \right] \mathrm{d}s \right], \tag{3.21}$$

where $\phi_t^{\boldsymbol{\nu}} \equiv P_t \Phi_t^{\boldsymbol{\nu}}$ is the real SDF.

Following Brennan and Xia (2002), Sangvinatsos and Wachter (2005) and Koijen et al. (2010), we assume that market incompleteness is mainly brought about by the innovation to the general price level. In particular, we assume that markets are nominally complete. This means that the SDF used to price the nominal bonds can be any of the family $\Phi_t^{\boldsymbol{\nu}}$, meaning that any choice for $\boldsymbol{\nu}$ would provide the same price for nominal bonds.

3.3 Preferences

In the portfolio allocation analysis we consider three different specifications for preferences: isoelastic utility over terminal wealth, over intermediate consumption and over intermediate consumption and real balances. As the first two are standard and almost universally used in asset allocation, in the derivation of the optimal portfolio strategy we focus on the third case, relegating respectively to Appendix D and C the first two cases. In a monetary economy, agents allocate their wealth to cash, risky assets and real balances. The cash is hold in a bank account and yields an interest rate, whilst real balances (currencies) are not remunerated. Consumers incur the opportunity cost of holding part of their wealth into real balances for various reasons. Amongst the traditional ones put forward in the monetary economics literature, one can invoke transaction/search costs saving or cash-in-advance constraints. A particularly useful shortcut to guarantee that money will be held at the optimum, although it is dominated by the cash in terms of remuneration, is the money-in-the-utility-function (MIUF) paradigm. This is the standing approach in this area of economics, whereby the preferences of the consumer are defined over consumption and real balances (Walsh, 2003).

We follow the macroeconomic literature and consider a long-term investor who derives utility at each time t from real consumption, c_t, and from holding real balances, m_t. For tractability, we choose a specification for preferences which features constant elasticity of substitution (CES) between consumption and real balances, namely:

$$u\left(c_t, m_t\right) = \frac{\left[\alpha c_t^{1-\frac{1}{\rho}} + (1-\alpha)\, m_t^{1-\frac{1}{\rho}}\right]^{\frac{1-\gamma}{1-1/\rho}}}{1-\gamma}, \tag{3.22}$$

where α is the share of consumption in the preferences, ρ is the intratemporal elasticity of substitution between consumption and real balances and γ is the curvature of the utility function. The latter parameter is the relative risk aversion in the case $\alpha = 1$, corresponding to the usual Constant Relative Risk Aversion (CRRA) case, but in a setting with money and non-separable intratemporal preferences, as it will be shown in the following, relative risk aversion is stochastically time varying.[9]

Assuming time additivity, the total utility from time t to time T (the investment horizon) can be written as:

$$U_t = \int_t^T e^{-\delta(s-t)} u\left(c_s, m_s\right) \mathrm{d}s. \tag{3.23}$$

Amongst the notable features of this setting for preferences, are the facts that: i) the traditional power utility (CRRA) case is nested ($\alpha = 1$), ii) intertemporal hedging will be driven not only by the usual consumption-to-wealth ratio but also the real money-to-wealth ratio, iii) risk aversion is stochastic and time varying. Indeed, relative risk aversion is given by:

$$RRA_t \equiv -\frac{c_t \frac{\partial^2 u}{\partial^2 c_t}}{\frac{\partial u}{\partial c_t}} = \frac{1}{\rho} + \left(\gamma - \frac{1}{\rho}\right) \frac{\alpha c_t^{1-\frac{1}{\rho}}}{\alpha c_t^{1-\frac{1}{\rho}} + (1-\alpha)\, m_t^{1-\frac{1}{\rho}}} \tag{3.24}$$

$$= \gamma - \left(\gamma - \frac{1}{\rho}\right) \frac{(1-\alpha)\, m_t^{1-\frac{1}{\rho}}}{\alpha c_t^{1-\frac{1}{\rho}} + (1-\alpha)\, m_t^{1-\frac{1}{\rho}}}. \tag{3.25}$$

From (3.24), as it could be noticed already in (3.22), it follows that the separability of preferences in consumption and real balances is obtained whenever $\gamma = \frac{1}{\rho}$. From (3.25) it can be noticed that the impact of money on relative risk aversion depends crucially on the difference between γ and $\frac{1}{\rho}$. Through its impact on consumption and real balances, monetary policy impacts relative risk aversion and hence the optimal allocation. Preferences with money-in-the-utility-function are a new ingredient in the portfolio choice problem and have been introduced in this paper in order to capture the monetary aspect of the economy, which is particularly important for the study of the impact of monetary

[9] An important particular case arises when the elasticity of substitution ρ is equal to 1, in which case the utility function has a Cobb-Douglas functional form: $u\left(c_t, m_t\right) = \frac{1}{1-\gamma}\left[c_t^{\alpha} m_t^{1-\alpha}\right]^{1-\gamma}$.

policy on the optimal allocation of a long-term investor.

3.4 Optimal strategy

To solve for the optimal portfolio choice, we use the martingale approach as developed in Karatzas et al. (1987), Cox and Huang (1989), Cox and Huang (1991), He and Pearson (1991) and Karatzas et al. (1991). Our method is directly inspired from the latter paper, since we allow for market incompleteness, meaning that traded assets cannot span all the uncertainty in the economy. The long-term investor's problem is to find the solution to:

$$\begin{cases} \min\limits_{\{\boldsymbol{\nu_t}\}_{t=0}^{t=T}} \max\limits_{\{c_t, m_t, \boldsymbol{\omega}_t\}_{t=0}^{t=T}} \mathrm{E}_0 \left[\int_0^T e^{-\delta s} u\left(c_s, m_s\right) \mathrm{d}s \right] \equiv J_0(w_0) \\ \text{s.t.} \quad \mathrm{E}_0 \left[\int_0^T \frac{\phi_s^{\boldsymbol{\nu}}}{\phi_0^{\boldsymbol{\nu}}} \left(m_s R_s + c_s\right) \mathrm{d}s \right] = w_0. \end{cases} \tag{3.26}$$

In the case where nominal markets are complete, the optimal portfolio strategy is given in the proposition below.

Proposition 1. *The optimal portfolio strategy is given by:*

$$\begin{aligned} \boldsymbol{\omega}_t =& \frac{1}{\gamma} \left(\boldsymbol{\Sigma}_Y' \boldsymbol{\Sigma}_Y\right)^{-1} \boldsymbol{\Sigma}_Y' \boldsymbol{\Lambda}_t^* + \left(1 - \frac{1}{\gamma}\right) \left(\boldsymbol{\Sigma}_Y' \boldsymbol{\Sigma}_Y\right)^{-1} \boldsymbol{\Sigma}_Y' \boldsymbol{\sigma}_P \\ & - \frac{1 - \frac{1}{\gamma}}{1 - \rho} \left(RRA_t - \frac{1}{\rho}\right)^{-1} \left(\boldsymbol{\Sigma}_Y' \boldsymbol{\Sigma}_Y\right)^{-1} \boldsymbol{\Sigma}_Y' \boldsymbol{\sigma}_{RRA_t} \\ & + \left(\boldsymbol{\Sigma}_Y' \boldsymbol{\Sigma}_Y\right)^{-1} \boldsymbol{\Sigma}_Y' \boldsymbol{\Sigma}_X \frac{\mathbf{F}_{\mathbf{X}}\left(t, \mathbf{X}_t; T\right)}{F\left(t, \mathbf{X}_t; T\right)}, \end{aligned} \tag{3.27}$$

where the time-varying relative risk aversion RRA_t, its volatility $\boldsymbol{\sigma}_{RRA_t}$ and the definition of the function $F\left(t, \mathbf{X}_t; T\right)$ are respectively given by:

$$RRA_t = \frac{1}{\rho} + \left(\gamma - \frac{1}{\rho}\right) \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^\rho R_t^{1-\rho}}, \tag{3.28}$$

$$\boldsymbol{\sigma}_{RRA_t} = - \frac{1 - \rho}{\gamma - \frac{1}{\rho}} \left(RRA_t - \frac{1}{\rho}\right)^2 \left[\frac{\gamma - RRA_t}{RRA_t - \frac{1}{\rho}} \frac{\alpha}{1 - \alpha} \right]^{\frac{\rho}{\rho - 1}} \boldsymbol{\Sigma}_X \mathbf{R}_1, \tag{3.29}$$

$$F\left(t, \mathbf{X}_t; T\right) \equiv E_t \left[\int_t^T e^{-\frac{\delta}{\gamma}(s-t)} \left[\frac{\phi_s^{\boldsymbol{\nu}^*}}{\phi_t^{\boldsymbol{\nu}^*}} \left[\frac{RRA_s - \frac{1}{\rho}}{RRA_t - \frac{1}{\rho}} \right]^{-\frac{1}{1-\rho}} \right]^{1 - \frac{1}{\gamma}} \mathrm{d}s \right], \tag{3.30}$$

where $\phi_t^{\boldsymbol{\nu}^*}$ is the real state-price density, given by:

$$\frac{\mathrm{d}\phi_t^{\boldsymbol{\nu}^*}}{\phi_t^{\boldsymbol{\nu}^*}} = - \left(R_t - \pi_t + \boldsymbol{\sigma}'_P \boldsymbol{\Lambda}_t\right) \mathrm{d}t - \left(\boldsymbol{\Lambda}_t' - \boldsymbol{\sigma}'_P\right) \mathrm{d}\mathbf{z}_t, \tag{3.31}$$

and where $\boldsymbol{\nu}^*$ is constant and given by:

$$\boldsymbol{\nu}^* = (1-\gamma)\left(\mathbf{I}_{n_B} - \boldsymbol{\Sigma}_Y\left(\boldsymbol{\Sigma}_Y'\boldsymbol{\Sigma}_Y\right)^{-1}\boldsymbol{\Sigma}_Y'\right)\boldsymbol{\sigma}_P. \tag{3.32}$$

The optimal consumption- and money-to-wealth ratios are given by:

$$\frac{c_t}{w_t} = \frac{1}{\gamma-\frac{1}{\rho}}\left[RRA_t - \frac{1}{\rho}\right]F\left(t,\mathbf{X}_t;T\right)^{-1}, \tag{3.33}$$

$$\frac{m_t}{w_t} = \frac{\left(\frac{\alpha}{1-\alpha}\right)^{\frac{\rho}{\rho-1}}}{\gamma-\frac{1}{\rho}}\left[\frac{\gamma - RRA_t}{RRA_t-\frac{1}{\rho}}\right]^{\frac{\rho}{\rho-1}}\left[RRA_t - \frac{1}{\rho}\right]F\left(t,\mathbf{X}_t;T\right)^{-1}. \tag{3.34}$$

The value function at time t is given by:

$$J_t\left(w_t\right) = \frac{w_0^{1-\gamma}}{1-\gamma}\alpha^{\frac{1-\gamma}{1-1/\rho}}\frac{\mathrm{E}_t\left[\int_t^T e^{-\frac{\delta}{\gamma}(s-t)}\left(\phi_s^{\boldsymbol{\nu}^*}\right)^{1-\frac{1}{\gamma}}\left(RRA_s-\frac{1}{\rho}\right)^{-\frac{1}{\gamma\rho}\frac{1-\gamma}{1-1/\rho}}\mathrm{d}s\right]}{\mathrm{E}_0\left[\int_0^T e^{-\frac{\delta}{\gamma}s}\left(\phi_s^{\boldsymbol{\nu}^*}\right)^{1-\frac{1}{\gamma}}\left(RRA_s-\frac{1}{\rho}\right)^{-\frac{1}{\gamma\rho}\frac{1-\gamma}{1-1/\rho}}\mathrm{d}s\right]^{1-\gamma}}. \tag{3.35}$$

Proof. See Appendix B □

The optimal portfolio strategy (3.27) is characterised by four components, where the first three are instantaneous components, independent from the investment horizon while the last one is horizon dependent. The first component is the usual myopic component maximizing the portfolio nominal Sharpe ratio, based on nominal volatilities and market prices of risk and rescaled by the preference parameter γ. The second component hedges the exposure of optimal wealth to inflation. This is due to the fact that there exists a nominal risk-less asset, but not a real risk-less one, hence perfect inflation hedging is not possible and the hedge ratio is given by a minimum variance ratio. The third term is directly related to the presence of money in the utility function. It hedges the fluctuations of relative risk aversion and is due to the fact that the utility function is not separable in consumption and real balances. It is driven mainly by the short-term interest rate and strongly depends upon the values of preference parameters, γ, α and ρ. If $\alpha = 1$ (utility depending on consumption only) or $\gamma = 1$ (myopic investor), then this term is identically equal to zero. The last component is an intertemporal hedging component as in Merton (1971).[10] The balance of the portfolio, that is the remaining fraction of wealth not allocated to risky assets, nor held as uninvested money, is equal to $1 - \boldsymbol{\omega}_t\mathbf{e}_n - m_t/w_t$ and is invested in the nominal risk-less asset at the rate R_t.

[10] If the columns of the volatility matrix of the state variables $\boldsymbol{\Sigma}_X$ belong to the range of $\boldsymbol{\Sigma}_Y$, as it has been assumed in this case, then the hedging of this component is perfect. This component vanishes when the horizon is infinitely short ($T \to 0^+$), as we show in equation (E.26) in Appendix E.3. The alternative decomposition of optimal portfolio choice introduced by Lioui and Poncet (2001) and Detemple and Rindisbacher (2010) applies here.

As far as monetary policy is concerned, it has an impact on the dynamics of nominal interest rates, therefore not only on the drift component of the dynamics of bond and stock prices, but also on bond volatilities and risk premia. Market prices of risk, instead, do not depend on monetary policy in our model. Monetary policy parameters will therefore have an impact on all terms of the optimal portfolio strategy (3.27): on all the four terms through the volatility matrix of the available assets, $\mathbf{\Sigma}_Y$, and, on top of that, on the third and fourth through its impact on the short-term interest rate dynamics and thus on the dynamics of relative risk aversion.

4 Estimation technique

In an affine term structure model, the bond yields are linear in the unobserved factors. Under mild conditions, the unobserved factors can be written in terms of observable yields. Since the macro factors (expected inflation and expected output gap) are also linear in the state variables, they could ultimately be written as a linear function of observed yields. This means that yields span all the macroeconomic risks in the economy, which has been strongly rejected in the data as forcefully documented in Joslin et al. (2013). The situation is even worse in our case since this would mean that bond yields span also the equity premium, which is also unlikely to be true.

The first estimation method that we propose, which we call Spanned Macro ($\mathcal{SM}$), implies the spanning of the macroeconomic variables by bond yields. We do so as this model has been and is still extensively used, and we thus use the findings as a benchmark. It is important to consider that, as we impose the spanning of expected inflation and output gap, rather than their realised counterparts as in most of the literature,[11] we already relax the requirement of a perfect spanning of the realised macroeconomic variables. We then develop a setting, which we call Unspanned Macro ($\mathcal{UM}$), which relaxes the spanning condition of expected macro variables. The estimation technique for $\mathcal{SM}$ is described in Subsection 4.2, that for $\mathcal{UM}$ in Subsection 4.3. The technical details of the estimation procedures are reported in Appendix F and G, respectively.

Independently of the estimation strategy, the presence of a Taylor rule raises an identification problem, as monetary policy parameters can not be easily attributed to observable variables. In the following paragraph we start by discussing this issue.

4.1 Identifying the Taylor rule

The Taylor rule (3.4) has three components: the expected inflation, the expected output gap and the monetary policy shock. Assuming all the components are affine functions of some state variables makes the implied short term nominal rate also affine function of

[11] See for example Ang and Piazzesi (2003), Ang et al. (2007) and Bikbov and Chernov (2010).

the state variables as shown in (3.7). As a consequence, using no arbitrage restrictions, one can identify $\mathbf{R}_1$ from observing bond yields. We have two additional observables that allows identifying $\boldsymbol{\pi}_1$ and $\boldsymbol{\chi}_1$, namely realised inflation and realised output gap. Since monetary policy shocks are not observable, we cannot identify the monetary policy related parameters η and ξ. This problem is well known in the literature and has been studied in a comprehensive way by Chernov et al. (2013).

While several routes have been followed in the literature to overcome this difficulty,[12] a natural one, as in Ang and Piazzesi (2003) and other works, seemed to impose that the residual term υ_t in the Taylor rule (3.4) conveys information not spanned by the macroeconomic variables driving the nominal interest rate. In other words, the monetary policy residual should not co-vary with expected inflation and expected output gap. Recall that the loadings of expected inflation, expected output gap and monetary policy residual on the Wiener processes are $\mathbf{\Sigma}_X\boldsymbol{\pi}_1$, $\mathbf{\Sigma}_X\boldsymbol{\chi}_1$ and $\mathbf{\Sigma}_X\boldsymbol{\upsilon}_1$, respectively. We thus require that:

$$(\mathbf{\Sigma}_X\boldsymbol{\pi}_1)'(\mathbf{\Sigma}_X\boldsymbol{\upsilon}_1) = 0 \tag{4.1}$$

$$(\mathbf{\Sigma}_X\boldsymbol{\chi}_1)'(\mathbf{\Sigma}_X\boldsymbol{\upsilon}_1) = 0 \tag{4.2}$$

Since $\mathbf{R}_1 = \boldsymbol{\upsilon}_1 + \eta\boldsymbol{\pi}_1 + \xi\boldsymbol{\chi}_1$, the monetary policy parameters are identified as follows:[13]

$$\begin{bmatrix} \eta \\ \xi \end{bmatrix} = \begin{bmatrix} \boldsymbol{\pi}_1'\mathbf{\Sigma_X}'\mathbf{\Sigma_X}\boldsymbol{\pi}_1 & \boldsymbol{\pi}_1'\mathbf{\Sigma_X}'\mathbf{\Sigma_X}\boldsymbol{\chi}_1 \\ \boldsymbol{\chi}_1'\mathbf{\Sigma_X}'\mathbf{\Sigma_X}\boldsymbol{\pi}_1 & \boldsymbol{\chi}_1'\mathbf{\Sigma_X}'\mathbf{\Sigma_X}\boldsymbol{\chi}_1 \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{\pi}_1'\mathbf{\Sigma_X}'\mathbf{\Sigma_X}\mathbf{R}_1 \\ \boldsymbol{\chi}_1'\mathbf{\Sigma_X}'\mathbf{\Sigma_X}\mathbf{R}_1 \end{bmatrix} \tag{4.3}$$

The economic intuition behind the identification procedure above is straightforward: we require the residual not to be correlated with the other variables intervening in the Taylor rule, in order for it to be a pure monetary policy shock. As shown above, the unobservable factors can be replaced by a linear combination of the economic variables (short term interest rate, expected inflation and expected output gap). It seems thus natural to purge this shock from the information already contained in the macro variables. It is useful to make a precision at this stage: the identification procedure does not impose any constraint on the correlations between the factors.

The uniqueness of the identified parameters might be an issue, as it is well known that invariant transformations of the affine term structure model may lead to observationally equivalent representations of bond prices. However, applying the definition of invariant

[12] See section 2.4 in Duffee (2013).

[13] Since the parameters are retrieved from other estimated parameters, the estimation errors of η and ξ are determined from the distributions of the estimators of $\mathbf{R}_1$, $\boldsymbol{\pi}_1$ and $\boldsymbol{\chi}_1$. In order to do so, we use a bootstrapping methodology, drawing 10^6 possible sets of parameters according to their joint distribution, assumed to be multivariate normal, and determining the standard deviation of the corresponding distribution of the two monetary policy parameters. Alternatively, we could have used the delta method, consisting in a linear approximation of the relation between the monetary policy coefficients and the effectively estimated parameters. These methods are reviewed in Efron and Tibshirani (1986).

transformation given in Dai and Singleton (2000),[14] it can be easily shown that all the terms appearing on the right hand side of equation (4.3), thus the solution (η, ξ), are immune to any invariant transformation of the model.

4.2 Identification with spanned macroeconomic factors ($\mathcal{SM}$)

We assume that three latent factors ($n_X = 3$) drive the term structure of interest rates. Since bond yields are linear functions of these factors,[15] the latent variables can always be written as linear functions of observable bond yields. Since in addition the macro variables are themselves linear in the unobserved factors, they can ultimately be written as linear function of observable bond yields. In other words, the macro risks are spanned by the bond market.

We assume that three yields are observed perfectly and additional bond yields are observed with errors. In our case, the 6-month, 2-year and 10-year zero coupon yields are observed without errors, whereas the 3-month, 1-year and 5-year are observed with errors. The other observables are the log of the stock index level, the log of the consumer price index and the output gap.

As pointed out in Dai and Singleton (2000), there exist many observationally equivalent specifications of an affine term structure model. Without any loss of generality, as far as the state variables dynamics are concerned, we assume that the long run means are zeros and that the matrix of speeds of mean reversion $\mathbf{\Theta}$ is lower triangular. The volatility matrix $\mathbf{\Sigma}_X$ is set equal to the identity.In this setting there are five independent sources of risk, we assume, again without loss of generality, that the first three components of the vector of innovations in the economy $\mathbf{z}_t$ are those of the state variables and therefore the fourth and fifth rows of $\mathbf{\Sigma}_X$ are filled with zeros. The state variable dynamics are therefore specified by the six parameters different from zero of the lower triangular speed of mean reversion matrix, $\mathbf{\Theta}$.

For what concerns the dynamics of the stock and the consumer price index, we assume without loss of generality that the vectors $\boldsymbol{\sigma}_S$ and $\boldsymbol{\sigma}_P$ have respectively the first four and five components to be non-null. The fourth component in $\mathbf{z}_t$ is thus a pure stock market risk and the fifth is pure inflation related idiosyncratic risk. Inflation-linked security prices (TIPS) have short historical track-record and are plagued with several imperfections (mainly lack of liquidity). We have therefore decided to avoid using them. This implies that it is not possible to determine the market price of unexpected inflation risk and that we have to make an assumption on its value. We assume the market price of the specific risk associated to the component of risk in the price index innovations which is not spanned by the other assets (corresponding to the fifth component of the vector

[14]See Appendix A in Dai and Singleton (2000).

[15]The affine relation between state variables and bond yields is described in Appendix A.

$\boldsymbol{\sigma}_P$), to be equal to zero. We follow in this Sangvinatsos and Wachter (2005), Koijen et al. (2010) and Campbell et al. (2013) amongst many others. Apart from this restriction, we keep the most general specification for the market prices of risk 5×1 column vector $\mathbf{\Lambda}_0$ and 5×3 matrix $\mathbf{\Lambda}_1$, leaving fully unrestricted the first four rows of each of these two and filling the fifth rows with zeros. This means that we estimate also the equity premium as a variable quantity, dependent on a linear combination of the three state variables.

The parameters describing the short-term rate, the scalar R_0 and the 3×1 column vector $\mathbf{R}_1$, can be identified as they appear in the yield pricing formulas and in the stock dynamics. The parameters describing the expected inflation, the scalar π_0 and the 3×1 column vector $\boldsymbol{\pi}_1$, can be identified as they appear in the price index dynamics. As the series for the realised output gap is available only with quarterly frequency, we decide to perform the estimation of the coefficients χ_0 and $\boldsymbol{\chi}_1$ after the estimation of all the other parameters by means of a linear regression, assuming that the realised output gap is given by the average expected output gap plus an error term, which magnitude is indicated with σ_ϵ^G. The expected output gap directly depends on the time series of the state variables, which can be computed from the yields time series thanks to the parameters that have been estimated.[16] As the observation errors of the 3-month, 1-year and 5-year yields are assumed to be Gaussian, sequentially and cross-sectionally independent, the imperfection of their observation can be described with 3 parameters, respectively σ_ϵ^1, σ_ϵ^2 and σ_ϵ^3.

Parameters are estimated using the quasi maximum likelihood technique as in Duffee (2002) and, in a richer setting considering equity returns and inflation, Sangvinatsos and Wachter (2005). The technicalities of the maximum likelihood estimation procedure for this setting can be found in Appendix F.

4.3 Identification with unspanned macroeconomic factors ($\mathcal{UM}$)

While early term structure models incorporating macro variables embedded an implicit assumption that bond yields span realised inflation and realised output gap, some authors tried to overcome this strong assumption by letting bond yields to span the expected value of the macro variables (see for example Kim and Wright (2005)). Even this assumption, implicit in the first setting described above, turned out to be too restrictive and rejected by the data systematically.

The question that we were facing is: how can we relax the spanning assumption while keeping the model parsimonious and tractable? While macro models have to deal with the spanning issue of macro variables, we have to deal with the spanning issue of the equity market risk premium in addition. One promising solution to the first spanning

[16] In Ang and Piazzesi (2003), a two step procedure is also used to estimate a dynamic term structure model with both yields and macro variables. Since the volatility matrix of the factors is diagonal, the pure innovation to the output gap do not impact the estimates for the state variables. We are thus is a similar setting as Ang and Piazzesi (2003) where the latent factors are orthogonal to the macro factors.

problem has been offered by Joslin et al. (2014). Instead of projecting the economy wide pricing kernel of the observed bond yields only, the idea is to project it on the information contained in macro variables as well as in the bond yields. This approach could be adapted to include the information from the equity market. The problem is that it is not easy to embed this VAR approach in a continuous time framework. We need the latter for tractability of the dynamic asset allocation problem which is the focus of this paper. We thus followed another route.

We assume that bonds are priced by two state variables only, whereas the equity risk premium is determined by a linear combination of all the three state variables. As in the previous setting, we choose a number of yields equal to the number of state variables pricing the bonds to be perfectly observed. In this case only two yields need to be observed without errors, for this purpose we choose the 6-month and the 5-year. As the third state variable is unobserved, it needs to be filtered. We decide to use the Kalman filter technique.[17] We also assume that the innovations of the third state variable can be dynamically hedged with two bonds and the stock, which is equivalent to saying that the volatility vector of the stock index, $\boldsymbol{\sigma}_S$, has only three non-zero components. We have therefore reduced to four the total number of independent sources of risk. The volatility vector associated with the consumer price index, $\boldsymbol{\sigma}_P$, is therefore fully determined by four components. The column vector $\boldsymbol{\Lambda}_0$ has therefore now only four components and the matrix $\boldsymbol{\Lambda}_1$ has size 4×3.

In order to impose the bond pricing to be dependent only on the first two state variables, it is enough to restrict the third component of $\mathbf{R}_1$ and the third elements of the first and second row of $\boldsymbol{\Lambda}_1$ to be equal to zero. As the number of sources of risk has been reduced, the market prices of risk column vector $\boldsymbol{\Lambda}_0$ and matrix $\boldsymbol{\Lambda}_1$ have now only four rows, with the fourth row filled with zeros, as the specific risk of unexpected inflation can not be identified, as for the setting described in Section 4.2. Likewise, all the rest of the market specification is exactly the same.

The estimates are obtained maximizing the likelihood function, which is a by-product of the Kalman filtering procedure. The expected output gap parameters are obtained, as described in the previous section for the $\mathcal{SM}$ setting, by linear regression. The technical details of the estimation procedure can be found in Appendix G.

5 Data

We estimate the model according to monthly U.S. data, starting from 31st January 1952 until 31st December 2013. We use zero coupon bond yields series for the following maturities: 3-month, 6-month, 1-year, 2-year, 5-year, 10-year. Yields until December

[17] See for example Duffee and Stanton (2004), Section 2.3.

1998 were available on the website of Gregory Duffee. The series have been extended until December 2013 using end-of-month 3M and 6M Treasury Bills daily data, available on the Federal Reserve Economic Data website (series GS3M and GS6M), and 1Y, 2Y, 5Y and 10Y zero-coupon yields fitted by Gürkaynak et al. (2007), available on the Federal Reserve website.

As a representative of the U.S. stock market, we consider the CRSP NYSE/Amex/NASDAQ/ARCA Value-Weighted Market Index, extracting the end-of-month data from the daily series. As price index, we take the Consumer Price Index for All Urban Consumers: All Items (CPIAUCSL), available with monthly frequency. As real variable for the Taylor rule, we compute the realised U.S. Output gap from the quarterly series of U.S. Real GDP and U.S. Real Potential GDP,[18] using the standard definition $(\mathrm{GDP}_{\mathrm{actual}} - \mathrm{GDP}_{\mathrm{potential}})/\mathrm{GDP}_{\mathrm{potential}}$; this is time series is available at a quarterly frequency only, whilst the others are all available at monthly frequency. Also CPI and GDP data are available on the Federal Reserve Economic Data website.

6 Estimation results

We first report the estimates of the model parameters under the two estimation settings and we compare the performance of the models as to the fitting of asset prices and macro variables. We then compute the impulse response functions to shocks in macroeconomic variables for some quantities of particular interest for the portfolio strategy. Finally, we perform a static analysis to assess the sensitivity of relevant economic quantities (risk premia, asset volatilities, correlations and maximum achievable Sharpe ration) to shifts in macroeconomic and monetary policy variables.

6.1 Parameter Estimates

The two panels in Table 1 show the parameter estimates and their standard errors for the two estimation settings proposed.

[Table 1 about here.]

For the $\mathcal{SM}$ setting, the estimates of the speeds of mean reversion of the latent factors are all statistically significant at the 5% level, with one value significantly higher than one ($\Theta^{33} = 1.9938$), another one close to 0.5 ($\Theta^{11} = 0.5224$) and the last one close to 0.1 ($\Theta^{22} = 0.1122$). Although we take into account the stock market and realised inflation when estimating the model, the estimates are close to those obtained by pure

[18] In particular, Real Potential GDP is the Congressional Budget Office (CBO) estimate of the output the economy would produce with a high rate of use of its capital and labour resources.

term structure models based on latent variables,[19] as well as to richer models assuming a constant equity risk premium.[20] For the $\mathcal{UM}$ setting, the diagonal of the matrix of speed of mean reversion is again fairly precisely estimated, whilst the off-diagonal terms have a higher relative standard error. The diagonal terms are lower than in the first setting, suggesting that the factors are more persistent.[21]

For both estimation settings, the estimates of the long-run means of the economic variables, R_0, π_0 and χ_0, are statistically significant at conventional significance levels. The main difference between the two setting relates to the long run mean of the expected output gap: it is -0.68% in the first setting (close to the empirical mean -0.57%) while it is -0.76% in the second setting. The 19bp difference is still reasonable relative to the standard deviation of the historical distribution of the output gap, equal to 296bp (Table 2). The reason of this difference is that, for a small sample, the factors are not necessarily centered around zero. The nominal short term interest rate loads on all the factors driving the nominal term structure in both settings (3 factors for $\mathcal{SM}$ and 2 for $\mathcal{UM}$). As to the market prices of risk, although some loadings are estimated imprecisely, one can reject the hypothesis that any of the market prices of risk is constant. In $\mathcal{SM}$, each market price of risk loads statistically significantly on one or two out of the three factors, included that for the stock. In $\mathcal{UM}$, both market prices of risk of the two idiosyncratic risks driving the nominal term structure load on the second factor, whilst the market price of risk of the stock loads on the first factor, which has a lower speed of mean reversion. A particularly relevant element of the matrix $\mathbf{\Lambda}_1$ in $\mathcal{UM}$ is the coefficient on the third row and third column, that is the coefficient linking the equity premium to the third state variable, that allows to capture the fraction of equity premium that is not explained by the information carried by nominal yields. This coefficient is negative, but its value is quite small and not statistically significant. This shows that the majority of the equity premium that can be predicted is in fact explained by nominal yields. This is an information that the $\mathcal{SM}$ setting can not provide, as the equity premium is by construction defined as a function of nominal yields only. In $\mathcal{UM}$, the market price of the stock market specific risk loads significantly only on the first factor driving the nominal term structure.[22,23] The stock

[19] In Table 3 page 949 of Joslin et al. (2011), a three-factor term structure model is estimated under ten different specifications and the diagonal of the matrix of the speed of mean reversion of the three factors has two terms less than one and one greater than one in absolute value across all the models. See also Duffee (2002) for similar findings.

[20] See Sangvinatsos and Wachter (2005), page 195, Table I, Panel B.

[21] When estimating a two factors term structure model, Koijen et al. (2010) report values for the diagonal elements very close to ours (see Table 1 page 753).

[22] Baele et al. (2010) study the drivers of the co-movements of bonds and stocks returns while Baker and Wurgler (2012) report that term structure related variables which predict bond returns also predict returns on bond-like stocks.

[23] Note that, as in Koijen et al. (2010), we leave the matrix of loadings of the market prices of risk on the factors unconstrained and about half of the parameters turn out to be not statistically significant. Some authors suggest to constrain the loadings of some of the market prices of risk on some factors, by setting them equal to zero (see Duffee (2002) and Sangvinatsos and Wachter (2005)).

and unexpected inflation volatility vectors, $\boldsymbol{\sigma}_S$ and $\boldsymbol{\sigma}_P$, are mostly dominated by their idiosyncratic components, that means their fourth and fifth components respectively in $\mathcal{SM}$ and their third and fourth components in $\mathcal{UM}$.[24]

Monetary policy parameters reveal an unambiguous domination of the expected inflation component in the Taylor rule. The expected inflation coefficient is estimated at 0.8376 in $\mathcal{SM}$ and 0.6932 in $\mathcal{UM}$, whilst the loadings on the expected output gap are respectively 0.3713 and 0.3565. All of these estimates are statistically significant at the 5% level. The relative weight of inflation ($\frac{\eta}{\eta+\xi}$) is similar for both settings and shows a higher weight on expected inflation, which is not surprising, also because our sample period includes the monetary experiment of the 80s, when the Fed was known to privilege price stability. For a sample period similar to ours, Campbell et al. (2013) find an estimate for the weight over realised output gap of 0.32 (0.21) and 1.08 (0.43) for the weight over realised inflation.[25] Similar findings have also been reported by Ang et al. (2011).[26] Interestingly thus, using either realised inflation and output gap, or their expectations, has little impact on the estimates of the weights of these macro variables in the Taylor rule. The fact that we obtain similar estimates for the two settings confirms the robustness of the identification of monetary policy parameters.

An important difference between the two settings relates to the fitting of the yields. Since the term structure in $\mathcal{UM}$ is described by only two factors, one may expect a deterioration in the pricing errors. This is indeed the case when comparing the pricing errors σ^i_ϵ of the yields observed imperfectly. In the first setting the errors for 3M, 1Y and 5Y yields all have a standard deviation of the order of one or two tenths of basis points. In the second setting, the errors for 3M, 1Y, 2Y and 10Y yields are clearly slightly higher, but are still equal to 30 basis points at most. The parameter σ^G_ϵ, measuring the standard deviation of the annualised difference between realised output gap and average expected output gap, is 2.25% for both settings.

Overall, the estimates in $\mathcal{SM}$ are comparable to those reported in the literature for similar settings, although we took into account equity return predictability, as well as inflation idiosyncratic risk. The $\mathcal{UM}$ offers a picture which differs mainly in terms of persistence of the latent factors and for a slight increase in bond pricing errors. Using these parameter estimates, we assess hereafter to what extent the two settings fit the moments of asset returns and macro variables.

[24]Similar findings have been reported by in Sangvinatsos and Wachter (2005) and Koijen et al. (2010).

[25]See Table 4. Since they include the lagged values of the monetary policy instrument, we consider their implied Taylor rule parameters, which are comparable to our figures.

[26]See Table 2 page 441 where they treat the case of monetary policy rules with constant parameters.

6.2 Moment Matching

We report in Table 2 the first and the second moments of the estimated quantities. Even though $\mathcal{SM}$ proves to be slightly more accurate, both settings fit fairly well the means and the standard deviations of bond yields from 3M to 10Y maturity, confirming that two factors are able to effectively explain the nominal term structure. In Appendix H, following Dai and Singleton (2002), we also perform a Monte Carlo analysis assessing the behavior of the two models specification in terms of average yield spreads, standard deviations of yield spreads and Campbell-Shiller long-rate predictive regression coefficient, verifying that there are no significant differences in terms of nominal bond yields dynamics between the two models. We also report the annualised volatilities of the returns on the price index (realised inflation) and of the stock index. The historical volatility of the stock market, estimated from monthly data, is 15.07%. The model-implied conditional equity volatility, discretised on a monthly basis and expressed in annual terms, is instead 15.01% for $\mathcal{SM}$ and 15.00% for $\mathcal{UM}$, which thus perform equally well on this dimension too. The conditional model-implied volatility is slightly lower than the unconditional historical measure, thanks to the contribution of market predictability. The value is lower for $\mathcal{UM}$ probably because this setting (very slightly) improves the predictability of equity returns. The conditional volatility of realised inflation is lower in $\mathcal{UM}$ (0.86% vs 0.90%), showing that the factor not driven by bond yields helps capturing information on expected inflation too. We also show the estimated long-run means of risk premia: in $\mathcal{SM}$ ($\mathcal{UM}$), for bonds, they range from 0.25% (0.19%) for the 3M zero-coupon bond to 2.50% (2.34%) for the 10Y zero-coupon bond, whilst the stock index average expected excess return is equal to 6.70% (6.63%). Finally, we notice that the model-implied volatilities of expected inflation and output gap, as well as the volatilities of bond and stock risk premia, are lower in the second setting, probably because in $\mathcal{UM}$ there is fewer overfitting of macroeconomic data.

[Table 2 about here.]

For what concerns the macroeconomic variables, Figure 1 shows the expected and realised output gap series. For both estimation settings, the two series clearly have a common pattern, even though the expected series does not predict the peaks in the effectively realised series. The coefficient of determination of the regression relating the realised output gap to the time-average of the expected output gap, for which we provide the details in Appendix F, is about 43% for both settings. The expected inflation time series seem in both cases to accurately follow the medium-frequency variations of realised inflation.

[Figure 1 about here.]

The three top graphs in Figure 2 respectively show the time series of model-implied 2Y/10Y bond and stock risk premia for both settings. Bond risk premia are very correlated and increase their variability with the maturity. Implied equity risk premium is on average higher than bond risk premia. Its time series is visibly correlated with bond risk premia (especially for the 10Y maturity) for $\mathcal{SM}$, whilst it seems to be less correlated in $\mathcal{UM}$. The bottom graph of Figure 2 shows the maximum instantaneous annualised Sharpe ratio achievable when the investable assets are the six zero-coupon bonds considered and the stock index. For $\mathcal{SM}$ ($\mathcal{UM}$), its value is typically around 1 (0.6), but it can attain higher peaks, reaching almost 4 (2.5), especially between the end of the 70s and the beginning of the 80s.

[Figure 2 about here.]

Tables 3a and 3b show model-implied conditional pairwise correlations between returns in bonds, stock index and CPI. Bond returns show strong pairwise correlations among bonds with different maturities. The stock index is also weakly positively correlated with bond returns. The price index is instead slightly negatively correlated with bond returns and very weakly positively correlated with the equity index. All of these results are consistent with the pairwise correlations calculated from historical data, as shown in Table 3c.

[Table 3 about here.]

Table 4 shows instantaneous correlations between asset risk premia and innovations in the economic state variables (rows), and asset returns, as well as CPI returns and innovations in the economic state variables (columns). Bond risk premia are, for longer maturities and for both estimation settings, negatively correlated with bond returns. This is somehow intuitive, as poor performance usually implies higher future expected excess returns. However, for the first setting, this is not the case for the stock index, as the parameter estimates imply a positive correlation between stock returns and stock premium. Also, there is a positive correlation between the expected output gap and asset returns, which is not consistent with the findings by Cooper and Priestley (2009), who find a negative relation between output gap and both bond and equity expected excess returns. For $\mathcal{UM}$, instead, the parameter estimates imply a more reasonable negative correlation between stock returns and stock premium. This is again due to the fact that the presence of a state variable that is not derived from bond yields allows to better capture the dynamics of stock risk premium. Furthermore, equity premium is positively correlated with returns of bond of all maturities and, again differently from $\mathcal{SM}$, there is in this case a negative correlation between the expected output gap and asset returns, which is consistent with the results in Cooper and Priestley (2009), who find a negative relation between output gap and both bond and equity expected excess returns.

The short-term nominal rate R is, for both settings and as intuition suggests, strongly negatively correlated with bond returns, especially for shorter maturity bonds. It is also slightly negatively correlated with stock returns and positively correlated with innovations in the CPI. The expected inflation π is also, in both estimation settings, strongly correlated with the short-term nominal rate R and shows qualitatively similar pairwise correlations with asset returns, though slightly smaller. Differences between the two settings can be noticed in pairwise correlations involving expected output gap χ. In $\mathcal{SM}$, is negatively correlated with bond returns and expected inflation, but, as Taylor rule's interpretation of short-term rate would suggest, χ is positively correlated with the short-term interest rate R. In $\mathcal{UM}$, instead, the output gap is negatively correlated only with short-duration bonds, but it becomes positively correlated for the riskier 10Y bonds and strongly positively correlated with the stock index. This seems reasonable, as a positive change in the economic output usually implies contemporaneous positive returns for risky assets. Furthermore, the expected output gap is in this case strongly positively correlated with expected inflation and more positively correlated with the short-term nominal rate.

[Table 4 about here.]

In conclusion, in this section we have compared the more traditional estimation setting $\mathcal{SM}$, where the three state variables are derived from bond yields, with the innovative setting $\mathcal{UM}$, which describes the yield curve with two state variables only and uses a third one to instead price, together with the other two state variables, the equity premium. This third state variable is estimated by means of the Kalman filter. We have shown that this setting still describes very well the dynamics of the yield curve and allows to enrich the information set carried by the state variables. As a consequence, we are able to better capture the dynamics of the equity premium, which is now less correlated with bond premia and, coherently with common knowledge, negatively correlated with equity returns. Furthermore, in this case the expected output gap is also positively correlated with stock returns, as intuition suggests, and is negatively correlated with the risk premia of the risky assets, coherently with the literature (Cooper and Priestley, 2009, Campbell et al., 2013).

6.3 Impulse Responses

In this section we study the impulse responses of the two models with respect to shocks on expected inflation and output gap. These responses, which are de-meaned with respect to the long-run stationary values, are computed starting from the steady state $\bar{\mathbf{X}}$ by applying a shock to one of the macroeconomic variables and observing the reaction of the model in the absence of the stochastic component in the dynamics of the state variables.

[Figure 3 about here.]

Figure 3 shows the impulse responses to a 1% positive shock on expected inflation over the short-term nominal rate, the expected inflation, the output gap and the real rate. For what concerns the interest rates, we show the impulse responses also in the cases where the monetary policy parameters are shifted in relative terms by -30% and $+30\%$. It is worth noting that, as these quantities are affine functions of the state variables, the response to a 1% negative shock would be exactly the opposite as shown in the graphs. Positive shocks in expected inflation imply an instantaneous shock of the same sign in the nominal rate R_t. This is intuitive because of the relation explained by the Taylor rule (3.4), which also implies that the responses have the same shape, but are more pronounced when the value of the coefficient η is higher. There is no significant difference in terms of immediate response between the two models, even though the decay in $\mathcal{UM}$ seems to be slower than in $\mathcal{SM}$. Shifts in the coefficient over the expected output gap, ξ, have very little effects in terms of dynamic response of the model to shocks on expected inflation. A difference between the two models is in the response shown by the expected output gap: whereas in $\mathcal{UM}$ there is a delayed negative response, which slowly decays to zero, in $\mathcal{SM}$ we initially notice a positive delayed inverse response and then a negative response. The response over the real rate, given by the difference between the responses over R and π, is dampened when the coefficient η is increased so that it is close to 1. For $\mathcal{UM}$, the response is also dampened when ξ is reduced.

[Figure 4 about here.]

The responses to shifts in χ (Figure 4) over the short-term rate are similar to those in π, but the amplitude of the response is smaller because the coefficient ξ is lower than η. The response of the real rate is not greatly affected by shifts in η, except for the speed of decay. Negative shifts to ξ, instead, seem to dampen the response of the real rate.

In general, the responses of the short-term rate to $\Delta\pi$ and $\Delta\chi$ are similar to those obtained by Ang et al. (2011),[27] even though we observe an initial overshoot only in $\mathcal{SM}$ for shocks on the expected output gap.

[Figure 5 about here.]

[Figure 6 about here.]

Figures 5 and 6 show the impulse responses of the risk premia and of the maximum obtainable Sharpe ratio. All risk premia respond negatively to positive shifts in π and χ (apart from a little inverse response to $\Delta\chi$ in $\mathcal{SM}$), as it is observed also in Ang et al. (2011).[28] The responses to $\Delta\pi$ seem to be smaller in $\mathcal{UM}$, whilst the responses to $\Delta\chi$

[27] See page 448, Figure 4, first two panels in the first row. In their analysis the amplitude of the shocks applied are equal to one standard deviation of the distribution of the relevant variables.

[28] See page 450, Figure 6, first two panels in the first row, showing the response of the 1Y and 5Y risk premia.

are smaller in $\mathcal{SM}$ than in $\mathcal{UM}$. It seems that the impact of monetary policy shifts over the impulse response of risk premia is not particularly important and, as η and ξ do not affect the market prices of risk, the impulse response of the maximum Sharpe ratio is not affected either. As the maximum Sharpe ratio is a quadratic function of the state variables and not an affine function as for the other quantities observed, differently from all the other quantities observed, the impulse response is not linear in the amplitude of the shock. That is why we show the responses for both 1% positive and negative shifts. For both settings, the response of the maximum Sharpe ratio has the opposite sign of the impulse $\Delta\pi$ and is greater for negative shifts. The same applies when the impulses are applied to χ, even though there is a slight initial positive inverse response in $\mathcal{SM}$ for $\Delta\chi = +1\%$.

6.4 Static Sensitivity Analysis

In this section, using the estimates obtained for the models $\mathcal{SM}$ and $\mathcal{UM}$, we perform static analyses assessing the effect of shifts of monetary policy and economic variables over the parameters that mostly affect the portfolio strategy and the welfare of the investor.

[Figure 7 about here.]

[Figure 8 about here.]

For $\mathcal{SM}$, Figure 7a shows the impact of relative shifts of η and ξ on asset volatilities, risk premia, correlation between bond and stock returns and bond betas with respect to stock returns. Whilst for the stock index volatility and risk premium are constant, bond volatilities and risk premia are increasing functions of η. The dependence is particularly strong, especially for longer-maturity bonds. The dependence of volatilities and risk premia over variations in ξ is instead decreasing, but much weaker than for variations of η. The correlations of bond returns with stock returns are increasing functions of η and decreasing functions of ξ, determining the same pattern for bond betas with respect to the stock. The directions of variation of the betas are consistent with those observed by Campbell et al. (2013),[29] and the directions of variation of bond volatilities are consistent with those observed in two out of three subperiods they consider, that is from 1960 to 1996. The sensitivities for the $\mathcal{UM}$ setting (Figure 8a) show similar patterns, even though the scale of variation of correlations and betas seem in this case to be much wider.

In Figures 7b and 8b we instead analyse the static effect of shifts in π and χ over the risk premia of the risky assets and over the maximum obtainable static Sharpe ratio of the portfolio. For both settings, the relation between risk premia and shifts in expected inflation is monotonically decreasing for all assets and the maximum Sharpe ratio curve

[29] See Table 8 in Campbell et al. (2013).

is locally decreasing around the base case value. Shifts in the initial expected output gap have a qualitatively similar effect on risk premia and maximum Sharpe ratio as shifts in expected inflation, but on a smaller scale. An exception is the behaviour of the 2Y bond premium in $\mathcal{SM}$, which is increasing in $\Delta\chi$, differently from what happens for the other assets and from the increasing behaviour shown in $\mathcal{UM}$.

7 Portfolio strategy

In this section we analyse the impact of monetary policy shifts on the optimal portfolio strategy and the welfare of a long-term investor. In the $\mathcal{SM}$ setting, the investor can trade in three linearly independent bonds (2Y, 5Y and 10Y) and the equity index in addition to the riskless (cash) asset. In the $\mathcal{UM}$ setting, the investor can trade two bonds (2Y and 10Y), the equity index and the riskless asset. In both settings, the nominal market is complete, whilst the real market is incomplete due to unspanned inflation risk.

We show the bonds and stock positions for the base case, where the parameter values are the estimates reported in Table 1 and the state variables are set at their long-run mean values. We then report the new asset positions and the welfare change relative to the base case after a $\pm1\%$ shock to expected inflation or expected output gap, as well as when η and ξ are subject to a relative change of $\pm30\%$.

We then treat three different portfolio choice problems: i) the terminal wealth case (pure asset allocation problem), ii) the intermediate consumption case and, finally, iii) the intermediate consumption and real balances case. We consider investment horizons up to 20 years and report the results for $\gamma = 4$.[30]

7.1 Utility over terminal wealth

In the case where utility depends on terminal wealth, the solution of the portfolio choice problem can be derived in closed form and is reported in Appendix D. We first discuss the findings for the $\mathcal{SM}$ setting, then those for the $\mathcal{UM}$ setting.

7.1.1 Findings for the $\mathcal{SM}$ setting

[Table 5 about here.]

The top panel of Table 5 reports the optimal portfolio strategy for the base case. As it can be immediately noticed, the positions in the three nominal bonds are huge for all maturities: the 2Y bond has optimal myopic positions of 1849%, the 5Y bond around -1193% and the 10Y bond around 305%. There are very strong horizon effects, since the 10Y bond position becomes 647% for an investment horizon of 20 years, more than

[30]Results for $\gamma = 8$ are in Appendix M.

twice as much as for a 2-year horizon. The stock position is equal to 69% and flat, as in the $\mathcal{SM}$ setting all the factors are spanned by bonds and the stock does not intervene in intertemporal hedging. Interestingly, the stock-bond mix is slightly impacted by horizon effects and seems to decrease with the horizon going from 8.8% for a 2-year investment horizon to 6.8% for 20 years investment horizon. The total bond position features smaller horizon effects than individual bond positions.

The results for the base case are close to those already reported in the earlier literature.[31] The huge positions are a consequence of the serious multicollinearity of bond yields when parameters are estimated from 3 perfectly observed yields and 3 bonds are then included in the optimal portfolio strategy. An important consequence is the huge importance of the myopic component of the strategy, which reduces substantially the relevance of the intertemporal hedging component.

In the second panel of Table 5, we report the new portfolio strategy when expected inflation increases or decreases by 1%. In first place we focus on the welfare change. In order to provide an intuitively understandable measure of differences in welfare relative to the base case, we report the change in initial wealth w_0 that would provide the same welfare in the base case as in the scenario with shifted parameter values.[32] A positive shock to expected inflation tends to increase the welfare when the horizon is 10 years or longer. The increase is about 0.25% for a 10-year horizon and 0.79% for 20 years. For short horizons, the impact is negative (−0.48% for 2 years and −0.20% for 5 years).

To understand the impact on welfare of a positive shock to expected inflation, it is helpful to consider the impulse response functions reported in Figure 3a. A positive shock to expected inflation tends to increase the nominal rate by less than the shock, which means that the real rate decreases in the short term. The same is true for the maximum Sharpe ratio, as shown in Figure 5a. Since the welfare is positively related to these two quantities, this explains the negative impact on the investor's welfare for short investment horizons. For the longer investment horizons (greater than 2 years), the real rate recovers and becomes even higher than before the shock. The maximum Sharpe ratio also follows a similar pattern. As a consequence, greater investment horizons allow the investor to capture this medium/long-term positive impact on the real rate and the maximum Sharpe ratio of a positive expected inflation shock. Hence the positive impact on the investor's welfare for investment horizons beyond 5 years.

A negative shock to expected inflation increases also the investor's welfare, but less so when the investment horizon is high. The increase in the investor's welfare is close to

[31]See Table VI, Panel B, page 211 in Sangvinatsos and Wachter (2005). See also Barillas (2010), Figure 5 page 45.

[32]The details of the calculation of this quantity are given in Appendix I. It is worth noting that, given the nature of the term structure model employed and the fact that nominal markets are complete, the welfare does not depend on the particular choice of the maturities of the bonds available for trade. This would not be the case if the model were based on a VAR of yields.

2.54% for a 2-year investment horizon while it is only close to 1.28% for a 20-year horizon. The intuition for this is similar to the one put forward for the positive expected inflation shock. A negative shock tends to decrease the nominal rate but less than the size of the expected inflation shock. As a consequence, the real rate is positively impacted. The maximum Sharpe ratio is also positively impacted in the short term. Both effects lead to an increased welfare for a short-term investor. The maximum Sharpe ratio recovers its initial value in a smooth way without experiencing any negative impact (see Figure 5a). After two years the effect of the shock disappears. The real rate also recovers, but not monotonically, as after a sharp increase in the short term, it shows a negative overshoot after one year and then slowly approaches its initial value. Longer investment horizons spread over a long time period the positive impact in the short term and capture the negative impact on the real rate. All this reduces the welfare gains from a negative shock to inflation for long investment horizons. The welfare impact is not negative for long investment horizons because the maximum Sharpe ratio is never significantly negative.

As to the portfolio strategy, a positive shock to expected inflation increases substantially the bond exposure while reducing significantly the stock position. The stock-to-bond ratio is reduced to about 3% from 7% in the base case and intertemporal hedging effects are larger. Symmetrically, a significant decrease in bond positions and an increase in the stock's after a negative shock to expected inflation are observed. The stock-to-bond ratio becomes in this case twice as large than in the base case.

The intuition behind these changes in the portfolio positions is as follows. A positive shock to expected inflation impacts negatively the risk premia of the bonds as well as the risk premium of the stock. The latter is even more impacted than the bonds since its risk premium decreases by 4% for a 1% shock to expected inflation, while the 2Y bond risk premium experiences a decrease of only 1%. This is why the position in the 2Y bond increases substantially relative to the other assets positions. Symmetrically, a negative shock to expected inflation tends to increase bond risk premia, but the stock risk premium is much more impacted than the 2Y bond risk premium. Hence, bond positions are reduced relative to the stock position, which becomes now twice as large as in the base case. It is worth noting that in our interpretation we focus on the risk premia of traded assets since, in our constant volatility setting, shocks to expected inflation have no effect on volatilities.

In the third panel of Table 5 we report the portfolio strategy when expected output gap increases or decreases by 1%. A positive shock reduces welfare by 0.90% for a 20-year horizon, while a negative shock increases welfare by 3.64%. A positive shock increases the nominal rate and the real rate. The maximum Sharpe ratio, on the other hand, increases and then decreases after 4 years. Hence the slight positive effects of a positive shock for short investment horizons (2 years) and then a negative effect, suggesting that the impact on the maximum Sharpe ratio dominates. A negative shock to expected output instead increases substantially the maximum Sharpe ratio and this leads to sizable welfare improvement for any horizon.

As to the portfolios strategy, it turns out that the stock risk premium is impacted about the same way as the bond risk premia (5Y and 10Y). Its position is thus not that affected and the bond positions are mostly adjusted to reflect the deterioration (increase) in the risk premia for positive (negative) shocks.

We can now turn to scrutinize the impact of monetary policy shifts on the investor's behavior. Recall that, by construction, shifts in monetary policy have no impact on market prices of risk. Therefore, any potential impact of monetary policy shifts on welfare are driven by the impact on the dynamics of the real interest rate.[33]

In the fourth panel of Table 5 we report the optimal positions and welfare variations for a positive and a negative changes to the expected inflation weight. A positive change is welfare improving and the welfare gain is as high as 4.46% for a 20-year investment horizon, whilst the welfare loss from a negative change is equal to -4.52%. A higher weight on expected inflation allows the nominal rate to better absorb shocks to expected inflation, thus reducing the shock on the real rate. Since the maximum Sharpe ratio is, by construction, not affected by monetary policy shifts, a positive shift to η is valuable and, symmetrically, a negative shift harmful. The effects of such shifts on the volatility of the real rate are negative, which reinforces the positive impact of the positive shifts in expected inflation weight. The position in the stock index is unchanged, as the investment opportunity subset that changes with η is only the nominal bond market, which constitutes in this case a complete market by itself.[34] Monetary policy shifts have no im-

[33]Recall that the real rate is given by:

$$\begin{aligned} r_t = R_t - \pi_t &= R_0 + (\boldsymbol{\upsilon}_1 + \eta\boldsymbol{\pi}_1 + \xi\boldsymbol{\chi}_1)' \mathbf{X}_t - \pi_0 - \boldsymbol{\pi}_1'\mathbf{X}_t \\ &= R_0 - \pi_0 + (\boldsymbol{\upsilon}_1 + (\eta - 1)\,\boldsymbol{\pi}_1 + \xi\boldsymbol{\chi}_1)' \mathbf{X}_t \end{aligned}$$

The instantaneous variance of changes in the real rate will thus be:

$$\mathbb{V}[\mathrm{d}r_t] = (\boldsymbol{\upsilon}_1 + (\eta-1)\,\boldsymbol{\pi}_1 + \xi\boldsymbol{\chi}_1)' \boldsymbol{\Sigma}_X'\boldsymbol{\Sigma}_X (\boldsymbol{\upsilon}_1 + (\eta-1)\,\boldsymbol{\pi}_1 + \xi\boldsymbol{\chi}_1)$$

Since by assumption $\boldsymbol{\Sigma}_X'\boldsymbol{\Sigma}_X = \mathbf{I}$ was assumed to be the identity and $\boldsymbol{\upsilon}_1'\boldsymbol{\pi}_1 = \boldsymbol{\upsilon}_1'\boldsymbol{\chi}_1 = 0$, then:

$$\mathbb{V}[\mathrm{d}r_t] = \boldsymbol{\upsilon}_1'\boldsymbol{\upsilon}_1 + (\eta-1)^2\,\boldsymbol{\pi}_1'\boldsymbol{\pi}_1 + 2\xi\,(\eta-1)\,\boldsymbol{\pi}_1'\boldsymbol{\chi}_1 + \xi^2\boldsymbol{\chi}_1'\boldsymbol{\chi}_1$$

This yields:

$$\begin{aligned} \frac{\partial\mathbb{V}[\mathrm{d}r_t]}{\partial\eta} &= 2\,(\eta-1)\,\boldsymbol{\pi}_1'\boldsymbol{\pi}_1 + 2\xi\boldsymbol{\pi}_1'\boldsymbol{\chi}_1 \\ \frac{\partial\mathbb{V}[\mathrm{d}r_t]}{\partial\xi} &= 2\,(\eta-1)\,\boldsymbol{\pi}_1'\boldsymbol{\chi}_1 + 2\xi\boldsymbol{\chi}_1'\boldsymbol{\chi}_1 \end{aligned}$$

For both settings, assuming the parameter values as in the base case, the first derivative is negative and the second is positive. The variance function is convex and the minimum volatility is obtained for $\eta = 1$ and $\xi = 0$.

[34]For the $\mathcal{SM}$ setting, $\boldsymbol{\Sigma}_Y$ is upper block-triangular, with a 3×3 block in the upper-left corner, and has full rank. It can be shown that the fourth row of the projection matrix $(\boldsymbol{\Sigma}_Y'\boldsymbol{\Sigma}_Y)^{-1}\boldsymbol{\Sigma}_Y'$, appearing

pact on the market prices of risk, and thus on the maximum Sharpe ratio, but do impact bond volatilities. As shown in Figure 7a, a positive shift in expected inflation weight increases bond volatilities and decreases their corresponding positions, hence increasing the stock/bond mix.

In the last panel of Table 5 we report the impact of shifts in the expected output gap weight. A positive shift is slightly welfare improving (0.42% for a 20-year horizon), while a negative shock is harmful (−0.46%). These effects are limited to few basis points for horizons up to 5 years and are still small for long horizons. Whilst the maximum Sharpe ratio is not affected by changes in ξ, the real rate is affected. The volatility of the real rate is (very slightly) decreasing in ξ, which would suggest a welfare variation going in the other direction. An interpretation of the welfare increase for a higher value of ξ and long horizons can be given looking at the impulse responses of the real rate in Figures 3a and 4a. Corresponding to a higher value for ξ, the long-term response of the real rate to expected inflation and output gap shocks is dampened, allowing a stabilisation of the real rate over long investment horizons. The portfolio positions are almost unaffected by shifts in ξ. Overall, shifts in monetary policy matter for long-term investors, and those related to expected inflation matter more than those related to expected output gap.

7.1.2 Findings for the $\mathcal{UM}$ setting

[Table 6 about here.]

Comparing the base case strategy in Table 6 to that in Table 5, one notices the following. The bond positions are substantially lower under the $\mathcal{UM}$ setting;[35] the 2Y bond myopic position has been divided by close to 4 (4.641 vs. 18.491), the total bond position by close to 2 (4.221 vs. 9.604) and the stock-bond mix has been multiplied by more than 2. Horizon effects are now substantial both in the bond positions and the stock position. For a 20-year investment horizon, the intertemporal hedging component in the total bond position relative to the myopic component is −61% (1.805/4.641 − 1), while it was only around +7% (10.241/9.604 − 1) in the $\mathcal{SM}$ setting. Intertemporal hedging in the stock position, which is 0 by construction in the $\mathcal{SM}$ setting, is in this case close to −13% (0.635/0.729 − 1).

in the portfolio strategy (3.27) and determining the optimal weight of the stock index, does not depend on the upper-left block of $\mathbf{\Sigma}_Y$ and has zeros as first three elements. This means that the stock does not contribute in the strategy relatively to risks exclusively controlled by the state variables, such as the interest-rate risk.

[35] Other studies allowing for an asset universe containing only two bonds deliver findings qualitatively similar to ours in different setting. For example, Table VI, Panel A page 211 in Sangvinatsos and Wachter (2005) reports reasonable positions for bonds whenever investors can trade two bonds and the stock. For this purpose, they assume that the nominal market is incomplete, whilst ours is always complete; they also assume constant equity premium whilst ours is time varying. Koijen et al. (2010) report reasonable positions thanks to the portfolio constraints faced by the investor.

Negative shocks to expected inflation or expected output gap tend to increase welfare. For a 2-year horizon, the increase in welfare after an expected inflation shock is 2.751%, and it is 2.122% in case of an expected output gap shock. The figures are 3.629% and 2.436%, respectively, for a 20-year horizon. Negative shocks to expected inflation increase the real rate and the maximum Sharpe ratio, hence the positive impact on welfare. Negative shocks to expected output gap reduce the real rate, but increase the maximum Sharpe ratio. The latter effect more than offsets the negative impact on the real rate, hence the positive welfare effect. The impact on the portfolio strategy are substantial: the stock position increases substantially, reflecting the increase in the stock risk premium, while the total bond position is relatively stable. Intertemporal hedging effects increase dramatically in individual bond positions, although the effect is attenuated when looking at the total bond position. The stock-bond mix doubles at a 20-year investment horizon. This again reflects the strong desirability of the stock after the increase in its premium following a negative shock.

A positive shock to expected inflation or expected output gap harms the investor's welfare. The effect is stronger for a positive shock to expected inflation: the welfare loss for a 20-year horizon is -2.260% (expected inflation) and -1.193% (expected output gap). In the case of expected inflation, the initial negative impact on the real rate is reinforced by the negative impact on the maximum Sharpe ratio, while in the case of the output gap, the positive impact on the real rate attenuates the impact of the negative shock on the maximum Sharpe ratio. Furthermore, positive shocks to expected inflation or expected output gap reduce the risk premium of the stock and hence its desirability. Hence the reduced position in the stock and the decline of the stock/bond mix. Horizon effects also diminish in the optimal portfolio strategy.

As shown in the fourth and fifth panels of Table 6, the impact of monetary policy shifts on welfare are substantial. For a 20-year investment horizon, positive (negative) shifts in expected inflation weight increase (decrease) the welfare by 10.496% (-9.703%). Positive (negative) shifts in expected output gap weight decrease (increase) the welfare by -2.507% (2.532%). Thus, the impact of relative shifts on the expected inflation weight are again stronger than shifts on the expected output gap weight. The risky asset positions are reduced for positive shifts of η, reflecting the increase of the asset volatilities.

The impact of shifts in the expected inflation weight follows the same logic as in the $\mathcal{SM}$ setting: when η is increased towards 1, the nominal rate reflects better expected inflation shocks, hence the impact of expected inflation shocks over the real rate are attenuated (both in terms of level and volatility), which is a valuable feature for a long-term investor. As to the impact of shifts on the weight of the expected output gap, in the $\mathcal{UM}$ setting a reduction of ξ corresponds to a decrease in the volatility of the real rate and, as it can be seen by the impulse responses of the real rate in Figures 3b and 4b, also dampens the response of the real rate to shocks in expected inflation and output gap.

7.1.3 Comparision between $\mathcal{SM}$ and $\mathcal{UM}$ settings

The analyses for both the $\mathcal{SM}$ and the $\mathcal{UM}$ settings show that shifts of the monetary policy weight on expected inflation are of prime importance for the long-term investor. An increase of η reduces the volatility of the real rate, which is welfare improving. The $\mathcal{UM}$ setting shows welfare impacts of monetary policy shifts twice as large as those observed under the $\mathcal{SM}$ setting for a 20-year investment horizon. In both settings the impact of shifts of the weight on the expected output gap is less important than the impact of shifts on the weight on expected inflation. Although in both cases a small reduction of ξ entails a reduction of the real rate volatility, the effects on welfare are different and the expected welfare improvement corresponding to a reduction of ξ is captured only in $\mathcal{UM}$.

7.2 Utility over intermediate consumption

For the sake of brevity, we report the impact of monetary policy shifts only for the $\mathcal{UM}$ setting. We consider an annualised time-preference parameter $\delta = 2.5\%$. The solution to the portfolio choice problem can be expressed in quasi-closed form only when nominal markets are complete, as described in Appendix C. This is the case when the investor has access to two bonds (for example, 2Y and 10Y) and the equity index.

[Table 7 about here.]

The first thing that can be noticed comparing Table 7 to Table 6 is that, in the presence of intermediate consumption, the intertemporal hedging components are smaller than in the case where utility derives from terminal wealth only. This effect is documented in Wachter (2002) and is due the fact that the whole path of the stream of consumption matters for investor's utility. There is a duration effect for the investment horizon, as utility depends on a weighted-average of the contributions given by consumption at each point in time, rather than a single contribution coming at maturity. The "equivalent" horizon is therefore much shorter than T, hence the intertemporal hedging component is less pronounced. Even in this case, but on a smaller scale than in the case of utility over terminal wealth, the intertemporal hedging component is such that there is an increase in the allocation of the 10Y bond and a diminution of the allocation in the 2Y bond with the investment horizon, so that the investor can benefit progressively more from the term-premium.

The consumption-to-wealth ratio varies very little upon shocks to expected inflation or expected output gap, as well as upon shifts in monetary policy. The effect of shifts in monetary policy parameters are similar to those obtained for the case where utility depends on terminal wealth, but with a slightly lower impact of the intertemporal hedging component on the portfolio allocation and, consequently, on the investor's welfare. Nevertheless, the presence of the intertemporal hedging component still significantly reduces the shifts in portfolio allocation corresponding to shifts in η, especially for longer horizons, with respect to the strong effects that η has over the myopic allocation. Again, we see that welfare increases with the coefficient η and is very slightly decreasing with ξ, but the effects are smaller than in the previous section. For example, for an investor with an investment horizon of 20 years, the positive (negative) impact on welfare of a positive (negative) shift in the expected inflation weight is 2.620% (−2.649) with utility over intermediate consumption, while it was 10.496% (−9.703) with utility over terminal wealth.

7.3 Utility over intermediate consumption and real balances

A direct observable consequence of the presence of money in the utility function is the third term appearing in the optimal portfolios strategy (3.27), related to the time-variation of relative risk aversion, but there is an impact also on the fourth term, representing the intertemporal hedging component. A complication of the presence of money is that the function F cannot be computed explicitly. Usually, when utility derives only from consumption, an exponential affine solution for F is obtained, as for example in Sangvinatsos and Wachter (2005). In our case, relative risk aversion is driven by the short-term nominal interest rate, which distribution is Gaussian, while the SDF is conditionally log-normally distributed. The only viable solution for an empirical implementation of the strategy is therefore the implementation of a simulation-based approach, along the lines of Detemple et al. (2003) and Cvitanic et al. (2003).

We use the Monte Carlo Malliavin Derivatives (MCMD) simulation method, described in Appendix E, using the simulation equations summarised in Appendix E.3. In order to assess the reliability of the method, in Appendix L we perform the calculation through MCMD of the portfolio strategy in the case of utility over consumption only ($\alpha = 1$), which can be compared to the results of the previous section, confirming that the two methods provide solutions that are very close to each other. In terms of preference parameters, for the coefficient measuring the relative weight of consumption and money, we consider as base case $\alpha = 0.96$,[36] whereas for the elasticity of intratemporal substitution between consumption and money, we take $\rho = 0.8$.

[Table 8 about here.]

First of all, we can compare the results in Table 8, obtained in the presence of money-in-the-utility-function, with those in Table 7, obtained when utility is from intermediate consumption only. The first noticeable difference between the strategies in the two cases is that, when money is taken into account, the presence of money affects bond and

[36]The range of values considered is in line with the literature on the topic (Finn et al., 1990, Holman, 1998, Lioui and Maio, 2013).

cash positions while the stock position is left unchanged. Since real balances expose to inflation risk like nominal cash positions, the latter is strongly affected by the presence of real balances and more so when the share of real balances is high.

This is because of the presence of a third instantaneous hedging term in the optimal strategy (3.27) that appears when relative risk aversion is time-varying. The volatility of relative risk aversion, $\boldsymbol{\sigma}_{\mathrm{RRA}_t}$, is proportional to the volatility of the short-term nominal interest rate, $\boldsymbol{\Sigma}_X \mathbf{R}_1$, as relative risk aversion varies because of the variable cost of money holdings, proportional to the nominal interest rate. This additional term is therefore given by bond positions that hedge the exposure to instantaneous changes in the interest rate. In particular, in the base case scenario, we notice that the total bond position is reduced with respect to the position in Table 7. This is because real balances carry a cost proportional to the interest rate: if the interest rate rises, the investor will be exposed to a higher cost for holding money, thus this position must be hedged with a position such that wealth increases in correspondence of a rising interest rate and vice versa. A reduction of the total bond position corresponds indeed to a contribution of positive exposure to nominal short-term rate changes. Furthermore, individual bond exposures (in absolute value) are slightly reduced, in order to compensate for the additional contribution of interest-rate risk carried by real balances.

The instantaneous hedging component arising from the presence of time-varying risk aversion, which is pretty sharp for short horizons, is partially offset by the intertemporal hedging component, making the strategies for $\alpha < 1$ and for $\alpha = 1$ more alike for longer maturities. In general, we can notice a lower tendency to speculatively allocate wealth into riskier bonds, which is reasonable considering that a smaller fraction of utility comes from wealth consumption. There is instead very little difference in terms of stock allocation. For what concerns the consumption plan, there is a reduction in the consumption-to-wealth ratio, due to the fact that there is a substitution effect between consumption and real balances. The last row of each panel indicates the money-to-wealth ratio, that is proportional to consumption, as from equation (3.34). m_0/w_0 is therefore decreasing with the maturity and the proportionality factor with respect to consumption depends on the interest rate, thus on instantaneous relative risk aversion. For the base case parameter values, the quantities c_0/w_0 and m_0/w_0 are of the same order. The amount of uninvested money, m_0, with respect to the case without money-in-the-utility-function, is partially financed by reducing the total position in bonds and partially with cash, which position is even more negative in this case.

The second panel of Table 8 shows the allocation when the relative weight of consumption (α) is increased or decreased with respect to its base case value. We notice that variations in the optimal solution are quite sharp. It comes with no surprise that for decreasing α, that is when money is given more relative importance, there is a decrease in consumption, an increase in real balances and an increase in the hedging demand specific to the presence of money in the utility function. In particular, there is an important variation of the instantaneous hedging contribution that makes the strategy deviate from the myopic solution obtained in the case of utility over consumption only. The case of utility over consumption is instead approached when α is increased and gets closer to 1.

The third panel of Table 8 shows instead a static comparative analysis with respect to the elasticity of substitution ρ. The sensitivity is lower than in the previous case, with an increasing importance of real balances with respect to consumption, thus of the corresponding hedging term, when ρ is decreased.

The effects of shifts in π and χ are qualitatively similar to those obtained in the case of utility over consumption for what concerns portfolio positions and welfare, but with some important differences, due to the fact that the strategy in this case strongly depends on the short-term rate. In fact, initial shifts in π and χ imply shifts in the short-term rate R in the same direction, according to the coefficients η and ξ. If the initial short-rate is lowered (negative $\Delta\pi$ or $\Delta\chi$), the total bond position is significantly decreased, as well as individual absolute bond exposures. If the short-term rate is instead increased (positive $\Delta\pi$ or $\Delta\chi$), the total bond position increases and the allocation gets closer to (but still lower than) that obtained in the case of utility over consumption only. This comes in parallel with the fact that, when the short-term rate is low, the money-to-wealth ratio is higher and the consumption-to-wealth ratio is lower than in the base case and vice versa. What happens is that, when R is low, the cost of holding real balances is low, the investor keeps more money uninvested and reduces the total position in bonds, as she holds an additional risk negatively correlated with the nominal rate. When instead the interest rate is high, she holds fewer real balances and the total bond position can increase.

In terms of sensitiveness with respect to monetary policy parameters, comparing Table 8 with Table 7, we can see that shifts in η have very similar effects on the portfolio strategy. The effects on welfare are also similar, even though slightly less pronounced, because of the smaller speculative positions into risky assets. The same conclusions can be drawn for shifts in ξ, where differences in wealth are in general smaller.[37]

8 Conclusions

In the present paper we have studied the portfolio choice problem for an individual long-term investor, deriving utility over real consumption and real balances, and analysed the effect of monetary policy shifts on her allocation.

We have expressed the nominal short-term rate in terms of a Taylor rule, allowing to study the impact of monetary policy parameters on portfolio strategy and welfare. It

[37]For short horizons the relative welfare changes are too little to be significant if compared to the noise of the Monte Carlo simulation. This might explain the figures with an opposite sign with respect to the expected trend.

appears that monetary policy has a significant impact on the optimal portfolio choice, particularly through the reactions to changes in expected inflation, and only secondarily through the reactions to changes in the expected output gap. A more actively conservative policy, that is a policy sharply reacting to inflationary or deflationary trends in the attempt of stabilizing prices, causes a significant increase in nominal bond volatilities and betas, implying an overall reduction of positions in risky assets, a shift towards longer-maturity bonds and an increase of welfare.

With respect to the existing asset allocation literature, we modify the specification of a traditional Gaussian dynamic term structure model by using a factor that helps pricing the time-varying equity premium. This factor complements the information carried by the yield-based factors and is filtered from equity returns and realised inflation. The consequence of this choice is that, differently from the market specification used by Sangvinatsos and Wachter (2005), there is an intertemporal hedging demand for the equity in the portfolio strategy, as time-variation of equity premium can not be hedged with bonds. This hedging demand is decreasing with the investment horizon, whilst we see an increase in the position of the longest maturity bond. This result is very different from other cases in the literature, such as in Wachter (2002), where it is assumed that the equity premium is perfectly anti-correlated with equity returns. In this case indeed it happens that the stock demand is sharply increasing with the horizon, whilst, in the context of our richer investment opportunity set, the investor prefers to increase the allocation in risky long-term bonds and slightly reduce the stock allocation.

Finally, the introduction of real balances in preferences entails a substitution effect between consumption and money holding, in the sense that the optimal investor decides to give up part of her consumption in order to finance the opportunity cost of holding a certain amount of cash. The amount of money kept uninvested is proportional to the instantaneous consumption rate, where the coefficient of proportionality is decreasing in the short-term nominal rate, representing the cost of holding real balances. In terms of optimal portfolio strategy, the introduction of money in the utility function, implying a time-variation of relative risk aversion, causes the appearance of an additional instantaneous hedging demand. This component reduces risky bond positions and corresponds to a positive exposure to the short-term nominal rate, aimed at hedging instantaneous changes in the cost of money. The absolute value of the intertemporal hedging demand is also reduced when the importance of real balances in preferences increases.

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Table 1: Parameter estimates for the two estimation settings.

(a) $\mathcal{SM}$ setting

Θ^{11}	Θ^{22}	Θ^{33}	Θ^{21}	Θ^{31}	Θ^{32}		η	ξ
0.5224	0.1122	1.9938	0.3634	−0.8670	−0.4560		0.8376	0.3713
(0.1597)	(0.0582)	(0.2907)	(0.1484)	(0.2869)	(0.2085)		(0.1151)	(0.0773)
R_0	π_0	χ_0	σ_ϵ^1	σ_ϵ^2	σ_ϵ^3			σ_ϵ^G
0.0443	0.0341	−0.0068	0.0020	0.0010	0.0011			0.0225
(0.0152)	(0.0093)	(0.0014)	(0.0001)	(0.0000)	(0.0000)			
$\mathbf{R}_1$	$\boldsymbol{\pi}_1$	$\boldsymbol{\chi}_1$	$\boldsymbol{\Lambda}_0$	$\boldsymbol{\Lambda}_1$			$\boldsymbol{\sigma}_S$	$\boldsymbol{\sigma}_P$
−0.0077	0.0012	−0.0225	0.2302	−0.5132	−0.0876	0.5541	0.0037	−0.0007
(0.0027)	(0.0039)	(0.0019)	(0.1029)	(0.1494)	(0.0544)	(0.2368)	(0.0059)	(0.0004)
0.0126	0.0106	−0.0048	−0.4131	−0.1873	0.0083	−0.6749	−0.0168	0.0012
(0.0012)	(0.0018)	(0.0008)	(0.0688)	(0.1780)	(0.0499)	(0.1074)	(0.0055)	(0.0003)
−0.0124	−0.0213	0.0072	0.3447	0.1686	0.2406	−0.6295	0.0052	0.0000
0.0014	(0.0024)	(0.0028)	(0.1763)	(0.1751)	(0.0935)	(0.2765)	(0.0058)	(0.0001)
			0.4109	−0.0126	−0.1978	0.4357	0.1488	0.0007
			(0.2077)	(0.0887)	(0.0738)	(0.2262)	(0.0039)	(0.0003)
			0	0	0	0	0	0.0087
								(0.0002)

(b) $\mathcal{UM}$ setting

Θ^{11}	Θ^{22}	Θ^{33}	Θ^{21}	Θ^{31}	Θ^{32}		η	ξ
0.1030	0.7876	0.1611	−0.3169	0.2355	−0.3441		0.6932	0.3565
(0.0537)	(0.1676)	(0.0764)	(0.1708)	(0.1746)	(0.2695)		(0.0992)	(0.1679)
R_0	π_0	χ_0	σ_ϵ^1	σ_ϵ^2	σ_ϵ^3	σ_ϵ^4		σ_ϵ^G
0.0446	0.0350	−0.0076	0.0024	0.0017	0.0019	0.0032		0.0225
(0.0173)	(0.0146)	(0.0014)	(0.0001)	(0.0000)	(0.0001)	(0.0001)		
$\mathbf{R}_1$	$\boldsymbol{\pi}_1$	$\boldsymbol{\chi}_1$	$\boldsymbol{\Lambda}_0$	$\boldsymbol{\Lambda}_1$			$\boldsymbol{\sigma}_S$	$\boldsymbol{\sigma}_P$
0.0166	0.0117	0.0053	−0.3971	0.1091	−0.5009	0	−0.0146	0.0015
(0.0018)	(0.0035)	(0.0009)	(0.0888)	(0.1284)	(0.0402)		(0.0056)	(0.0003)
−0.0082	−0.0140	−0.0094	0.1403	0.2182	−0.4206	0	0.0049	0.0003
(0.0037)	(0.0030)	(0.0018)	(0.1223)	(0.1420)	(0.1649)		(0.0062)	(0.0004)
0	−0.0063	0.0085	0.4333	−0.2356	0.2291	−0.0465	0.1494	0.0008
	(0.0011)	(0.0008)	(0.1780)	(0.0946)	(0.1662)	(0.0687)	(0.0039)	(0.0003)
			0	0	0	0	0	0.0083
								(0.0002)

Θ^{ij} are the elements of the lower-triangular mean reversion matrix for the state variables dynamics in equation (3.3). R_0 and the vector $\mathbf{R}_1$ are the parameters determining the affine relation between the state variables and the short-term rate that are described in the Taylor rule equation (3.7). π_0 and the vector $\boldsymbol{\pi}_1$ are the parameters determining the affine relation between the state variables and the expected inflation described in the second equation in (3.5). χ_0 and the vector $\boldsymbol{\chi}_1$ are the regression coefficients determining the affine relation between the state variables and the output gap described in the third equation in (3.5). The vector $\boldsymbol{\Lambda}_0$ and the matrix $\boldsymbol{\Lambda}_1$ are the parameters specifying the affine relation between the state variables and the vector of market prices of risk, described in equation (3.2). $\boldsymbol{\sigma}_P$ is the volatility vector of the price index dynamics appearing in equation (3.9). $\boldsymbol{\sigma}_S$ is the volatility vector of the stock index appearing in equation (3.11). The terms η and ξ are the weights of expected inflation and output gap, appearing in the Taylor rule of equation (3.6), derived on the basis of the equations (4.3). The terms σ_ϵ^i are the standard deviations of the observation errors associated to the imperfectly observed yields. All parameters are estimated by maximum-likelihood, except for the regression coefficients relating expected output-gap to state variables and the monetary policy parameters, computed as in equation (4.3). The values in brackets are the standard errors of the estimates.

Table 2: Mean values and standard deviations of relevant historical and estimated data.

(a) $\mathcal{SM}$ setting

Time series	Mean value		Standard deviation	
	Estimation	Data	Estimation	Data
3M ZC yield	4.74%	4.66%	3.13%	3.08%
6M ZC yield	4.85%	4.85%	3.14%	3.14%
1Y ZC yield	5.03%	5.06%	3.13%	3.14%
2Y ZC yield	5.29%	5.29%	3.09%	3.09%
5Y ZC yield	5.69%	5.70%	2.90%	2.90%
10Y ZC yield	6.08%	6.08%	2.70%	2.70%
Equity returns			15.01%	15.07%
Realized inflation			0.90%	1.09%
Realized output gap		−0.57%		2.96%
3M ZC risk premium	0.25%		0.21%	
6M ZC risk premium	0.46%		0.41%	
1Y ZC risk premium	0.77%		0.83%	
2Y ZC risk premium	1.16%		1.68%	
5Y ZC risk premium	1.78%		3.83%	
10Y ZC risk premium	2.50%		6.61%	
Equity risk premium	6.70%		6.17%	
Nominal risk-free rate	4.61%		3.13%	
Expected inflation	3.53%		2.16%	
Expected output gap	−0.57%		1.99%	

(b) $\mathcal{UM}$ setting

Time series	Mean value		Standard deviation	
	Estimation	Data	Estimation	Data
3M ZC yield	4.77%	4.66%	3.16%	3.08%
6M ZC yield	4.85%	4.85%	3.14%	3.14%
1Y ZC yield	5.00%	5.06%	3.10%	3.14%
2Y ZC yield	5.25%	5.29%	3.04%	3.09%
5Y ZC yield	5.70%	5.70%	2.90%	2.90%
10Y ZC yield	6.02%	6.08%	2.64%	2.70%
Equity index returns			15.00%	15.07%
Realized inflation			0.86%	1.09%
Realized output gap		−0.57%		2.96%
3M ZC risk premium	0.19%		0.15%	
6M ZC risk premium	0.36%		0.29%	
1Y ZC risk premium	0.65%		0.59%	
2Y ZC risk premium	1.08%		1.22%	
5Y ZC risk premium	1.78%		3.23%	
10Y ZC risk premium	2.34%		6.25%	
Equity index risk premium	6.63%		5.95%	
Nominal risk-free rate	4.68%		3.19%	
Expected inflation	3.51%		2.36%	
Expected output gap	−0.57%		1.96%	

The tables show first and second moments of the distributions of relevant historical time series and their parameter-implied counterparts. Means and standard deviations of bond yields are computed from the historical data time series and the corresponding values implied by the filtered time series of the state variables, $\mathbf{X}_t$, and the estimated parameters, through equation (A.5). For stock returns and realised inflation, the historical quantities are obtained annualising the standard deviation of monthly returns, whilst the estimated ones are computed as the total model-implied conditional volatility. The average risk premia depend on the estimated parameters and are equal to $\mathbf{A}_1(\tau)\,\boldsymbol{\sigma}'_X \mathrm{E}(\boldsymbol{\Lambda}^*_t)$, where $\mathrm{E}(\boldsymbol{\Lambda}^*_t)$ is the average value of the market prices of risk vector, implied by equation (3.2). Average nominal risk-free rate, expected inflation and expected output gap are computed equivalently. Their standard deviations, as well as those for risk premia are computed from the time series implied by the filtered time series of the state variables and the estimated parameters.

Table 3: Conditional correlation matrix of asset returns, as implied by estimated parameters and calculated from data.

(a) $\mathcal{SM}$ setting

	3M ZCB	6M ZCB	1Y ZCB	2Y ZCB	5Y ZCB	10Y ZCB	Stock	CPI
3M ZCB	1.000							
6M ZCB	0.987	1.000						
1Y ZCB	0.921	0.971	1.000					
2Y ZCB	0.782	0.869	0.962	1.000				
5Y ZCB	0.615	0.718	0.847	0.948	1.000			
10Y ZCB	0.510	0.589	0.692	0.790	0.928	1.000		
Stock	0.116	0.115	0.107	0.092	0.086	0.091	1.000	
CPI	−0.203	−0.205	−0.195	−0.171	−0.148	−0.141	0.050	1.000

(b) $\mathcal{UM}$ setting

	3M ZCB	6M ZCB	1Y ZCB	2Y ZCB	5Y ZCB	10Y ZCB	Stock	CPI
3M ZCB	1.000							
6M ZCB	0.996	1.000						
1Y ZCB	0.982	0.993	1.000					
2Y ZCB	0.922	0.946	0.977	1.000				
5Y ZCB	0.680	0.724	0.798	0.908	1.000			
10Y ZCB	0.467	0.520	0.613	0.767	0.966	1.000		
Stock	0.109	0.110	0.109	0.103	0.078	0.055	1.000	
CPI	−0.215	−0.222	−0.227	−0.227	−0.195	−0.157	0.037	1.000

(c) Sample correlations from data time-series

	3M ZCB	6M ZCB	1Y ZCB	2Y ZCB	5Y ZCB	10Y ZCB	Stock	CPI
3M ZCB	1.000							
6M ZCB	0.950	1.000						
1Y ZCB	0.856	0.950	1.000					
2Y ZCB	0.759	0.867	0.954	1.000				
5Y ZCB	0.596	0.713	0.828	0.923	1.000			
10Y ZCB	0.472	0.582	0.687	0.789	0.924	1.000		
Stock	0.046	0.105	0.113	0.092	0.079	0.097	1.000	
CPI	−0.105	−0.110	−0.110	−0.121	−0.141	−0.135	0.010	1.000

Panels (a) and (b) report one-month conditional correlations between the innovations of nominal risky assets and the economic state variables, as implied by the estimated parameter values in table 1. Panel (c) shows unconditional correlations of bond, stock and CPI realised returns, calculated from the monthly data time series.

Table 4: Instantaneous correlations of risk premia and economic variables with asset returns, as implied by estimated parameters

(a) $\mathcal{SM}$ setting

	3M ZCB	6M ZCB	1Y ZCB	2Y ZCB	5Y ZCB	10Y ZCB	Stock	CPI	R	π	χ
3M ZCB RP	0.338	0.199	−0.039	−0.320	−0.557	−0.562	0.012	0.033	−0.477	−0.803	0.349
6M ZCB RP	0.375	0.231	−0.014	−0.300	−0.512	−0.473	0.023	0.025	−0.519	−0.858	0.448
1Y ZCB RP	0.407	0.262	0.015	−0.271	−0.460	−0.387	0.032	0.017	−0.553	−0.895	0.520
2Y ZCB RP	0.440	0.296	0.050	−0.234	−0.414	−0.327	0.038	0.010	−0.584	−0.918	0.545
5Y ZCB RP	0.517	0.379	0.137	−0.150	−0.349	−0.291	0.045	−0.002	−0.651	−0.941	0.471
10Y ZCB RP	0.615	0.491	0.265	−0.023	−0.265	−0.280	0.049	−0.017	−0.731	−0.939	0.290
Stock RP	0.733	0.618	0.404	0.129	−0.075	−0.048	0.072	−0.045	−0.836	−0.994	0.348
R	−0.986	−0.948	−0.840	−0.652	−0.464	−0.393	−0.105	0.118	1.000	0.846	0.046
π	−0.746	−0.634	−0.424	−0.159	0.018	−0.041	−0.080	0.052	0.846	1.000	−0.405
χ	−0.176	−0.288	−0.443	−0.551	−0.422	−0.067	0.010	0.052	0.046	−0.405	1.000

(b) $\mathcal{UM}$ setting

	3M ZCB	6M ZCB	1Y ZCB	2Y ZCB	5Y ZCB	10Y ZCB	Stock	CPI	R	π	χ
3M ZCB RP	0.438	0.386	0.278	0.064	−0.369	−0.601	0.037	0.026	−0.488	−0.756	−0.702
6M ZCB RP	0.471	0.420	0.314	0.101	−0.334	−0.571	0.041	0.019	−0.520	−0.776	−0.715
1Y ZCB RP	0.519	0.470	0.366	0.156	−0.281	−0.524	0.046	0.009	−0.567	−0.805	−0.732
2Y ZCB RP	0.577	0.530	0.429	0.224	−0.214	−0.464	0.052	−0.003	−0.622	−0.838	−0.749
5Y ZCB RP	0.642	0.597	0.501	0.303	−0.134	−0.390	0.059	−0.017	−0.684	−0.871	−0.766
10Y ZCB RP	0.669	0.626	0.533	0.337	−0.097	−0.356	0.062	−0.023	−0.710	−0.883	−0.772
Stock RP	0.901	0.876	0.817	0.677	0.302	0.042	−0.040	−0.102	−0.923	−0.894	−0.839
R	−0.998	−0.994	−0.974	−0.902	−0.632	−0.405	−0.101	0.139	1.000	0.864	0.648
π	−0.842	−0.816	−0.756	−0.615	−0.249	0.000	−0.406	0.049	0.864	1.000	0.527
χ	−0.622	−0.593	−0.531	−0.394	−0.066	0.143	0.558	0.101	0.648	0.527	1.000

The table reports parameter-implied instantaneous correlations between asset risk premia and returns in the tradable assets and the CPI, as well as innovations in the economic variables. The estimated parameter values are as in table 1a. The correlation between two generic quantities A and B is calculated as $\boldsymbol{\sigma}_A'\boldsymbol{\sigma}_B / \left(\|\boldsymbol{\sigma}_A\| \, \|\boldsymbol{\sigma}_B\|\right)$. Risk premia volatilities for zero-coupon bonds are given by $\boldsymbol{\Sigma}_X \boldsymbol{\Lambda}_1' \boldsymbol{\Sigma}_X \mathbf{A}_1'(\tau)$, whereas the equity risk premium volatility can be expressed as $\boldsymbol{\Sigma}_X \boldsymbol{\Lambda}_1' \boldsymbol{\sigma}_S$. Zero-coupon return volatility vectors are equal to $\boldsymbol{\Sigma}_X \mathbf{A}_1'(\tau)$. The volatilities in innovation in short-term rate, expected inflation and expected output gap are respectively given by $\boldsymbol{\Sigma}_X \mathbf{R}_1$, $\boldsymbol{\Sigma}_X \boldsymbol{\pi}_1$ and $\boldsymbol{\Sigma}_X \boldsymbol{\chi}_1$.

Table 5: Optimal portfolio for $\mathcal{SM}$ setting and utility over terminal wealth.

	Base case				
	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$
2Y bond	18.491	14.747	15.391	16.330	17.515
5Y bond	−11.932	−9.875	−10.813	−12.099	−13.739
10Y bond	3.045	3.030	3.834	5.005	6.465
Stock	0.694	0.694	0.694	0.694	0.694
Cash	−9.297	−7.596	−8.105	−8.930	−9.935
Total bond	9.604	7.902	8.412	9.236	10.241
Stock/bond	0.072	0.088	0.082	0.075	0.068

	$\Delta\pi=-0.01$					$\Delta\pi=+0.01$				
	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$
2Y bond	13.424	4.384	5.038	5.967	7.144	23.559	25.110	25.744	26.693	27.886
5Y bond	−7.109	−1.430	−2.427	−3.700	−5.329	−16.756	−18.319	−19.199	−20.498	−22.148
10Y bond	2.404	1.597	2.419	3.576	5.027	3.685	4.462	5.249	6.433	7.904
Stock	1.035	1.035	1.035	1.035	1.035	0.353	0.353	0.353	0.353	0.353
Cash	−8.754	−4.586	−5.065	−5.879	−6.877	−9.840	−10.606	−11.146	−11.982	−12.993
Total bond	8.719	4.551	5.030	5.844	6.842	10.488	11.254	11.794	12.629	13.641
Stock/bond	0.119	0.227	0.206	0.177	0.151	0.034	0.031	0.030	0.028	0.026
$\Delta\widetilde{w}_0/w_0$ (%)	–	2.542	2.278	1.828	1.283	–	−0.477	−0.200	0.247	0.791

	$\Delta\chi=-0.01$					$\Delta\chi=+0.01$				
	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$
2Y bond	21.474	17.548	18.469	19.440	20.632	15.509	11.946	12.313	13.221	14.397
5Y bond	−16.627	−15.879	−17.480	−18.821	−20.472	−7.238	−3.871	−4.146	−5.376	−7.005
10Y bond	5.078	5.797	6.887	8.087	9.557	1.011	0.263	0.780	1.923	3.373
Stock	0.703	0.703	0.703	0.703	0.703	0.684	0.684	0.684	0.684	0.684
Cash	−9.629	−7.170	−7.580	−8.409	−9.420	−8.966	−8.022	−8.631	−9.451	−10.450
Total bond	9.925	7.466	7.876	8.706	9.717	9.282	8.338	8.947	9.767	10.766
Stock/bond	0.071	0.094	0.089	0.081	0.072	0.074	0.082	0.076	0.070	0.064
$\Delta\widetilde{w}_0/w_0$ (%)	–	1.987	2.675	3.093	3.640	–	0.033	−0.014	−0.379	−0.902

	$\Delta\eta/\eta=-30\%$					$\Delta\eta/\eta=+30\%$				
	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$
2Y bond	27.911	21.774	22.804	24.340	26.268	13.604	11.092	11.518	12.160	12.983
5Y bond	−19.418	−15.618	−17.024	−18.897	−21.259	−8.252	−7.006	−7.665	−8.644	−9.916
10Y bond	5.086	4.699	5.717	7.164	8.978	2.060	2.189	2.855	3.861	5.111
Stock	0.694	0.694	0.694	0.694	0.694	0.694	0.694	0.694	0.694	0.694
Cash	−13.273	−10.548	−11.191	−12.301	−13.680	−7.106	−5.968	−6.403	−7.071	−7.872
Total bond	13.580	10.854	11.497	12.607	13.986	7.412	6.274	6.709	7.377	8.178
Stock/bond	0.051	0.064	0.060	0.055	0.050	0.094	0.111	0.103	0.094	0.085
$\Delta\widetilde{w}_0/w_0$ (%)	–	−0.264	−1.024	−2.430	−4.520	–	0.263	1.021	2.425	4.462

	$\Delta\xi/\xi=-30\%$					$\Delta\xi/\xi=+30\%$				
	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$
2Y bond	17.602	14.010	14.546	15.242	16.119	19.233	15.372	16.111	17.263	18.715
5Y bond	−10.435	−8.642	−9.399	−10.272	−11.390	−13.221	−10.950	−12.055	−13.711	−15.821
10Y bond	2.430	2.515	3.238	4.228	5.457	3.585	3.489	4.370	5.709	7.384
Stock	0.694	0.694	0.694	0.694	0.694	0.694	0.694	0.694	0.694	0.694
Cash	−9.291	−7.577	−8.079	−8.891	−9.880	−9.290	−7.604	−8.121	−8.954	−9.972
Total bond	9.597	7.883	8.385	9.198	10.186	9.597	7.910	8.427	9.261	10.278
Stock/bond	0.072	0.088	0.083	0.075	0.068	0.072	0.088	0.082	0.075	0.068
$\Delta\widetilde{w}_0/w_0$ (%)	–	−0.010	−0.046	−0.126	−0.455	–	0.009	0.040	0.109	0.415

The first columns report the myopic component of the optimal allocation, whereas the following columns correspond to investment horizons respectively equal to 2Y, 5Y, 10Y and 20Y. The portfolio strategy is computed considering the case where utility derives from terminal wealth only and the investor can trade in 3 bonds and the stock index. Parameters are those estimated in Table 1a and $\gamma=4$.

Table 6: Optimal portfolio for $\mathcal{UM}$ setting and utility over terminal wealth.

	Base case				
	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$
2Y bond	4.641	2.316	1.943	1.854	1.805
10Y bond	−0.420	0.165	0.353	0.544	0.787
Stock	0.729	0.712	0.685	0.656	0.635
Cash	−3.950	−2.193	−1.981	−2.053	−2.226
Total bond	4.221	2.481	2.296	2.397	2.592
Stock/bond	0.173	0.287	0.298	0.274	0.245

	$\Delta\pi=-0.01$					$\Delta\pi=+0.01$				
	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$
2Y bond	4.086	0.309	−0.157	−0.248	−0.297	5.196	4.322	4.043	3.955	3.906
10Y bond	0.199	1.103	1.310	1.502	1.744	−1.040	−0.773	−0.603	−0.415	−0.170
Stock	0.907	0.895	0.869	0.839	0.818	0.551	0.528	0.501	0.472	0.452
Cash	−4.192	−1.308	−1.022	−1.093	−1.265	−3.707	−3.078	−2.941	−3.013	−3.187
Total bond	4.285	1.413	1.153	1.254	1.447	4.156	3.550	3.440	3.541	3.736
Stock/bond	0.212	0.634	0.754	0.669	0.565	0.133	0.149	0.146	0.133	0.121
$\Delta\widetilde{w}_0/w_0$ (%)	–	2.751	3.311	3.557	3.629	–	−1.507	−1.965	−2.193	−2.260

	$\Delta\chi=-0.01$					$\Delta\chi=+0.01$				
	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$
2Y bond	4.381	0.672	0.202	0.111	0.062	4.902	3.960	3.684	3.596	3.548
10Y bond	0.090	0.976	1.182	1.379	1.629	−0.931	−0.645	−0.476	−0.292	−0.055
Stock	0.933	0.923	0.898	0.868	0.847	0.525	0.501	0.473	0.443	0.423
Cash	−4.403	−1.570	−1.282	−1.358	−1.538	−3.496	−2.816	−2.681	−2.748	−2.915
Total bond	4.471	1.647	1.385	1.490	1.691	3.971	3.315	3.208	3.305	3.492
Stock/bond	0.209	0.560	0.648	0.583	0.501	0.132	0.151	0.147	0.134	0.121
$\Delta\widetilde{w}_0/w_0$ (%)	–	2.122	2.246	2.224	2.436	–	−1.003	−1.034	−1.000	−1.193

	$\Delta\eta/\eta=-30\%$					$\Delta\eta/\eta=+30\%$				
	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$
2Y bond	5.932	2.862	2.348	2.252	2.239	3.781	1.961	1.680	1.591	1.515
10Y bond	−0.654	0.090	0.298	0.479	0.708	−0.275	0.205	0.382	0.582	0.836
Stock	0.788	0.750	0.713	0.678	0.657	0.684	0.682	0.664	0.639	0.617
Cash	−5.065	−2.702	−2.359	−2.409	−2.604	−3.190	−1.848	−1.726	−1.812	−1.969
Total bond	5.278	2.952	2.646	2.731	2.947	3.506	2.166	2.062	2.173	2.351
Stock/bond	0.149	0.254	0.269	0.248	0.223	0.195	0.315	0.322	0.294	0.263
$\Delta\widetilde{w}_0/w_0$ (%)	–	−0.251	−1.233	−3.676	−9.703	–	0.250	1.235	3.756	10.496

	$\Delta\xi/\xi=-30\%$					$\Delta\xi/\xi=+30\%$				
	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$
2Y bond	5.009	2.468	2.073	2.003	1.980	4.320	2.184	1.830	1.724	1.653
10Y bond	−0.495	0.144	0.337	0.523	0.764	−0.357	0.182	0.366	0.560	0.805
Stock	0.692	0.688	0.667	0.641	0.618	0.763	0.734	0.702	0.669	0.649
Cash	−4.206	−2.300	−2.077	−2.167	−2.362	−3.726	−2.101	−1.898	−1.953	−2.107
Total bond	4.514	2.612	2.410	2.526	2.744	3.962	2.366	2.197	2.284	2.458
Stock/bond	0.153	0.263	0.277	0.254	0.225	0.193	0.310	0.319	0.293	0.264
$\Delta\widetilde{w}_0/w_0$ (%)	–	0.014	0.155	0.715	2.532	–	−0.015	−0.157	−0.719	−2.507

The first columns report the myopic component of the optimal allocation, whereas the following columns correspond to investment horizons respectively equal to 2Y, 5Y, 10Y and 20Y. The portfolio strategy is computed considering the case where utility derives from terminal wealth only and the investor can trade in 2 bonds and the stock index. Parameters are those estimated in Table 1b and $\gamma = 4$.

Table 7: Optimal portfolio for $\mathcal{UM}$ setting and utility over intermediate consumption.

	Base case				
	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$
2Y bond	4.641	3.118	2.540	2.284	2.153
10Y bond	−0.420	−0.047	0.128	0.253	0.370
Stock	0.729	0.721	0.709	0.694	0.679
Cash	−3.950	−2.793	−2.376	−2.231	−2.202
Total bond	4.221	3.071	2.668	2.537	2.523
Stock/bond	0.173	0.235	0.266	0.273	0.269
c_0/w_0	–	0.533	0.239	0.145	0.103

	$\Delta\pi=-0.01$					$\Delta\pi=+0.01$				
	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$
2Y bond	4.086	1.567	0.701	0.343	0.167	5.196	4.675	4.384	4.229	4.143
10Y bond	0.199	0.793	1.028	1.176	1.302	−1.040	−0.888	−0.774	−0.670	−0.563
Stock	0.907	0.903	0.892	0.877	0.863	0.551	0.540	0.526	0.510	0.496
Cash	−4.192	−2.262	−1.622	−1.396	−1.332	−3.707	−3.327	−3.135	−3.069	−3.075
Total bond	4.285	2.359	1.730	1.519	1.469	4.156	3.787	3.609	3.558	3.579
Stock/bond	0.212	0.383	0.515	0.577	0.587	0.133	0.142	0.146	0.143	0.139
c_0/w_0	–	0.540	0.243	0.148	0.106	–	0.529	0.236	0.143	0.102
$\Delta\widetilde{w}_0/w_0$ (%)	–	1.787	2.505	2.875	3.085	–	−0.943	−1.405	−1.677	−1.840

	$\Delta\chi=-0.01$					$\Delta\chi=+0.01$				
	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$
2Y bond	4.381	1.912	1.054	0.697	0.522	4.902	4.329	4.030	3.873	3.786
10Y bond	0.090	0.670	0.903	1.051	1.179	−0.931	−0.765	−0.649	−0.545	−0.440
Stock	0.933	0.929	0.919	0.905	0.891	0.525	0.513	0.498	0.482	0.468
Cash	−4.403	−2.512	−1.876	−1.653	−1.592	−3.496	−3.077	−2.879	−2.810	−2.814
Total bond	4.471	2.582	1.957	1.748	1.701	3.971	3.564	3.381	3.328	3.346
Stock/bond	0.209	0.360	0.470	0.517	0.524	0.132	0.144	0.147	0.145	0.140
c_0/w_0	–	0.539	0.242	0.147	0.105	–	0.530	0.237	0.144	0.103
$\Delta\widetilde{w}_0/w_0$ (%)	–	1.431	1.872	2.011	2.094	–	−0.674	−0.879	−0.930	−0.969

	$\Delta\eta/\eta=-30\%$					$\Delta\eta/\eta=+30\%$				
	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$
2Y bond	5.932	3.933	3.153	2.809	2.641	3.781	2.582	2.139	1.941	1.833
10Y bond	−0.654	−0.178	0.036	0.176	0.297	−0.275	0.030	0.179	0.296	0.409
Stock	0.788	0.768	0.747	0.726	0.708	0.684	0.685	0.679	0.668	0.657
Cash	−5.065	−3.523	−2.936	−2.711	−2.647	−3.190	−2.297	−1.996	−1.905	−1.899
Total bond	5.278	3.755	3.189	2.984	2.939	3.506	2.612	2.318	2.237	2.242
Stock/bond	0.149	0.204	0.234	0.243	0.241	0.195	0.262	0.293	0.299	0.293
c_0/w_0	–	0.533	0.238	0.143	0.101	–	0.533	0.240	0.146	0.105
$\Delta\widetilde{w}_0/w_0$ (%)	–	−0.084	−0.413	−1.168	−2.649	–	0.084	0.410	1.159	2.620

	$\Delta\xi/\xi=-30\%$					$\Delta\xi/\xi=+30\%$				
	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$
2Y bond	5.009	3.346	2.716	2.446	2.316	4.320	2.920	2.387	2.144	2.012
10Y bond	−0.495	−0.087	0.100	0.229	0.345	−0.357	−0.014	0.150	0.273	0.389
Stock	0.692	0.692	0.684	0.672	0.660	0.763	0.748	0.731	0.713	0.697
Cash	−4.206	−2.951	−2.500	−2.347	−2.321	−3.726	−2.655	−2.268	−2.129	−2.098
Total bond	4.514	3.259	2.816	2.675	2.661	3.962	2.906	2.537	2.416	2.401
Stock/bond	0.153	0.212	0.243	0.251	0.248	0.193	0.258	0.288	0.295	0.290
c_0/w_0	–	0.533	0.239	0.145	0.104	–	0.533	0.239	0.144	0.103
$\Delta\widetilde{w}_0/w_0$ (%)	–	0.004	0.039	0.175	0.530	–	−0.004	−0.040	−0.178	−0.538

The first columns report the myopic component of the optimal allocation, whereas the following columns correspond to investment horizons respectively equal to 2Y, 5Y, 10Y and 20Y. The portfolio strategy is computed considering the case where utility derives from intermediate consumption only and the investor can trade in 2 bonds and the stock index. Parameters are those estimated in Table 1b and $\gamma = 4$.

Table 8: Optimal portfolio for $\mathcal{UM}$ setting and utility over intermediate consumption and real balances.

	Base case ($\alpha = 0.96$, $\rho = 0.8$)				
	$T = 0$	$T = 2$	$T = 5$	$T = 10$	$T = 20$
2Y bond	3.976	2.696	2.175	1.930	1.764
10Y bond	−0.345	−0.035	0.106	0.218	0.356
Stock	0.729	0.721	0.708	0.693	0.679
Cash	−	−2.866	−2.205	−1.972	−1.892
Total bond	3.631	2.662	2.281	2.148	2.120
Stock/bond	0.201	0.271	0.311	0.323	0.320
c_0/w_0	−	0.511	0.229	0.138	0.098
m_0/w_0	−	0.483	0.216	0.131	0.093

	$\alpha = 0.94$					$\alpha = 0.98$				
	$T = 0$	$T = 2$	$T = 5$	$T = 10$	$T = 20$	$T = 0$	$T = 2$	$T = 5$	$T = 10$	$T = 20$
2Y bond	3.721	2.535	2.039	1.798	1.647	4.259	2.877	2.330	2.083	1.901
10Y bond	−0.317	−0.030	0.098	0.205	0.344	−0.377	−0.040	0.114	0.232	0.369
Stock	0.729	0.721	0.708	0.693	0.679	0.729	0.721	0.709	0.693	0.679
Cash	−	−2.895	−2.145	−1.877	−1.797	−	−2.836	−2.277	−2.084	−2.003
Total bond	3.404	2.505	2.137	2.003	1.990	3.882	2.836	2.444	2.315	2.270
Stock/bond	0.214	0.288	0.331	0.346	0.341	0.188	0.254	0.290	0.299	0.299
c_0/w_0	−	0.502	0.225	0.136	0.097	−	0.520	0.233	0.141	0.100
m_0/w_0	−	0.668	0.299	0.181	0.129	−	0.278	0.125	0.075	0.054

	$\rho = 0.6$					$\rho = 0.99$				
	$T = 0$	$T = 2$	$T = 5$	$T = 10$	$T = 20$	$T = 0$	$T = 2$	$T = 5$	$T = 10$	$T = 20$
2Y bond	3.968	2.728	2.285	2.096	1.928	3.985	2.606	1.875	1.478	1.319
10Y bond	−0.344	−0.036	0.105	0.219	0.357	−0.346	−0.030	0.109	0.216	0.352
Stock	0.729	0.721	0.708	0.693	0.679	0.729	0.721	0.708	0.693	0.679
Cash	−	−2.902	−2.317	−2.141	−2.058	−	−2.775	−1.906	−1.517	−1.442
Total bond	3.623	2.692	2.390	2.315	2.285	3.638	2.576	1.984	1.694	1.671
Stock/bond	0.201	0.268	0.296	0.299	0.297	0.200	0.280	0.357	0.409	0.406
c_0/w_0	−	0.510	0.228	0.138	0.098	−	0.511	0.229	0.139	0.099
m_0/w_0	−	0.489	0.219	0.132	0.094	−	0.477	0.214	0.129	0.092

The first columns report the myopic component of the optimal allocation, whereas the following columns correspond to investment horizons respectively equal to 2Y, 5Y, 10Y and 20Y. The portfolio strategy is computed considering the case where utility derives both from intermediate consumption and real balances, and the investor can trade in 2 bonds and the stock index. Parameters are those estimated in Table 1b and $\gamma = 4$.

Table 8 (continued)

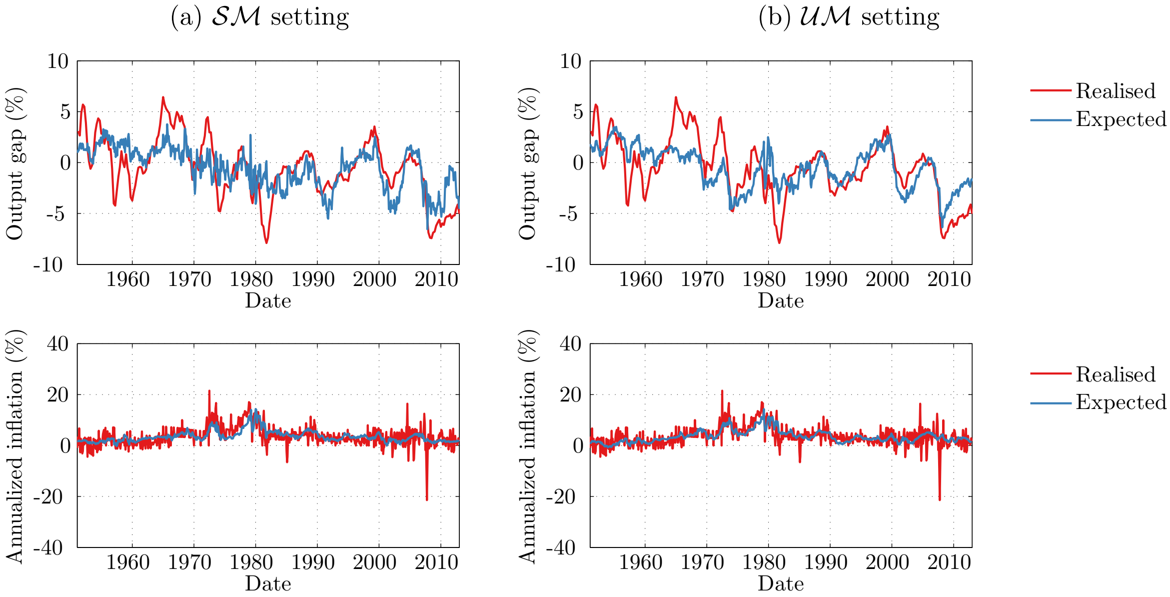
	$\Delta\pi = -0.01$					$\Delta\pi = +0.01$				
	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$
2Y bond	3.324	1.061	0.270	−0.056	−0.292	4.605	4.310	4.070	3.915	3.799
10Y bond	0.285	0.810	1.010	1.142	1.298	−0.973	−0.879	−0.799	−0.708	−0.582
Stock	0.907	0.903	0.891	0.876	0.862	0.551	0.539	0.526	0.510	0.495
Cash	–	−2.335	−1.423	−1.115	−0.978	–	−3.398	−2.988	−2.833	−2.795
Total bond	3.609	1.872	1.280	1.086	1.007	3.632	3.431	3.272	3.207	3.217
Stock/bond	0.251	0.482	0.696	0.807	0.857	0.152	0.157	0.161	0.159	0.154
c_0/w_0	–	0.517	0.232	0.141	0.101	–	0.507	0.227	0.137	0.097
m_0/w_0	–	0.560	0.252	0.153	0.109	–	0.428	0.191	0.115	0.082
$\Delta\widetilde{w}_0/w_0$ (%)	–	2.211	2.793	3.099	3.395	–	−1.291	−1.596	−1.756	−1.902

	$\Delta\chi = -0.01$					$\Delta\chi = +0.01$				
	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$
2Y bond	3.670	1.468	0.692	0.364	0.112	4.277	3.925	3.660	3.500	3.399
10Y bond	0.170	0.685	0.884	1.017	1.176	−0.860	−0.755	−0.674	−0.583	−0.460
Stock	0.933	0.929	0.919	0.904	0.891	0.525	0.513	0.498	0.482	0.467
Cash	–	−2.604	−1.728	−1.427	−1.279	–	−3.136	−2.687	−2.522	−2.494
Total bond	3.840	2.153	1.576	1.381	1.288	3.416	3.171	2.986	2.917	2.939
Stock/bond	0.243	0.431	0.583	0.655	0.692	0.154	0.162	0.167	0.165	0.159
c_0/w_0	–	0.516	0.231	0.140	0.100	–	0.509	0.227	0.138	0.098
m_0/w_0	–	0.522	0.234	0.142	0.101	–	0.453	0.202	0.123	0.087
$\Delta\widetilde{w}_0/w_0$ (%)	–	1.561	1.900	2.016	2.218	–	−0.769	−0.837	−0.811	−0.857

	$\Delta\eta/\eta = -30\%$					$\Delta\eta/\eta = +30\%$				
	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$
2Y bond	5.268	3.544	2.847	2.508	2.250	3.116	2.076	1.670	1.501	1.388
10Y bond	−0.580	−0.167	0.015	0.144	0.298	−0.199	0.046	0.156	0.257	0.389
Stock	0.787	0.767	0.747	0.725	0.707	0.685	0.685	0.679	0.668	0.657
Cash	–	−3.628	−2.825	−2.507	−2.346	–	−2.290	−1.721	−1.558	−1.529
Total bond	4.688	3.377	2.862	2.651	2.548	2.916	2.121	1.826	1.758	1.777
Stock/bond	0.168	0.227	0.261	0.274	0.277	0.235	0.323	0.372	0.380	0.370
c_0/w_0	–	0.510	0.228	0.137	0.097	–	0.511	0.229	0.139	0.100
m_0/w_0	–	0.483	0.216	0.130	0.091	–	0.484	0.217	0.132	0.095
$\Delta\widetilde{w}_0/w_0$ (%)	–	−0.024	−0.272	−0.931	−2.339	–	0.069	0.358	1.074	2.548

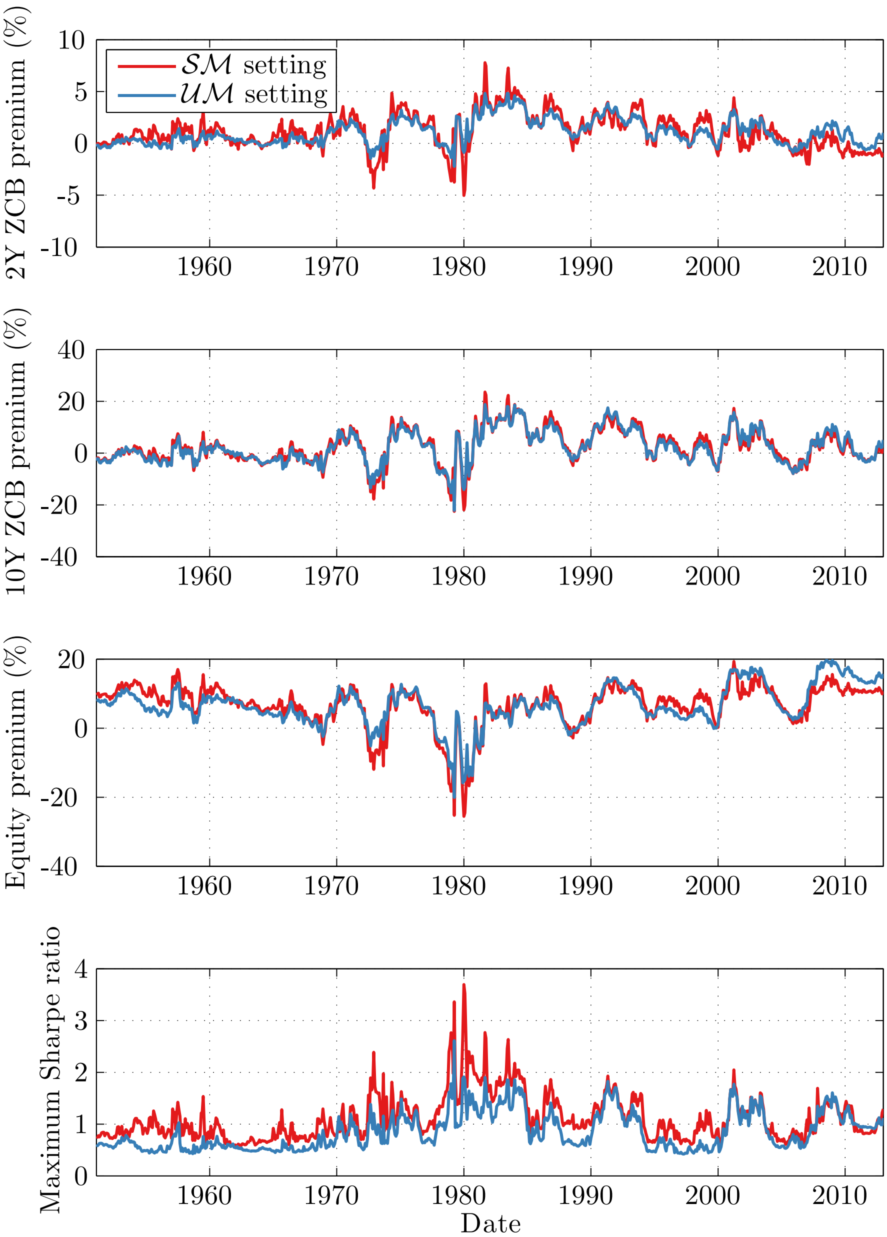
	$\Delta\xi/\xi = -30\%$					$\Delta\xi/\xi = +30\%$				
	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$	$T=0$	$T=2$	$T=5$	$T=10$	$T=20$
2Y bond	4.344	2.934	2.367	2.110	1.929	3.655	2.482	2.000	1.769	1.599
10Y bond	−0.420	−0.074	0.082	0.199	0.341	−0.283	−0.002	0.124	0.230	0.370
Stock	0.693	0.692	0.684	0.672	0.660	0.763	0.748	0.731	0.712	0.696
Cash	–	−3.035	−2.349	−2.113	−2.024	–	−2.712	−2.072	−1.842	−1.758
Total bond	3.924	2.860	2.449	2.310	2.270	3.373	2.480	2.124	1.999	1.969
Stock/bond	0.176	0.242	0.279	0.291	0.291	0.226	0.302	0.344	0.356	0.353
c_0/w_0	–	0.511	0.229	0.138	0.099	–	0.511	0.229	0.138	0.098
m_0/w_0	–	0.483	0.216	0.131	0.093	–	0.483	0.216	0.131	0.093
$\Delta\widetilde{w}_0/w_0$ (%)	–	−0.007	0.020	0.153	0.530	–	0.034	0.046	−0.026	−0.323

Figure 1: Output gap and expected inflation



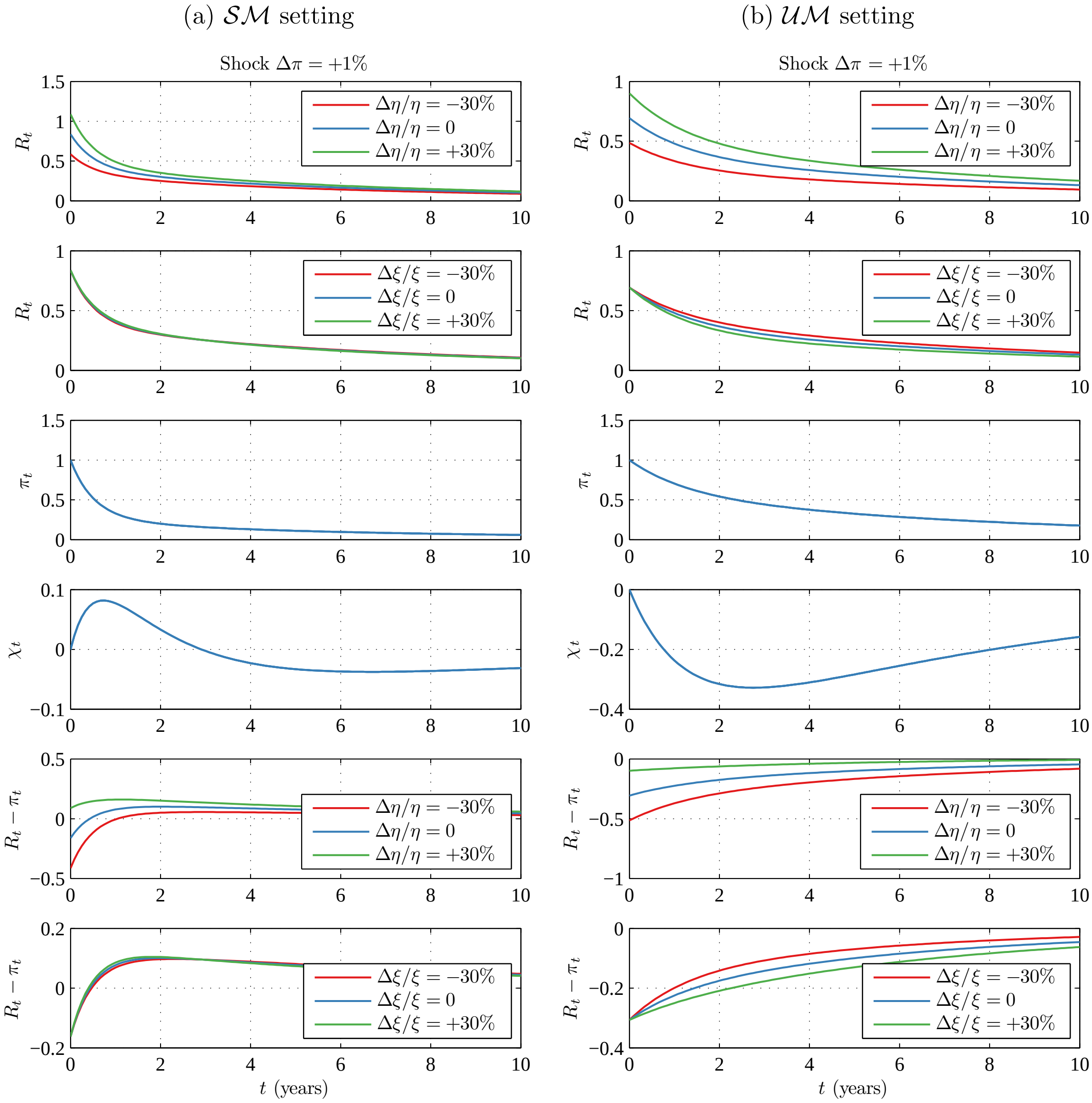
The upper graphs represent the output gap time series and the series implied by implied by the filtered time series of the state variables $\mathbf{X}_t$ through the third equation in (3.5). The lower graphs represent the realised inflation (logarithmic returns of the consumer price index) and the filtered time series of expected inflation, π_t, reconstructed using the second equation in (3.5).

Figure 2: Risk premia of bonds and equity index, as well as maximum implied Sharpe ratio



The first three graphs represent the risk premia of the 2Y and 10Y nominal zero-coupon bonds, as well as of the stock index, estimated with the two different settings proposed. "$\mathcal{SM}$ setting" denotes the setting where three factors are derived from nominal bond yields. "$\mathcal{UM}$ setting" denotes instead the setting where two factors are derived from nominal bond yields and one factor is filtered from equity returns. The lower graph represents the maximum achievable Sharpe Ratio at each point in time. Denoting with $\mathbf{\Sigma}$ the volatility matrix associated to the set of available assets, the maximum Sharpe ratio achievable at each moment in time depends on the instantaneous Sharpe ratio of the assets and can be expressed as $[(\mathbf{\Sigma}'\mathbf{\Lambda}(\mathbf{X}_t))'(\mathbf{\Sigma}'\mathbf{\Sigma})^{-1}(\mathbf{\Sigma}'\mathbf{\Lambda}(\mathbf{X}_t))]^{1/2}$.

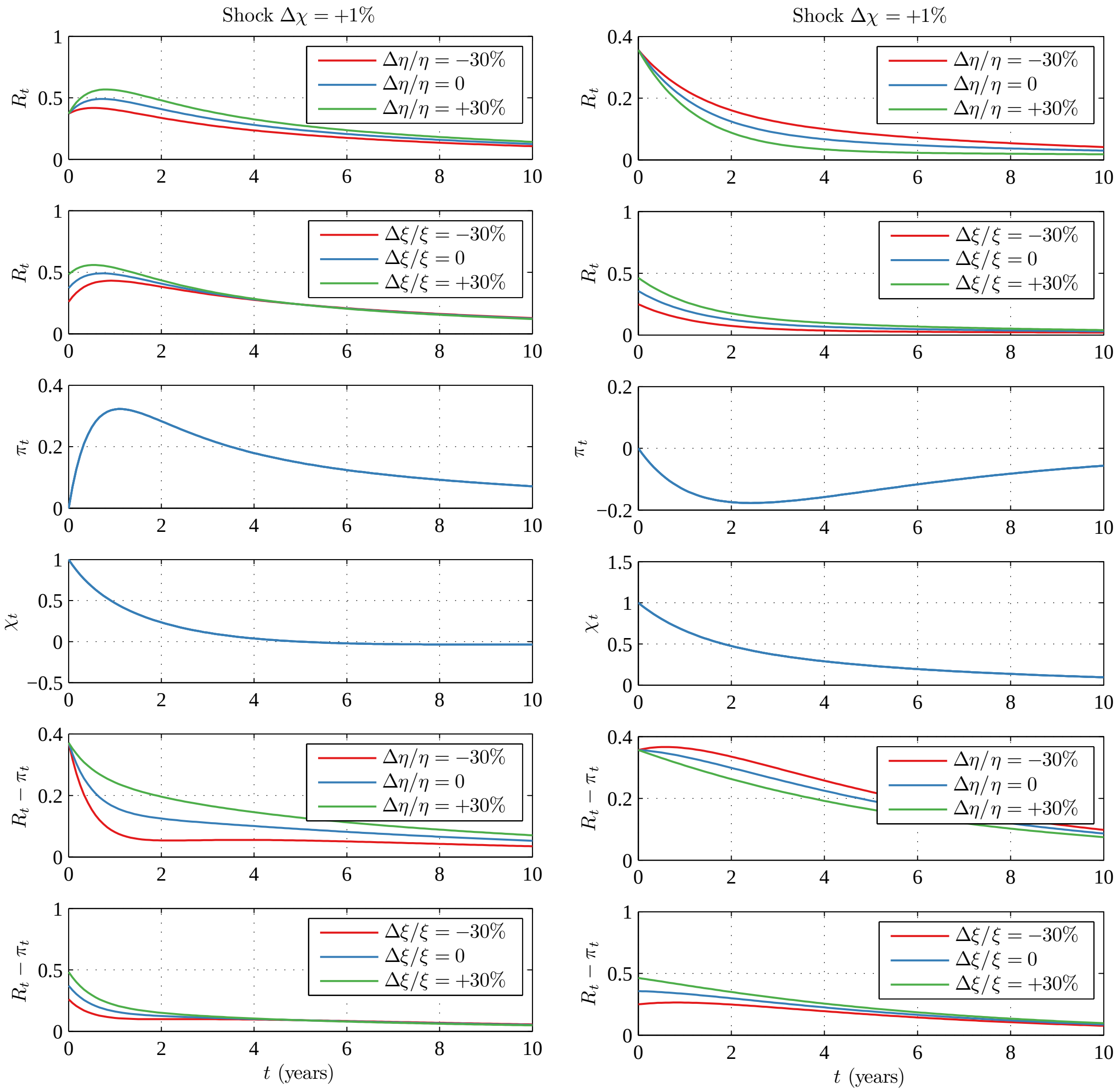
Figure 3: Impulse response of economic variables to expected inflation shock.



The graphs show the impulse responses of the system to a 1% positive shock on the expected inflation, $\Delta\pi$, over the short-term nominal rate R, the expected inflation π, the output gap χ and the real rate $R - \pi$. The graphs regarding the short-term nominal rate and the real rate show the responses for the base case values of monetary policy parameters η and ξ, as well as to the case where these values are subject to relative shifts equal to $\pm 30\%$. Panel (a) is referred to the $\mathcal{SM}$ setting, Panel (b) to the $\mathcal{UM}$ setting.

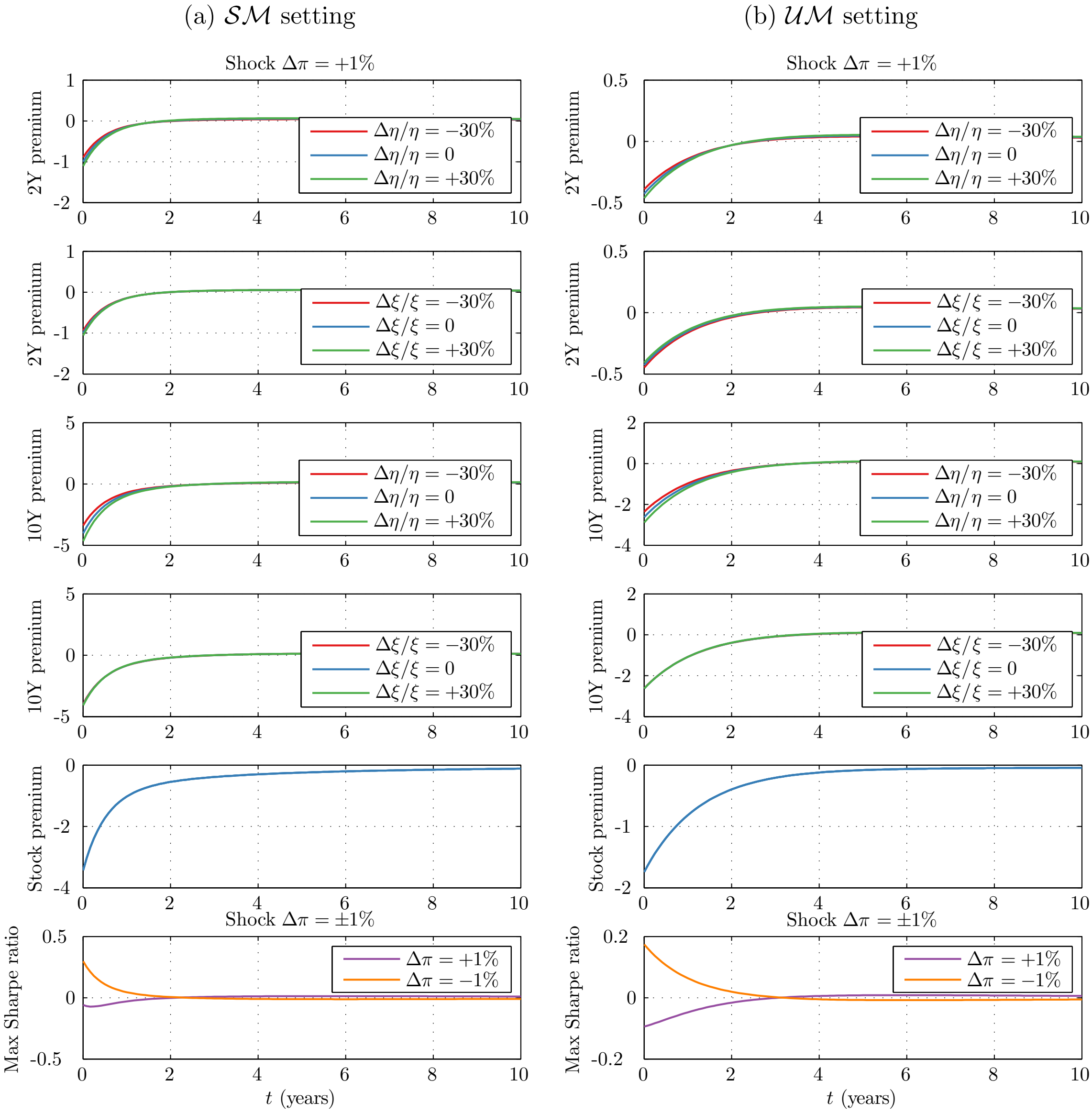
Figure 4: Impulse response of economic variables to expected output gap shock.

(a) $\mathcal{SM}$ setting (b) $\mathcal{UM}$ setting



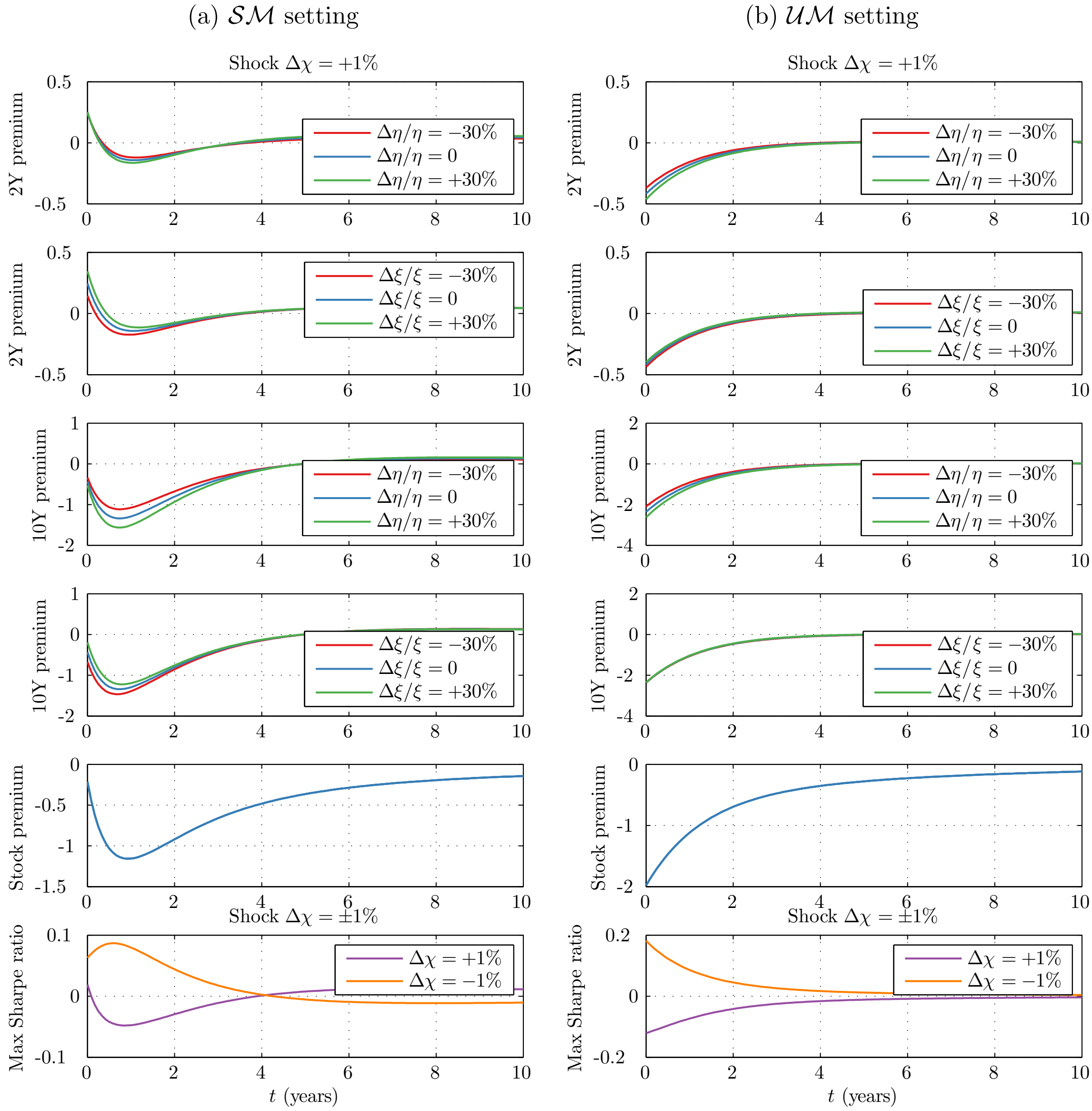
The graphs show the impulse responses of the system to a 1% positive shock on the expected output gap, $\Delta\chi$, over the short-term nominal rate R, the expected inflation π, the output gap χ and the real rate $R - \pi$. The graphs regarding the short-term nominal rate and the real rate show the responses for the base case values of monetary policy parameters η and ξ, as well as to the case where these values are subject to relative shifts equal to $\pm 30\%$. Panel (a) is referred to the $\mathcal{SM}$ setting, Panel (b) to the $\mathcal{UM}$ setting.

Figure 5: Impulse response of risk premia and maximum Sharpe ratio to expected inflation shock.



The first five rows of graphs show the impulse responses of the system to a 1% positive shock on the expected inflation, $\Delta\pi$, over the risk premia of the risky assets (2Y and 10Y nominal bonds, equity index). The last row of graphs show the impulse responses of the system to 1% positive and negative shocks on the expected inflation over the maximum Sharpe ratio obtainable when the nominal market is complete. The graphs regarding the bond risk premia show the responses for the base case values of monetary policy parameters η and ξ, as well as to the case where these values are subject to relative shifts equal to $\pm 30\%$. Panel (a) is referred to the $\mathcal{SM}$ setting, Panel (b) to the $\mathcal{UM}$ setting.

Figure 6: Impulse response of risk premia and maximum Sharpe ratio to expected output gap shock.

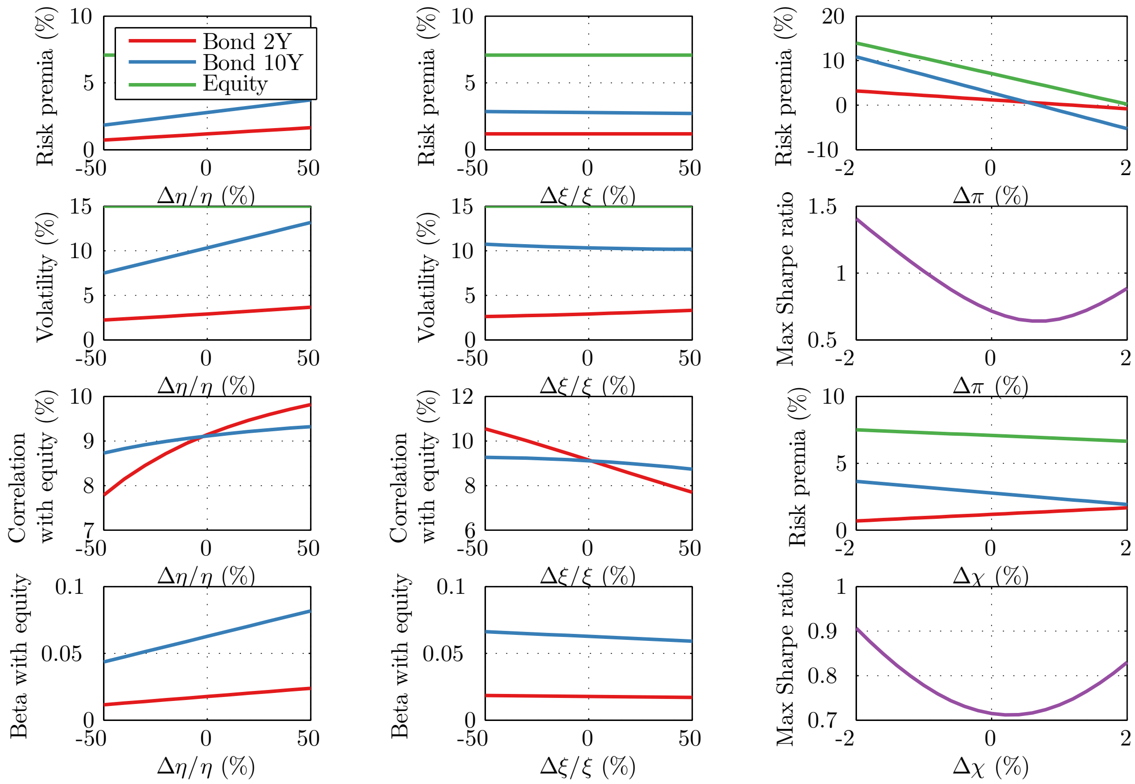


The first five rows of graphs show the impulse responses of the system to a 1% positive shock on the expected output gap, $\Delta\chi$, over the risk premia of the risky assets (2Y and 10Y nominal bonds, equity index). The last row of graphs show the impulse responses of the system to 1% positive and negative shocks on the expected output gap over the maximum Sharpe ratio obtainable when the nominal market is complete. The graphs regarding the bond risk premia show the responses for the base case values of monetary policy parameters η and ξ, as well as to the case where these values are subject to relative shifts equal to $\pm 30\%$. Panel (a) is referred to the $\mathcal{SM}$ setting, Panel (b) to the $\mathcal{UM}$ setting.

Figure 7: Static sensitivity analysis for the $\mathcal{SM}$ setting.

(a) Sensitivity of risk premia, volatilities, correlations and betas to monetary policy shifts

(b) Sensitivity of risk premia and maximum Sharpe ratio to shifts of the economic state variables

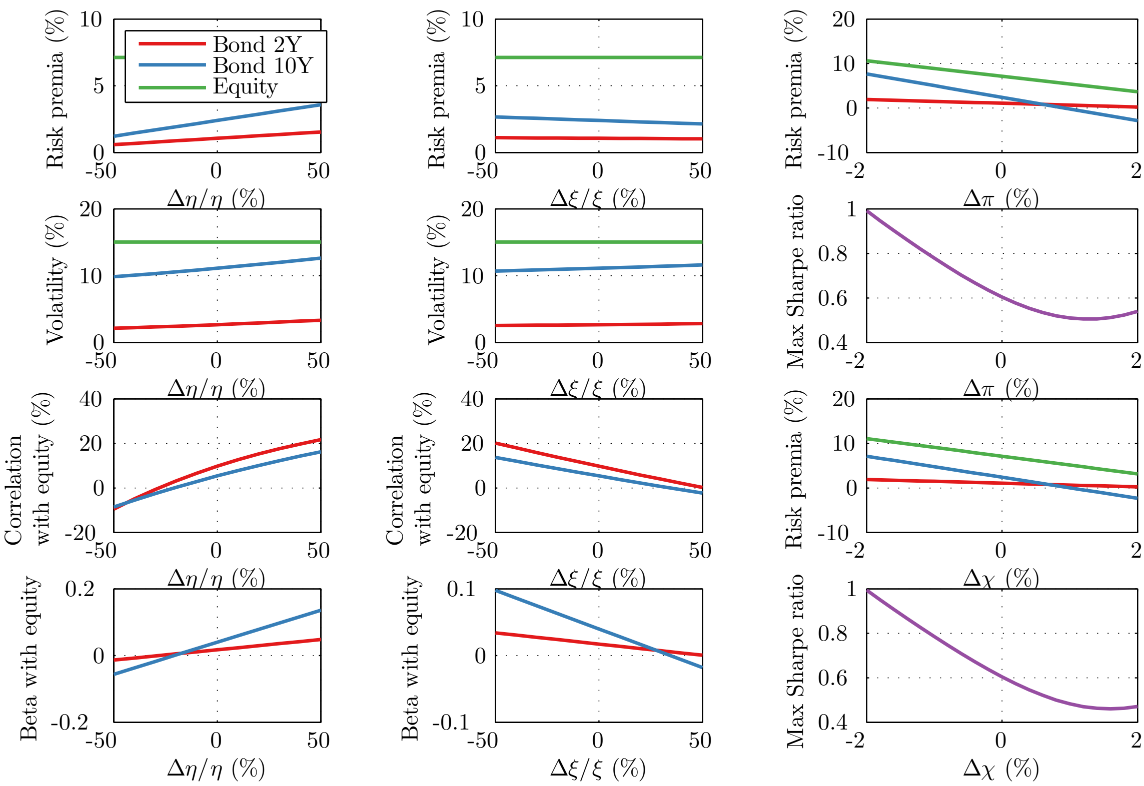


For the $\mathcal{SM}$ specification, the graphs in the first panel report the sensitivity of risk premia, volatilities, correlations and betas (with respect to the equity), of the nominal bonds (2Y and 10Y) and the stock index, to shifts in the parameters describing the monetary policy (weight of expected inflation η and weight of expected output gap ξ) with respect to their estimated values. The graphs in the second panel show the sensitivity of the risk premia and of the maximum Sharpe ratio achievable to shifts in the economic state variables (expected inflation π and output gap χ) with respect to their long-run means. In each of the three graphs only one economic state variable is shifted, whereas the other is kept equal to its long-run mean. The maximum Sharpe ratio is calculated considering as investable universe three bonds (2Y, 5Y and 10Y) and the stock index.

Figure 8: Static sensitivity analysis for the $\mathcal{UM}$ setting.

(a) Sensitivity of risk premia, volatilities, correlations and betas to monetary policy shifts

(b) Sensitivity of risk premia and maximum Sharpe ratio to shifts of the economic state variables



For the $\mathcal{UM}$ specification, the graphs in the first panel report the sensitivity of risk premia, volatilities, correlations and betas (with respect to the equity), of the nominal bonds (2Y and 10Y) and the stock index, to shifts in the parameters describing the monetary policy (weight of expected inflation η and weight of expected output gap ξ) with respect to their estimated values. The graphs in the second panel show the sensitivity of the risk premia and of the maximum Sharpe ratio achievable to shifts in the economic state variables (expected inflation π and output gap χ) with respect to their long-run means. In each of the three graphs only one economic state variable is shifted, whereas the other is kept equal to its long-run mean. The maximum Sharpe ratio is calculated considering as investable universe two bonds (2Y and 10Y) and the stock index.